Ilkogretim Online - Elementary Education Online, 2020; 19 (2): pp. 958-981
http://ilkogretim-online.org.tr
doi:10.17051/ilkonline.2020.695840

An investigation of preservice mathematics teachers' teaching processes about "procedural and conceptual knowledge" related to division with fractions[1]

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Abstract. In this study, the changes in the behaviors of elementary mathematics teacher candidates towards teaching "Procedural and Conceptual Knowledge" related to division with fractions were investigated during a three-tier teaching experiment. Six preservice teachers participated in the study. In addition, in the first cycle, 26 students participated in the study, and 26 students participated in the second cycle. The data of the study consists of camera recordings, interviews, lesson plans prepared by preservice teachers and pictures of student notebooks. Themes, categories, and some criteria were generated from the data. The presentation of the findings was supported by the data collected from these various sources, and the triangulation method was used to validate findings. As a result of the study, we observed a decrement in the amount of time that preservice teachers used for rule based teaching and an increment in the amount of time for teaching meaningful mathematics.

Keywords: Teaching procedures and concepts, division with fractions, three-tiered teaching experiment

Received: 31.12.2018 Accepted:04.10.2019 Published: 15.03.2020

INTRODUCTION

Mathematical competence can be defined as a set of skills necessary to use mathematics effectively in both daily life and school. Kilpatrik, Swafford, and Findell (2001) stated that mathematical competence consists of five interrelated elements. These are expressed as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Milgram and James (2005) and Schoenfeld (2007) stated that these definitions were generally accepted to explain mathematical competence. It is stated that these definitions of competence are not independent skills but are interrelated. Conceptual understanding, which is one of the sub-dimensions of mathematical competence, is explained as the ability to understand mathematical concepts and to associate them with each other and with operations. Procedural fluency is defined as knowing which operations will be used in certain situations and being able to use them effectively, accurately, and flexibly. These classifications are highlighted in the curricula of many countries (e.g., Australian National Curriculum Board, 2009; Ministry of National Education, 2013, MacGregor, 2013; Ministry of National Education, 2009; National Council of Teachers of Mathematics, 2013). In addition, these competence definitions were used both in designing textbooks (e.g., Glencoe Math, 2013; Glencoe Math, 2014; Math Connects, 2012) and as a reference in the assessment process (e.g., PISA, 2012).

The definitions of conceptual understanding and computational fluency, which are among the sub-dimensions of mathematical competence, are presented after detailed discussion by mathematics educators. The definitions of "conceptual and procedural knowledge" provided by Hiebert and LeFevre (1986) constitute one of the most important points of the discussion. In this study conducted by Hiebert and LeFevre (1986), conceptual and procedural knowledge classifications are provided by emphasizing concepts and operations in mathematics. In some

[1] This study was produced from the doctoral dissertation in which the first author completed at Hacettepe University, Institute of Educational Sciences.

studies (e.g., Hiebert and LeFevre, 1986; Hiebert and Wearne, 1986), conceptual knowledge is expressed as a network of rich relationships between different knowledge. In order to develop conceptual knowledge in mathematics, it is emphasized that the relationships between concepts should be focused. For example, a student who knows two different concepts can establish the relationship between these concepts over time. The abundance of these relationships express the richness of conceptual knowledge. Procedural knowledge is defined in two parts: The first is part expressed as the rules and procedures used to solve mathematical problems, while the second part of procedural knowledge is defined as knowledge about conventional symbols and forms of expression in mathematics.

Procedural and Conceptual Knowledge

According to the classifications provided by Hiebert and LeFevre (1986), conceptual knowledge can be seen as a type of knowledge or a relational structure consisting of a network of information with very rich relationships. Procedural knowledge is defined as both mathematical symbols and some rules and procedures used in problem solving. In general, a person who understands a concept is expected to easily perform all the procedures and operations associated with it (Macievejewski, Mgombeloand & Savard, 2011). The discussions on "conceptual and procedural knowledge" also focus on the points where these two types of knowledge interact. Hiebert and LeFevre (1986) stated that associating procedural and conceptual knowledge will lead to understanding the underlying facts of the procedures, so that students will realize which procedures can be used more effectively. In a study conducted by Byrnes and Wasik (1991), it was found that the students who had deep conceptual knowledge realized their mistakes more easily while applying their procedural knowledge to the problems. Similarly, Carpenter (1986) showed that only the students who had advanced conceptual knowledge were able to perceive whether the results were logical or not while doing operations with fractions.

