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Report

The HyOpt model

Input data and mathematical formulation

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ABSTRACT

This report gives a description of the mathematical model underlying the HyOpt model and the associated input data. HyOpt is an optimization model for the design and evaluation of energy systems with a focus on hydrogen-based technology. Given a structure of the energy system plus expected energy demands and costs, the model decides which elements should be included and with what capacity, in order to optimize a given objective—typically maximizing the net present value of the whole modelled system. This problem is formulated as a mixed integer linear optimization problem and solved used using a commercial solver.

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1 Overview

HyOpt is an optimization model for the design and evaluation of energy systems including hydrogen-based technology. Given a structure of the energy system plus expected energy demands and costs, the model decides which elements should be included and with what capacity, in order to optimize a given objective—typically maximizing the net present value of the whole modelled system. In addition to the proposed structure, the model also decides (dynamic) operation of the elements and reports investment- and operational costs over a specified time horizon.

HyOpt consists of three parts: an Excel front end, an SQLite database, and the optimization model itself, written in the FICO™ Mosel optimization language. The typical workflow starts by specifying the input in the Excel front end. This includes the list of the proposed network elements and their properties, the time structure and all required time series. Then one runs Mosel, which reads the input data from Excel or the database, constructs an optimization-model instance, solves it using the FICO™ Xpress solver, and then pushes the results back to the Excel file and the database. Thereafter, one can study the results in several automatically-generated tables and charts.

2 Model structure and parameters

All elements in the modelled system are represented as *nodes* with some specified properties. For each pair of nodes, we can then allow flow of some *products* between them. Possible products include hydrogen (compressed or liquid), oxygen, electricity, water, natural gas, etc. The flow can be optionally allowed only in some periods, which allows for modelling of downtime, or availability of a transport between two nodes (such as a ship).

The main goal of the model is to find which nodes should be installed, at what time, and with what capacity. Note that the capacity can be modelled either in power (flow), typically for production nodes, or in energy (volume) for storage nodes.

2.1 Time periods

The time horizon of the model is divided into *strategic periods*, typically one year or longer. All infrastructure investments happen at the start of these periods. In addition, the available capacity gets updated, to reflect ageing. Each strategic period includes a sequence of *operational periods*, in which the model uses the infrastructure. The total length of the operational periods might be shorter than the strategic period they correspond to. In such a case, the operational results get scaled up to the desired length. For example, if the strategic period is one year and the operational periods cover one week of operations, the resulting volumes, costs and incomes will be multiplied by 52.

At the moment, the model allows for only one operational sequence per strategic period. However, a future revision is planned that will allow for multiple sequences so we can, for example, have operations for four weeks, one for each season of the year.

The temporal sets are as follows:

- T^{SP}: set of strategic time periods, $T^{SP} = 1, \ldots, T$

- T^{OP}_{sp}: set of operational time periods per strategic period, $T^{OP}_{sp} = 1, \ldots, \omega_{sp}$

In addition, different parameters are used to describe time in HyOpt:

- $T^{\text{unit}} \in \mathbb{N}$: time unit, it is the basic time unit, measured in seconds

- $\Delta T^{\text{OP}}_{sp,op} \in \mathbb{R}$: duration of operational period, in multiples of T^{unit}

- $\Delta T^{\text{OP,h}}_{sp,op} \in \mathbb{R}$: duration of operational period, in hours

- $\Delta T_{sp}^{\mathrm{SP}} \in \mathbb{R}$: duration of strategic periods, as a multiple of the whole operational horizon

- $\Delta T_{sp}^{\mathrm{SP,y}} \in \mathbb{R}$: duration of strategic periods, in years

- $\Delta T_{sp}^{\mathrm{SP,h}} \in \mathbb{R}$: duration of strategic periods, in hours

- $T_{sp} \in \mathbb{R}$: start time of strategic periods, in years from the model start

2.2 Nodes, products and flows

Nodes are the main building block of the model, since they represent every element of the system. They can be divided into plants, markets, storages and transport. The sets used by the model for the nodes are:

- N: set of all nodes n (of all types)

- I: set of plant nodes i

- S: set of storage nodes s

- R: set of transport nodes r

- M: set of market nodes m

All nodes have a capacity that can be altered through investments in additional capacity, removal of capacity or degradation. The capacity modelled varies between the different node types

- Plants: maximal production rate per hour (either in terms of a specified output product or (combination of) input product(s))

- Storages: maximal storage level

- Transport nodes: maximal outflow per hour

- Markets: maximal delivery to market or purchases in market per hour

The unit used for measuring capacity will be product dependent. Storages and markets have a single product and the unit will be based upon it. For a plant the unit of the capacity defining product will be used.

The model works with a set of products, P. These products are produced in plant nodes or bought in a market node, stored in storage nodes, transported through transport nodes and sold/delivered in market nodes. Products flow between nodes with the possibility of flow controlled by the following parameters:

$Can_{n_1,n_2,p}^{\mathrm{flow}}$	whether a product p can flow between the nodes (0/1)
$Link_{n_1,n_2,p,sp,op}$	whether there can be flow between the nodes in the time period (0/1)

The flow of product is given in units depending on the product: energy products (Power and Heat) are measured in MWh, while all gases and liquids are measured in kilograms.

2.3 Data parameters

2.3.1 Economic parameters

The net present value of the whole system is calculated based on a given discount rate. For discounting operational values throughout a strategic period, we use an adjusted average value.

- D: discount rate

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- Y: discount-rate multiplier for one year

$$Y = \frac{1}{1 + \frac{D}{100}} \tag{1}$$

- T_{sp}: start of the strategic period in years. The first period is assumed to start at 0.

- D_{sp}: discount-rate multiplier for the start of a strategic period

$$D_{sp} = Y^{T_{sp}}, \qquad \forall sp \in T^{SP} \tag{2}$$

- D_{sp}^{avg}: average discount-rate multiplier for a strategic period (used for operations)

$$D_{sp}^{\text{avg}} = \frac{Y^{T_{sp}} - Y^{T_{sp} + \Delta T_{sp}^{\text{SP,y}}}}{\frac{\ln(1 + \frac{D}{100})}{\Delta T_{sp}^{\text{SP,y}}}}, \qquad \forall sp \in T^{SP} \tag{3}$$

2.3.2 General node parameters

All nodes share some common specifications related to capacity, investments and costs, the most important being:

- existing capacity, if any

- maximum allowed capacity, both in total and per new addition

- CAPEX, both fixed and capacity-dependent

- OPEX, both fixed and capacity- and volume-dependent

- lifetime and maintenance information

The following table describes parameters available for all nodes. Most of the input is optional, though some costs should be provided to prevent the model from installing unlimited capacities.

