

The Effect of Role Origin on Bidding Behavior in an Asymmetric All-Pay Auction: An Experiment[†]

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Abstract: This paper reports the results of an experiment concerning whether different origins of bidder role affect bidding behavior in the two-person all-pay auction in which ties are broken in favor of one role (strong bidder) over the other (weak bidder). Two role allocation procedures are compared. In the first one, the roles are allocated at random. In the second one, the roles are allocated based on their performance in a simple real-effort task. The data provided no evidence to confirm the effect of role origin on bidding behavior in the current asymmetric all-pay auction.

Keywords: Role Origin; Asymmetric All-Pay Auction; Gender; Experiment

JEL Classification: C92, C72

1. Introduction

In experimental economics randomization is a commonly used technique in order to ensure each subject an equal chance of being assigned to each of different roles, such as advantageous and disadvantageous roles. However, there are two potential issues about this common practice. First, randomization may undermine external validity. A casual observation suggests that the right to become an advantageous role is not always obtained by luck but often by prior efforts or skills.¹ Second, and more importantly, how people behave may be sensitive to how they obtain roles. For example, Hoffman et al. (1994) detected the possibility that role origin matters in economic decision-making situations. They conducted the ultimatum game experiment in which the right to become the proposer role was allocated either by luck or by the score on a general knowledge quiz.

[†] This paper is dedicated to Professor Kunio Kama on the occasion of his retirement from Soka University. Generous financial support from Soka University is gratefully acknowledged.

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¹ For example, consider the election contest in which two candidates, an incumbent and a challenger, are running for office. Even if they are expected to divide the vote evenly, the incumbent would end up with an election victory due to officeholder benefits, such as greater name recognition and voters' status-quo bias. To establish the right to enjoy officeholder benefits, the incumbent usually has to exert substantial prior efforts; he has to accumulate more experience in office, engage in fundraising events, and make his presence felt among his constituents.

They found that the latter case induced the proposers to significantly lower their proposals.

The proposer is known to be in the stronger position as previous experimental research on ultimatum games shows that the proposer tends to earn 50% to 70% of the pie. The finding of Hoffman et al. (1994) hints that the earned right to be in an advantageous position may serve to legitimize the right to exploit its advantages even more. Then, a natural question is whether this finding can be replicated in other economic decision-making situations where role differences are clearly defined, such as asymmetric competitions.

This paper reports on an experiment designed to explore the effect of role origin on bidding behavior in the two-person all-pay auction in which one role (strong bidder) is more advantageous than the other (weak bidder). Role origin refers to the procedure that allocates the two bidder roles to subjects. The experiment consists of two treatments. In one treatment, subjects compete in a simple real-effort task in order to secure the advantageous role in the auction. In the other treatment, the two roles are allocated randomly.

The current experiment is designed in that in theory bidders' bidding strategies remain the same between the treatments; subjects should bid according to the same mixed strategy. However, the effect of role origin, if such exists, may induce subjects to bid differently in the earned treatment. For example, a subject earning the strong bidder role by effort forms a feeling of entitlement to the prize and a subject in the weak bidder role respects the opponent's exerted effort. Then, both bidder roles may bid non-competitively.²

The paper proceeds as follows. Section 2 presents a model of the two-person asymmetric all-pay auction and its unique equilibrium solution. Section 3 describes an experiment to examine the effect of role origin on bidding behavior in the all-pay auction. Section 4 summarizes the results. Section 5 concludes.

2. Model

Risk-neutral strong and weak bidders, indexed by s and w , compete over a single, indivisible prize v . Its valuation is the same for both bidders. Each bidder independently chooses a bid b_i , $i \in \{s, w\}$, from the common discrete strategy space $B = \{0, \varepsilon, 2\varepsilon, \dots, c\}$, where $\varepsilon > 0$ and c is the common bid cap. v and c are assumed to be multiples of ε such that $v > c > \varepsilon$.³

Ties are broken asymmetrically; the strong bidder wins the prize if $b_s \geq b_w$, and the weak bidder

² It is well known in the theoretical literature of contests that when asymmetry between contestants is sufficiently large, they expend their resources non-competitively. This phenomenon is called the *discouragement effect* (Konrad, 2009). In the current setting, however, theory suggests that this phenomenon does not occur.