In the literature, there are four different explanations about the relationship between procedural and conceptual knowledge (Haapasalo & Kadijevich, 2000). The first is the view that procedural knowledge is necessary but not sufficient for conceptual knowledge (Gray & Tall, 1993; Kitcher, 1983; Kline, 1980; Sfard, 1991; Vergnaud, 1990). The second view is that conceptual knowledge is necessary but not sufficient for procedural knowledge (Brynes & Wasik, 1991). Researchers in the third view argue that effective procedural knowledge is necessary and sufficient for conceptual knowledge (Byrnes & Wasik, 1991; Haapasalo, 1993; Hiebert & Lefevre, 1986). Researchers in the fourth view state that procedural and conceptual knowledge is completely separate from each other and that there is no interaction between them (Nesher, 1986; Resnick & Omanson, 1987).

According to the researchers in the first view, computational information consitutes a part of conceptual information. This view was supported by many researchers who considered concepts as processes (Dubinsky & Harel, 1992; Gray and Tall, 1994; Kaput, 1982; Sfard, 1991). In order to fully understand a concept, it is emphasized the necessity of information for the application of that concept. For researchers of this view, operations are logical reasoning with a cause and effect relationship, and thus operations constitute a field of application, so application is an important part of concepts (Sfard, 1991). Similarly, some researchers who advocates "mathematics education should focused on historical development" support this view. Accordingly, students should be taught in the same way that mathematics developed in history (Lauritzen, 2012; Sfard, 1991). According to this view, a concept should be taught to the individual in the same way as it developed in the history of mathematics. For example, the concept of number emerged at the end of the counting process. In order to operate in rational numbers, integers are required. Hence, unless a student develops his / her operational knowledge, he / she cannot fully understand the concept (Sfard, 1991).

The researchers in the second view state that conceptual knowledge is a supportive feature of procedural knowledge, but it is not sufficient by itself (Byrnes and Wasik, 1991). One of the contributions of conceptual knowledge to procedural knowledge is that it makes easy to detect some transactional errors. Byrnes and Wasik (1991) state that operational errors are mostly due to the unknown meaning of mathematical symbols. Accordingly, operations and procedures are only meaningful when considered together with the related concept. A common error in the literature is that adding nominators and denominators without using common denominator rule while performing the addition of fractions. This error has been considered by some researchers as a conceptual error, it has been considered as an operational error by some other researchers. According to this second view, students who have advanced conceptual knowledge in fractions can realize that the result of addition is incorrect that uses a wrong procedure. In addition, they can realize where they made operational errors (Aytekin, 2012; Lauritzen, 2012).

The researchers in the third view state that effective procedural knowledge is necessary and sufficient for conceptual knowledge. Researchers in this view report that students' operational errors can be explained by the lack of conceptual knowledge (Byrnes & Wasik, 1991). In other words, insufficient conceptual knowledge will necessarily lead to operational errors (Lauritzen, 2012). Thus, it is concluded that procedural and conceptual knowledge develops simultaneously. This view has been supported by correlation tests that aim to measure procedural and conceptual knowledge (Aytekin & Toluk Uçar, 2014).

Researchers who argue that procedural and conceptual knowledge are not related show that some students have a high level of conceptual knowledge while they are operationally weak (Lauritzen, 2012). Contrary to this situation, some students may have a high level of procedural knowledge and have a low level of conceptual knowledge. Resnick and Omanson (1987) conducted a study to examine whether students with conceptually high knowledge perform computationally high performance. They found that the students who understood the conceptual meaning of the subtraction made many transaction errors in the subtraction. Zucker (1984) did not find a statistically significant relationship between understanding decimal notations and operational success, although the number of their sample was quite big. Of course, it should not be concluded that there is no relationship between these two types of information (Lauritzen, 2012). However, the absence of a statistically significant relationship between these two types of knowledge supports the view that these knowledge should be handled independently.