Name	Description
Ext_n	whether the capacity of the node can be changed or not
Bin_n^{inv}	whether capacity is for a single unit or not
$Loss_n^{\mathrm{Y}}$	capacity lost per year due to degradation of a node
$\mu_{n,sp^I,sp}$	multiplier for how much capacity installed in strategic period sp^I is available in the period sp; calculated from $Loss_n^{\mathrm{Y}}$
Add_n^{max}	maximum new capacity that can be installed per node
Add_n^{min}	minimum new capacity that can be installed per node
Cap_n^{max}	maximum total capacity that can be installed per node
Cap_n^{min}	minimum total capacity that can be installed per node
Cap_n^{ex}	existing capacity per node
$Capex_n^{\mathrm{fix}}$	fixed CAPEX of a node
$Capex_n^{\mathrm{var}}$	variable CAPEX scaling with the installed capacity
$Opex_n^{\mathrm{fix}}$	fixed yearly opex per node
$Opex_n^{\mathrm{rel}}$	relative yearly OPEX (as % of CAPEX)
$Opex_n^{\mathrm{var}}$	variable OPEX with respect to installed capacity per node
$C_n^{\mathrm{rem,fix}}$	fixed part of removal cost
$C_n^{\mathrm{rem,var}}$	variable (per-capacity) part of removal cost
$C_n^{\mathrm{rem,rel}}$	removal cost, as a fraction of CAPEX
Val_n^{rest}	rest value, as a fraction of CAPEX

The nodes come in several types, which differ in function and therefore have some type-specific input data. The following subsections discusses the data input for the different node types.

2.3.3 Production Plants

A production plant is a node that converts one or several input products into one or more output products. Typical examples are electrolysers, fuel cells and turbines. For these, we must provide a production function describing the amount of each output as a function of the input. In the simplest case, this is given by a simple multiplier, but we also allow for the conversion rate to be a piecewise-linear function of capacity utilization, as long as this function is decreasing.[1] In addition, wind turbines and photo-voltaic (PV) panels need production profiles derived from wind and irradiance data.

The parameters used for the production plants are the following:

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[1]Modelling of efficiency that increases with capacity utilization is possible, but it would make the model too difficult to solve.

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Name	Description
P_i^{in}	set of products that are input of the plant, $P_i^{in} \subseteq P$
P_i^{out}	set of products that are produced by the plant, $P_i^{out} \subseteq P$
P_i^{cap}	the product that describes the capacity of a plant, $P_i^{cap} \in P_i^{in} \cup P_i^{out}$
P_i^{main}	the main out product used for e.g. unit costs, $P_i^{main} \in P_i^{out}$
Adj_i	whether the production of a plant is adjustable or not
$Fix_i^{load,\%}$	production capacity fixed to a given fraction of installed capacity
$Spills_{i,p}$	whether the plant can spill product p
$C_{i,p}^{spill}$	the cost for spilling product
C_i^{unit}	cost per produced unit
Pow_i^{cons}	power consumption per produced unit

Production functions and modes:

Name	Description
D_i	production modes for plant i
$\mu_{i,d,p}$	multipliers for linear production function
$L_{i,d,p}$	intervals for efficiency-linearisation for production plants
$\lambda_{i,d,p,l}$	size of the interval
$\pi_{i,d,p,l}$	multiplier in the interval
$Prod_{i,sp,op}^{rate}$	production rate per capacity installed

2.3.4 Markets

Markets are nodes with load, demand and/or supply for a particular product. Note that we distinguish between load, which has to be delivered, and demand, whose satisfaction is optional. For each of the three values, we have to provide a time series (profile) with volumes and prices (typically hourly data). In addition, we may relax the absolute requirement that all load has to be delivered and instead specify penalty for non-delivered load and/or required regularity as a minimal fraction of the load that has to be delivered.

The basic market parameters are as follows

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Name	Description
P_m^{M}	the product of the market
M_p	all markets for product p, $M_p = \{m \in M : P_m^{\mathrm{M}} = p\}$
$L_{m,sp,op}^{\mathrm{oad}}$	load curve, i.e., demand that must be delivered
$D_{m,sp,op}^{\mathrm{emand}}$	maximal market demand
$S_{m,sp,op}^{\mathrm{upply}}$	maximal market supply
$Reg_m^{\min}$	minimum regularity in deliveries for each strategic period
$Price_m^{\mathrm{sell}}$	the income from selling product
$Price_m^{\mathrm{buy}}$	the price for buying product
$Price_m^{\mathrm{load}}$	the income from satisfying load
Val_m^{loss}	the cost incurred for unmet load requirements

2.3.5 Storages

Storage nodes store a single product between time periods. Typical examples are compressed-hydrogen storage and batteries. Storage levels have to stay below the invested capacity. They have the following additional input parameters, all optional:

- minimal fill level serving as a buffer, either in absolute volume or as a fraction of installed capacity

- maximal rate of filling/charging and emptying/discharging, measured either in volume unit per hour (for hydrogen storage) or in the fraction of capacity per hour (for batteries)

- efficiency of filling and emptying (typically used for batteries)

The parameters needed to model the storages are described in the table below:

Name	Description
$S_s^{\mathrm{init},\%}$	initial storage fill (in %) at the start of each strategic period
C_s^{yclic}	whether the storage should have a cyclic behaviour (0/1)
$S_s^{\min}$	minimum storage level as an absolute value
$S_s^{\min,\%}$	minimum storage level as a percentage of the capacity
R_s^{in}	maximum fill speed, as an absolute value
R_s^{out}	maximum empty speed, as an absolute value
$R_s^{\mathrm{in},\%}$	maximum fill rate relative to capacity
$R_s^{\mathrm{out},\%}$	maximum empty rate relative to capacity
ρ_s^{in}	efficiency during filling/charging
ρ_s^{out}	efficiency during emptying/discharging
C_s^{loss}	cost per unit loss

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2.3.6 Transport

Transport nodes are used for cases where we need more control over the flow between nodes. Since we do not include edges as separate entities in the model, the products simply flow between the specified pairs of nodes, without any limits or costs. This is sufficient in most cases, but sometimes we want to limit the flow, put a cost on it, or model transmission losses. In these cases, we can put a transport node between the two connected nodes. As an example, consider a charging or filling station, where the flow capacity depends on the price of the station and, in case of a battery charger, there are some losses in the process.

The total flow through the node will at all times be restricted by the invested capacity of the node. In addition we have the following parameters that can be used in the modelling of the transport nodes

Name	Description
$Tran_{n,p}^{\text{loss}}$	transport losses as a share of flow into the node

3 Optimization model

3.1 Decision variables

There are two types of decision variables in the model: *strategic variables* for decisions done at the beginning of the strategic periods, and *operational variables* for all decisions in the operational periods.

