The Effects of Wind on the Loading and Vibration of Stone Pinnacles

S.H. COCKING[(1)], S. PRICE[(2)] and M.J. DEJONG[(3)]

[(1)] *MEng Student, Department of Engineering, University of Cambridge, Cambridge, CB2 1PZ, United Kingdom, sam.cocking@cantab.net*

[(2)] *Consultant, Price & Myers, 37 Alfred Place, London, WC1E 7DP, United Kingdom, SPrice@pricemyers.com*

[(3)] *Senior Lecturer, Department of Engineering, University of Cambridge, Cambridge, CB2 1PZ, United Kingdom, mjd97@cam.ac.uk*

ABSTRACT

Following the collapse of a tympanum pinnacle at Beverley Minster in Yorkshire, a research project was undertaken to investigate the wind forces which act on stone pinnacles. A survey was conducted and the most common failure modes were identified, which highlighted the importance of dynamic forces in addition to the static drag force. Further, the potential impact of decorative crockets on these forces was of interest.

Both static and dynamic forces on pinnacles were investigated through a series of wind tunnel tests. The results demonstrate the relative magnitude of these forces, and that the decorative crockets do appreciably affect both the drag force and wind-induced vibration. The experimental data was used to derive general relationships for wind forces acting on stone pinnacles for potential use in engineering practice.

KEYWORDS: Wind, drag, vortex-induced vibration, pinnacles, historic masonry

NOTATIONS

A	frontal area
c_d	drag coefficient
c_{dyn}	dynamic force coefficient
D	characteristic width
F	drag force
F_{dyn}	dynamic force
F_{min}	minimum value of the dynamic force
f_n	natural frequency
$F_\omega(x)$	dynamic force per unit height
h	height
$m(x)$	mass per unit height
Sc	Scruton number
v	wind speed
y_{max}	maximum modal deflection
α	dynamic response factor
δ_s	structural damping
ρ_{air}	density of air
$\varphi(x)$	normalised mode shape

1. INTRODUCTION

In early 2013, one of the main tympanum pinnacles at Beverley Minster came down in a gale. Failures such as this are not uncommon, with as many as 40 pinnacles collapsing each year. While nobody was unfortunate enough to have been present for the failure at Beverley, this is not always the case and a number of fatalities have occurred over the years. Figure 1 shows a view of the tympanum at Beverley Minster, between the two western towers.

A number of theoretical questions were raised during the investigation into this collapse. Of these, the most fundamental concerned the nature of the wind force which acts on a pinnacle. The wind force involves a static drag component and induces a significant dynamic response, and has historically not been well understood. In addition, the effect of decorative crockets (the ornamental protrusions often seen adorning the edges of pinnacles) on these forces is not known. Could it be that these crockets serve a practical purpose in disturbing the flow, thus inhibiting the formation of vortices? Could medieval stonemasons have intended these crockets to serve more than an aesthetic purpose?

The wind forces acting on a stone pinnacle have been investigated in a research project at the Cambridge University Engineering Department (CUED). Tests were undertaken in CUED's Markham wind tunnel in order to experimentally establish relationships for these forces. Currently a designer wishing to assess the wind loading on a pinnacle would have few options other than a full Computational Fluid Dynamics (CFD) analysis. Although CFD is a highly useful tool, it is desirable to have a simple hand calculation against which its results can be verified. A primary objective of this work was to establish relationships to satisfy that need.

2. PINNACLE CONSTRUCTION

Pinnacles must be designed with reinforcement to prevent overturning due to wind loading, although the nature of this reinforcement can be quite varied. Original construction often involved a central socket, roughly 2.5cm in diameter, being carved into the masonry blocks. After the pinnacle had been erected, this socket was filled with molten lead, which set to provide lead pins between each pair of stones. The malleability of the lead was ideal for this purpose since it permitted some slight movement at each joint without causing stress concentrations and compromising the overall integrity of the pinnacle. However, corrosion over time has been a problem for lead dowels and this, combined with the need to work at height with molten lead, means that this form of construction is no longer employed.

When a pinnacle is built or replaced today, the dowel is typically constructed using one of two methods. In both cases the dowel is made of non-ferrous metal, typically stainless steel. A continuous dowel is occasionally used, with some manner of bolted baseplate, and may be tensioned up to provide a pre-stress. However, for any but the largest pinnacles this approach is highly conservative and so is rarely seen in practice.