Teaching procedural and conceptual knowledge

Byrnes and Wasik (1991) state that conceptual and operational knowledge include different mental processes. Students expecting conceptual knowledge focus more on associations, while students expecting procedural knowledge use these procedures as a means to achieve some of their goals. Järvelä and Haapasalo (2005) divided the students into three categories according to the types of knowledge they expected. These three categories are procedural-oriented students, conceptual-oriented students, and students expecting operational rules without conceptual basis. Accordingly, a procedural-oriented student first chooses to learn the procedures and then associated rules. However, this student's conceptual knowledge can also develops over time. On the other hand, a conceptual-oriented student strengthens his / her procedural knowledge by focusing on the concepts and inferring from these concepts. The students who focus only on the procedural knowledge try to memorize the rules of operations by practicing without focusing on understanding the conceptual basis. A similar classification was made by Entwistle and Tait (1990). They stated that some students focused on memorizing rules about operations instead of focusing on meanings. It is stated that some students tried to combine new knowledge with previous knowledge and paid attention to making sense of new concepts by looking for relationships. Marton and Säljö (1976) and Entwistle and Tait (1990) stated that there are two different situations according to students’ learning styles. The first situation is to focus on the meaning that is expressed as learning in order to associate with prior knowledge. The second is the superficial approach, where attention is paid only to the key points without making any association between the concepts. This second approach is mostly mentioned as memorizing rules about concepts and processes. It has been observed that teachers in the first classes of primary education tend to teach more rules about the procedures, and in the advanced classes, especially at the university level, the definition of the concept is provided and then practiced on similar problems (Haapasalo, 1993). These two approaches negatively affect students' beliefs about doing mathematics. Kadijevich and Haapasalo (2001) noted that the relationships between procedural and conceptual knowledge can be promoted through learning activities. Not only the quality of the teaching activity, but also the student's approach to learning affects the speed of this association process. Some students focus on memorizing rules about procedures rather than seeking conceptual relationships (Lauritzen, 2012). These students' expectations should be directed to understanding the meaning of operations and concepts rather than memorizing the rules about operations.

In this study, the changes in the behaviors of preservice elementary mathematics teachers' towards teaching the concepts and operations related to division with fractions were investigated during a three-tiered teaching experiment process. For this purpose, in the process, our aim was to guide preservice teachers to meaningful teaching instead of teaching the concepts and procedures using rules as described above. It has been considered that some positive and negative pedagogical behaviors of preservice teachers may affect this process. Therefore, we expected that some positive pedagogical behaviors of preservice teachers will improve and some negative behaviors will decrease.

There are several reasons that lead to this study. The first reason is about mathematics teacher training. In the literature, it is stated that while teaching a subject, most of the mathematics teachers teach by memorization instead of teaching the meanings of the operations (Lauritzen, 2012). The second reason is about teaching division with fractions. In the literature, it is stated that this subject is quite rich in conceptual terms (Tirosh, 2000); however, students have difficulty in making sense of this subject (Işıksal, 2006; Ma, 1999), and teachers have difficulty in teaching it (Utley and Redmond, 2008). A third reason for choosing the topic of division in fractions is that this topic is suitable to provide meaningful teaching by associating the rules with concepts by using invert and multiply and common denominator algorithms. However, in the process of "three-tiered teaching experiment," there was no study on how teacher candidates developed teaching by fractions.

In Turkey, with respect to conceptual and procedural knowledge, in the "Primary Mathematics Curriculum" prepared in 2009 stated that "This program emphasizes the concepts related to mathematics, the relationships between the concepts, the meaning of the operations and the acquisition of the processing skills (MEB, 2009, p: 8). It is seen that there is a similar approach in the secondary school mathematics curriculum which was updated and put into practice in 2013 (MEB, 2013). This curriculum stated that the focus of the program is on learning areas formed by concepts and relationships. Moreoever, it is stated that the conceptual approach requires more time to form the conceptual foundations of knowledge about mathematics. Thus, conceptural approach requires establishing relationships between conceptual and procedural knowledge and skills. In parallel with these views, this study may contribute to how the division of fractions can be taught with the conceptual approach mentioned above and how teachers should be trained in this way.

