PFSD-2018 IOP Publishing
IOP Conf. Series: Journal of Physics: Conf. Series **1145** (2019) 012036 doi:10.1088/1742-6596/1145/1/012036

Mathematical investigation of pressure pulsations characteristics and natural acoustic frequencies in the gas-dynamic channel

A A Glazunov, I V Eremin, K V Kostyushin, K N Zhiltsov[1] and I M Tyryshkin

Research Institute of Applied Mathematics and Mechanics of *Tomsk State University*,
36 Lenina Avenue, Tomsk 634050, Russian Federation.

[1]Corresponding author: konstantin@niipmm.tsu.ru

Abstract. Paper presents a numerical simulation of the occurrence of flow instability and pressure self-oscillations for a complex configuration of the gas-dynamic tract in combustion chamber. Unsteady axisymmetric two-dimensional Navier-Stokes equations are used for mathematical modelling of compressible one-phase medium. To simulate turbulence, the *k*-ε and *LES* models were used. Fast Fourier Transform (FFT) determined the frequency spectrum of pressure pulsations in the combustion chamber. It is shown that in the case of a simple geometry of the free gas cavity in combustion chamber, both models of turbulence make it possible to determine the spectrum of the natural acoustic frequencies. Using the LES model in the case of complex geometry makes it possible to predict the hydrodynamic structure of a flow accurately. The flow, in this case, has an intensive vortex generation. Formation of small-scale vortex occurs in the near-wall regions and large eddies in the core of a flow. Frequency of large eddies formation can be combined with the natural acoustic frequencies of combustion chamber and can affect the amplitude of pressure pulsations.

1. Introduction

The processes occurring in the combustion chamber of solid rocket motor are characterized by high pressures and burning rates of a fuel, a complex composition of the combustion products [1-4]. Self-oscillation of operating parameters exceeding the specified limits can lead to unstable operation of the engine with it subsequent possible destruction. The main sources of this instability can be both unsteady combustion of the charge, and gas-dynamic instability of large-scale vortex structures [3].

The development of instability in the free volume of a combustion chamber (CCh) occurs under the influence of disturbance forming pressure waves [4]. Such a pressure waves propagate in the longitudinal direction with frequencies close to the natural acoustic frequencies of a combustion chamber. The natural acoustic frequencies of pressure oscillations in the combustion chamber are usually in range $f \approx 100 - 1000$ Hz. In the case of a complex combustion chamber configuration in modern solid rocket motor (SRM) the flow of combustion products during the engine operation is a complex structure with the presence of stagnant zones. A complex time-varying geometry leads to a gas-dynamical instability in the flow, its possible breakdown and intensive formation of large-scale vortex. Vortices generate acoustic signals that affect to the main flow. When the frequencies of vortex formation coincide with the natural acoustic oscillations in combustion chamber, frequency “capture” take place and then the magnitude of pressure pulsations will increase.

Recently, a large number of works have been carried out to study the nature of the appearance of unstable operating regimes in SRM chambers. Methodological recommendations were developed to determine the natural oscillations for the combustion chambers [5, 6]. For example, in paper [3], studies of the hydrodynamic nature of low-frequency oscillations caused by the instability of large-scale vortex structures in the main gas flow were made. In [7-9] was presented the results of a study of the influence of unsteady solid fuel combustion conditions on pressure fluctuations in combustion chambers. In paper [10] a model of the intra-chamber process in a solid rocket motor was proposed and tested. The phenomenon of inertia of gas-phase processes in a combustion wave was taken into account. The papers [11, 12] are devoted to a detail investigation of a vortex dynamics in the SRM channels of various configurations and discus their effect on the magnitude of pressure oscillations. In work [13] numerical calculations of the flow in a two-chamber SRM with the arising low-frequency pressure oscillations are given. It should be noted that in all studies rocket engines with solid fuel charge of various configurations are considered [14, 15]. This is related with the need to realize a given pressure curve in the combustion chamber. Therefore, the established methods for determining the natural acoustic frequencies of the combustion chambers in order to determine the source of pressure oscillations require refinement, taking into account the geometry of the charge.