An alternative, preferred by stonemasons, is to use shorter stub dowels between pairs of masonry blocks. This allows the reinforcement to be fitted as the pinnacle is constructed so that there is never a time when the blocks are unreinforced. The stub dowels are bedded in a soft material, usually lime mortar.

Over time it is common to observe that joints in a pinnacle will become weak or "soft". This may either be because of corrosion, if a ferrous dowel is present, or because the lime mortar in which stub dowels have been embedded is beginning to degrade. In any case it is often very difficult to assess the nature and state of reinforcement from external inspection of a pinnacle. The same cover of stone which acts to protect the reinforcement from corrosion also prohibits any direct examination.

The most commonly observed repair work for a soft joint is to fit a metal cramp. This is essentially an oversized metal staple which is embedded into the blocks either side of the joint, prohibiting movement of the joint and offering an alternate load path. Over the years lead, iron and steel cramps have been used, but today it is most likely that a cramp will be made from stainless steel plate, bent at its ends. Typical dimensions are 150mm long by 25mm wide, with an embedment of roughly 30mm. A plate thickness of 3mm is preferred since it can be bent into shape using hand tools. The cramp is often embedded in either lead or lime mortar; this both prevents the formation of stress concentrations in the stone and stops the cramp from coming loose.

In assessing the wind forces which act on a pinnacle, it is important to consider the load path which these forces will follow. The critical points of this load path will be where forces are transferred between masonry blocks and either dowels or cramps. The vast majority of pinnacles which fail unexpectedly early in their lives can be categorised into one of two common failure modes. These are pull-out failure of the dowel and fracture of the stone, initiating from a dowel socket or the embedment point of a cramp. In order to avoid these failure modes a number of specifications are recommended. Firstly, the embedment of dowels must be sufficient to ensure full load transfer. Second, non-ferrous metals should be used to avoid corrosion of the dowel, which can also lead to pull-out failure. Third, the lime mortar bed in the dowel socket must be adequately thick to prevent the formation of stress concentrations in the stone.

Similar rules may be stated for the fitting of cramps. Static load tests are currently underway at CUED to determine the capacity of cramp connections, though this is not discussed further in this paper.

3. WIND TUNNEL TESTS

The static wind forces acting on a pinnacle can be expressed using the standard drag equation:

$$F = \left(\frac{1}{2}\rho_{air} v^2\right) c_d A \tag{1}$$

where the bracketed term is the dynamic pressure of the air, A is the frontal area presented to the wind and c_d is the drag coefficient, which is dependent on geometry. To establish the value of c_d for a pinnacle, testing of a model pinnacle was carried out in the Markham wind tunnel at CUED.

3.1 Model

A 1:2 length scale model of the tympanum pinnacle at Beverley Minster was designed and constructed (see Figure 2). The model is only of the pinnacle and makes no attempt to capture the surrounding fabric of Beverley Minster. The objective was to obtain results for the pinnacle itself, which would be more broadly applicable to any pinnacle. The designer would then need to consider what a likely local maximum wind speed might be for a specific scenario. In the case of Beverly, the nearby fabric will funnel the air between the towers, and thus increase its local speed around the pinnacle that collapsed.

Although the geometry of the model has been based on Beverley Minster, most pinnacles take the form of a square-based truncated conical section. The level of tapering varies and at Beverley the pinnacles are notably slender, with a half angle of approximately 3 degrees; however the key aspect of the geometry will be the sharp corners of the cross-section, which are common across all but the most eroded pinnacles. Further testing would be required to investigate the impact of varying the taper.

The model crockets were also designed to be as general as possible, by simplifying their geometry. The size and spacing of crockets can vary significantly but the values at Beverley, which have been applied to the model crockets, are reasonably standard.

The model pinnacle itself is a timber shell, mounted on a central aluminium rod by means of internal steel connections. The rod extends slightly below the base of the pinnacle to allow for some rotation before connecting to a rigid baseplate on the floor of the tunnel. It is instrumented at two locations with sets of strain gauges. A National Instruments data logger was used to continuously read strain data throughout the tests. By ensuring that the aluminium remains in its linear-elastic range and assuming that the rod, below the internal connections, behaves as a cantilever in flexure, it is possible to back-calculate the moments at these locations. Applying statics, this information is sufficient to determine the base shear and moment of the model pinnacle.

3.2 Wind Tunnel

Testing was carried out in the Markham wind tunnel at CUED. This tunnel can achieve an upper wind speed approaching 60m/s, though practically slightly less due to blockage effects. In order to match the Reynolds number of the flow, and based on the 1:2 length scale of the model, the results from these tests correspond to an upper wind speed of nearly 30m/s in the real case. However, since no drag crisis was observed in the results, it is likely that the drag coefficients are invariant of Reynolds number and are therefore also valid at higher wind speeds. This is often true for geometries with sharp corners.