It is known that beliefs of individuals significantly affect the choices they have made in their lives (Hofer and Pintrich, 1997; Pajares, 1992). Many studies reported that students' and teachers' beliefs about mathematics shape their behaviors in the process of learning and teaching (Abrosse, Clement, Philipp, Chauvot, 2004; Pajares, 1992; Picker ve Berry, 2000; Raymond, 1997; Schoenfeld, 1989; Thompson, 1984; Thompson, 1992; Toluk Uçar, Pişkin, Akdoğan ve Taşçı, 2010). For example, students who believe that learning mathematics passes through memorizing similar solutions focus more on memorizing operational information in the learning process (Toluk Uçar, Pişkin, Akdoğan & Taşçı, 2010).

The current study focused on the procedural and conceptaul teaching of preservice teachers. However, convincing them to the necessity of meaningful instruction is needed to guide them from a rule-based instruction to a meaningful instruction. At each stage of this study, detailed information is given about how teacher beliefs affected their procedural and conceptual teaching and approaches to students. In addition, during the study, we tried to develop positive beliefs of the preservice teachers, and the effect of the researcher's orientation has been included. There are quite different views on the relationship between operational and conceptual knowledge. Researchers in each view provide evidence to support their own ideas and include study results. This current study has been carried out with the view that procedures and concepts are inseparably related to each other and that they are born from each other. We observed that this point of view was very consistent in the process, and it was confirmed in the last interviews with all participants of the study. In this respect, it is thought to contribute to the discussions in literature about how the relationship between procedural and conceptual knowledge are related.

We investigated the following study question: "How do the preservice elementary mathematics teacher' behaviors change when teaching the concepts and procedures related to division with fractions during the process of a three-tiered teaching experiment". Changes in these behaviors are examined in two parts: The first part is rule-based teaching behaviors that define procedures by memorizing without focusing on the meaning of operations and concepts. The second part is meaningful teaching behaviors that focus on the meanings behind operations and concepts. In the study, we investigated whether preservice mathematics teachers' focuses on the rule-based teaching behaviors decreased or not, and whether their focuses on meaningful teaching behaviors increased or not.

METHODS

A three-tiered teaching experiment method was used in the study as shown in Table 1. The three-tiered teaching experiment method is also called "multi-tiered teaching experiment" in the literature. Students, preservice teachers, and researchers are represented by the term "tiered" which gives the method its name. Teaching experiment is a study method that is based on examining how a teacher develops appropriate conditions, taking into account his / her influence, how changes occur in the students in order to make a better teaching (Cobb and Steffe, 1983; Hunting, 1983; Steffe, 1984). In the teaching experiment study, the teacher has the role of both researcher and teacher (Steffe, 1991). Due to the nature of the teaching experiment, the teacher can take some measures in the process and make changes in the teaching environment in order to enable students to learn better. The impact of these changes on students' progresses needs to be evaluated together with the teacher's own role (Steffe, 1991). In addition, there are two interrelated teaching experiments in this study. The first of these is the teaching experiments that a researcher has conducted to improve the teaching of preservice teachers' division of fractions.

The second is the teaching experiments conducted by preservice teachers in order to make sense of the division of fractions with 6th grade students. In this three-tiered teaching experiment, the aim was to examine the changes in the behaviors of preservice elementary mathematics teachers' towards teaching the concepts and operations related to division in fractions. Although the study was conducted in three tiers as a whole, this paper only focuses on the development of preservice teachers. Therefore, the development of students and researchers was excluded from the main focus of this study. It was thought that it would be more appropriate to report these two tiers in another study since the changes occurred in the students due to the influence of the researcher, and the preservice teachers required further interpretation.

This study differs from case studies in that the researchers actively participate in the process to influence preservice teachers and have an influence on them. In the regular case studies, researchers aim to observe the phenomena in their natural environment without intervening in the events. In this study, researcher and preservice teachers discussed the problem of how to teach division in fractions better (using meaningful and appropriate pedagogy). However, unlike action study, this study examines the development of preservice teachers. The process designed within the scope of the study differs from the lesson study in terms of "independent planning" and "examination of individual progress" of preservice teachers.

Akın and Kabael (2016) stated that teaching experience studies are included in a separate class within qualitative study methods. However, in teaching experiments, researchers stated that those experiments were useful for mathematics education study because they involved the participants' mental processes. Steffe and Thompson (2000) also stated that researchers could examine the situations that affect students' cognitive, emotional and conceptual developments and how these characteristics are shaped in the process, so that mathematics education researchers can be involved in the mathematical learning of students from the first hand.