In present paper, using the FLUENT application package was testing the method of determining the natural frequencies of the free gas cavity in combustion chambers. For mathematical modeling, the low-Reynolds model of turbulence k-ε [16] or the model of Large Eddy Simulation [17] solved together with the equations of gas dynamics. A study of the flow structure gas dynamic shows that intensive vortex generation in the case of using *LES* model can make a significant contribution to the position of the first modes and to the amplitude of acoustic pressure oscillations.

2. Computational model

The axisymmetric problem of combustion products outflow solved in a volume of the combustion chamber and nozzle block. Combustion products are a highly enthalpy gas blown from a burning surface with a constant flow rate. The gas is a single-phase compressible medium whose thermodynamic properties correspond to the equilibrium parameters of a two-phase mixture of combustion products in a typical mixed solid fuel of [14]. The scheme of the problem statement present in figure 1. The shape of the solid fuel charge includes an axisymmetric radial slot. The arrows indicate the injection of combustion products from the burning surface.

2.1. Computation domain and boundary conditions

The grid size of the computational domain during calculations was more than 3×10^6 cells. The grid consists of rectangular elements and is creating in the GAMBIT preprocessor. At the wall boundaries of the combustion chamber and the nozzle, an additional grid thickening is provided to satisfy the $y+$ condition for the corresponding turbulence model.

At the inlet boundary simulates the process of solid fuel combustion on a stationary portion of the pressure curve and set the boundary condition of constant mass flow, enthalpy and direction of the velocity vector along the normal to the surface. On solid walls no slip conditions are established, symmetry conditions on the symmetry axis. On the nozzle exit used the condition of supersonic gas outflow ($p_{\text{atm}} = 0$). The geometric parameters of calculation area present in table 1.

2.2. Numerical method

Using the FLUENT 6.3 software package numerical studies were carried out. FLUENT successfully used to calculate the tasks of gas dynamics and internal ballistics. To calculate the equations of continuity, momentum and energy, a coupled solver is used, which allow obtain a stable solution. Reynolds averaged Navier-Stokes equations are closed by the k-ε turbulence model. Navier-Stokes equations are calculated directly for large vortex structures in the case when using Large Eddies Simulation model. If the size of the vortices is smaller than the size of the calculated cell, the sub-grid model of Smagorinsky-Lilly is used [16]. The second order approximation for time and space are used for difference schemes. Preliminary calculations showed that the integration time step equal to $1 \cdot 10^{-5}$sec allows to trace the pressure pulsations in combustion chamber to obtain a high-resolution signal spectrum.

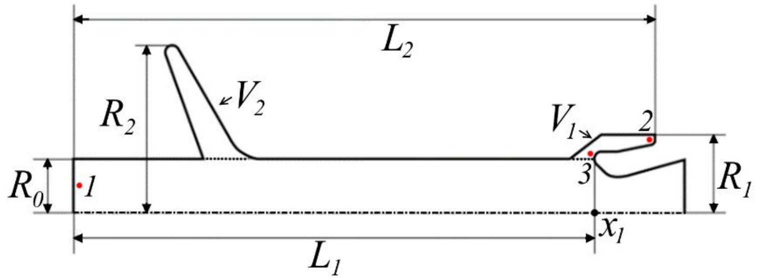

Figure 1. Schematic view of computation domain.

Table 1. Geometric dimensions of the calculation area.

Parameter	Notation	Value (m)
Length of the CCh before the nozzle	L_1	1.47 m
Full length of CCh	L_2	1.64 m
Radius of cylindrical section of CCh	R_0	0.152 m
Radius of the recessed part	R_1	0.22 m
Radius of the radial slot	R_2	0.47 m
Volume of the recessed part	V_1	0.043 m^3
Volume of the radial slot	V_2	0.107 m^3
Nozzle inlet	x_1	-0.064 m

The pressure pulsations are recorded on the 3 virtual points established in the calculation area. Point No. 1 is located at the front bottom of the combustion chamber, points No. 2 and 3 in the recessed portion of the nozzle (figure 1). Mathematical modeling of the gas dynamic tract work was carried out in two stages. At the first stage, the calculation was carried out until the flow field was established in the gas-dynamic tract at a pressure of 5 atm. In the second stage the injection of the working gas from the combustion surface increases sharply in order to obtain an oscillatory kind of process. The flow field obtained in the first stage used here as initial conditions. The total physical simulation time for all the calculations was 0.5 seconds. Such a period is sufficient to determine the characteristics of pressure pulsations in combustion chamber using the Fast Fourier Transform method.

