Achieving Meaningful Efficiency in Coercion-Resistant, Verifiable Internet Voting

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Abstract: In traditional voting schemes with paper, pens and ballot-boxes, appropriate procedures are put in place to reassure that the result of the tally is correct. Considering that in internet voting errors or fraud will generally scale over a much greater fraction of votes, the demand to get strong reassurances as well, seems more than justified. With the ambition of offering a maximum degree of transparency, so-called *verifiable* schemes have been proposed. By publishing the relevant information, each voter may verify that her vote is included in the final tally and that accepted votes have been cast using proper voting material. Remarkably, this can be done while giving strong guarantees regarding the secrecy of the ballot at the same time. On the negative side, high transparency will generally make it easier for voters to demonstrate how they voted, e.g. towards a coercer. In this paper we propose an internet voting protocol that is verifiable and simultateously makes it practically impossible for vote-buyers or coercers to elicit the voters' behaviour. We compare its efficiency with existing work under equal degrees of coercion-resistance using an appropriate measure (δ). The contribution of our scheme lies in its efficiency during the most critical phases of the voting procedure, i.e. vote casting and tallying. Moreover, during these phases efficiency is insensitive to the desired degree of coercion-resistance.

1 Introduction

The secrecy of the ballot serves as a means to protect citizens from external influence that pressures them into casting a vote that does not reflect their personal preference. The key to achievement lies in preventing that citizens demonstrate how they voted. In traditional paper-based schemes, precautions may require voters to fill out their ballots on site, possibly even in an isolated booth. Thus voters get the privacy it takes to render any information they take out of the polling-station meaningless. Particularly, they cannot provide a coercer with a *receipt*, i.e. the information it takes to reveal the ballot they have cast. In internet voting, the quest for receipt-free and simlutaneously voter-verifiable systems is still ongoing. In a first phase some propositions have been made that rely on strong assumptions, such as the existence of untappable channels [HS00] prior to voting event. (In practice voters would need to register in person each time they are asked to vote using the internet.) In 2005, Juels et al. achieved a breakthrough by proposing a receipt-free and yet verifiable protocol under strongly reduced trust assumptions [JCJ05] (henceforth refered to as *the* JCJ protocol). Remarkably their scheme is not only receipt-free but also highly resistant to coercers who want to push voters into handing out their credentials, voting at random or abstaining from casting a ballot. Schemes that succeed at circumventing these attacks of coercion are called coercion-resistant.[1] For putting these advances in security into practice, Juels et al. still need to make strong assumptions regarding computational power of the tallying servers, which yield an implementation of JCJ infeasible for large-scale elections, as shown in [CCM08].

Since 2005 there were a number of propositions that take the work of Juels et al. as a starting point and aim at rendering coercion-resistant internet voting practical - notably while preserving the security features of JCJ [Ar08, ABR10, CH11, SKH11, SHK11]. With one exception, the propositions need to be configured to achieve high degrees of coercion-resistance at the cost of efficiency.[2]. The price is always payed either by the voter or the tallying servers who still have to perform lots of computing. The present paper too proposes a protocol which is parametrizable regarding coercion-resistance. However, the price for a high degree of coercion-resistance is only payed during the setup-phase, i.e. the phase which is least time critical. Notably, the computations related to the setup phase specific to a vote only (*post-registration*) needs to be completed only after the last vote has been cast. Typically we may expect voting phases to be long enough for post-registration to complete, thus allowing the first vote to be cast just after the last voter has registered. Casting votes is just as fast as in JCJ and tallying becomes drastically faster. We hereby address the general notion that userfriendliness and the possibility to obtain the election results early are preconditions to the successful introduction of internet voting.

In Section 2 we give an understanding of how coercion-resistance can be measured and in which respect the JCJ protocol is coercion-resistant. After presenting our protocol in Section 3 we compare its effciency with the known proposals from the literatue in Section 4 . Finally we make concluding remarks in Section 5 .

2 Coercion-Resistance and its Quantification

[1]As it is common in the technical literature, we do not distinguish between vote-buyers (people who give) and coercers (people who take). As far as we are concerned, a coercer is an algorithm designed to obtain the information it takes to reveal whether a voter has adhered to some predefined instructions.

[2]The only exception is the protocol proposed in [ABRTY10]. However, the scheme does not provide the same degree of verifiability as JCJ. This special case will be revisited in the context of Section 3.4 and Section 4.

There exists a variety of definitions of coercion-resistance. [KTV10] gives a nice overview of the various approaches. In their 2005 protocol proposition, Juels et al. include their own particular notion. The paper proves the protocol to be coercion-resistant in terms of their definitions. Subsequent JCJ-related protocols that were introduced under a formal view on coercion-resistance, have essentially done so under this model, or under slight technical adaptations.