3.1.1 Strategic variables

The only 'real' strategic decisions are what capacity to add to, or remove from, the nodes in the network. In addition, there is an option to use binary variables for marking whether we invest or de-invest at the node. This is required for calculation of fixed CAPEX. On the other hand, all binary variables increase the computational complexity of the model, so they are typically used for larger investments with significant fixed costs.

In addition to variables modelling these decisions, there are several 'technical' variables for keeping track of installed capacity, including capacity loss due to ageing or calculation of maximal power during the strategic period, if required by the power-tariff evaluation. A complete list of the strategic variables are given in the following table:

Name	Description	
$\mathbf{cap}_{n,sp}^{\text{add}}$	nominal capacity added at the period start	*Investment*
$\mathbf{add}_{n,sp}$	binary indicator for added capacity > 0	
$\mathbf{cap}_{n,sp}^{\text{rem}}$	nominal capacity removed at the period start	
$\mathbf{rem}_{n,sp}$	binary indicator for removed capacity > 0	
$\mathbf{cap}_{n,sp}$	available capacity during a strategic period	*Capacity*
$\mathbf{cap}_{n,sp}^{\text{nom}}$	nominal capacity during strategic period	
$\mathbf{hascap}_{n,sp}$	binary indicator for capacity > 0	
$\mathbf{cap}_{n,sp^I,sp}^{\text{rem}}$	capacity from sp^I removed at the period start	*Capacity with*
$\mathbf{cap}_{n,sp^I,sp}^{\text{nom}}$	capacity from sp^I still installed in period	*investment time*
$\mathbf{hascap}_{n,sp^I,sp}$	binary indicator for installed capacity > 0	
$\mathbf{block}_{n,sp}^{\text{add}}$	number of blocks to invest in (integral)	*Discrete*
$\mathbf{block}_{n,sp}^{\text{rem}}$	number of blocks to remove (integral)	*investments*

Note that all continuous variables are assumed non-negative. It is possible to restrict the strategic periods for which it is possible to make investments by setting $Can_{sp}^{\text{inv}} = 0$ for the periods without investment

$$Can_{sp}^{\text{inv}} = 0 \implies \mathbf{add}_{n,sp} = 0 \text{ and } \mathbf{rem}_{n,sp} = 0, \qquad \forall n \in N, sp \in T^{SP}. \tag{4}$$

This implication is not directly included in the technical implementation of the model. There we only create variables as needed. Thus $\mathbf{add}_{n,sp}$ and $\mathbf{rem}_{n,sp}$ variables, and the associated $\mathbf{cap}_{n,sp}^{\text{add}}$ and $\mathbf{cap}_{n,sp}^{\text{rem}}$, will not be created if $Can_{sp}^{\text{inv}} = 0$. The variables are kept in the model formulation provided here to make the presentation simpler. A similar convention is used also for other implications of the same type.

Not all nodes allow a capacity extension

$$Ext_n = 0 \implies \mathbf{add}_{n,sp} = 0 \text{ and } \mathbf{rem}_{n,sp} = 0, \qquad \forall n \in N, sp \in T^{SP}. \tag{5}$$

The number of blocks to invest in is only tracked if relevant (i.e. $Cap_n^{\text{block}} > 0$)

$$Cap_n^{\text{block}} = 0 \implies \mathbf{block}_{n,sp}^{\text{add}} = 0 \text{ and } \mathbf{block}_{n,sp}^{\text{rem}} = 0, \qquad \forall n \in N, sp \in T^{SP}. \tag{6}$$

3.1.2 Operational variables

Operational variables are used for modelling of the system's operation. This includes:

- production of the production nodes

- loads delivered to, or obtained from, the market nodes

- storage levels at the storage nodes

- flows between the nodes

In addition, there are extra variables required for modelling of the piecewise linear production profiles, where provided, plus several different book-keeping variables. Since there are typically many operational periods for every strategic one, we avoid operational variables requiring binary or integer variables to keep the solution time down. The table below provides the basic operational variables.

Name	Description
$\mathbf{prod}_{i,p,sp,op}$	Production for each operational period in a plant
$\mathbf{slack}_{i,p,sp,op}$	Variable used to allow for production spill
$\mathbf{inv}_{s,sp}^{\text{init}}$	Initial storage inventory at the start of each strategic period
$\mathbf{inv}_{s,sp,op}$	Inventory at the end of each operational period
$\mathbf{vol}_{m,sp,op}^{\text{sell}}$	Volume sold in the market
$\mathbf{vol}_{m,sp,op}^{\text{buy}}$	Volume bought in the market
$\mathbf{load}_{m,sp,op}^{\text{del}}$	Delivery of required load in the market
$\mathbf{load}_{m,sp,op}^{\text{lost}}$	Lost/unsatisfied load
$\mathbf{flow}_{n_1,n_2,p,sp,op}$	Flow of products between nodes
$\mathbf{flow}_{i,d,sp,op}^{\text{in}}$	Combined inflow for production nodes

These variables will only be available for relevant node types. In addition there can be parameters that restrict the availability of certain variables. For flow variables in particular, it must be explicitly be specified that flow of a product is possible between two nodes. Otherwise, the flow is fixed to zero

$$Can_{n_1,n_2,p}^{\text{flow}} = 0 \implies \mathbf{flow}_{n_1,n_2,p,sp,op} = 0, \qquad \forall sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{7}$$

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As for the strategic variables, in the implementation of the model we handle these implications by only creating flow variables as needed. Some links only allow flow in certain periods, e.g. a ferry is connected to a land-based battery only in a restricted set of operational periods

$$Link_{n_1,n_2,p,sp,op} = 0 \implies \mathbf{flow}_{n_1,n_2,p,sp,op} = 0. \tag{8}$$

For production and transport nodes it is possible to state if a product is compatible with the node or not through the parameter $P_{n,p}^{\text{comp}}$. For production nodes this limits the products that can be produced

$$P_{i,p}^{\text{comp}} = 0 \implies \mathbf{prod}_{i,p,sp,op} = 0, \qquad \forall sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{9}$$

For transport nodes this restricts the products that can flow through the node

$$P_{r,p}^{\text{comp}} = 0 \implies \mathbf{flow}_{n,r,p,sp,op} = 0 \text{ and } \mathbf{flow}_{r,n,p,sp,op} = 0, \qquad \forall n \in N, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{10}$$

3.1.3 Other variables

In addition to the above variables, the model can involve other variables for more detailed modelling or for covering other aspects not handled by the basic modelling.