3. Methods of research

3.1. Analytical methodology

The study of natural acoustic frequencies oscillations from analytical point of view imply the interaction of pressure waves with the combustion surface and the walls of the combustion chamber. In this case, in accordance with [4] the equation for determining the natural acoustic frequencies is written as:

$$f = \frac{a}{2}\sqrt{\left(\frac{n}{L}\right)^2 + \left(\frac{2\alpha_{mk}}{D}\right)^2}, \tag{1}$$

where a – sound of speed in gas; L – length of combustion chamber; D – diameter of combustion chamber; n – natural number, which determines the mode of longitudinal oscillations; m, k – natural numbers that define radial and tangential modes.

According to [4], it is assumed that in combustion chambers, which length is much larger than other dimensions, the pressure fluctuations that significantly affect to operation of the combustion chamber arise at the frequencies of the longitudinal modes. Thus, equation (1) can be written:

$$f_L = \frac{a}{2L}\cdot n = \frac{\sqrt{\gamma(RT)_{av}}}{2L}\cdot n, \tag{2}$$

where $(RT)_{av}$ – mean value in chamber.

3.2. Calculation of pressure pulsations and frequency analysis

Pressure pulsations are determined at virtual points as follows:

$$p_i' = p_i - p_{ch},$$

where p_i – local changes of pressure at virtual points, p_{ch} – mean value of pressure over the volume of the gas cavity at the current time.

Received signals of the pulsating pressure components are decomposed into the frequency spectrum by Fast Fourier Transform:

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt,$$

where $f(t)$ – continuous signal of the original function, $F(\omega)$ – frequency signal.

4. Results and discussion

Using the method mention above, numerical studies to determine the natural frequencies of gas cavity in combustion chamber with fillers of complex shape have been carried out. Figure 2 shows the results of calculations comparing the influence of the turbulence model *k*-ε with *LES* model on the flow character for the initial configuration (figure 1). It can be seen that at the stage of a sharp increase the injection of combustion products the flow characters are very different. Particular, the dynamics of large-scale vortex structures is well illustrated for the *LES* model. Figure 3 shows the distribution of pressure in combustion chamber at the stationary section of the pressure curve in the section along the axis from the front bottom to the inlet of the nozzle. Curves No. 1 and 2 shows the pressure distribution calculated from *k*-ε model at moment t_1 and t_2, and curves No. 3 and 4 similar pressure distributions calculated by *LES* model. The distribution of pressure along the axis is essentially non-monotonic in the case of *LES*, which a consequence of vortex structure of the flow. Therefore, application solutions based on the averaged models does not give a complete picture of the flow characteristics, and eliminates the possibility of determining the source of pulsation. This does not allow working out methods to deal with fluctuations.

Figure 4 shows the results of frequency analysis of signals from virtual points *1-3* for calculating the flow with *k*-ε model. We can see that the position of the first acoustic mode coincides for the points located at the front bottom and in the recessed portion of the nozzle. The amplitude of pulsations of the first mode at point *1* is about 430 Pa. At point's No. *2-3* in the recessed portion of the nozzle, the amplitude is 880 Pa. Such a difference in the pulsation amplitudes we can explain by the conclusions obtained in [3]. Vorticity area is creating in the stagnant zone that occurs in the recessed portion of the nozzle. The dimensions of the contact discontinuity of the potential and vortex flow fields determine the vortex frequency of the stagnant zone. Therefore, the first mode of natural acoustic frequencies and frequencies of vortex oscillations of the stagnant zone are superimposed, which leads to a local increase in the pulsation amplitude. It is important to note that there is a mode on virtual sensor No. 1 whose oscillation amplitude coincides with the first acoustic mode. Its frequency is about 700 Hz. The presence of this oscillation mode has explained by the structure of the flow in the free gas cavity. In this case, there is an expiration of a dense stream of gas from the radial slot. This leads to the appearance of acoustic oscillations with a higher frequency.