All proposed protocols foresee the same defense strategy for the voter subjected to coercion: She hands out a fake credential to the adversary and casts the ballot of her choice through the anonymous channel using her real credential. In short, according to JCJ a protocol is coercion-resistant if an active non-adaptive adversary cannot distinguish dealing with the defense strategy from obtaining the real credential, with a non-negligible probability of success. In order to prove coercion-resistance of the JCJ protocol, the authors need to assume that along with the published result, the difference Γ between the number of cast votes n and the number of the ones that are actually counted (due to using a valid voting credential) gives the adversary no advantage at succeeding with coercion (*adversarial uncertainty*). As we will argue, adversarial uncertainty will always be low enough to allow coercion, even without any quantitative prior knowledge regarding Γ.

In [KTV10], Küsters et al. introduce their notion of a measure for quantifying coercion-resistance. They define the degree of coercion-resistance δ as the probability that the (reasonable) adversary will accept a run given that the voter submits to coercion minus the probability that the adversary will accept a run given that the voter applies the defense strategy. [3] They point out that there are opportunities of coercion already on the base of the expected and the effective tally, i.e. attacks that apply even in an ideal system. In that sense in JCJ it seems well justified to assume adversarial uncertainty with regard to the expected tally. However Γ is a value specific to coercion-resistant internet voting schemes. On one hand, since these schemes are not in practice yet, adversarial uncertainty with regard to Γ is to be expected in real life. On the other hand, since also voters are uncertain about Γ, the coercer can still launch an attack that grounds on a wild guess $\Gamma = c$: He can offer money in case $\Gamma \leq c$ or scratch the car if $\Gamma > c$. The reasonable voter will then submit to coercion if she believes that the vote cast with the fake credential would cause Γ to exceed c by 1. Since in a scheme that is meant to be coercion-resistant there is no reason to actually take advantage of using fake credentials, c might initially be chosen relatively small, thus yielding δ correspondingly high.

Given the exclusion of Γ from adversarial uncertainty, some parametrizable JCJ-related protocols can be configured to achieve a degree of coercion-resistance that depends solely on the estimated Γ. However in this case, the parameters have to be chosen such that there remain no meaningful gains in efficiency as compared with JCJ. In any case it

[3]If a vote buyer offers a voter 100 dollars for a vote when using a system that allows no defense strategy, the voter may expect to get the full reward when submitting to coercion and nothing otherwise. Intuitively speaking, δ signifies the fraction of the 100 dollars voters may in average expect to additionally get from a vote buyer when submitting to coercion as opposed to applying a defense strategy in a δ-coercion resistant system. Obviously, small δ values are what we are looking for.

seems that accelerating JCJ through parametrization inherently comes along with some loss in coercion-resistance. Nevertheless, this needs to be considered legitimate, knowing that JCJ were not coercion-resistant if not assuming adversarial uncertainty regarding Γ either. Finally, it cannot be estimated, whether coercion based on Γ promises less success than coercion based on the loss of coercion-resistance inherent to accelerating JCJ.

The protocol we are about to introduce is δ-coercion resistant in a parameter β. We will compare its performance with the others, under parameters β that yield equal degrees of coercion-resistance δ, where δ signifies the reduction of coercion-resistance compared with the JCJ-protocol. Remarkably, unlike Γ, we are able to quantify δ for each of the protocols.

3 Protocol

Due to space constraints, we are not able to introduce JCJ beforehand. Instead we will indicate relevant divergencies from JCJ within our exposition. Due to the strong relation between both protocols, we find this approach to be justified. After showing the basic idea behind our protocol in Section 3.1 and presenting the applied cryptographic primitives in Section 3.2 , in Section 3.3 we start off by introducing a basic version of our protocol. It already holds strong security features. In Section 3.4 we will propose some slight enhancements due to improve verifiability. We chose this step-by-step approach for the sake of readability. We will informally justify the δ-coercion resistance within the exposition of our protocol, i.e. assuming the ideality of the applied cryptographic primitives. The formal security proof is left for future work.

3.1 The Idea

Our scheme foresees the same defense strategy for voters under coercion as JCJ and the other verifiable coercion-resistant protocols known from the literature: They hand out an invalid credential and cast a vote to the public bulletin board (PB) using their real credential. The protocol should not enable the coercer to decide whether having obtained an invalid or a real credential, despite verifiability. Evidently this requires the voters' ability to cast votes to PB an arbitrary number of times, regardless of whether using real or invalid credentials.[4] As a consequence PB may contain multiple votes cast using the same credential and votes cast with an invalid credential. Coercion-resistant protocols thus all need to include steps *remove duplicates* and *authorize votes*, prior to decryption.