Name	Description	
$\mathbf{prod}_{i,d,p,sp,op}^{\text{mode}}$	production in each mode	*Operation mode*
$\mathbf{mode}_{i,d,p,sp,op}$	binary variables to decide the operation mode of the plant	
$\mathbf{prod}_{i,d,p,sp,op}^{\text{mode,adj}}$	production adjusted for active mode	
$\mathbf{flow}_{i,sp,op,d,p,l}^{\text{in,pw}}$	intervals for input flow	*Piecewise linear*
$\mathbf{prod}_{i,sp,op,d,p,l}^{\text{pw}}$	values for piecewise production	*production*
$\mathbf{em}_p$	total product emissions	*Emission*
$\mathbf{del}_p^{\text{tot}}$	total market deliveries of product	

3.2 Constraints

Most of the constraints in the model are 'technical' constraint that model the production profiles, ensure continuity of flow balance, keep track of storage levels, etc.

In addition, there is an optional policy constraints for putting the limit on yearly CO2 emissions, either in absolute values or as a required reduction from a specified base level.

3.2.1 Capacity modelling

All nodes have an available capacity that is fixed throughout each strategic period. This capacity can increase through investing in additional capacity or it can decrease due to active removal or degradation effects.

The nominal capacity can be found by summing over nominal capacity from earlier investment periods

$$\mathbf{cap}_{n,sp}^{\text{nom}} = \sum_{sp^I \le sp} \mathbf{cap}_{n,sp^I,sp}^{\text{nom}}, \qquad \forall n \in N, sp \in T^{SP}. \tag{11}$$

Knowing the investment period for the nominal capacity, the available capacity can be found by assuming a known degradation profile

$$\mathbf{cap}_{n,sp} = \sum_{sp^I \le sp} \mu_{n,sp^I,sp} \cdot \mathbf{cap}_{n,sp^I,sp}^{\text{nom}}, \qquad \forall n \in N, sp \in T^{SP}. \tag{12}$$

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In addition there may be lower bounds on the nominal capacity

$$\mathbf{cap}_{n,sp}^{\text{nom}} \geq Cap_n^{\text{min}}, \qquad \forall n \in N, sp \in T^{SP}. \tag{13}$$

Nodes can be classified as one of two types with regard to capacity expansion. They can be of type *binary investment* where only one unit can be available at any time and the capacity has to be removed before investment in new capacity. For these nodes we can have both lower and upper limits on new capacity investments, as well as fixed costs both for adding, having and removing capacity. The other option is nodes with *continuous investment* that allows arbitrary capacity updates with an upper limit in each strategic period.

Binary investment nodes For the nodes with binary investment ($Bin_n^{\text{inv}} = 1$), the added capacity must be between upper and lower limits if investing

$$\mathbf{cap}_{n,sp}^{\text{add}} \leq Add_n^{\text{max}} \cdot \mathbf{add}_{n,sp}, \qquad \forall n \in N, sp \in T^{SP}, \tag{14}$$

$$\mathbf{cap}_{n,sp}^{\text{add}} \geq Add_n^{\text{min}} \cdot \mathbf{add}_{n,sp}, \qquad \forall n \in N, sp \in T^{SP}. \tag{15}$$

Modelling of indicator variable for the removal of capacity

$$\mathbf{cap}_{n,sp}^{\text{rem}} \leq Cap_n^{\text{max}} \cdot \mathbf{rem}_{n,sp}, \qquad \forall n \in N, sp \in T^{SP}. \tag{16}$$

If capacity is removed for a node with binary investment, all capacity must be removed

$$\mathbf{cap}_{n,sp}^{\text{rem}} \geq \mathbf{cap}_{n,sp-1}^{\text{nom}} - Cap_n^{\text{max}} \cdot (1 - \mathbf{rem}_{n,sp}), \qquad \forall n \in N, sp \in T^{SP}, \tag{17}$$

Nominal capacity installed in period sp^I available in period sp

$$\mathbf{cap}_{n,sp^I,sp}^{\text{nom}} = \begin{cases} \mathbf{cap}_{n,sp}^{\text{add}}, & sp^I = sp \\ \mathbf{cap}_{n,sp^I,sp-1}^{\text{nom}} - \mathbf{cap}_{n,sp}^{\text{rem}}, & sp^I < sp \end{cases}, \qquad \forall n \in N, sp^I, sp \in T^{SP} : sp^I \leq sp, \tag{18}$$

where $\mathbf{cap}_{n,0}^{\text{nom}} = Cap_n^{\text{ex}}$. Modelling of indicator variables to show if a period has capacity greater than zero

$$\mathbf{hascap}_{n,sp} = \mathbf{hascap}_{n,sp-1} + \mathbf{add}_{n,sp} - \mathbf{rem}_{n,sp}, \qquad \forall n \in N, sp \in T^{SP}, \tag{19}$$

where $\mathbf{hascap}_{n,0} = 1$ if $Cap_n^{\text{ex}} > 0$ and 0 otherwise. In addition, we have limits on the maximum allowed nominal capacity installed

$$\mathbf{cap}_{n,sp}^{\text{nom}} \leq Cap_n^{\text{max}} \cdot \mathbf{hascap}_{n,sp}, \qquad \forall n \in N, sp \in T^{SP}. \tag{20}$$

For some applications it is necessary to track how long installed capacity has existed (lifetime modelling and maintenance). For these nodes, the constraints (19) and (20) are replaced with the following

$$\mathbf{hascap}_{n,sp^I,sp} = \mathbf{hascap}_{n,sp^I,sp-1} - \mathbf{rem}_{n,sp}, \qquad \forall n \in N, sp \in T^{SP}, sp^I \in T^{SP,I} : sp^I \leq sp, \tag{21}$$

where $\mathbf{hascap}_{n,sp^I,0} = 1$ if $Cap_n^{\text{ex}} > 0$ and 0 otherwise, and

$$\mathbf{cap}_{n,sp^I,sp}^{\text{nom}} \leq Cap_n^{\text{max}} \cdot \mathbf{hascap}_{n,sp^I,sp}, \qquad \forall n \in N, sp, sp^I \in T^{SP} : sp^I \leq sp. \tag{22}$$

If the capacity is installed in discrete units (identified by $Cap_n^{\text{block}} > 0$), this can be modelled by the following constraints

$$\mathbf{cap}_{n,sp}^{\text{add}} = Cap_n^{\text{block}} \cdot \mathbf{block}_{n,sp}^{\text{add}}, \qquad \forall n \in N, sp \in T^{SP}, \tag{23}$$

$$\mathbf{cap}_{n,sp}^{\text{rem}} = Cap_n^{\text{block}} \cdot \mathbf{block}_{n,sp}^{\text{rem}}, \qquad \forall n \in N, sp \in T^{SP}. \tag{24}$$

