Good mathematics teaching at lower primary school level

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This paper explores Norwegian lower primary teachers' views about good mathematics teaching as revealed in a focus group interview at the end of a two-year school-based professional development program. Analyses of the empirical data indicate three main categories of findings: the teachers' facilitation of learning, the students' thinking in and about mathematics and the use of teaching aids in teaching. The results are discussed in relation to other Nordic studies and possible implications are also provided.

Keywords: Mathematics teachers' discussion, lower primary school, good mathematics teaching.

Introduction and theoretical background

This study investigates teacher views about good mathematics teaching. Several studies attempt to identify the components of good mathematics teaching without finding a clear answer (Cai, Kaiser, Perry, & Wong, 2009; Franke, Kazemi, & Battey, 2007; Hiebert & Grouws, 2007; Kilpatrick, Swafford & Findell, 2001). A challenge is that cultural as well as political differences influence mathematics teaching. Views about the role of the teacher, about the subject in school and society, and about learning differ across cultures (Cai et al., 2009). In the Chinese context, for instance, mathematics teaching is teacher-oriented and exam-oriented, and teachers are more focused on the students and their learning than on themselves and their teaching (Li, 2011). In the Nordic context, Fauskanger, Mosvold, Valenta and Bjuland (2018) conducted a study in which upper primary school teachers' views on good mathematics teaching were revealed through group interviews at the start of a major professional development project. The teachers referred to their role as teaching facilitators by having good structure, classroom management, and the possibility to differentiate using different types of assignments, which both motivates the students and invites more and diverse solutions. According to Fauskanger et al. (2018), good teaching was also about motivated, engaged, creative and curious students. In another study, Fauskanger (2016) investigated views on the ingredients of good mathematics teaching among lower and upper primary school mathematics teachers who participated in a professional development program. These teachers felt that student response was the most decisive factor for high quality teaching. They emphasized teacher qualities such as enthusiasm and attitude towards the subject rather than the teachers' own knowledge. Hemmi and Ryve (2015) studied Swedish and Finnish teacher educators' views of good mathematics teaching through focus group interviews and individual interviews. There were many apparent similarities between Sweden and Finland, but the Finnish teacher educators emphasized clear presentation of mathematics for the whole class, routines for mental arithmetic and homework, and clear learning goals for each class, while the Swedish teacher educators referred to the relationship with each individual child, building on the students' capabilities and finding mathematics in everyday situations. In three studies carried out among Finnish student teachers (at lower primary school level), Kaasila and Pehkonen (2009) looked

at students teachers' views of good mathematics teaching. They believed that teachers needed to be goal-oriented, listen to the students' thinking and show flexibility when unexpected episodes arise. The student teachers pointed out that teachers should have knowledge of varied work methods, base their teaching on the students' day-to-day experiences and have a particular focus on problem solving. Continuous assessment and development of socio-mathematical norms were considered important elements of good mathematics teaching.

Teaching mathematics is complex and researchers have attempted to distinguish the different aspects to identify main practices. These are referred to as core practices (McDonald et al., 2013) or high-leverage practices (Forzani, 2014). This study focusses on the Nordic context, therefore core practices are not discussed further.

The study in this paper is based on a group of Norwegian teachers at lower primary school level who, together with a teacher educator in a focus group interview, reflected on their own mathematics teaching at the end of a two-year school-based mathematics professional development program. The content in this development program were decided by the headmaster in cooperation with representatives of the mathematics teachers at the school. Among the themes were numeracy, different approaches to the four arithmetical operations and how to lead productive mathematical discussions. This study does not measure the effect of the program, but it can be assumed that the teachers' descriptions of good mathematics teaching has been influenced by them trying out exercises and activities in their own classes and by improved research-based knowledge of mathematics didactics throughout the two-year period. To my knowledge of research in the field, few studies have examined Norwegian lower primary school teachers' descriptions of good mathematics teaching. On this basis, the study seeks to answer the following research question: What might Norwegian lower primary school teachers' views about good mathematics teaching look like? Teaching refers to the interaction between teachers and students relating to subject matter. Cohen, Raudenbush and Ball (2003) describe this interaction as the instructional triangle.