³ Otsubo (2015) characterized a complete set of Nash equilibrium in mixed strategies for the model with no binding bidding cap. In a sharp contrast to the current model, there exist both a unique symmetric equilibrium and a continuum of asymmetric equilibria.

wins it otherwise. Thus, the weak bidder has to outbid the strong bidder. Their payoff functions are given as follows:

$$\pi_s(b_s, b_w) = \begin{cases} v - b_s & \text{if } b_s \geq b_w \\ -b_s & \text{otherwise} \end{cases}$$

and

$$\pi_w(b_w, b_s) = \begin{cases} v - b_w & \text{if } b_w > b_s \\ -b_w & \text{otherwise} \end{cases}$$

where $\pi_i(\cdot)$ is bidder i 's payoff function. For any pure-strategy profile at least one bidder has an incentive to unilaterally deviate her strategy, no pure-strategy equilibrium exists.

In the mixed extension of the game, denote by (σ_s, σ_w) a profile of mixed strategies, where σ_i is bidder i 's mixed strategy, i.e., a probability distribution over B , and $\sigma_i(b)$ is the probability assigned by σ_i to a pure strategy $b \in B$. Then, there exists a unique equilibrium in mixed strategies (σ_s^*, σ_w^*) characterized by

$$\sigma_s^*(b) = \begin{cases} \frac{\varepsilon}{v} & \text{if } b \in \{0, \dots, c - \varepsilon\} \\ 1 - \frac{c}{v} & \text{if } b = c \end{cases}$$

and

$$\sigma_w^*(b) = \begin{cases} 1 - \frac{c}{v} & \text{if } b = 0 \\ \frac{\varepsilon}{v} & \text{if } b \in [\varepsilon, \dots, c] \end{cases}$$

with equilibrium payoffs $v - c$ for the strong bidder and 0 for the weak bidder.⁴ In equilibrium, the expected bids are $c - \frac{c(c+\varepsilon)}{2v}$ for the strong bidder and $\frac{c(c+\varepsilon)}{2v}$ for the weak bidder, respectively. Thus, the expected sum of the bids is always equal to c . The probability that the strong bidder wins the prize is $1 - c \frac{(c+\varepsilon)}{2v^2}$, which is larger than 0.5. Due to the unfair tie-breaking rule, the strong bidder always has a higher chance of winning the prize than the weak bidder.

3. Experiment

3.1 Design

There were two treatments in the experiment. In the random treatment, strong and weak bidder roles were randomly allocated in that half of the subjects played the strong bidder role and the remaining subjects played the weak bidder role. In the earned treatment, on the other hand, the two roles were allocated according to their performance in a simple real-effort task similar to the counting zeros task introduced by Abeler et al. (2011). There were 20 questions in this task,

⁴ The proof is available upon request.

each of which consists of about 200 integers ranging from 0 to 9. Subjects were given six minutes to count zeros in these questions. They were ranked based on the number of questions they answered correctly and then divided into two equal-size groups, the high score group and the low score group. Subjects in the former group played the strong bidder role, whereas those in the latter group played the weak bidder role.

The same set of parameter values was adopted in both treatments; $c = 800$, $v = 1000$, and $\varepsilon = 200$.⁵ In equilibrium both bidders use the same mixed strategy that assigns probability 0.2 to each of the bids 0, 200, 400, 600, and 800. The probability that the strong bidder wins the prize is 0.6.⁶ Their expected bids are equal to 400.

One may notice that both bidders can assure their equilibrium payoffs by choosing their maximin strategies, namely 800 for the strong bidder and 0 for the weak bidder. The current all-pay action belongs to a class of *unprofitable games* (Harsanyi, 1966); games in which maximin strategies do not coincide with the unique equilibrium strategies, yet yield the same payoff as the equilibrium strategies. Several game theorists conjectured that the unprofitability of the equilibrium undermines its plausibility as a predictor (for example, Harsanyi, 1966; Aumann and Maschler, 1972).