Continuous investment nodes For nodes with continuous investment ($Bin_n^{\text{inv}} = 0$), we have an upper limit on added capacity, but no lower limit

$$\mathbf{cap}_{n,sp}^{\text{add}} \leq Add_n^{\max}, \qquad \forall n \in N, sp \in T^{SP}. \tag{25}$$

For continuous updates we need to track the age of the installed capacity to model the degradation, and therefore we need to track the age of the removed capacity through separate $\mathbf{cap}_{n,sp^I,sp}^{\text{rem}}$ variables. The removed capacity in each period is found by summing over all investment periods

$$\mathbf{cap}_{n,sp}^{\text{rem}} = \sum_{sp^I \in T^{SP,I}:sp^I \leq sp} \mathbf{cap}_{n,sp^I,sp}^{\text{rem}}, \qquad \forall n \in N, sp \in T^{SP}. \tag{26}$$

It is not possible to remove more capacity than what was added in each investment period

$$\mathbf{cap}_{n,sp^I,sp}^{\text{rem}} \leq \mathbf{cap}_{n,sp^I}^{\text{add}}, \qquad \forall n \in N, sp^I, sp \in T^{SP} : sp^I < sp, \tag{27}$$

and capacity can not be removed in the same period as it was added

$$\mathbf{cap}_{n,sp^I,sp^I}^{\text{rem}} = 0, \qquad \forall n \in N, sp^I \in T^{SP}. \tag{28}$$

Nominal capacity with investment year can then be tracked

$$\mathbf{cap}_{n,sp^I,sp}^{\text{nom}} = \begin{cases} \mathbf{cap}_{n,sp}^{\text{add}}, & sp^I = sp \\ \mathbf{cap}_{n,sp^I,sp-1}^{\text{nom}} - \mathbf{cap}_{n,sp^I,sp}^{\text{rem}}, & sp^I < sp \end{cases}, \qquad \forall n \in N, sp, sp^I \in T^{SP} : sp^I \leq sp, \tag{29}$$

where $\mathbf{cap}_{n,sp^I,0}^{\text{nom}} = Cap_n^{\text{ex}}$. The total installed nominal capacity can not exceed the maximum allowed

$$\mathbf{cap}_{n,sp}^{\text{nom}} \leq Cap_n^{\max}, \qquad \forall n \in N, sp \in T^{SP}. \tag{30}$$

3.2.2 Operational constraints

Production Depending on the type of production plant, the capacity constraints are different. Note that for each production plant, p, there is a given product, P_i^{cap}, for which capacity is given. If this product is an out-product of the node, the capacity limit is for the production of the product. Otherwise, the capacity limit will be on the total inflow of the product. A production plant are required to operate at or below its capacity

$$\Delta T_{sp,op}^{\text{OP,h}} \cdot Prod_{i,sp,op}^{\text{rate}} \cdot \mathbf{cap}_{i,sp} \geq \begin{cases} \mathbf{prod}_{i,p,sp,op}, & p \in P_i^{out} \\ \sum_{n \in N} \mathbf{flow}_{n,i,p,sp,op}, & \text{otherwise} \end{cases},$$
$$\forall i \in I, p \in \{P_i^{\text{cap}}\}, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{31}$$

A production plant can be non-adjustable, i.e., it always produces at its capacity. For plants of this type, the inequality in the constraint above is replaced with an equality.

For the case of an "External source" (which do not have capacity, just production rate):

$$\mathbf{prod}_{i,p,sp,op} = \Delta T_{sp,op}^{\text{OP,h}} \cdot Prod_{i,sp,op}^{\text{rate}}, \qquad \forall i \in I, p \in \{P_i^{\text{cap}}\}, sp \in T^{SP}, op \in T_{sp}^{OP} \tag{32}$$

For nodes where production capacity is a given fraction of available capacity:

$$\Delta T_{sp,op}^{\text{OP,h}} \cdot Prod_{i,sp,op}^{\text{rate}} \cdot Fix_i^{\text{load},\%} \cdot \mathbf{cap}_{i,sp} = \begin{cases} \mathbf{prod}_{i,p,sp,op}, & p \in P_i^{out} \\ \sum_{n \in N} \mathbf{flow}_{n,i,p,sp,op}, & \text{otherwise} \end{cases},$$
$$\forall i \in I, p \in \{P_i^{\text{cap}}\}, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{33}$$

The model allows multiple inputs by converting all inputs to one combined flow, either by adding all inputs together, or by fixing their ratios to a given 'main input'. If a value for $\eta_{i,d,p}$ is provided for all input products in P_i^{in}, we combine all input flows using the given multipliers

$$\mathbf{flow}_{i,d,sp,op}^{in} = \sum_{p \in P_i^{in}} \sum_{n \in N} \eta_{i,d,p} \cdot \mathbf{flow}_{n,i,p,sp,op}, \qquad \forall i \in I, d \in D_i, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{34}$$

If values for $\eta_{i,d,p}$ are provided for all but one input product $\bar{p}$, this will be regarded as the main product and will be used as basis for calculating required inflows of other products

$$\mathbf{flow}_{i,d,sp,op}^{in} = \sum_{n \in N} \mathbf{flow}_{n,i,\bar{p},sp,op}, \qquad \forall i \in I, d \in D_i, sp \in T^{SP}, op \in T_{sp}^{OP}, \tag{35}$$

$$\sum_{n \in N} \mathbf{flow}_{n,i,p,sp,op} = \eta_{i,d,p} \cdot \mathbf{flow}_{i,d,sp,op}^{in}, \qquad \forall i \in I, p \in P_i^{in} \setminus \bar{p}, d \in D_i, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{36}$$

Each production plant can, in theory, operate in more than one *mode*. For example, an air-conditioning unit can provide either heating or cooling. Therefore, all production variables have a 'mode' index, even for nodes with only one mode.