Methodological approach

The empirical data used in this study is from a focus group interview with seven lower primary school teachers at a school that has completed a two-year professional development program for mathematics teachers. The interview included two teachers from each of the years one to year three and one from year four. Two were men and five were women. Two of these were experienced preschool teachers who have worked at lower primary level for about 15 years. The others were primary and lower secondary teachers with between 15 to 30 ECTS credits in mathematics and between four and 20 years of experience from primary and lower secondary school. The school has three teachers on each of the four years of lower primary school.

The participants were informed of the topic of the focus group interview in advance. They were asked to discuss and reflect on their own mathematics teaching on the basis of their experiences from competence raising and what they had tried out, and their definition of good mathematics teaching. The interview lasted one hour and was recorded and transcribed in full.

Transcripts from the focus group interviews were analyzed using conventional content analysis (Fauskanger & Mosvold, 2015; Hsieh & Shannon, 2005) used in studies that attempt to describe a phenomenon in order to better understand it. The phenomenon described in this study is mathematics teaching. In conventional content analysis, inductive codes are linked to suitable categories, as shown in a table in Figure 1. The interview subjects are referred to as R1 to R7. The transcribed interviews were placed in a table with rows containing individual statements, such as R3 in Figure 1, key words from these, inductive codes and categories. The material was analyzed twice with a two-month interval to prevent categories being overlooked. One of the challenges of conventional content analysis is not obtaining a complete understanding of the context because of categories being left out (Hsieh & Shannon, 2005).

Category	Inductive code	Examples of individual comments
The teachers' facilitation of learning	Communication in the classroom	R3: 'Some years ago, if I spent much too much time on a conservation, it felt like "when are we going to do the maths?"'
	Representation – particularly transitions between representations	R4: 'because we've used manipulatives before too... And the transition from using manipulatives to actually drawing up maths problems [...]'
The students' thinking in and about the subject of mathematics	The students' thinking in the subject of mathematics	R5: 'show them that there is more than one way of working it out, several strategies.'
	The students' thinking about the subject of mathematics	R3:'The challenge is that there are a few students in the class that you don't manage to engage in the conversation, that only really become involved when they are given the maths problem in the book'.
Subject resources in the facilitation	Textbook	R6: 'And then I suppose it's very safe. You probably very much trust that those who have written the textbooks know what we need to get through and...it's also related to time pressure sometimes, that it's easy.'
	Type of task: open, explorative, tasks related to daily activities	R2: '...to see the maths in everything around us. Grasp the everyday situations.'

Table 1: Codes and categories

Results

The analysis of the empirical data led to three main categories of findings: teachers' facilitation of learning, students' thinking in and about mathematics and use of teaching aids in teaching. These three main categories are sometimes related. For example, the students' thinking in and

about a mathematics exercise might be connected with the teacher's facilitation of learning through communication in the classroom. This is in line with the description of teaching in the instruction triangle as an interaction between subject matter, the students and the teacher (Cohen et al., 2003). Through the focus group interview, the teachers emphasized increased awareness of several areas at the same time as they still had challenges in a number of these areas. When the results are presented, both challenges and increased awareness are shown in each category.