The extent to which a subject is willing to undertake such a risky mixed strategy depends on the subject's risk attitude. Suppose that the strong bidder chooses his maximin strategy, he will earn a sure payoff of 200. If his bid falls to, say 600, his potential payoff rises to 400, but whether or not he will receive this payoff relies on what the weak bidder does. In the presence of such strategic uncertainty, he would form a belief that the probability of getting this payoff is less than one. A risk-loving strong bidder would still be willing to choose a lower bid, whereas a risk-averse strong bidder would bid 800 for a sure payoff of 200. By the similar argument, a risk-loving weak bidder would be willing to bid higher, whereas a risk-averse weak bidder would bid 0. It is of particular importance to measure subjects' risk attitudes in order to interpret the all-pay auction data. Therefore, prior to playing the all-pay auction, subjects were given a simple lottery choice task inspired by Holt and Laury (2002) for elicitation of subjects' risk preferences.

3.2 Procedures

A total of seventy-six undergraduate students from various majors enrolled at Soka University in Tokyo, Japan, were recruited from an online bulletin board on the university's portal site. Thirty-six of them participated in the random treatment and the rest the earned treatment. The numbers of males and females participated in the experiment were 53 and 23, respectively. These treatments

⁵ The currency unit of these values is Japanese Yen. When the experiment was conducted, the USD/JPY currency exchange rate ranged approximately from 108 yen to 109 yen.

⁶ A tie occurs with probability 0.2.

had four sessions each, and no subject was allowed to take part in more than one session.⁷ Each session lasted about 90 minutes, including instructions and payment.

Once all subjects were seated, they began to read instructions for Stage 1 silently at their own pace.⁸ Then, an experimenter read the instructions aloud to induce their common knowledge. The experimenter answered questions individually.

In Stage 1, subjects were presented with a list of ten choice problems between a gamble with payoffs of 300 yen or 0 yen (called as “Option A”) and a certain payoff of 140 yen (called as “Option B”). Table 1 displays the ten paired lottery choice problems used in Stage 1. Subjects were instructed to state which option, A or B, they would prefer for each problem. The instructions clearly explained that the earnings of Stage 1 would be determined by playing one of the lottery choice problems drawn randomly at the end of the session. Notice that as moving down the table, the probability of the high payoff p_h in Option A decreases. Though everyone is expected to choose Option A in the 1st choice problem, when p_h becomes sufficiently low, subjects are expected to switch over to Option B. For example, a risk-neutral subject should cross over to Option B in the 7th choice problem whereas a risk-averse subject should switch to Option B before the 7th choice problem. At the end of Stage 1, subjects were asked to answer individual characteristics that may have an impact on risk preferences, such as gender (Croson and Gneezy, 2009), academic major, and the number of years attended at Soka University.

After completion of Stage 1, subjects were given new instructions for Stage 2 and asked to read the instructions silently at their own pace. Just as in Stage 1 the experimenter read the instructions aloud and answered questions individually. This stage began with assigning the two roles, the strong bidder (called as “Player A”) and the weak bidder (called as “Player B”), to subjects through

Problem No.	Option A	Option B
1	300 yen with probability 1, 0 yen with probability 0	140 yen
2	300 yen with probability 0.9, 0 yen with probability 0.1	140 yen
3	300 yen with probability 0.8, 0 yen with probability 0.2	140 yen
4	300 yen with probability 0.7, 0 yen with probability 0.3	140 yen
5	300 yen with probability 0.6, 0 yen with probability 0.4	140 yen
6	300 yen with probability 0.5, 0 yen with probability 0.5	140 yen
7	300 yen with probability 0.4, 0 yen with probability 0.6	140 yen
8	300 yen with probability 0.3, 0 yen with probability 0.7	140 yen
9	300 yen with probability 0.2, 0 yen with probability 0.8	140 yen
10	300 yen with probability 0.1, 0 yen with probability 0.9	140 yen

Table 1: The ten paired lottery choice problems in Stage 1

⁷ Each session accommodated either 8 or 10 subjects.

⁸ Instructions are available upon request.

one of two role assignment procedures described above. After role assignment, each subject was randomly paired with a subject of the opposite role. Subjects had no way of knowing the identity of their opponents. Before playing the auction game, subjects completed the quiz to make sure their understanding of the instructions.