Having established the inflow used as basis for calculating production, the model can either use a linear production function or a piecewise linear production function. For the simplest version with a linear production function we have

$$\mathbf{prod}_{i,d,p,sp,op}^{mode} = \mu_{i,d,p} \cdot \mathbf{flow}_{i,d,sp,op}^{in}. \tag{37}$$

The more advanced solution with a piecewise linear function is modelled using a separate production and inflow variable for each interval. This will only work for a concave piecewise production function, ensuring that higher efficiency intervals are used first.

$$\mathbf{flow}_{i,d,sp,op}^{in} = \sum_{l \in L_{i,d,p}} \mathbf{flow}_{i,sp,op,d,p,l}^{in,pw} \tag{38}$$

$$\mathbf{prod}_{i,d,p,sp,op}^{mode} = \sum_{l \in L_{i,d,p}} \mathbf{prod}_{i,sp,op,d,p,l}^{pw} \tag{39}$$

For each interval we have a separate conversion factor

$$\mathbf{prod}_{i,sp,op,d,p,l}^{pw} = \pi_{i,d,p,l} \cdot \mathbf{flow}_{i,sp,op,d,p,l}^{in,pw} \tag{40}$$

If the product used for capacity, P_i^{cap}, is an out-product of the plant, we have that

$$\mathbf{prod}_{i,sp,op,d,p,l}^{pw} \leq \Delta T_{sp,op}^{OP,h} \cdot \lambda_{i,d,p,l} \cdot \mathbf{cap}_{n,sp} \tag{41}$$

Note that this only works if p is the capacity defining product. If the capacity is given with regard to an input product, we have that

$$\mathbf{flow}_{i,sp,op,d,p,l}^{in,pw} \leq \Delta T_{sp,op}^{OP,h} \cdot \lambda_{i,d,p,l} \cdot \mathbf{cap}_{n,sp}. \tag{42}$$

Having established the production for each mode, we can find the correct production that may depend on the mode. If the plant has only one mode d, production is set to equal the production in this mode

$$\mathbf{prod}_{i,p,sp,op} = \mathbf{prod}_{i,d,p,sp,op}^{mode}. \tag{43}$$

If a plant has more than one mode, we need to adjust the production depending on the mode of the plant, i.e. we need the value $\mathbf{prod}_{i,d,p,sp,op}^{mode,adj} \leq \mathbf{prod}_{i,d,p,sp,op}^{mode} \cdot \mathbf{mode}_{i,d,p,sp,op}$, which can be modelled with the following constraints

$$\mathbf{prod}_{i,d,p,sp,op}^{mode,adj} \leq \mathbf{prod}_{i,d,p,sp,op}^{mode} \tag{44}$$

$$\mathbf{prod}_{i,d,p,sp,op}^{mode,adj} \leq \Delta T_{sp,op}^{OP,h} \cdot Cap_i^{max} \cdot \mathbf{mode}_{i,d,p,sp,op} \tag{45}$$

$$\mathbf{prod}_{i,d,p,sp,op}^{\text{mode,adj}} \geq \mathbf{prod}_{i,d,p,sp,op}^{\text{mode}} - \Delta T_{sp,op}^{\text{OP,h}} \cdot Cap_n^{\max} \cdot (1 - \mathbf{mode}_{i,d,p,sp,op}) \tag{46}$$

The total production can then be found be summing production over all modes

$$\mathbf{prod}_{i,p,sp,op} = \sum_{d \in D_i} \mathbf{prod}_{i,d,p,sp,op}^{\text{mode,adj}}. \tag{47}$$

Note how multi-modal nodes require binary variables $\mathbf{mode}_{i,d,p,sp,op}$ in every operational period, so they will make the model much harder to solve.

For each plant i, the production must balance the out-flow for all out-products, or it can be spilled (if allowed)

$$\mathbf{prod}_{i,p,sp,op} - \mathbf{slack}_{i,p,sp,op} = \sum_{n \in N} \mathbf{flow}_{i,n,p,sp,op} \tag{48}$$

If the production process has a power consumption given per produced unit of the main out product, p, this must be balanced by power inflow

$$Pow_i^{\text{cons}} \cdot \mathbf{prod}_{i,p,sp,op} = \sum_{n \in N} \mathbf{flow}_{n,i,\text{Power},sp,op} \tag{49}$$

Storages   Storage levels can not exceed the invested capacity. This is required both for the initial storage levels as well as storage levels throughout the operational periods

$$\mathbf{inv}_{s,sp}^{\text{init}} \leq \mathbf{cap}_{s,sp}, \qquad \forall s \in S, sp \in T^{SP}, \tag{50}$$

$$\mathbf{inv}_{s,sp,op} \leq \mathbf{cap}_{s,sp}, \qquad \forall s \in S, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{51}$$

Each storage can only store one product, p, and for this product we require a mass balance taking into consideration losses both when inserting and removing product from storage

$$\mathbf{inv}_{s,sp,op} = \mathbf{inv}_{s,sp,op-1} + \sum_{n \in N} \rho_s^{\text{in}} \cdot \mathbf{flow}_{n,s,p,sp,op} - \sum_{n \in N} 1/\rho_s^{\text{out}} \cdot \mathbf{flow}_{s,n,p,sp,op} \tag{52}$$

where $\mathbf{inv}_{s,sp,0} = \mathbf{inv}_{s,sp}^{\text{init}}$.

Since each strategic period typically consists of operational parts covering only parts of the strategic period, there can be a need to synchronize inventory levels at the first and last operational period (ω_{sp}) with a cyclical storage

$$\mathbf{inv}_{s,sp}^{\text{init}} = \mathbf{inv}_{s,sp,\omega_{sp}}, \qquad \forall s \in S, sp \in T^{SP}. \tag{53}$$

It is also possible to require a specified fill share at the beginning of each strategic period

$$\mathbf{inv}_{s,sp}^{\text{init}} = S_s^{\text{init},\%} \cdot \mathbf{cap}_{s,sp}, \qquad \forall s \in S, sp \in T^{SP}. \tag{54}$$

The minimum storage levels can be constrained both in absolute and relative terms

$$\mathbf{inv}_{s,sp,op} \geq S_s^{\min}, \qquad \forall s \in S, sp \in T^{SP}, op \in T_{sp}^{OP}, \tag{55}$$

$$\mathbf{inv}_{s,sp,op} \geq S_s^{\min,\%} \cdot \mathbf{cap}_{s,sp}, \qquad \forall s \in S, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{56}$$

Similarly, the filling and emptying speed can be constrained in both absolute and relative terms

$$\sum_{n \in N} \mathbf{flow}_{n,s,p,sp,op} \leq \Delta T_{sp,op}^{\text{OP,h}} \cdot R_s^{\text{in}}, \qquad \forall s \in S, sp \in T^{SP}, op \in T_{sp}^{OP}, \tag{57}$$

$$\sum_{n \in N} \mathbf{flow}_{s,n,p,sp,op} \leq \Delta T_{sp,op}^{\text{OP,h}} \cdot R_s^{\text{out}}, \qquad \forall s \in S, sp \in T^{SP}, op \in T_{sp}^{OP}, \tag{58}$$

$$\sum_{n \in N} \mathbf{flow}_{n,s,p,sp,op} \leq \Delta T_{sp,op}^{\text{OP,h}} \cdot R_s^{\text{in},\%} \cdot \mathbf{cap}_{s,sp}, \qquad \forall s \in S, sp \in T^{SP}, op \in T_{sp}^{OP}, \tag{59}$$

$$\sum_{n \in N} \mathbf{flow}_{s,n,p,sp,op} \leq \Delta T_{sp,op}^{\text{OP,h}} \cdot R_s^{\text{out},\%} \cdot \mathbf{cap}_{s,sp}, \qquad \forall s \in S, sp \in T^{SP}, op \in T_{sp}^{OP}. \tag{60}$$