Teachers' facilitation of learning

The teachers seemed to use more whole class conversations and dialogue in mathematics teaching after participating in the professional development. They also said that it was challenging to engage the students in subject-related talks. The teachers viewed the dialogues with the students and between students as an aid to developing the students' thinking: 'Kind of building a bridge between the terminology they have and... sort of new knowledge' (R3). This remark may indicate a view of learning in which the students develop new knowledge from already established terms. The same teacher had started using learning pairs and felt that the students gave each other ideas that were useful to the subsequent conversation with the whole class. The teachers did not feel that the class failed if they spent time on discussion and deviated from the class plan (R1). R6 reported that they often used to think 'Oh no, now I have to get the other part done,' where the other part referred to solving exercises in the textbook. This can mean that the teacher thought more about quality and what led to learning than quantity, as in solving lots of math problems in the mathematics teaching. R3 described the use of dialogues in teaching as a *quantum leap* in relation to before the professional development program. In communication with the students, the teachers expressed that they had become more precise in their use of terms, as described by R1: 'addition and subtraction, and not plus and minus.'

R4 specified what was meant by more dialogue in the following example. Previously, the date and day were written on the board in the morning assembly, while the content was now more mathematical: 'Who's birthday is next? How many days are there until...? How long ago was Christmas?' The teachers developed math problems from the information that emerged, and the students were encouraged to develop their own problems.

Several teachers found dialogues to be challenging for both students and teachers in mathematics classes. Students needed to practice talking and explaining their thoughts. Some students asked (R2): 'Can't we just do a task?' The teachers stated that they needed to learn what questions to ask in order to elicit student thinking. To address some of the challenges described by the teachers in my study, it will be necessary to develop classroom norms and relations that are in line with several of the high-leverage practices (Forzani, 2014).

The teachers in the study taught at lower primary level and found it important to use various representations, such as concrete manipulatives and semi-concrete manipulatives, drawings, verbal representations and written representations in the form of math problems and numbers intended to help more students to understand more. They expressed great awareness of the use of new representations such as sketches of blank number lines: "Blank number line. Open number line. I think it's almost been revolutionary. I use it in nearly every possible context, very positive to use," (R6). The transition from concrete representation to abstract ideas was

challenging for the students, according to several of the teachers. R4 gave an example where she lined the students up at the front of the class to show doubles and halves. For the students to understand what numbers represented half and double, the teachers felt that they had improved their knowledge as to what questions to ask in order for the students to see the connection between the practical and the written parts. The teachers believed that this transition was important (R3). This indicated that they found it important to facilitate students' learning and how their current abilities could be related to what they were going to learn.

The students' thinking in and about the subject of mathematics

This category was also concerned with communication in mathematics teaching. When students explain their thoughts, it takes place in a communication situation. The teachers felt that the students must be given time to think and ask questions and that they, as teachers, should not feel that the students should rather be solving written math problems. By letting the students show their thoughts when solving problems, the teachers could emphasize that mistakes can be positive in that they can help the teachers and students to understand. "And understanding kind of how they think, and going into it and understanding a bit more why things are wrong and why it is hard, I think is very important" (R4). This showed that knowing about common student mistakes and ways of thinking was important for the teacher. According to the teachers, the students also became aware of there being more than one way of reaching the solution. They believed that the students acquired a better understanding by explaining their thoughts since this formed a 'bridge' between the terminology the students already had and new knowledge.

The teachers gave examples of their students' remarks when thinking *about* the subject of mathematics: "Oh yes, now I understand it." This expressed a sense of mastery. However, the teachers also described the challenges relating to students' different understandings of the mathematics subject. As mentioned earlier, it can be a challenge to get the students to talk in mathematics classes precisely because they are of the impression that mathematics means solving lots of math problems. R3 explained it in the following way: "The challenge is that there are a few students in the class that you don't manage to engage in the dialogue, that only really become involved when they are given the math problem in the book." She also believed that this particularly applied to students who were quick at calculations and those who were not particularly motivated in the subject of mathematics. This may indicate a view that mathematics is about quickly solving lots of math problems.

Subject matter/resources

When the teachers described the content of their own teaching, the main topics of discussion were the textbook and different types of tasks (often aside from the book). They expressed an increased awareness in relation to both.