As in JCJ, our protocol divides the authorities put in charge of the voting system among *registrars* and *talliers*. Regarding corruption by a coercive adversary, we advise the reader to assume all registrars and a majority of talliers to be trustworthy. This could be

[4]If the number of accepted votes were limited, the coercer could test the received credential for validity by counting the number of times he can use it for casting votes.

weakened by requiring all registrars to be trustworthy only during the registration step and during the other phases by assuming that each voter knows a registrar of which she knows that he will not participate in a coercive attack against her. This weakening requires no change to the proposed protocol and the reasoning strictly follows [JCJ05]. Regarding *verifiability* (defined in [JCJ05] as *strong verifiability*) none of the authorites need to be trusted. The definition requires voters to be able to detect the exclusion of legitimate votes, changes to legitimate votes and the inclusion of multiple votes cast with the same credential. In Section 3.4 we will change this definition as well and give more power to voters at verification under the notion of *improved verifiability* (the features of which are also mentioned in [JCJ05], however not formalized), such that they can additionally verify that all credentials used to cast votes are assigned to eligible voters, whereas the basic protocol would only allow voters to verify this given respective trustworthy majorities of registrars and talliers. To achieve *improved verifiability* in the full protocol, we will enhance the basic protocol accordingly in Section 3.4 . The conclusion will be that our scheme reaches δ-coercion resistance and a degree of verifiability equal to the JCJ-scheme, notably under equal assumptions regarding the authorities and adversarial power. After showing the applied primitives, we are ready to introduce our protocol.

3.2 Cryptographic Primitives

The new scheme applies the following cryptographic primitives. The ones not employed by the JCJ protocol are identified accordingly. At justifying coercion-resistance and verifiability in the course of our exposition, we assume primitives to be ideal. **Multiparty ElGamal Cryptosystem with Threshold.** We propose all ciphertexts to be ElGamal over a pre-established multiplicative cyclic group $(\mathcal{G}_q, \cdot, 1)$ of order q, for which the decisional Diffie-Hellman problem (DDHP) is considered to be hard. [5] Assuming no decryption, ElGamal ciphertexts are meant not to disclose any information of the encrypted plaintext, even in the event that the plaintext-space is small and in the presence of other ciphertexts.

We also propose the application of a multiparty computation scheme derived from [Pe91, GJK99] to preserve the confidentiality of encrypted values throughout the protocol. Thus, malicious decryption is only possible in the event of a conspiring majority (the number depends on the chosen threshold) of group members, i.e. registrars or talliers.

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[5] We thus follow Civitas [5], which basically instantiates the JCJ protocol. However they do deviate in the choice of the underlying cryptosystem. The reason behind JCJ choosing a modified version of ElGamal (M-ElGamal) lies in the reasoning in their security proof. Although we could allow our protocol to adopt M-ElGamal as well, we adhere to ElGamal, thus making its performance more easily comparable to most of the other known proposals for coercion-resistant internet voting. Further, the question whether to choose ElGamal or M-ElGamal does not seem sensitive to the design of a particular verifiable voting protocol, but rather to the desired security reassurances of the cryptosystem itself. Notably, recently ElGamal has been proved to have the beneficial IND-CCA1 property (resistance against non-adaptive chosen ciphertext attacks) just as much as M-ElGamal [Li11]. Underlying our informal security argumentation within the protocol description, we assume that the plaintexts of all ciphertexts are unconditionally hidden, even when the plaintext space is restricted, and given the ideality of the remaining primitives.

Verifiable Mix-Nets. Trustworthy mix-nets take an ordered set of ciphertexts and output re-randomized encryptions in random order such that the link is infeasible to be retrieved. They are implemented as a sequence of shuffles, each performed by a distinct mix-node. The link between elements from input and output is only retrieved in the event of all nodes conspiring. Correctness of execution is proved using NIZKP.

Plaintext Equality Test PET. Given two ElGamal encryptions E_1 and E_2, the algorithm returns $true$ if the plaintexts are equal and $false$ otherwise. This is done by checking whether the decryption of $(E_1/E_2)^z$ equals 1 for a random value $z \in \mathbb{Z}_q$. [JJ00] PET is verifiable and reveals no non-negligible information on the plaintexts.

Additional Primitive M-PET. Unlike JCJ the new scheme relies on an additional method for efficiently testing the equality among the elements encrypted by a set of ciphertexts as described in [We08]. Clearly, applying PET pair-wise on all elements of the set will result in quadratic running time. This is exactly the approach chosen in JCJ-protocol and the reason for its inefficiency during the tallying stage.