Transport As transport nodes only function as a transmission node for the flow, we have a simple flow balance that accounts for possible transport losses

$$\sum_{n' \in N} \mathbf{flow}_{n,n',p,sp,op} = (1 - Tran^{\text{loss}}_{n,p}) \cdot \sum_{n' \in N} \mathbf{flow}_{n',n,p,sp,op}. \tag{61}$$

The total flow into the transport node is limited by its capacity (shared by all compatible products)

$$\sum_{n' \in N, p \in P} \mathbf{flow}_{n',n,p,sp,op} \leq \Delta T^{\text{OP,h}}_{sp,op} \cdot \mathbf{cap}_{n,sp}. \tag{62}$$

Markets The total delivery from the market and the volume bought must be below the capacity

$$\mathbf{load}^{\text{del}}_{m,sp,op} + \mathbf{vol}^{\text{sell}}_{m,sp,op} \leq \Delta T^{\text{OP,h}}_{sp,op} \cdot \mathbf{cap}_{m,sp}, \qquad \forall m \in M, sp \in T^{SP}, op \in T^{OP}_{sp}, \tag{63}$$

$$\mathbf{vol}^{\text{buy}}_{m,sp,op} \leq \Delta T^{\text{OP,h}}_{sp,op} \cdot \mathbf{cap}_{m,sp} \qquad \forall m \in M, sp \in T^{SP}, op \in T^{OP}_{sp}. \tag{64}$$

The volumes delivered and bought must be below the maximum supply and demand

$$\mathbf{vol}^{\text{buy}}_{m,sp,op} \leq S^{\text{upply}}_{m,sp,op}, \qquad \forall m \in M, sp \in T^{SP}, op \in T^{OP}_{sp}, \tag{65}$$

The required load in the market must be satisfied or we will have lost load that will incur a penalty

$$\mathbf{load}^{\text{del}}_{m,sp,op} = L^{\text{oad}}_{m,sp,op} - \mathbf{load}^{\text{lost}}_{m,sp,op}, \qquad \forall m \in M, sp \in T^{SP}, op \in T^{OP}_{sp}. \tag{66}$$

There can be hard constraints on the overall regularity in the load deliveries

$$\sum_{op \in T^{OP}_{sp}} \mathbf{load}^{\text{del}}_{m,sp,op} \geq Reg^{\text{min}}_m \cdot \sum_{op \in T^{OP}_{sp}} L^{\text{oad}}_{m,sp,op}, \qquad \forall m \in M, sp \in T^{SP}. \tag{67}$$

The flow balance for the market is separated into two parts to ensure no flow through the market

$$\sum_{n \in N} \mathbf{flow}_{n,m,p,sp,op} = \mathbf{load}^{\text{del}}_{m,sp,op} + \mathbf{vol}^{\text{sell}}_{m,sp,op}, \qquad \forall m \in M, p \in \{P^{\text{M}}_m\}, sp \in T^{SP}, op \in T^{OP}_{sp}, \tag{68}$$

$$\sum_{n \in N} \mathbf{flow}_{m,n,p,sp,op} = \mathbf{vol}^{\text{buy}}_{m,sp,op}, \qquad \forall m \in M, p \in \{P^{\text{M}}_m\}, sp \in T^{SP}, op \in T^{OP}_{sp}. \tag{69}$$

3.3 Objective function

The model decides the optimal decisions in order to optimize a given objective. The objective is typically maximizing the net present value of the whole modelled system (reflecting both the costs and incomes for the system). The net present value is calculated by discounting costs and income for a specified time horizon and discount rate.

At the strategic level there can be CAPEX associated with adding capacity and removing capacity

$$\mathbf{capex}^{\text{add}}_{sp} := \sum_{n \in N: Bin^{\text{inv}}_n = 1} Capex^{\text{fix}}_n \cdot \mathbf{add}_{n,sp} + \sum_{n \in N} Capex^{\text{var}}_n \cdot \mathbf{cap}^{\text{add}}_{n,sp} \tag{70}$$

$$\mathbf{capex}^{\text{rem}}_{sp} := \sum_{n \in N: Bin^{\text{inv}}_n = 1} C^{\text{rem,fix}}_n \cdot \mathbf{rem}_{n,sp} + \sum_{n \in N} C^{\text{rem,var}}_n \cdot \mathbf{cap}^{\text{rem}}_{n,sp} \tag{71}$$

In addition there can be fixed and relative OPEX associated with the installed capacity

$$\mathbf{opex}^{\text{rel}}_{sp} := \sum_{n \in N} Opex^{\text{rel}}_n \cdot \mathbf{cap}^{\text{inst}}_{n,sp} \tag{72}$$

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where

$$\mathbf{cap}_{n,sp}^{\text{inst}} := Capex_n^{\text{fix}} \cdot \mathbf{hascap}_{n,sp} + Capex_n^{\text{var}} \cdot \mathbf{cap}_{n,sp}^{\text{nom}}. \tag{73}$$

The variable and fixed costs are related to nominal capacity and the availability of capacity

$$\mathbf{opex}_{sp}^{\text{var}} := \sum_{n \in N} Opex_n^{\text{var}} \cdot \mathbf{cap}_{n,sp}^{\text{nom}}, \tag{74}$$

$$\mathbf{opex}_{sp}^{\text{fix}} := \sum_{n \in N : Bin_n^{\text{inv}} = 1} Opex_n^{\text{fix}} \cdot \mathbf{hascap}_{n,sp}. \tag{75}$$

At the operational level there is a cash flow associated with the operations of the nodes. For production plants there are unit costs for production and costs associated with spill of products

$$\mathbf{prodCost}_{sp} := \sum_{i \in I, op \in T_{sp}^{OP}} \left(C_i^{\text{unit}} \cdot \mathbf{prod}_{i,P_i^{\text{main}},sp,op} + \sum_{p \in P} C_{i,p}^{\text{spill}} \cdot \mathbf{slack}_{i,p,sp,op} \right). \tag{76}$$