In relation to the types of tasks, R4 explained that she no longer made booklets containing extra tasks, but used open-ended and problem-solving tasks that the students could work on over time. She also stated that, "I hope they have become better at thinking at least, to sort of, solve problems." This may imply that the teachers felt that investigation and problem solving were key elements of students' understanding, and thereby of good mathematics teaching. The

teachers also told that they discussed mathematics teaching with colleagues more than earlier, because the tasks were challenging. At lower primary level, the teachers gave the students notebooks where they could draw and write problems and solutions themselves. An open exercise for year one students was the hundred-day party where the mathematical topics the teachers covered were even numbers, odd numbers, ten friends, bridging through ten, counting, subtraction and addition. The teachers in my study explained that they had become more alert to the mathematics in everything around them, which could be linked to the types of exercises. R2 commented: "Grasp the everyday situations. And get them into what's related to mathematics in the class." When the teachers in my study seem to have increased awareness of using mathematics in all subjects, this might relate to the fact that the basic skill of calculation in all subjects had been a theme in the professional development program. R7 summed up what she thought good mathematics teaching was in the following way: "When the students understand when and how they can use their knowledge of mathematics in everyday life."

When the textbook was raised as a topic, there was some disagreement among the teachers. R7 told that she has become "critical to the textbooks, and I don't completely trust that the textbooks necessarily meet all the learning goals." R2 has become more aware of being freer in relation to the textbook, while R6 finds the textbook safe. This shows that teachers can disagree about the textbook's role in mathematics teaching.

According to the research question, the teachers' views about good mathematics teaching was described in the three main categories of findings in this section. Some of them were related to results in other Nordic studies, as discussed in the next session, but also to the content of the two-year professional program. This paper does not assess the effect of the program, but the teachers' views about good mathematics teaching can be influenced by this content and improved research-based knowledge. The teachers emphasized the use of open-ended and problem solving tasks and acceptance of communication and dialogues to facilitate learning.

Discussion

There were both similarities and differences between how good mathematics teaching was described by the teachers in this study as compared with other Nordic studies (Fauskanger, 2016; Fauskanger et al., 2018; Hemmi & Ryve, 2015; Kaasila & Pehkonen, 2009) that have examined good mathematics teaching.

The Norwegian teachers' descriptions of dialogs in lower primary school teaching were similar to those of other studies. Fauskanger et al. (2018) pointed out that facilitating conversation and discussion in mathematics teaching was a key aspect of students' learning at the same time as such conversation could inform the teacher about the students' thinking. International research highlights classroom discussion in mathematics teaching (e.g., Franke et al., 2007).

Based on the empirical data, students' thinking appeared to influence the planning of mathematics teaching for the participating teachers. Similar to the Finnish study the findings of this study documented that the teacher needs to listen to the students in order to understand their way of thinking (Kaasila & Pehkonen, 2009). The student teachers in Hemmi and Ryve's (2015)

study believed that Swedish teachers build on an extreme expression of constructivism and were therefore more student-focused, while the Finnish referred to whole class discussion.

Selecting open tasks and investigative activities that are motivating and give the students opportunities to show several solutions was emphasized by both Fauskanger et al. (2018) and Hemmi and Ryve (2015). Connecting mathematics with everyday life seemed to be important both in the Swedish and Finnish education studies (Hemmi & Ryve, 2015; Kaasila & Pehkonen, 2009) which are in line with the findings of my study.

In this concluding discussion, I will highlight one characteristic of good teaching that the teachers focused on and one characteristic that previous research has highlighted, but that was not mentioned in my focus group interview.

During the focus group interview, the teachers believed that using several different representations and working on the transition between these both contributed to good teaching and entailed a challenge. Using different representations has not been included in the characteristics of good mathematics teaching in other Nordic studies. This could of course mean that the use of representations and the transition between representations were included in some of the other categories in these studies. However, it may also be explained by the fact that the teachers in my study were teachers at lower primary school level and here, the need to use different representations was greater, and the transition from concrete to abstract thinking was more difficult than for older students. Work on expressing math problems or amounts in numbers after using manipulatives were considered particularly difficult for this age group.