The model was tested at tunnel wind speeds ranging from 30-60m/s, and in two different directions. In direction 1, the model was positioned such that the leading face of the pinnacle was perpendicular to the air flow. Rotating the pinnacle about a vertical axis by 45 degrees gives direction 2; the windward and leeward points were then at corners. Both orientations, when viewed in plan, have lines of symmetry in the windward and cross-wind directions.

4. STATIC DRAG FORCES

Back-calculation from the strain data readily yielded the static drag forces. The data obey the relationship in Equation (1), in which the drag force varies linearly with the square of the wind speed. Figure 3 demonstrates this relationship.

Since the frontal areas were known for the model, general drag coefficients c_d may be obtained, so that Equation (1) may now be applied to a general pinnacle of similar geometry. These drag coefficients, along with the model's frontal areas, are presented in Table 1.

The results in Table 1 are categorised according to the orientation of the model to the air flow. It is interesting to compare the values of c_d with those that have been previously found for similar geometries. For instance, a vertical cantilever of height h and square cross-section D x D can be viewed as the non-tapering analogue of a pinnacle without crockets. If such a cantilever is chosen so that its aspect ratio h / D matches the pinnacle, then its drag coefficients are found to be roughly 1.5 in direction 1 and 1.2 in direction 2 [1]. The corresponding values in Table 1 are both approximately 16% lower, so it can be seen that the effect of tapering the pinnacle is to reduce its drag coefficient. While this may not be a hugely surprising result, it is intriguing to note that the reduction is uniform in both directions. It is also interesting to note that the effect of the crockets is substantial in direction 2, but negligible in direction 1. Further testing would be needed to fully establish the effect of tapering on the wind flow and resulting wind forces.

5. DYNAMIC FORCES

5.1 Vortex-Induced Vibration

Vortex-induced vibration of a pinnacle gives rise to dynamic forces. Each time a vortex is shed it does work on the pinnacle, imparting energy which must then be dissipated through internal structural damping. In general, the rate at which work is done on the pinnacle is not equal to the rate at which energy is dissipated, which results in a "beating" effect as the amplitude of the dynamic force varies with time [2]. Overall, the effect is a high frequency sinusoidal force of varying amplitude, with components in both the windward and lateral directions. A time history of one such force, taken from one of the wind tunnel tests, is presented in Figure 4.

It is necessary to simplify the information in this plot in order to look for relationships in the data. The horizontal lines in Figure 4 indicate the mean and maximum amplitudes of the dynamic force over the 20 second sample time. The ratio of the maximum and mean amplitudes turns out to be normally distributed in all conditions and so a good value of this ratio for use in design is given by $\mu+3\sigma$, where μ is the mean value of the ratio and σ is its standard deviation. Because of the properties of the normal distribution, there is a probability of 0.15% of this ratio being exceeded. It is therefore possible to derive expressions for the mean amplitude and relate these back to its maximum likely value, by multiplying by the appropriate ratio. These ratios are presented in Table 2.

It can be seen in Figure 4 that the ratio of maximum to mean force is roughly 3.6 for this test, somewhat below the values in Table 2. However, when this is compared with other tests (the equivalent of simply testing for a longer sample time) it is found that the ratio quickly increases towards the values given in Table 2. This highlights the importance of testing for a sufficiently long time period when dealing with a randomly fluctuating force.

Plotting the mean and maximum amplitudes against the square of the wind speed indicates a linear relationship in both cases, similar to results for the static drag force. However, there is more variation in this data, so the line of best fit no longer establishes a "safe" design equation. Instead it is necessary to fit an envelope above the data so that, at a given wind speed, this predicts the highest likely amplitude of the dynamic force. There is more spread in the data for the maximum amplitudes, so it is more convenient to establish design envelopes for the mean amplitudes and to then relate these back to the maximum amplitudes through applying the ratios in Table 2.