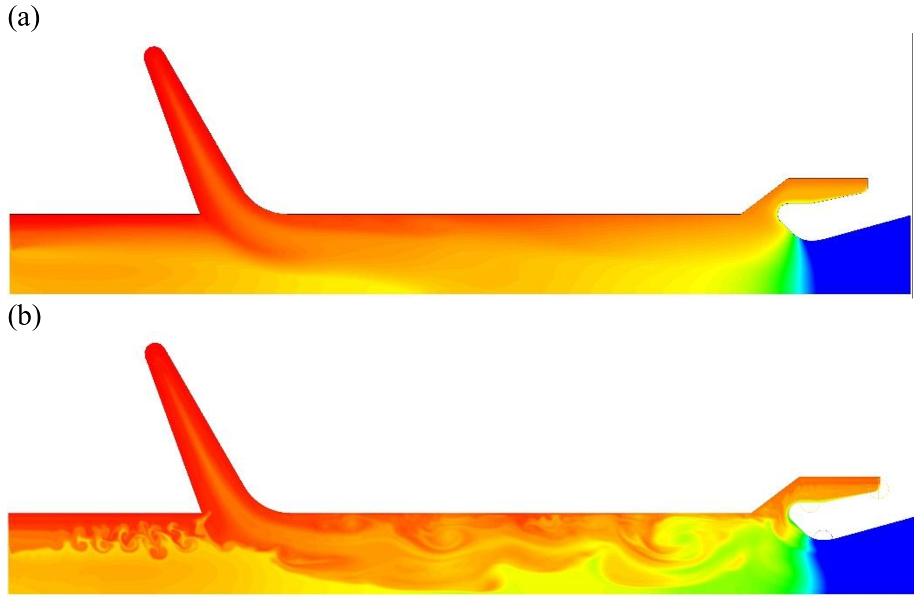

Figure 2. Density field distribution for *k*-ε model (a) and *LES*-model (b) at initial time.

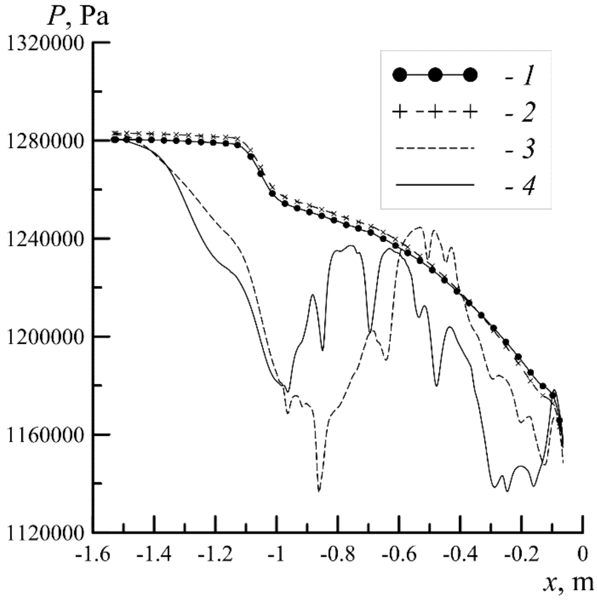

Figure 3. Pressure distributions in longitudinal cross-section of cylindrical part of the combustion chamber: *1* – *k*-ε model at $t = t_1$; *2* – *k*-ε model at $t = t_2$; *3* – *LES* model at $t = t_1$; *4* – *LES* model at $t = t_2$.

Figure 5 shows the results of frequency analysis from virtual points No. *1* (figure 5-*a*) and No. 2 (figure 5-*b*) for the calculation by *LES* model. The data at point No. *3* was omitted, since the spectra and amplitudes of pressure pulsations are almost identical to point No. *2*. Amplitude of the first acoustic mode is two times higher than in the previous calculation. On the virtual sensor No. *1* it's about 850 Pa, and at the point No. *2* – 1850 Pa. Frequency spectrum form of the pressure pulsations calculating by *LES* is different significantly from the *k*-ε model (figures 4, 5).