Given ciphertexts $X_1, \ldots X_n$, the modified PET (M-PET) raises all values to a random value $z \in \mathbb{Z}_q$, and decrypts them to obtain the blinded plaintexts $x_1^z = DEC(X_1^z), \ldots, x_n^z = DEC(X_n^z)$. The blinded plaintexts can be efficiently compared for equality for instance by sequentially saving them in a hash-table. If a collision is detected, the algorithm returns $true$, $false$ otherwise. M-PET reveals no non-negligible information of the plaintexts, given that the discrete logarithm of any plaintext x_i is unknown in the base of any plaintext x_j, $1 \leq i < j \leq n$.

Communication Channels. There is a public board PB which is used as a *public broadcast channel*. Voters post their votes to PB and the authorities post all output of the tallying phase to PB. For the sake of simplicity we also assume that all public information, including public values from the employed PKI, is accessible on PB. Further there is an *untappable, authenticated channel* from the registrars to the voters to hand the voters their credentials. Finally an anonymous channel is in place to allow casting votes anonymously to PB.

Non-Interactive Zero-Knowledge Proofs NIZKP. To provide verifiability, many computations throughout the protocol need to come along with non-interactive zero-knowledge proofs. They allow voters to prove knowledge of a plaintext, proving plaintext membership of a given sub-domain of $\mathcal{G}_q$, authorities proving correct execution of PET, M-PET, correct mixing, encryption and decryption. We rely on the Fiat-Shamir heuristic for secure non-interactivity, i.e. negligible knowledge-errors and overwhelming witness-hiding.

3.3 Basic Protocol

Pre-Registration. The talliers jointly establish a multiparty ElGamal threshold PKI, publish their public key ε on PB and keep their shares of the corresponding private key to themselves. The registrars jointly establish a number of $\beta \cdot N_+$ random credentials, where β denotes the security parameter underlying the degree of coercion-resistance δ, and N_+ denotes the maximum expected number of individual voters ever to participate at elections hosted by the voting system. The credentials are tuples of the form (σ, i), whereas we use the terms σ-credential and i-credential to refer to the respective components. Each component is random from $\mathcal{G}_q$, and only computable if the registrars maliciously co-operate. They jointly encrypt and post to PB each of the two components $(\mathrm{E}_\varepsilon(\sigma, \alpha_\sigma), \mathrm{E}_\varepsilon(i, \alpha_i))$ and memorize their share of the randomnesses α_σ and α_i, both random from Z_q. We call the resulting list of encrypted credential components the *credential-pool*. Finally, they pass all $E_\varepsilon(i, \alpha_i)$ through a mix-net and the talliers decrypt the output to form the list $\mathcal{UNL} < i >$, i.e. the list of i-credentials, the elements of which are **unl**inkable to the *credential-pool* by the coercer. The pre-registration step is needed only prior to the first election hosted by the voting system. Since valid i-credentials need to be made public later in the protocol, the list $\mathcal{UNL} < i >$ is meant to enable voters, as in JCJ, to lie about their credential already directly after registering. The *credential-pool* however will be processed at a later stage to allow the exclusion of votes cast with an invalid credential.

Registration. The voter-roll is initialized as an empty list on PB. After successful authentication for registration, the registrars choose an unassigned ciphertext tuple from the *credential-pool* and post it to the voter-roll along with an identifier of the voter. They hand voters their credential (σ, i), along with a proof that the credential corresponds with the ciphertext tuple. As all computations by registars and talliers, this procedure is conducted by the means of multi-party computation, such that only a malicious collusion can compute the secret, i.e. the plaintexts. The proof is implied by one proof from each registrar computed by the respective partial knowledge of the randomnesses α_σ and α_i. Finally, the voter secretly chooses random elements $\hat{\sigma} \in \mathcal{G}_q$ and $\hat{\imath} \in \mathcal{UNL} < i >$. Whenever the coercer asks the voter to hand out her credential, she lies and hands out $(\hat{\sigma}, \hat{\imath})$. In the basic version of the protocol, the *voter-roll* only serves as a reference for locating the unassigned credentials from the *credential-pool* and for identifying the credentials to be retained in case voters loose eligibility.

Post-Registration. The registrars pass all ciphertext tuples $(\mathrm{E}_\varepsilon(\sigma, \alpha_\sigma), \mathrm{E}_\varepsilon(i, \alpha_i))$ of the *credential-pool* to a mix-net. From the output, the talliers decrypt the second component, i.e. the ciphertexts containing i-credentials. We call the resulting list $\mathcal{UNL} < E_\varepsilon(\sigma), i >$, as the coercer cannot link its elements neither to the credential-pool, nor to the non-anonymous voter-roll. The post-registration step needs to be completed only prior to tallying, i.e. the phase in which voters cast their votes can be used for this step. Thereby the negative impact of the time-consuming mixnets is mitigated, or even fully compensated, given that the voting phase is sufficiently long.