Plots showing the mean amplitudes of the dynamic forces, under various conditions, are given in Figure 5. These plots are for the case when crockets are present, the most relevant for pinnacles. Lines of best fit are shown in addition to proposed design envelopes, which take the following general form:

$$F_{dyn} = \left(\frac{1}{2}\rho_{air}v^2\right)c_{dyn}A + F_{min} \qquad (2)$$

The first half of this expression is identical to the earlier Equation (1) for the drag force, apart from the replacement of c_d with a dynamic force coefficient c_{dyn}, while the parameter F_{min} provides a vertical translation that accounts for variation in the data. F_{min} is assumed to be invariant of wind speed, which certainly seems to be the case in Figure 5, and is assumed to scale with frontal area. Further tests would be needed to verify this, however a quick thought experiment can suggest that this is plausible. The work done on the pinnacle by the wind as a vortex is shed, which is the source of the dynamic force, will depend on the dynamic pressure of the air flow. This pressure acts over a certain portion of the side of the pinnacle during vortex separation, although it results in vibration in both the lateral and windward directions. If the length scale of the pinnacle is increased by some factor l, then the area over which the dynamic pressure acts is increased by l^2. Therefore the size of the dynamic force is also increased by l^2. Since the frontal area of the pinnacle clearly grows by the same factor, it can be surmised that the dynamic force as a whole should scale with the frontal area.

5.2 Effect of the Dynamic Response

The force coefficient c_{dyn} in Equation (2) is the dynamic analogue of the static drag coefficient c_d. While the latter only accounts for geometric influences, c_{dyn} is more nuanced. Not only does it account for geometrical influences, it also represents the effect of the structure's dynamic, or inertial, response. If a structure deflects as a vortex is shed then this deflection will have an impact on the work done by the shedding of the next vortex, affecting the dynamic forces experienced by the pinnacle in the future.

The dynamic response of the pinnacle will depend on its inertial properties and it is highly unlikely that these will be the same as those of the model. Therefore it is necessary to scale the model's design equations before they can be applied to a real stone pinnacle.

If by chance these properties were the same then an inertial formulation of the dynamic force would produce the same results for both the stone pinnacle and the model, once the latter had been suitably scaled up in size. EC1 uses just such an approach to account for forces resulting from vortex-induced vibration, which is described by the following equation [3]:

$$F_{\omega}(x) = m(x).\,(2\pi f_n)^2.\,\phi(x).\,y_{max} \qquad (3)$$

This force is per unit height of the structure and is essentially a reworking of Hooke's Law. The mass per unit height multiplied by the square of the natural frequency gives the modal stiffness, while the normalised mode shape multiplied by the maximum modal displacement gives the modal displacement at location x.

Once this force has been found for both the model and the real tympanum pinnacle, the ratio of these will give a dynamic response factor α which must be applied to the model's design equations to correct them for the real case.

The mass per unit lengths of the tympanum pinnacle and model may be readily estimated. As a first approximation they may both be considered as constant and are roughly

62.7kg/m and 4.0kg/m respectively, although in reality for a stone pinnacle this will vary linearly with height. Since mass per unit length scales with area, the model value needs to be increased by a factor of 4.

The modeshapes for the two cases may be assumed to be similar and so can be neglected in the ratio. The natural frequency of the model was found during the wind tunnel tests, while that of the tympanum pinnacle was obtained through measurement using a laser vibrometer. By chance the values were similar, and so natural frequency can also be neglected in this calculation.

The maximum modal displacement y_{max} may be estimated from the Scruton number, using the relationship in Equation (4). The expression for the Scruton number itself is given in Equation (5).

$$y_{max} \approx \frac{1.5D}{Sc} \tag{4}$$

$$Sc = \frac{2\delta_s m(x)}{\rho_{air} D^2} \tag{5}$$

In these expressions, D is a characteristic length measurement, in this case the pinnacle's side length at location x, and δ_s is the structural damping of the system. This was measured for the model in a free vibration test but could only be estimated, at 2%, for the tympanum pinnacle. The values of y_{max} may now be calculated and are found to be 0.58mm for the tympanum pinnacle and 7.0mm for the model. y_{max} will scale with length for the model and so must be increased by a factor of 2.

Overall, this results in a dynamic response factor of α = 0.162. Therefore, the model predicts higher dynamic forces than would be observed in a stone pinnacle and so the design envelopes need to be factored down. This is hardly surprising; the timber model exhibited a notably dramatic dynamic response which is simply not observed in the real case of a stone pinnacle.

5.3 Final Design Equations for the Dynamic Forces

Now that all necessary corrections have been made, the final design equations for dynamic forces can be proposed. These represent the highest expected amplitudes of the dynamic forces that would be observed for a stone pinnacle of geometry roughly similar to the tympanum pinnacle at Beverley Minster. They are presented in Table 3.

These equations assume that the frontal area of the pinnacle is equal to that of the model. For a general pinnacle of frontal area A the designer would need to multiply these equations by the ratio A / A_{model}. The values of A_{model} are given in Table 1.