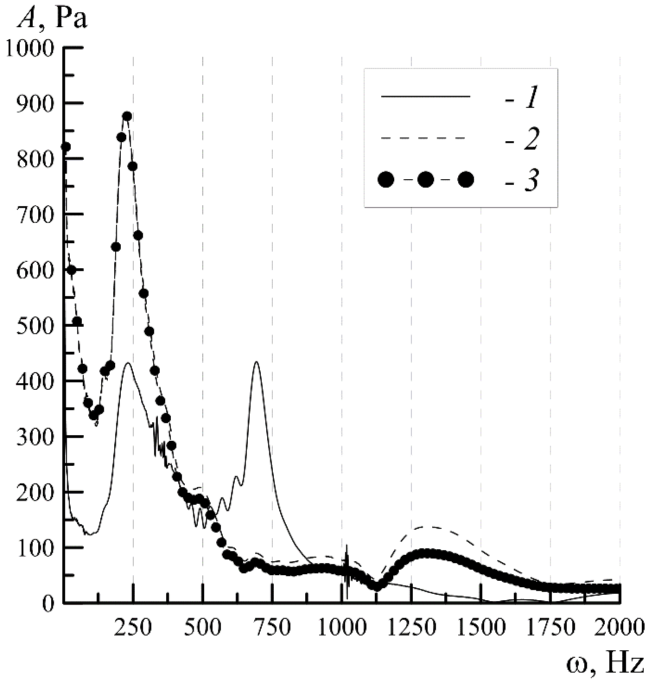

Figure 4. Frequency range of pressure oscillations on virtual point No. *1-3*, numerical modelling by *k*-ε.

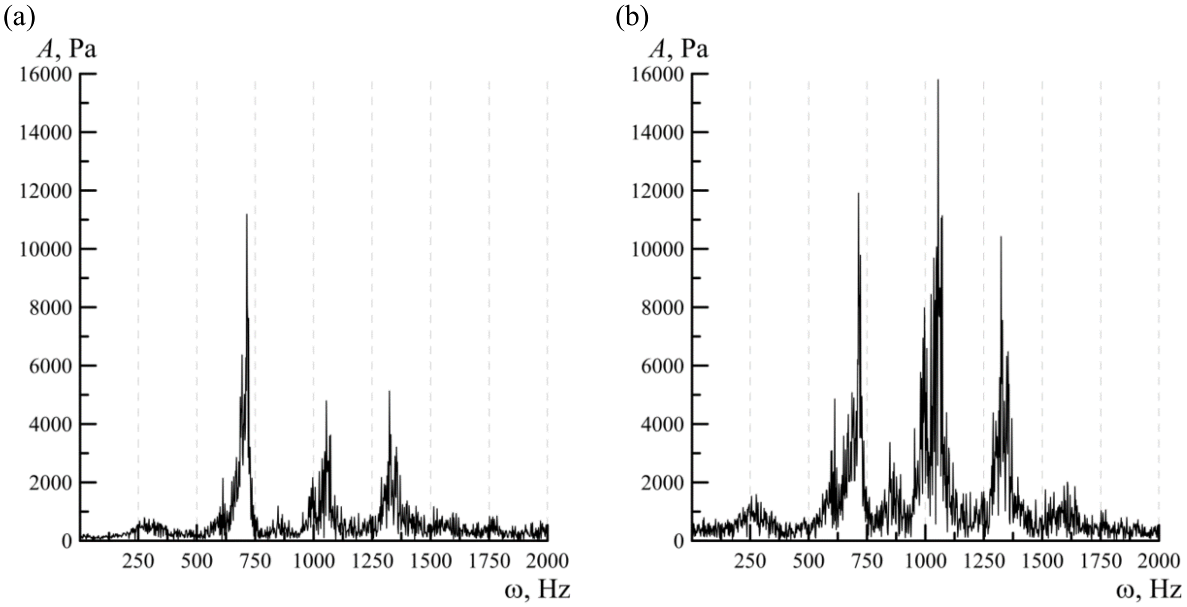

Figure 5. Frequency range of pressure oscillations on virtual sensors No. *1* (a) and *2* (b), numerical modelling by *LES*