Teaching experiments consist of successive teaching and clinical interview stages. Within the scope of the study and according to the stages that the study has, clinical interviews can be conducted at the beginning, middle or end of the study. In this study, we aimed at increasing the preservice teachers' behaviors related to meaningful teaching rather than teaching as a rule while teaching procedures and concepts. Researcher took an active role in every stage of the process and conducted clinical interviews with preservice teachers at each stage.

Table 1: *General structure of the three-tiered teaching experiment study**

Tiers of the study	Duties of individuals in each tier	Data Collection Style
Tier 3 **Researcher**	• Designing the study process in order to improve the knowledge and skills of preservice teachers. • In this process, to create a suitable environment for the analysis of possible student-teacher behaviors • Collaborate with preservice teachers to test the usefulness of plans, activities developed by preservice teachers and to make appropriate revisions. • To guide preservice teachers to reflective thinking to develop themselves.	Video and Audio Recordings Written Texts of Teachers
Tier2 **Teacher candidates**	• To cooperate with the researcher to aim for a better teaching • Give feedback to other friends and the researcher to develop a better teaching plan • To think about the applications conducted to improve students' conceptual understanding and to constantly improve his/her knowledge on this subject.	Video and Audio Recordings Written Texts of Teachers
Tier1 **Grade 6 students**	• To participate in the studies carried out within the scope of the study based on conceptual understanding both individually and with group friends. • To explain how they solve their thoughts and problems in their activities during the teaching process	Video and Audio Recordings Written Texts of Grade 6 Students

Note *: The tiers proposed by Lesh and Kelly (2000, page 198) were adapted by the researchers for this study.

Participants

Six third year preservice teachers from a mathematics teaching department of a state university in the Central Anatolia Region were included in the study on a voluntary basis. Within the scope of the study, "27 sixth grade students included in the first cycle, and 26 sixth grade students included in the second cycle" in order to provide the preservice teachers with real teaching experiences. The study was conducted in the second semester of 2014-2015 academic year. The preservice teachers who participated in the study had completed most of the mathematics content courses and had also taken the “Instructional Technologies and Material Design” and Special Teaching Methods I ”courses and continued to take the“ Special Teaching Methods II” course. The study was carried out according to the general framework given in Figure 1. Permission was obtained from the parents for the students to participate in the study. Most of the volunteered students attended in the teaching phase of the study. In accordance with the recommendations of the experts, if students were present in the classroom on the specified day and time, the preservice teacher was allowed to teach.

Table 2. *Preservice teachers and students participating in the study*

Group	Teacher Candidate	1st Cycle Students	1st Cycle Students
1.Group	Çağla	Sevil Esra Rıfat Aysel	Sevil Esra Rıfat Aysel Mehmet
	Faruk	Leyla Sıla Betül Hayriye	Leyla Sıla Aslı
	Büşra	Sevgi Halit Yasin Mehmet	Sevgi Halit Yasin Mehmet
2.Group	Feyza	Bahri Efe Halime Seda Emel	Bahri Efe Halime Seda
	Mesut	Melek Murat Emine Selim Demet	Melek Murat Emine Selim
	Zeliha	Melih Beyza Selin Selami İlkay	Melih Beyza Selin Selami
Total	6	27	24

In accordance with the scientific ethics rules, the real names of the preservice teachers and students who participated in the study were not used both in the table above and in the following stages.