A characteristic of good mathematics teaching mentioned in both Fauskanger et al. (2018) and Kaasila and Pehkonen's (2009) studies, is the structure of classes, with clear learning goals and classroom management. This was not mentioned, nor asked about, in the lower primary school teachers' focus group interview. This does not mean that it is not important to the lower primary school teachers in this study; it is perhaps more important here than in other years. However, the teachers might see classroom management and clear learning goals as such obvious factors that they did not mention them explicitly when they described mathematics teaching.

At the end of the professional development program the teachers felt that they had not only become more aware of dialogues with the students, but they also had more conversations and reflections among themselves. They feel that such discussion and reflection provide support and inspiration for their teaching. Knowledge sharing among the teachers may therefore contribute to long-term competence raising and deserve further research. Effect studies of professional development also need to be researched further.

References

Cai, J., Kaiser, G., Perry, B., & Wong, N. (Eds.). (2009). *Effective mathematics teaching from teachers' perspektives: National and cross-national studies*. Rotterdam: Sense Publishers.

Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25*(2), 119–142. doi:10.3102/01623737025002119

Fauskanger, J. (2016). Matematikklæreres oppfatninger om ingrediensene i god matematikkundervisning. *Acta Didactica Norge, 10*(3), 1–18.

Fauskanger, J., & Mosvold, R. (2015). En metodisk studie av innholdsanalyse - med analyser av matematikklæreres undervisningskunnskap som eksempel. *Nordic Studies in Mathematics Education, 20*(2), 79–96.

Fauskanger, J., Mosvold, R., Valenta, A., & Bjuland, R. (2018). Good mathematics teaching as constructed in Norwegian teachers' discourses. In E. Norèn, H. Palmèr, & A. Cooke (Eds.), *NORMA 17 – Nordic Research in Mathematics Education* (Vol. Skrifter från SMDF, nr 12 Göteborg: Svensk förening för matematikdidaktisk forskning, pp. 239─249). Göteborg.

Forzani, F. M. (2014). Understanding "core practices" and "practice-based" teacher education: Learning from the past. *Journal of Teacher Education, 65*(4), 357–368. doi:10.1177/0022487114533800

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (Vol. 1, pp. 225–256). Charlotte, NC: Information Age Publishing.

Hemmi, K., & Ryve, A. (2015). Effective mathematics teaching in Finnish and Swedish teacher education discourses. *Journal of Mathematics Teacher Education, 18*(6), 501–521. doi:10.1007/s10857-014-9293-4

Hiebert, J. S., & Grouws, D. A. (2007). The Effects of Classroom Mathematics Teaching on Students' Learning. In F. K. J. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 371–404). Charlotte, NC: Information Age Publishing.

Hsieh, H. F., & Shannon, S. E. (2005). Three Approaches to Qualitative Content Analysis. *Qualitative health research, 15*(9), 1277–1288.

Kaasila, R., & Pehkonen, E. (2009). Effective mathematics teaching in Finland through the eyes of elementary student teachers. In J. Cai, G. Kaiser, B. Perry, & N. Wong (Eds.), *Effective mathematics teaching from teachers' perspectives: National and Cross-National Studies* (pp. 203–216). Rotterdam: Sense Publishers.

Kilpatrick, J., Swafford, J., Findell, B. (2001). *Adding it up: helping children learn mathematics*. Washington, DC: National Academy Press.

Li, Y. (2011). Elementary teachers' thinking about a good mathematics lesson. *International Journal of Science and Mathematics Education, 9*(4), 949–973. doi:10.1007/s10763-010-9263-y

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for a common language and collective activity. *Journal of Teacher Education, 64*(5), 378–386. doi:10.1177/0022487113493807