Vote Casting. The voter selects the representation c of her prefered candidate(s) from a set $\mathcal{C} \subset \mathcal{G}_q$ we assume to be available on PB. To cast the vote, she uses the anonymous channel and posts the two ciphertexts $A = \mathrm{Enc}_\varepsilon(\sigma, \alpha_A)$ and $B = \mathrm{Enc}_\varepsilon(c, \alpha_B)$ to the voting-board on PB, along with her i-credential in plaintext. The voter aditionally needs to post one non-interactive zero-knowledge proof (NIZKP) per cipher-text. The first one requires voters to prove their knowledge of σ. This is done indirectly by proving knowledge of α_A. We thereby exclude the attempt to cast an illegitimate vote by copying and re-randomizing σ-ciphertexts from PB undetectedly.[6]. The other proof shows that $c \in \mathcal{C}$. Since each authorized vote on the voting board will be decrypted during the tallying phase, requiring the second proof prevents coercers from forcing voters to select $c \not\in \mathcal{C}$ according to some prescribed pattern, thus obtaining a receipt (*italian attack*) [Di07] or from using the talliers as a decryption oracle to obtain σ-credentials for subsequent votes.

Apart from casting the i-credential this step is exactly the same as in JCJ. Although the coercer has no means of deciding to which among the uncontrolled voters the i-credentials refer to, he still gains a quantifiable advantage at coercion. Recall that the voter under coercion had to choose an arbitrary value $\hat{\imath}$ from $\mathcal{UNL} < i >$ and pretend that this were his i-credential. The reasonable coercer will therefore observe the voting-board to find out whether someone has cast a vote using $\hat{\imath}$. If this is the case, the coercer will conclude that $\hat{\imath}$ is in fact an i-credential that belongs to another voter and that the voter under coercion has revealed a false credential. [7] The probability that a voter is unfortunate enough to choose such a value as $\hat{\imath}$ is less than $\frac{1}{\beta}$. The further exhibition of our protocol shows that the coercer gains no additional information useful for distinguishing the behaviour of the voter under coercion. This will lead to the conclusion that our scheme is indeed δ-coercion resistant, whereas $\delta = \frac{1}{\beta}$. [8]

Tallying. In the beginning of the tallying step, the voting board contains tuples of votes (A, B, i) that may have been cast with wrong proofs, that were cast with the same credential as other votes (we call these votes *duplicates*), or that hold A- or i-components that do not correspond with a valid credential (σ, i) from $\mathcal{UNL} < E_\varepsilon(\sigma), i >$. Prior to decryption and counting, these invalid votes need to be excluded.

First, votes with wrong proofs, and votes with i-credentials that are not contained as the second component of an element enlisted by $\mathcal{UNL} < E_\varepsilon(\sigma), i >$ are marked and excluded from further processing. In order to efficiently remove duplicates, the talliers consider only votes not cast with a distinct i-credential and apply $\mathrm{M} - \mathrm{PET}$ on the A-

[6]Due to this measure, votes cannot be cast by stealing the credentials of other voters, given a trustworthy majority of registrars (a majority could still compute σ and i) and talliers (a majority could compute the private decryption key and decrypt $sigma$-credentials from list $\mathcal{UNL} - (E_\varepsilon(\sigma), i)$

[7]Note, that this conclusion can only by drawn in the strict model proposed by JCJ, where it is assumed that exactly one voter is under coercion and that invalid credentials are only used to the degree of achieving adversarial uncertainty regarding Γ. If we now allow the coercer to believe that the vote cast with $\hat{\imath}$ as the i-credential is a fake vote (one with an invalid σ-credential), coercion will become even more difficult. However, we adhere to the strict model proposed in the JCJ paper.

[8]The precise value of δ is $\frac{N_+ - 1}{\beta N_+ - 1}$. Firstly, this is always smaller than $\frac{1}{\beta}$ and secondly, the difference is very small and irrelevant for reasonable N_+. We thus justify the facilitation of saying $\delta = \frac{1}{\beta}$.

components of votes cast with the same i-component.[9] At this stage a last-vote-counts or a first-vote-counts policy is enforced. Note, that the steps described so far could also be performed each time a vote is posted, i.e. prior to the tallying stage.