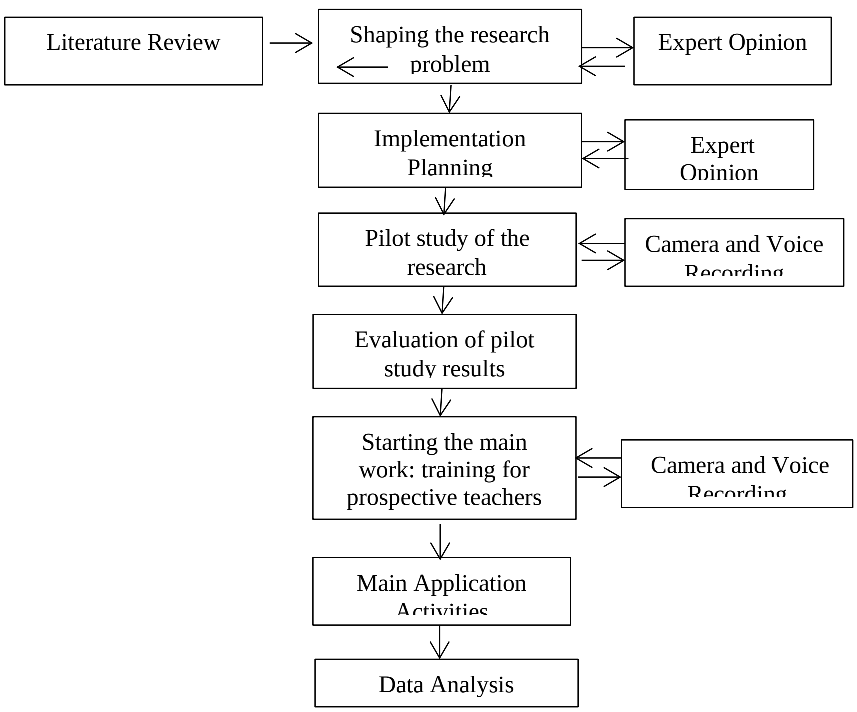

FIGURE 1. *General framework of the study*

The main implementation of the study was framed according to the data obtained from the pilot study. The steps of the actual study are shown in detail in Table 3. After the first cycle, the second cycle was started. In the first cycle, the prepared teaching plans are related with the objectives of "dividing a natural number by a fraction and dividing a fraction by a natural number". The teaching plans in the second cycle related to the objective of dividing one fraction by another.

Table 3. *Stages of actual implementation and data collection*

Stages	Applications in study stages	Data collection method
1	Main application teacher training	Audio recording
2	Preparing instructional plan independently	Receiving and collecting plans
3	Group discussion about plans	Audio recording
4	Revision of plans	Receiving and collecting plans
5	Discussion of revised reasons	Audio recording
6	Last revised if necessary	Receiving and storing plans
7	Teaching 6th grade students	Video recording and collecting notebooks
8	One-on-one interviews with preservice teachers	Audio recording
9	Group discussion on teaching	Audio recording
10	Data analysis and reporting	Audio recording
11	Return to step 1 for the second cycle	

Data Collection Tools

Interviews were conducted as it was necessary to determine what the preservice teachers' thoughts were about the process. In order to examine the changes in their teaching, the videos were recorded, and their lesson plans were examined.

Interviews

Two types of interviews were conducted, one-to-one and group interviews. Each interview was recorded with a voice recorder and transcribed in verbatim. Before the interviews, the researcher determined the questions to be asked to the preservice teachers. These questions focused on understanding whether or not the meaning of the procedures was actually learned and whether or not the meaning of the concepts were actually learned. Similarly, what the meaning of the taught procedures or concepts was and whether the teacher's teaching was understood as a rule by the student. The questions posed to all preservice teachers to cover these issues. In addition, spontaneous situations that occurred during the interview were also discussed. The interviews were formed as semi-structured interviews. In the group interviews, the researcher used an approach that aimed at detecting the students' ideas, letting them argue with each other about the subject, making suggestions about the points provided by each person, and making self-criticism about their own actions.

Lesson Plans

In both cycles of the study, each teacher candidate prepared a teaching plan and revised them after group discussions. For example, some of the preservice teachers who prepared a rule-driven plan by only using the processing algorithm in fractions, after these discussions, they tended to include meaningful teaching activities into their lesson plans through using models. On the other hand, in some stages, there were also preservice teachers who wanted to implement a significant portion of the initial plan without changing it. At the end of the two cycles, the researcher collected a total of 12 teaching plans and their revised versions to use as study data. In lesson plans, there were statements about the planned activity, teacher role, student role, and measurement and evaluation methods. In this way, the researcher who foresaw teaching plans had the opportunity to direct student and teacher roles to a student-centered teaching.