Subjects played the auction game only once. Each subject received an endowment of 1000 yen and independently and simultaneously chose her bid, which had to be a multiple of 200 yen between 0 yen and 800 yen inclusive. After all subjects submitted their bids, the experimenter collected them and then privately informed each subject of whether or not she won and how much she earned in this stage.

At the end of the session, the experimenter determined the earnings for Stage 1 by using a bingo cage in front of subjects. The bingo cage contained 10 balls numbered from 1 to 10. The experimenter turned the bingo cage twice with replacement. The first draw determined which paired lottery choice problem to be selected for payment, and the second draw determined the outcome of Option A in the selected choice problem.⁹ Then, the experimenter calculated the total earnings subjects had accumulated and paid them privately in cash. The average individual earnings were 1511 yen, including a show-up fee of 300 yen.¹⁰

3.3 Hypothesis

Differences in role origins do not alter the unique equilibrium, which serves to establish the following hypothesis.

Hypothesis. *Holding the player role constant, there is no difference in the bid distribution between the two role origins.*

4. Results

4.1 Lottery Choice Task (Stage 1)

This section begins with reporting results of the lottery choice task in Stage 1. It is worthwhile noting that three subjects who switched back and forth between the two options, as they moved down the problems in Table 1. Although these subjects stayed in the session until the end, their data were removed from the following analyses since their behavior is considered inconsistent. Therefore, the updated dataset includes only seventy-three subjects.

The proportions of Option A choices made by all subjects are shown by the solid line with

⁹ Suppose that the first draw is seven and the second draw is four. This means that the seventh paired lottery choice problem will be selected for payment. Those who chose Option A will earn 300 yen and those who chose Option B will automatically earn 140 yen.

¹⁰ The regional minimum hourly wage was 958 yen at the time of the experiment.

circles in the left panel of Figure 1, where the problem number is listed on the horizontal axis. The dash line with crosses represents the predicted behavior for a risk-neutral subject. As expected, all subjects were expected to choose Option A in the very first question. The proportion of Option A choices decreases in the problem number. That the solid line is above the dashed line for the questions 7-10 indicates the presence of risk-loving subjects. About 8% of the subjects never switched to Option B. The right panel of Figure 1 displays the proportions of Option A choices by gender. It shows that female subjects switched to Option B, i.e., safe choice, earlier than male subjects, which hints that male subjects are more risk-seeking than female subjects.

The problem numbers in which subjects switched from Option A to Option B can be used to estimate their risk aversion levels. Hereafter, assume that subjects were maximizing the expected value of the following CRRA utility function:

$$u(x) = \frac{x^{1-r}}{1-r} \quad (r \neq 1),$$

where x is the amount of money, and r is a relative risk aversion parameter. For each question, the value of this parameter can be determined by making the subject indifferent between Option A and Option B.¹¹

To estimate the distribution of r , an interval regression model is employed.¹² To allow for heterogeneity regarding risk attitude, subject i 's relative risk aversion r_i is modeled as a linear function of three individual characteristic variables. The dummy variable $male_i$ takes on the value 1

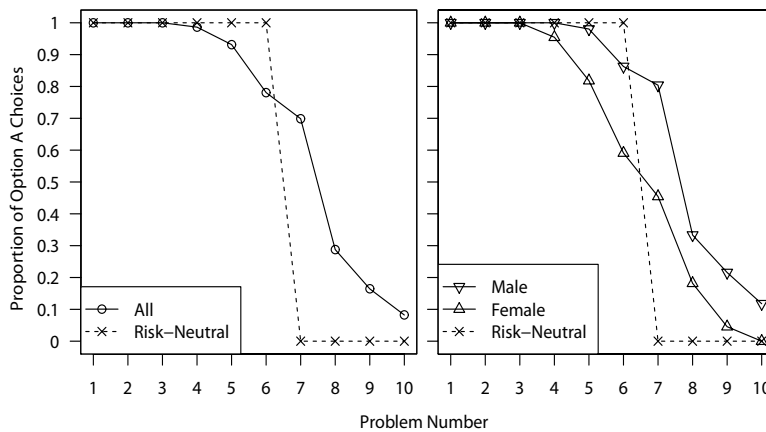


Figure 1: Proportion of Option A choices for each of the ten paired lottery choice problems

11 For example, $r = 0.707$ for the third question and $r = 0.532$ for the fourth question. If the subject switched to Option B in the fourth question, her risk aversion parameter lies between 0.532 and 0.707. A subject is risk-averse if $r > 0$, risk-neutral if $r = 0$, and risk-seeking if $r < 0$.