To authorize votes, the i-credentials are used to link the A- and B-components of the votes with the encrypted σ-credentials from $\mathcal{UNL} < E_\varepsilon(\sigma), i >$ to form tuples $(E_\varepsilon(\sigma), A, B)$. These tuples are passed to a mixnet. We call the output $\mathcal{UNL} < E_\varepsilon(\sigma), A, B >$, since its elements are unlinkable neither to $\mathcal{UNL} < E_\varepsilon(\sigma), i >$ and the voter-roll, nor to the votes on the voting board. For each element, the talliers apply PET on the first two components. If the algorithm returns *true*, A is an encryption of a valid σ-credential. In that case, the corresponding ciphertext B is decrypted and counted in the tally, otherwise the vote is excluded from further processing. Note, that since votes are being assessed for the validity of σ-credentials encrypted by the A-component, we may not apply $\mathrm{M-PET}$ at this stage - such an approach would allow the coercer to check the validity of $\hat{\sigma}$ by the means of another vote cast by him with an A-component encrypting e.g. $\hat{\sigma}^2$, i.e. a value the logarithm of which is known in base $\hat{\sigma}$. The basic protocol is illustrated in fig. 1.

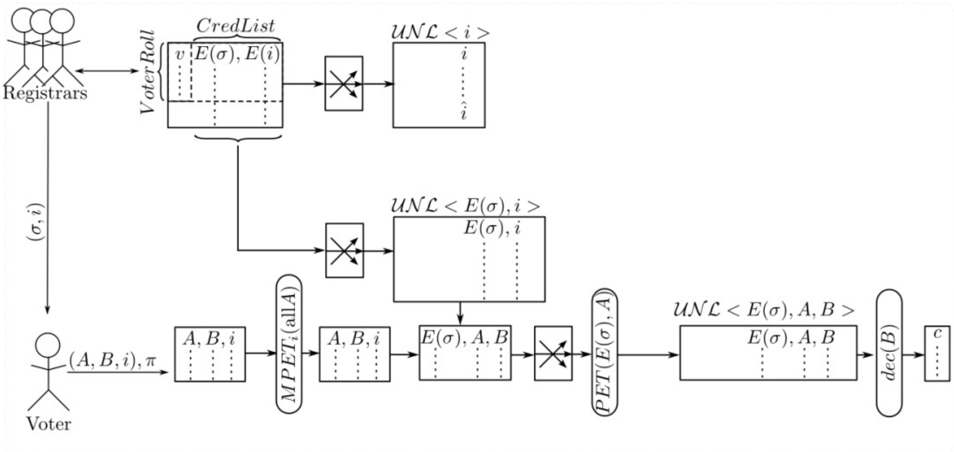

Fig. 1 Basic Protocol

Credential Retention. As implied above, our scheme aims at allowing voters to re-use the same credential (σ, i) at numerous voting events. We therefore need to provide a mechanism that disallows voters to cast votes after loosing eligibility, for instance when they leave the voting district. Removing their credential from the *credential-pool* at post-registration is clearly not an option, since the coercer could verify the validity of the previously received i-credential by observing whether the value still appears on $\mathcal{UNL} < E_\varepsilon(\sigma), i >$ after the post-registration step of the following election. The

[9] We hereby adhere to the approach proposed by Smith and Weber. However unlike Smith / Weber, we apply $\mathrm{M-PET}$ only at removing duplicates, not at authorizing votes as proposed by them. Since at the current stage we do not check the validity of the values encrypted by A, and since the coercer does not know the discrete logarithm of any valid σ-credential in the base of any other, the coercer learns nothing that is useful for his attack.

protocol therefore defines credential-retention by having the registrars compute a new σ-credential and replace $(\mathrm{E}_{\varepsilon}(\sigma, \alpha_{\sigma}))$ on the *credential-pool* with an encryption of this new value. The encryption of the ι-credential however remains the same. Finally, the voter's ID on the *voter-roll* is marked as non-eligible. The new credential in the *credential-pool* is marked and may not be assigned to new voters, since the coercer would know the true value of the ι-credential, in case it previously belonged to a voter controlled by him. Clearly, moving voters will not be able to use their retained credential for voting, since such votes would be discarded at *vote authorization*. Just as all unassigned credentials in the *credential-pool*, the new credential can only be used for voting unnoticed in the event of colluding registrars or talliers (a case to be ruled out in the full protocol).

Now we observe whether credential retention gives the adversary an advantage at judging if the voter who lost eligibility previously lied to him. We consider the two cases where the voter had submitted to coercion and where she applied the defense strategy. In the first case, the coercer would expect the distribution of Γ, i.e. votes not to be counted, to remain the same and the number of counted votes to decrease by one. In the second case, he would also expect Γ to decrease by one. This is exactly the distinguishing factor we need to assume irrelevant by the means of *adversarial uncertainty* when proving coercion-resistance of the JCJ-protocol, i.e. independent of credential retention.