Video Recordings

All of the preservice teachers' instructions were recorded by the researcher. Prior to all teaching, the camera was placed by the researcher in an appropriate place to cover the board and the whole classroom and necessary arrangements were made. One instruction lasted approximately 2 hours (120 minutes). Each preservice teachers taught the subject twice, and since there were six preservice teachers in the original study, the researcher had collected approximately 24 hours of video recording. The researcher showed the videos of each preservice teachers to the group members during the group discussion stage. Thus, he tried to exchange ideas about how a better teaching could be. The information that the videos would be examined in this way from a scientific point of view was informed to the preservice teachers, students themselves, and the parents of the students from whom written consents were obtained at the beginning of the study.

The Data Analysis

In this three-tiered teaching experiment, we examined how the preservice elementary mathematics teachers' teaching procedures and concepts about fractions change during the study. In the process of data analysis, the main data source was video recordings. In addition, lesson plans, interview records, and student books were analyzed to support the findings. In order to determine the themes and categories that were related with our study purposes, the video recordings of the preservice teachers' teaching were divided into sections according to their plans. Hence, when examining the video recordings, the plans prepared by the preservice teachers were taken into consideration. Thus, the differences between the planned teaching and actual teaching were observed. The researcher first analyzed the data by using his notes and then re-watching the videos to ensure their consistency. Two main themes emerged in the study. These are (1) the teaching procedures and concepts, (2) positive and negative factors affecting teaching. The analysis criteria for "the teaching procedures and concepts" are given in Table 4. Positive and negative factors affecting teaching are not presented in this article.

The following example is provided to explain the situations coded as meaningful teaching. By drawing a model of the 1/2 : 1/4 operation, if the preservice teachers showed how many quarters are in the half were evaluated as meaningful teaching. Only the activities where the result is 2 by using the invert and multiply rule were coded as teaching as a rule. Let us give an example for teaching the meaning of invert and multiply operation for 1/2: 1/4 operation. Assuming that there are 4 quarters in a whole, and showing this with a figure, then it was coded as meaningful instruction to emphasize the amount of quarters in 1/2 . In addition, it has been noted here that 1/2 times 4 is to find half of 4. The teaching of "invert and multiply" as teaching as a rule is coded as applying only the rule without finding the meaning and finding the result using the operations. At the beginning of this study, we observed that most of the preservice teachers knew the teaching in the form of rules, but few knew the logical rationale behind them. The study, in terms of its design, provides a very rich development opportunity as preservice teachers to learn from each other, students' learning processes and the researcher.

Table 4. *Themes for procedures and concept teaching*

Themes	Categories	Benchmarks
Meaningful Teaching	Meaningful teaching of division concept Meaningful teaching of invert and multiply rule Meaningful teaching of common denominator rule Meaningful teaching of equivalent fraction concept Teaching of simplification in fractions Meaningful teaching of expansion of fractions Meaningful teaching of compound fraction conversion	Necessary preliminary information about meaningful teaching of procedures and concepts is included in the teaching. Each stage in the concepts to be taught is meaningful. It was noted that preservice teachers' use of a model would not be considered as meaningful teaching alone and the other stages should be also meaningful.
Teaching as a Rule	Teaching division as a rule Teaching invert and multiply as a rule Teaching common denominator as a rule Teaching the concept of equivalent fraction as a rule Teaching of simplification in fractions as a rule Teaching of expansion of fractions as a rule Teaching of compound fraction conversion as a rule	In the explanations about what procedures and concepts meant through the model, it is considered whether the necessary preliminary information is taught in a meaningful way. If the explanations made by the preservice teacher on the model were not based on a justification and included the steps and directives to be applied, this instruction is considered as a rule teaching.

We examined whether there was a decrease in the use of rules during the preservice teachers' teaching, and whether there was an increase in the number of meaningful teaching. As previously said, each preservice teacher taught twice to a group of 6th grade students. In their first teaching, they taught to divide a natural number by a unit fraction, a unit fraction by a natural number, a natural number by a fraction and a fraction by a natural number. In their second teaching, they taught dividing two fractions. Teachings of the preservice teachers were recorded on videos. The researcher analyzed these videos by dividing them into sections according to the teaching order. In each section, the categories related to the identified themes were supported by one-to-one interviews with the preservice teachers. Thus, in the data analysis, the reliability of the study was increased by using a triangulation method.