4.1. Determining the natural frequencies of a free gas cavity

Method for determining the natural frequencies by equation (2) works well for cylindrical combustion chambers. When the charge configuration is complicated, it becomes a source of gas dynamic disturbances in the flow. Then the use of the analytic equation (2) is not possible. For example, the natural frequency of acoustic oscillations for the configuration shown in figure 1 and determined from (2) is 332 Hz. The results of numerical calculations by k-ε model show that the first mode of oscillation for this configuration is ~ 225-230 Hz. The calculation by the *LES* model given the result of ~ 250-270 Hz. Therefore, it is necessary to correct equation (2) for determining the natural frequencies of a free gas cavity of complex shape. Figure 6 shows the calculation area for developing the calculation methodology using the geometric approach. Approach based on bringing the gas cavity of combustion chamber to a cylindrical shape while full volume and cross-sectional area of the cylindrical portion correspond to the initial configuration.

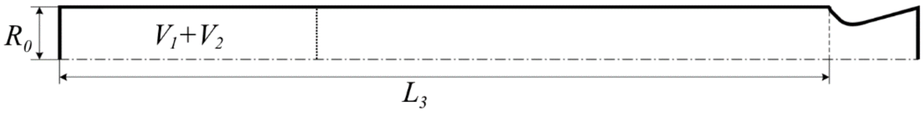

Figure 6. Testing computational domain for methodical calculations.

Figure 7 shows the results of calculations for the test configurations using k-ε models (figure 7-*a*) and *LES* (figure 7-*b*). On virtual points located at the forward bottom of the combustion chamber showed the result of processing signals. When calculating by k-ε model (figure 7-*a*), the 1st mode of oscillation is in the range 220-225 Hz, which corresponds to the same model calculation for the initial configuration (figure 1). Calculating by *LES* model (figure 7-*b*), the 1st mode of oscillation moves to a level above ~250 Hz, which also agrees with the calculation of the same model for the initial configuration (figure 1). A similar situation is observing for the second acoustic mode. For calculation by k-ε model, it is ~421 Hz, for calculation by *LES* model ~480-500 Hz. Amplitudes of acoustic modes are almost identical. The vortex flow pattern when using the *LES* model has a strong influence on higher frequencies (ω > 600 Hz). An important feature is the fact that the interaction of an acoustic wave with vortices leads to a frequency "hopping", which is confirm with the results of [2-4].