3.4 Full Protocol and Improved Verifiability

Evidently, the basic protocol complies with the definition of *verifiability* in the JCJ paper - it allows to detect the exclusion of legitimate votes, changes to legitimate votes and the inclusion of multiple votes cast with the same credential. Notably the definition already captures the commonly quoted requirement imposed on verifiable systems, i.e. that voters need to be able to verify that their vote has been cast as intended, recorded as cast and tallied as recorded. Regarding verifiability, our basic scheme is thus not less powerful than the well-known coercion-resistant scheme by Araújo et al. [ABR10, AFT07, Ar08]. However, the JCJ paper mentions that it may be desirable for any election observer to verify, that credentials have only been assigned to voters whose names are on a published roll. Indeed, the JCJ-protocol provides this kind of verifiability. However our basic protocol only does, when assuming trustworthy majorities among registrars and talliers. In order to ensure that one can detect the event where registrars or talliers collude to cast votes with a credential enlisted by the *credential-pool* but not by the *voter-roll*, we propose an enhancement to the tallying step.

In the tallying step prior to decryption, the *voter-roll* is passed to a mixnet which outputs the list $\mathcal{UNL} < E_{\varepsilon}(\sigma) >$. The coercer cannot link the entries of this list to the entries of the voter-roll. After votes from $\mathcal{UNL} < E_{\varepsilon}(\sigma), A, B >$ with A-components that encrypt an invalid σ-credential have been excluded from further processing (at vote authorization as described above), the talliers apply $\mathrm{M}-\mathrm{PET}$ on all A-components of $\mathcal{UNL} < E_{\varepsilon}(\sigma), A, B >$ and all entries in $\mathcal{UNL} < E_{\varepsilon}(\sigma) >$. If for an A-component of $\mathcal{UNL} < E_{\varepsilon}(\sigma), A, B >$ no collision is detected with any of the entries of the $\mathcal{UNL} < E_{\varepsilon}(\sigma) >$, the corresponding vote is obviously cast with a credential that corresponds with an entry in the $credential - pool$ that has not been assigned to any voter. These votes are excluded from further processing, i.e. their B-components are not decrypted. The full protocol is illustrated in fig. 2. Note, that since all input values to $M - PET$ are encryptions of valid σ-credentials, no discrete logarithm of any value in the base of any other is known. Therefore the coercer does not get any advantage, and it is justified to apply $M - PET$.

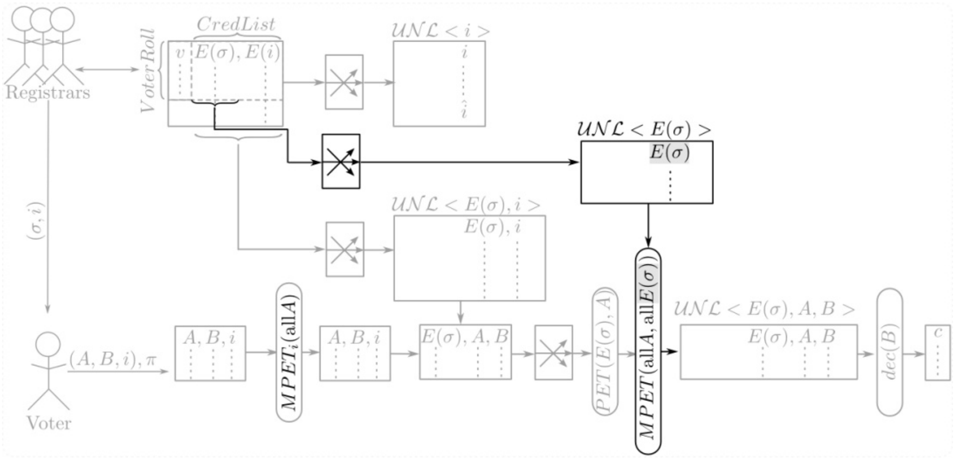

Fig. 2 Enhancement to the basic protocol to achieve full protocol

4 Efficiency

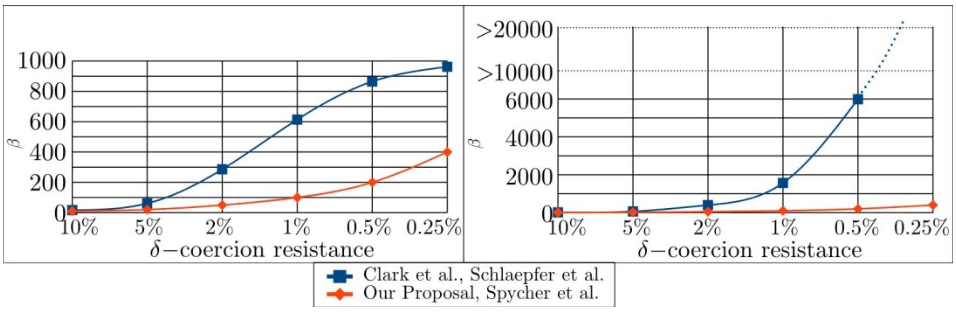

Fig. 3 The two drawings show the parameter β in dependence of the degree of coercion-resistance δ. The diagram on the left shows the case for 1000 voters and 1000 votes on the voting board, the one on the right 100000 voters and 100000 votes on the voting board.