6. IMPLICATIONS FOR REAL PINNACLES

6.1 Effects of the Dynamic Forces

The amplitudes of these dynamic forces are small compared to the size of the static forces which pinnacles must also withstand. However it is the frequency of loading which makes the dynamic forces relevant. This frequency is high and a pinnacle will undergo extensive cycles of loading over the course of its lifetime.

Eventually, when combined with material degradation, this dynamic loading may lead to the opening of joints. If a lead dowel is present, then this will allow ingress of deleterious materials such as water which will accelerate the dowel's corrosion.

If non-ferrous stub dowels have been used then these will have been bedded in a soft material, most likely lime mortar, which will degrade under cyclic loading. Dowel pull-out is a commonly reported mode of pinnacle failure and will be made more possible by the weakening of the mortared joints, which will eventually be unable to withstand the larger static wind loads.

Therefore, dynamic forces are important to the overall stability of a stone pinnacle.

6.2 The Impact of Crockets

The presence of crockets does increase the static drag force. The reason for this is twofold; not only does the drag coefficient increase slightly when crockets are added but they also result in a larger frontal area being presented to the wind. However, for a properly constructed pinnacle that has not yet notably degraded this increase is not hugely important, since the pinnacle should be more than capable of safely carrying these loads.

Nevertheless, crockets also decrease the amplitude and variation of the most critical dynamic forces. Therefore there is arguably a net benefit to the presence of crockets. The frequency of dynamic loading is very high and so even a slight decrease in the amplitude of this force could cause an increase in the life of the pinnacle.

7. CONCLUSIONS

Wind tunnel tests have established design equations for both the static drag and dynamic vortex-induced forces which act on a general stone pinnacle. These can be applied to any pinnacle of reasonably similar geometry to the tympanum pinnacles at Beverley Minster and require only knowledge of the pinnacle's frontal area and the local wind speed.

Furthermore, it has been shown that the presence of crockets has the potential to increase the length of a pinnacle's life, by lowering the magnitude of the dynamic forces which act on it. These dynamic forces contribute to the pinnacle's deterioration over time and therefore remain important to the structural engineer, even though they are small in size.

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Figure 1   A view of the tympanum at Beverley Minster