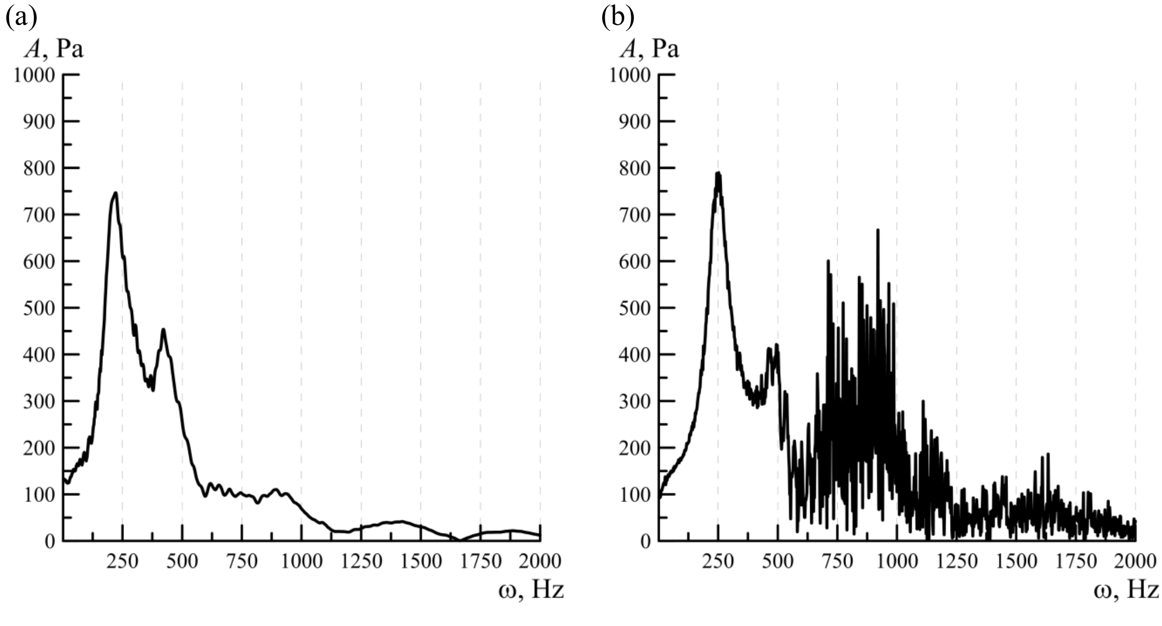

Figure 7. Frequency range of pressure oscillations for testing computational area.

The results indicate that the change of free gas cavity shape within our geometric approach does not lead to a shift of the natural acoustic frequency, in contrast of equation (2), the main parameter of which is the linear length of the combustion chamber. Change of the geometric shape of a charge affects only the pulsations amplitude and gas-dynamic structure of the flow. Bringing the complex shape of the initial charge to the cylindrical form is in good agreement with the analytical equation (2). Thus, it is possible to correct the analytical equation (2) for determining the longitudinal modes of pressure oscillations in complex shape gas cavities, based not on the actual length of the combustion chamber, but on the total free volume and present it as:

$$f_L = \frac{\sqrt{\gamma (RT)_{av}}}{2L_{eff}} \cdot n, \qquad L_{eff} = \frac{V_{tcv}}{\pi R_0^2}, \tag{3}$$

where V_{tcv} – total gas cavity volume.

Table 2 presents the generalized calculation results for two calculated areas. Numerical studies using the turbulence model *k*-ε show very similar results for both configurations, but the difference with the analytical method for each configuration separately is very significant. The calculation carried out by *LES* model for the initial configuration shows differences from both calculation methods that is explain by the complex vortex structure of the flow and the direct interaction of the main flow with the vortex disturbance. The geometric approach (3) proposed in this paper gives good results in determining the position of the natural acoustic frequencies. The difference with the calculated data is explained by the fact that the estimation is based on the geometric dimensions of the region and does not take into account the actual picture of the flow in the free volume of combustion chamber.

Table 2. The position of the 1st natural acoustic mode of free gas cavity.

	Equation (2), Hz	*k*-ε model, Hz	*LES* model, Hz	Equation (3), Hz
Initial configuration (Figure 1)	332	~228	~250-270	240
Testing configuration (Figure 6)	240	~222	~242	

5. Conclusion

The paper presents the results of numerical modeling of solid fuel rocket motor operation to determine the characteristics of pressure pulsations. The calculations were carried out using two numerical approaches: the Reynolds averaged Navier-Stokes equations with equations of *k*-ε turbulence model; calculation of the full Navier-Stokes equations with of Large Eddy Simulation model (*LES*). The frequency-amplitude spectra of pressure pulsations are compared for an axisymmetric configuration of a complex shape of free gas cavity in combustion chamber for both numerical approaches. The results show that using numerical approach based on the averaged equations allows us to estimate the position of a first acoustic mode of the combustion chamber natural acoustic frequency with sufficient accuracy. Methodical studies of different combustion chamber configurations allowed to correct the existing analytical technique (2) within the geometric approach (3) for free gas cavities of complex shape.

The calculation of complete Navier-Stokes equations with LES model has shown that the nature of the flow is significantly different from the calculation of Reynolds averaged approach. A pronounced vortex character leads to a significant increase of pressure pulsations amplitude. Given a complex shape of the combustion surface, the development of a calculation technique that allows to simulate of a gas-dynamic flow structure with vortices is an important link in identifying the causes of instability and preventing possible crash in operation of the rocket engine.

Acknowledgments

This research was supported by “The Tomsk State University competitiveness improvement programme” grant №8.2.12.2018.