We now present the efficiency properties of our protocol through comparison with the schemes known from the literature. In the schemes by Clark et al. [CH11] and Schläpfer et al. [SHK11], voters associate their vote with non-anonymous information on PB that refers to themselves. In order to mislead coercers they randomly choose a set of other voters they additionally associate their vote with, thus forming an anonymity set of size β.[10] In the case of Clark et al., the *computation time on the voter's platform* scales in the parameter β. Particularly the number of modular exponentiations is $4 \cdot \beta + 10$, assuming a set C of two candidates to choose from. However, the tallying stage remains unaffected by the parameter and efficient, i.e. it is equally efficient as our basic protocol. The tallying time of our full protocol takes slightly longer, depending on the size of the mix-net, but not more than twice as long. In Schläpfer et al. the *tallying time* scales in β, i.e. a mix-net during the tallying stage will need to perform $48 \cdot \beta \cdot N$ modular exponentiations, where N denotes the number of cast votes and when assuming four mix-nodes. The scheme by Spycher et al. [SKH11] does not rely on anonymity sets. Instead the registrar who enjoys the voter's trust even after registration, assigns her an average number of β votes, under uniform distribution, cast with a false credential. Clearly this will also scale the time of tallying, $156 \cdot \beta \cdot n + 156 \cdot N$ is the number of modular exponentiation due to the most expensive steps, where n denotes the number of voters. Fig.3 shows the choice of β in dependence on the desired degree of coercion-resistance for the schemes with a corresponding parameter.[11] The scheme by Araújo et al. [ABR10] is by nature efficient at all stages and coercion-resistant with $\delta = 0$. However, as shown in Section 3.4 , it gives no means to verify whether authorities have created illegitimate credentials and cast extra votes.

We conclude that our protocol is efficient at both vote-casting and tallying. It does scale over β, however only during the non-critical pre-registration and post-registration steps. We therefore omit exact quantification. Further our protocol allows high levels of coercion-resistance, even under relatively small parameters. Since the pre-registration step may be conducted independent from the voting procedures, it will not have a negative impact on the elections. Also the post-registration step can begin right after last voter has registered and only needs to end prior to tallying. The phase when citizens cast their votes should give enough time for completion.

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[10]In both cases coercion-resistance of degree $\delta = 0$ can be achieved by selecting $\beta = n$, where n is the number of voters. Moreover, it is sufficient for coerced voters to hide their votes in the anonymity set of size n, assuming adversarial uncertainty regarding the number of such votes. However this is a strong requirement, given large n.

[11]In Section 3.3 we have shown that the coercion-resistance of our scheme follows $\delta = \frac{1}{\beta}$. It is easy to see that the same relation applies to the scheme by Spycher et al. as well. In the case of the protocols that rely on anonymity sets we have followed the definition from [KTV10]. To obtain δ, we need to compute $\sum_{r \in R} Prob(r|(\sigma, i)) - Prob(r|(\hat{\sigma}, \hat{\imath}))$, where the condition in the first term signifies submission to coercion, the condition in the second one signifies applying the defense strategy. R denotes the set of results (i.e. the number of votes assigned to the voter under coercion) that the coercer would accept. Note, that inherent to assuming a reasonable coercer the difference within the sum is inherently never negative. $\mathrm{Prob}(r|(\sigma, i)$ we compute as $F_1(r)$, where F_1 is the distribution function of a binomial distribution with N trials and a success probability of $\frac{\beta-1}{n-1}$, where N denotes the number of cast votes and n the number of voters. $\mathrm{Prob}(r|(\hat{\sigma}, \hat{\imath}))$ we compute as $F_2(r-1)$, where F_2 again is the distribution function of a binomial distribution, this time with $N-1$ trials.

5 Conclusion

The verifiable JCJ-protocol offers coercion-resistance under conditions that disallow an implementation for large-scale elections. The proposed solutions that followed either compromise verifiability or require a trade-off between the degree of coercion-resistance and efficiency during the critical phase of tallying or the process of casting a vote. Our proposal too requires more computation than conservative verifiable schemes. However we have shown that compared with other schemes, the factor that scales the computation time is small for relatively high degrees of coercion-resistance. Moreover, the expensive computations specific to coercion-resistance can be performed while the polls are open, i.e. while nobody is waiting.

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