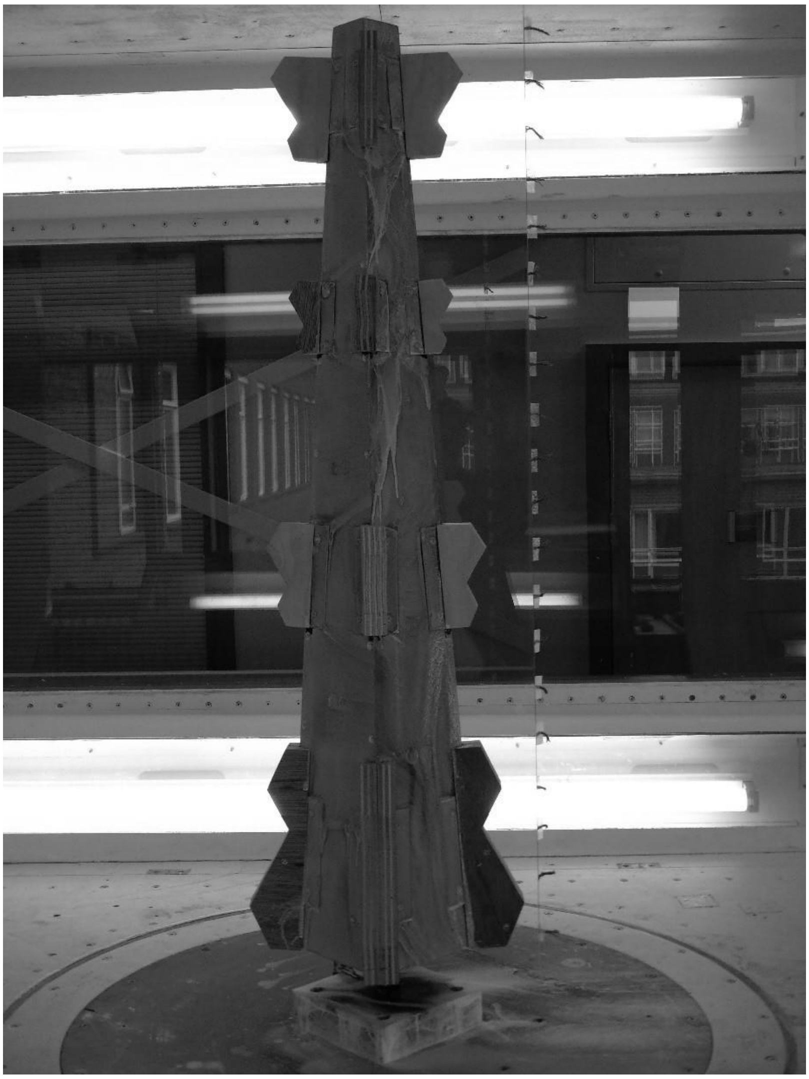


Figure 2   Model pinnacle in the Markham wind tunnel

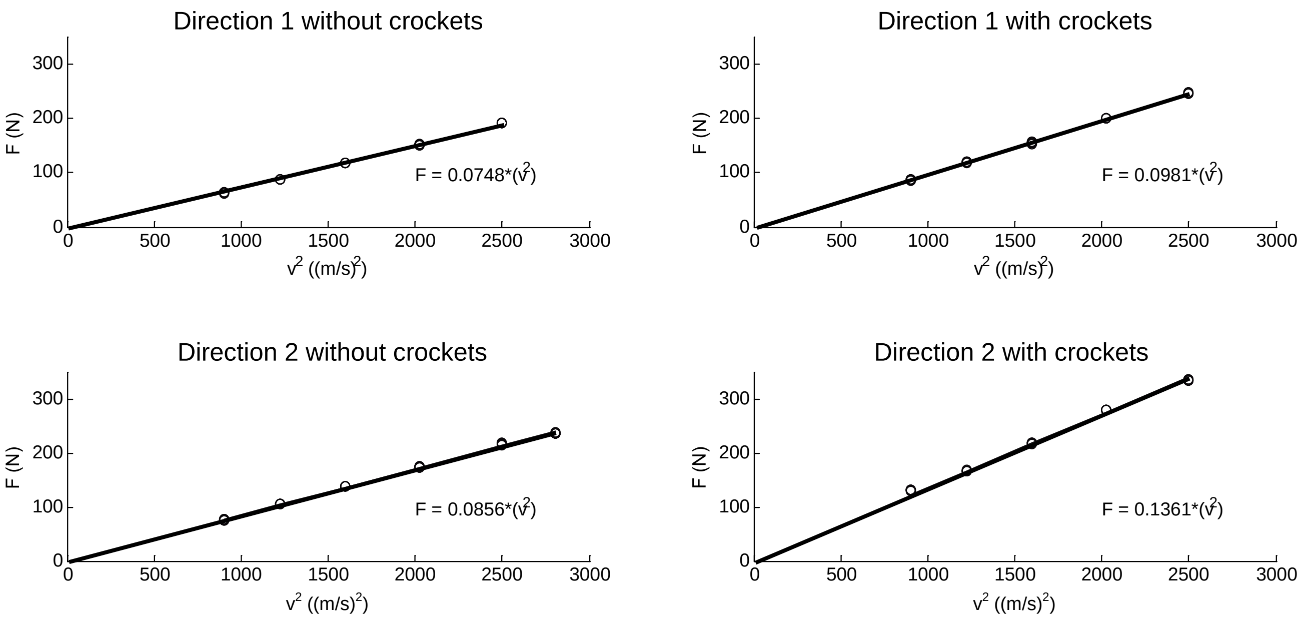


Figure 3 Plots of the static drag forces