12 According to Harrison and Ruström (2008), an interval regression model was first proposed by Collier and Williams (1999) for a multiple price list (MPL) experimental task. The estimation technique in this subsection follows Chapter 6 of Moffatt (2015).

if subject i is male and 0 otherwise. The other dummy variable $econ_i$ takes on the value 1 if subject i majors in Economics and 0 otherwise. The variable $year_i$ is the number of years subject i attended Soka University. A model incorporating heterogeneity with respect to risk attitude is

$$r_i = x_i \beta + \eta_i,$$

where $x_i = [1 \text{ male}_i \text{ econ}_i \text{ year}_i]$, $\beta = [\beta_0 \beta_1 \beta_2 \beta_3]^T$, and $\eta_i \sim N(0, \sigma^2)$. This model can be written as

$$r_i \sim N(x_i \beta, \sigma^2).$$

Suppose that subject i 's risk aversion parameter lies between a lower bound l_i and an upper bound u_i . Given $r_i \sim N(x_i \beta, \sigma^2)$, this event happens with probability

$$L_i = \Phi\left(\frac{x_i \beta - u_i}{\sigma}\right) - \Phi\left(\frac{x_i \beta - l_i}{\sigma}\right),$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Thus, the log-likelihood function can be written in the form of $\sum_i \ln L_i$.

Table 2 reports the maximum likelihood estimates of β and σ .¹³ The variable *male* is significant, and its coefficient is negative. Neither *econ* nor *year* is significant at conventional levels of significance. These results confirm past experimental evidence that male subjects are significantly more risk-seeking than female subjects.¹⁴

	Coefficient	95% Confidence Interval
(Intercept)	0.218	[-0.280 0.714]
<i>male</i>	-0.491**	[-0.839 -0.147]
<i>econ</i>	0.187	[-0.137 0.514]
<i>year</i>	-0.064	[-0.197 0.067]
σ	0.674***	[0.570 0.812]
Log-likelihood: -162.32		
Number of observations: 73		

*: $p < 0.05$, **: $p < 0.01$, ***: $p < 0.001$

Table 2: Maximum likelihood estimates of β and σ

¹³ R package `bbmle` includes `mle20` function which was used to implement the maximum likelihood estimation of the interval regression model.

¹⁴ For example, see Charness and Gneezy (2012).

4.2 All-Pay Auction (Stage 2)

Table 3 presents the observed frequency distributions of bids separately in terms of role origin and role.¹⁵ The mean bids differed for the Player A role (341.2 yen for the random treatment versus 260 yen for the earned treatment), whereas the mean bids were the same for the Player B role (177.8 yen for both treatments). To test the effect of role origin, the two observed frequency distributions of bids were compared separately for each player role. A permutation test of the two-sample Kolmogorov-Smirnov statistic failed to reject the null hypothesis of no difference for each player role at any conventional level of significance.¹⁶ The data provided no evidence of the effect of role origin on bidding behavior.

A further inspection of the table reveals two other findings that defy the theoretical implications. First, the bid distributions are not uniform and rather right-skewed except the Player A's bid distribution in the random treatment. Consequently, their mean bids are clearly lower than the predicted mean bid of 400 yen. To formally compare the predicted and observed distributions, a one-sample discrete Cramér-von Mises goodness-of-fit test was invoked.¹⁷ The rightmost column of Table 3 reports the results in terms of p -values. Only for the bid distribution of Player A subjects in the Random treatment did the test fail to reject the null hypothesis that the bids came from the discrete uniform distribution. Second, Player A subjects on average bid higher than Player B subjects. This finding seems to confirm a mild discouragement effect. However, neither of these differences was statistically significant; a two-tailed Wilcoxon rank-sum test failed to reject the null hypothesis of no difference in mean bid for each pairwise comparison.¹⁸

The analysis of Stage 1 confirmed the existence of gender difference in risk attitudes; male

15 In Stage 1 there were three subjects who switched back and forth between the two options as they moved down the problems in Table 1. Although these subjects stayed in the session until the end, their data were removed since their behavior is considered inconsistent. The updated dataset includes 51 males and 22 females.