In the market nodes there is both an income from selling product and meeting the required load

$$\mathbf{income}_{sp} := \sum_{m \in M, op \in T_{sp}^{OP}} \left(Price_m^{\text{sell}} \cdot \mathbf{vol}_{m,sp,op}^{\text{sell}} + Price_m^{\text{load}} \cdot \mathbf{load}_{m,sp,op}^{\text{del}} \right), \tag{77}$$

and a cost for buying product or not meeting the required load

$$\mathbf{cost}_{sp} := \sum_{m \in M, op \in T_{sp}^{OP}} \left(Price_m^{\text{buy}} \cdot \mathbf{vol}_{m,sp,op}^{\text{buy}} + Val_m^{\text{loss}} \cdot \mathbf{load}_{m,sp,op}^{\text{lost}} \right). \tag{78}$$

Storage nodes have potential losses that can incur costs

$$\mathbf{store}_{sp}^{\text{loss}} := \sum_{s \in S} [C_s^{\text{loss}} \cdot (1 - \rho_s^{\text{in}}) \cdot \sum_{n \in N, op \in T_{sp}^{OP}} \mathbf{flow}_{n,s,p,sp,op} + \tag{79}$$

$$C_s^{\text{loss}} \cdot (1/\rho_s^{\text{out}} - 1) \cdot \sum_{n \in N, op \in T_{sp}^{OP}} \mathbf{flow}_{s,n,p,sp,op}] \tag{80}$$

To find the total net present value we sum up over all strategic periods with a suitable discount rate

$$\mathbf{npv} := - \sum_{sp \in T^{SP}} D_{sp} \cdot (\mathbf{capex}_{sp}^{\text{add}} + \mathbf{capex}_{sp}^{\text{rem}})$$

$$- \sum_{sp \in T^{SP}} D_{sp}^{\text{avg}} \cdot \Delta T_{sp}^{\text{SP,y}} \cdot (\mathbf{opex}_{sp}^{\text{rel}} + \mathbf{opex}_{sp}^{\text{var}} + \mathbf{opex}_{sp}^{\text{fix}})$$

$$- \sum_{sp \in T^{SP}} D_{sp}^{\text{avg}} \cdot \Delta T_{sp}^{\text{SP,y}} \cdot \mathbf{store}_{sp}^{\text{loss}}$$

$$+ \sum_{sp \in T^{SP}} D_{sp}^{\text{avg}} \cdot \Delta T_{sp}^{\text{SP}} \cdot (\mathbf{income}_{sp} - \mathbf{cost}_{sp} - \mathbf{prodCost}_{sp}).$$

4 Model extensions

4.1 Emissions

It is possible to set a requirement on emission reduction. Currently, this is only implemented for CO_2-emissions related to power deliveries. The total emissions of a product is defined as total production spill plus total deliveries to markets for the product

$$\mathbf{em}_p = \sum_{i \in I, sp \in T^{SP}} \Delta T_{sp}^{\text{SP}} \cdot \sum_{op \in T_{sp}^{OP}} \mathbf{slack}_{i,p,sp,op} \tag{81}$$

$$+ \sum_{m \in M_p, sp \in T^{SP}} [\Delta T_{sp}^{\text{SP}} \cdot \sum_{op \in T_{sp}^{OP}} (\mathbf{vol}_{m,sp,op}^{\text{sell}} + \mathbf{load}_{m,sp,op}^{\text{del}})], \qquad \forall p \in P \tag{82}$$

The total market deliveries of a product are given as

$$\mathbf{del}_p^{\text{tot}} = \sum_{m \in M_p, \, sp \in T^{SP}} \Delta T_{sp}^{\text{SP}} \cdot \left(\sum_{op \in T_{sp}^{OP}} \mathbf{vol}_{m,sp,op}^{\text{sell}} + \mathbf{load}_{m,sp,op}^{\text{del}} \right), \qquad \forall p \in P. \tag{83}$$

For a given product p we can require a minimum emission reduction, $Reduce_p^{\%}$, compared to a given base emission, $Base_{p,\bar{p}}^{\text{em}}$, for the production of product $\bar{p}$

$$\mathbf{em}_p \leq (1 - Reduce_p^{\%}) \cdot Base_{p,\bar{p}}^{\text{em}} \cdot \mathbf{del}_{\bar{p}}^{\text{tot}}, \qquad \forall p \in P. \tag{84}$$

4.2 Lifetime modelling and rest value

It is possible to include lifetime modelling for nodes. This can either be based on calendar time or on an approximation of use time for the node. The following parameters are used in the modelling

- $T^{\text{hor,y}}$: Time horizon in years

- H_n^{y}: Lifetime of equipment in years (time-based)

- H_n^{h}: Lifetime of equipment in hours (usage-based)

- Cap_n^{util}: Average capacity utilization for the plant

- Val_n^{rest}: Rest value of a node, as a fraction of CAPEX

If the node has a time based lifetime, we set the value of installed capacity to zero if its age exceeds the lifetime

$$H_n^{\text{y}} > 0 \implies \mathbf{cap}_{n,sp^I,sp}^{\text{nom}} = 0 \text{ and } \mathbf{hascap}_{n,sp^I,sp} = 0, \qquad \forall sp \in T^{SP} : T_{sp} - T_{sp^I} > H_n^{\text{y}}. \tag{85}$$

It is impossible to model use time precisely as this would require too many binary variables. Instead an approach based on production and a fixed average capacity utilization is used for plant nodes. Usage based lifetime is not available for other node types. It is also restricted to nodes with binary investment as the binary variables are needed to restrict the production to zero after lifetime is reached. For all plants of this type with $H_n^{\text{h}} > 0$, we force capacity to zero if the produced volume exceeds the volume produced over its lifetime with average utilization

$$\sum_{sp \in sp^I \dots sp^R - 1} \Delta T_{sp}^{\text{SP}} \cdot \sum_{op \in T_{sp}^{OP}} \mathbf{prod}_{i,P_i^{\text{main}},sp,op} \leq H_n^{\text{h}} \cdot Cap_n^{\text{util}} \cdot \mathbf{cap}_{n,sp^I}^{\text{add}} +$$
$$H_n^{\text{h}} \cdot Cap_n^{\text{max}} \cdot (1 - \mathbf{hascap}_{n,sp^I,sp^R}), \qquad \forall sp^I, sp^R \in T^{SP} : sp^R > sp^I. \tag{86}$$

All nodes can be given a rest value at the end of the planning horizon, given as a relative value compared to the installed capacity at the horizon end

$$\mathbf{rest}^{\text{val}} := \sum_{n \in N} D_T \cdot Val_n^{\text{rest}} \cdot \mathbf{cap}_{n,T}^{\text{inst}}, \qquad \forall n \in N. \tag{87}$$

This value is added to the overall net present value

$$\mathbf{npv} := \mathbf{npv} + \mathbf{rest}^{\text{val}}. \tag{88}$$