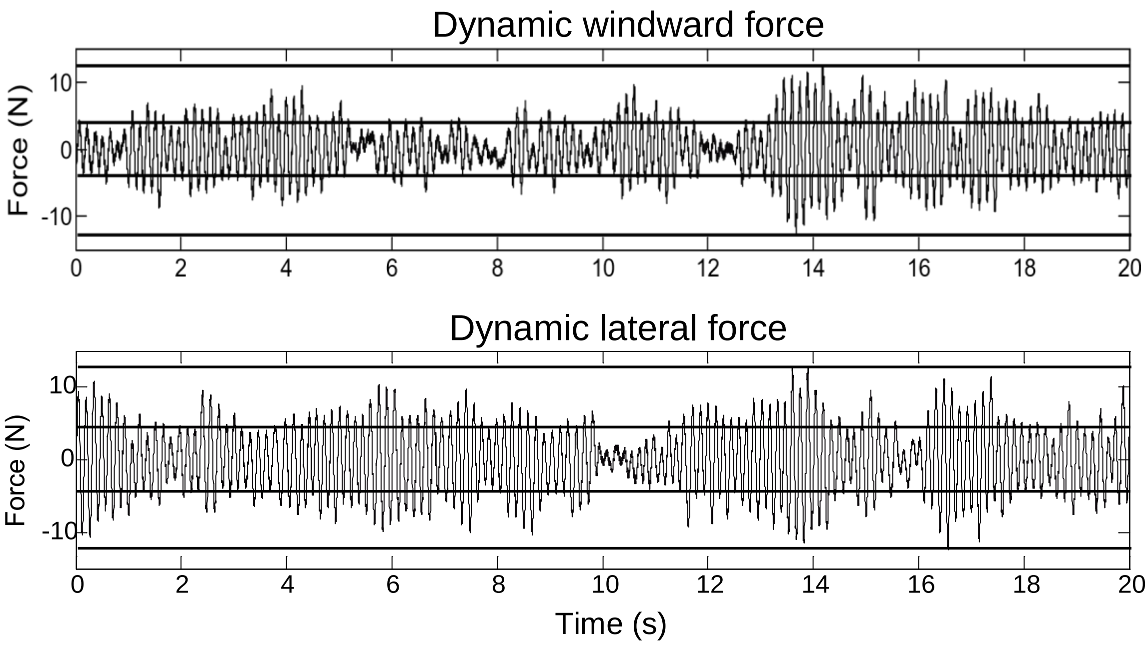
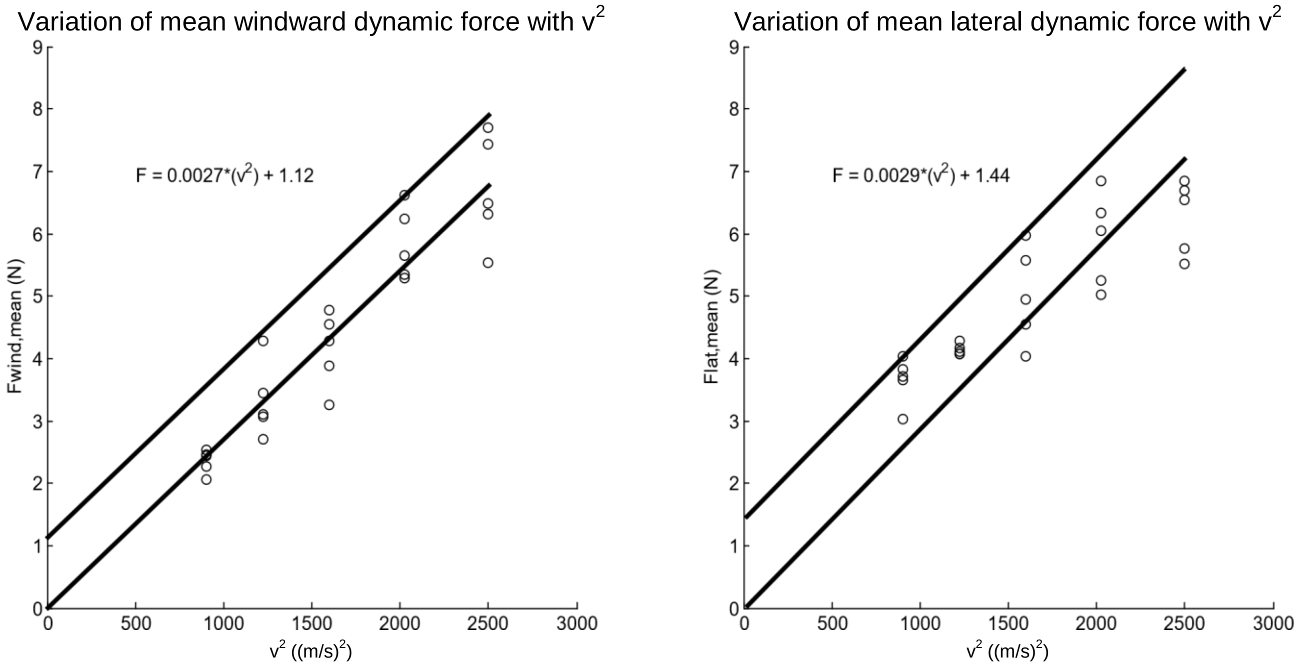


Figure 4 Time history of a dynamic force, resulting from vortex-induced vibration

Direction 1:



Direction 2:

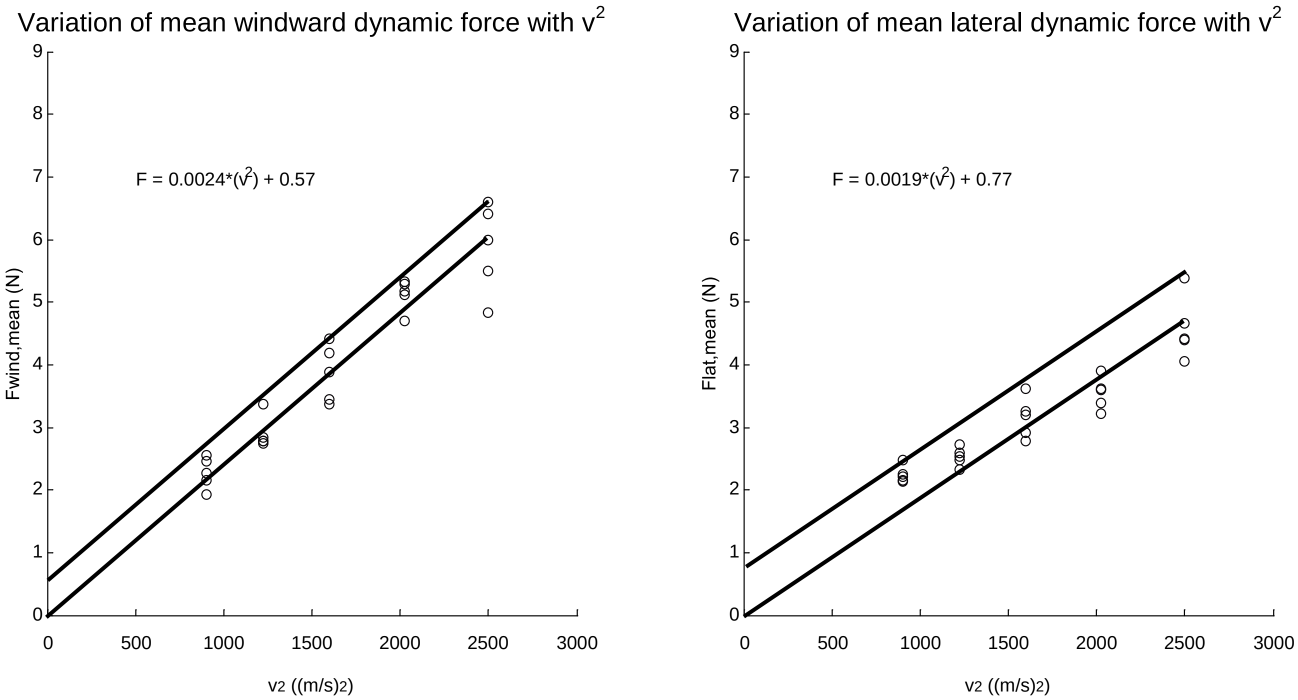


Figure 5 Plots of the mean amplitude of the dynamic forces, for the case when crockets are present

Table 1
Results for the static drag forces: Frontal areas A_{model} and drag coefficients c_d

Conditions:	**Frontal area A_{model} [m^2]:**	**Drag coefficient c_d:**
Direction 1 without crockets	0.100	1.25
Direction 1 with crockets	0.130	1.26
Direction 2 without crockets	0.141	1.01
Direction 2 with crockets	0.183	1.24

Table 2
Ratios of maximum amplitude to mean amplitude for the dynamic forces

Conditions:	**Windward dynamic forces** $F_{windward, max}$ / $F_{windward, mean}$:	**Lateral dynamic forces** $F_{lateral, max}$ / $F_{lateral, mean}$:
Direction 1 without crockets	5.07	5.01
Direction 1 with crockets	4.92	4.65
Direction 2 without crockets	5.49	4.43
Direction 2 with crockets	5.23	4.59

Table 3
Final design equations for the highest likely amplitudes of the dynamic forces

Conditions:	**Design Equations:**
Direction 1 without crockets	$F_{windward, max} = 0.0016v^2 + 1.28$
	$F_{lateral, max} = 0.0021v^2 + 2.00$
Direction 1 with crockets	$F_{windward, max} = 0.0022v^2 + 0.89$
	$F_{lateral, max} = 0.0022v^2 + 1.09$
Direction 2 without crockets	$F_{windward, max} = 0.00013v^2 + 0.45$
	$F_{lateral, max} = 0.00009v^2 + 0.37$
Direction 2 with crockets	$F_{windward, max} = 0.0020v^2 + 0.48$
	$F_{lateral, max} = 0.0014v^2 + 0.57$