16 The two-sample Kolmogorov-Smirnov test statistic was used to implement a permutation test of equal bid distributions. The observed test statistics were 0.2559 for the Player A role and 0.1667 for the Player B role. The test was done for the Player A role as follows. A sample of $n = 17$ bids for the random treatment and a sample of $m = 20$ bids for the earned treatment were pooled into one sample. Then, chose 17 bids out of the pooled sample at random to the random treatment without replacement. The remaining 20 bids were assigned to the earned treatment. The test statistic was computed based on this permutation resample. This permutation resampling was repeated 10000 times to form the permutation distribution of the statistic, i.e., the resampling distribution of the statistic under the null hypothesis. The p -value was estimated by locating the observed statistic on this distribution. A similar procedure was taken for the Player B role. The estimated p -values were 0.254 for Player A and 0.607 for Player B.

17 R package dgof includes the Cramér-von Mises goodness-of-fit test. For more information, see Arnold and Emerson (2011).

18 R package exactRankTests was used to perform the Wilcoxon rank-sum test. The p -values were 0.1713 for the random treatment and 0.5342 for the earned treatment, respectively.

Role Origin	Role	Bid (yen)					Mean	Median	<i>p</i> -value
		0	200	400	600	800			
Random	Player A	5	5	1	2	4	341.2	200	0.3522
	Player B	8	5	4	1	0	177.8	200	< 0.001
Earned	Player A	11	1	3	1	4	260	0	0.0101
	Player B	11	4	0	0	3	177.8	0	< 0.001

Table 3: Bid distribution by role origin and role

Role	Gender	Bid (yen)					Mean	Median
		0	200	400	600	800		
Player A	Male	12	5	2	1	3	208.7	0
	Female	4	1	2	2	5	442.9	500
Player B	Male	13	8	4	1	2	192.9	200
	Female	6	1	0	0	1	125	0

Table 4: Bid distribution by gender and role

subjects were significantly more risk loving than female subjects. This finding suggests the following two additional hypotheses. First, for the role of Player A, male subjects on average bid lower than female subjects. Second, for the role of Player B, male subjects on average bid higher than female subjects.

Table 4 reports the frequency distributions of bids separately in terms of gender and role. Holding role constant, the table shows different bidding patterns between males and females. For the Player A role, male subjects bid lower than female subjects; the mean bid of male subjects is 208.7 yen whereas that of female subjects is 442.9 yen. The latter is significantly higher than the former at a 5% significance level, based on a one-tailed Wilcoxon rank-sum test (p -value = 0.02594). For the Player B role, male subjects on average bid slightly higher than female subjects (192.9 yen versus 125 yen). A one-tailed Wilcoxon rank-sum test failed to reject the null hypothesis of no difference at any conventional significance level (p -value = 0.1276).

5. Conclusion

This paper contributes to the experimental literature of all-pay auctions (Dechenaux et al., 2014) by shedding some light on whether differences in role origin influence bidding behavior in the all-pay auction in which two bidder roles were treated asymmetrically in case of a tie. The experiment manipulated how these two roles were allocated to subjects. No evidence was found to reject the hypothesis of no difference in bidding behavior between when the right to acquire the strong bidder role is earned and when it is windfall. Instead, the results show that females bid in a more risk-averse manner than males.

There are two possible reasons why the manipulation of role origin failed to induce significant variations in bidding behavior. First, differences in the two role allocation procedures may be neither large nor clear enough to give birth to different behavioral rules indicating how each role should bid. Second, differences between the two bidder roles may be too small to recognize due to the set of parameter values used in the experiment; in equilibrium, both bidders should bid in the same manner, and the unfair tie breaking rule comes into play with probability 0.2. Revising these aspects of the design is left for future research.

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