References

[1] Timnat Y M 1987 *Advanced Chemical Rocket Propulsion* (London: Academic Press)

[2] Erokhin B T 1991 *Theory of SRM intrachamber process and design* [in Russian – Teoriya vnutrikamernykh protsessov i proyektirovaniya RDTT]. (Moscow: Mashinostroyeniye)

[3] Sukhinin S V and Akhmadeev V F 1993 Hydrodynamic Source of Oscillations in Combustion Chambers. *Combustion, Explosion, and Shock Waves* **29(6)** 38-46

[4] Prisnyakov V F 1984 *Dynamics of rocket engines in solid fuel* [in Russian – Dinamika raketnykh dvigateley tverdogo topliva]. (Moscow: Mashinostroyeniye)

[5] Vasenin I M, Krainov A Yu, Lipanov A M and Shrager E R 2015 Method for Direct Numerical Simulation of Turbulent Gas Flows in Curvilinear Coordinates *Computational Mathematics and Mathematical Physics* **55(5)** 883-890 DOI: 10.1134/S0965542515050176

[6] Hirschberg L, Schuller T, Collinet J, Schram C and Hirschberg A 2018 Analytical model for the prediction of pulsations in a cold-gas scale-model of a Solid Rocket Motor *Journal of Sound and Vibration* **419** 452–468 DOI: 10.1016/j.jsv.2018.01.025

[7] Poryazov V A and Krainov A Yu 2017 Mathematical model and calculation of the unsteady combustion rate of the metallized solid rocket propellants *Tomsk State University Journal of Mathematics and Mechnics* [Vestnik Tomskogo Gosudarstvennogo Universiteta, Matematika i Mekhanika – in Russian] **50** 99-111. DOI: 10.17223/19988621/50/9

[8] Su W, Wang N, Li J, Zhao Y and Yan M 2014 Improved method of measuring pressure coupled response for composite solid propellants *Journal of Sound and Vibration* **333** 2226-2240 DOI: 10.1016/j.jsv.2013.12.003

[9] Minkov L L, Shrager E R and Kiryushkin A E 2016 Two Approaches for Simulating the Burning Surface in Gas Dynamics. *Key Engineering Materials* **685** 114-118 DOI: 10.4028/www.scientific.net/KEM.685.114

[10] Larionov B I and Kuzmenko A V 2016 Study of pressure fluctuations in a solid rocket motor chamber in quasi-stationary operation mode [Issledovaniye pul'satsiy davleniya v kamere tverdotoplivnogo raketnogo dvigatelya na kvazistatsionarnom uchastke raboty – in Russian]. *Combustion and Explosion* **9(4)** 116-125

[11] Zhang Q, Li J, Wang W and Wang N 2011 Numerical Analysis on Oscillation Characteristics in a Tailpipe Nozzle Solid Rocket Motor. *Journal of Spacecraft and Rockets* **48(1)** 103-109 DOI: 10.2514/1.48867

[12] Zhang Q, Wang N, Li J, Su W and Zhang Y 2015 Effect of the head cavity on pressure oscillation suppression characteristics in large solid rocket motors. *Technological Sciences*. **58(7)** pp 1250-1262. DOI: 10.1007/s11431-015-5834-z

[13] Egorov M Y and Egorov Ya V 2005 Numerical study of low-frequency acoustic instability in a two-chamber SRM [Chislennoye issledovaniye nizkochastotnoy akusticheskoy neustoychivosti v dvukhkamernom RDTT – in Russian]. *Mathematical modeling of system and process* [Matematicheskoye modelirovaniye sistem i protsessov – in Russian] **13** 101-109

[14] Alemasov V E, Dregalin A F and Tishin A P 1980 *The theory of rocket engines* [in Russian – Teoriya raketnykh dvigateley] (Moscow: Mashinostroyeniye)

[15] Kostyushin K, Kagenov A, Eremin I, Zhiltsov K and Shuvarikov V 2017 Gas flow calculation method of a ramjet engine. *AIP Conference Proceedings* **1899** 060012 DOI: 10.1063/1.5009883

[16] Launder B E and Spalding D B 1972 *Lectures in Mathematical Models of Turbulence* (London: Academic Press)

[17] Volkov K N and Emelyanov V N 2008 *Modeling of large eddies in turbulent flows calculations* [in Russian – Modelirovaniye krupnykh vikhrey v raschetakh turbulentnykh techeniy] (Moscow: Fizmatlit)