RESULTS

In this section, the findings of the educational activities conducted by six preservice teachers with sixth grade middle school students are presented. According to the study question, in the analysis and presentation of the data, we focused on how preservice teachers developed during the three-tiered teaching experiment. In this context, teaching of preservice teachers have improved by observing and interpreting instructional videos they had conducted, sometimes from the researcher, and sometimes from each other. In this context, the findings of the preservice teachers are presented according to the themes related to the teaching of procedural and conceptual knowledge in Table 5 because we aimed at collecting inferences about how their teaching changes according to these themes. In the presentation of the findings, each theme is presented by providing an example of the changes in the preservice teachers' thoughts about their own teaching. In addition, at the end of the section, a general table on the number of themes and categories in two cycles is presented.

Sample cases about teaching the concept of division in meaningful teaching and teaching as a rule.

Çağla decided to use material when she saw that the students could not make modeling. However, instead of giving the materials to the students, she only used it herself in front of the class. She wanted everyone to observe her. She showed 4/3 and 1/6 fractions with her materials, then she modeled how many 1/6 were in 4/3.

> The researcher: You chose to use material without giving feedback to everyone. You saw the mistakes in their notebooks. Did you choose to use material instead of fixing them, as it would take too long to fix them one by one?
> Çağla: So, it was a bit. I am getting out of here anyway.

Looking at the statements of the preservice teacher at this point, it is understood that the concept of division is directed towards meaningful teaching, even though she only uses the materials without giving them to the students as seen below.

> Çağla: We modeled 4/3 fraction with our material. This part of the material shows a whole. The other shows 1/3.
> Esra: Yes.
> Çağla: We have 1/6 pieces here. How many 1/6 pieces we use when we put them on.
> Esra: Eight
> Çağla: Eight, right? You saw that, did not you. Count it.
> Esra: 2, 4, 6
> Metin: Nine
> Çağla: Count again.
> Metin: 1, 2, 3, 4, 5, 6, 7, 8. I did not see that.
> Çağla: Do you understand? I just told you, what did your friend do now? Your friend has equalized the denominators of fractions. Why is that? Because it must be made of the same piece.

The teaching activity made by Mesut can be given as an example of teaching as a rule of division concept.

> The researcher: In the question you asked the students, there were four siblings and the context of sharing the land.
> Mesut: Emine already solved the question and modeled it. She actually solved the question.
> The researcher: Yes, she modeled the land and divided it into four.
> Mesut: But Demet did not understand. I already showed Demet how the problem was solved.
> The researcher: Emine drew a figure and divided it into four. Then she split one of the pieces into two. She shaded a part. You said why do not you divide the other pieces into two equal parts. You said divide those pieces in half. Is that a true statement? Why did you ask her to divide other pieces?
> Mesut: To divide the whole.
> The researcher: The question you asked was to divide a 1/4 fraction into two, right?
> Mesut: We need to find out how many parts the whole consists of. I could have discovered him there, but I told him.

As shown above, Mesut asked the students to model the question. He invited a student to the blackboard to solve the question. Although Mesut invited the student in front of the blackboard, he did not expect her to solve the question by her own. He gave step-by-step instructions on what to do. The student completed the modeling and applied exactly what the teacher said at each stage. The student was seated in her place without coming up with a new idea. This process is considered as a rule-based teaching due to the precise directions given, even though modeling is done.

Sample cases about teaching the division with inverting multiplication in meaningful and rule-based form

Mesut tried to make the students realize the meaning of the invert and multiply rule in the teaching process. The conversations with Mesut during the clinical interview that happened at the end of the instruction are given below.

> The researcher: You told Murat that he has a 1/3 fraction, you have 3 whole, right? Then you asked how many 1/3 there are in 3. Demet answered 9. Melek said I can visualize it.
> Mesut: Demet said there is no need to model them.
> The researcher: Demet said there is no need for modeling.
> Mesut: Demet preferred to learn by memorizing.
> The researcher: What did Melek mean when she said I can visualize it without using the inverse multiply rule?
> Mesut: She said that she was imagining that model in her mind, but Demet used the inverse multiply rule.
> The researcher: Why Demet could not visualize?
> Mesut: Because she got used to it.

During the clinical interviews with Mesut, it was reminded that Melek, one of the students, said that she could visualize the problem. However, Demet stated that there was no need to draw a model and emphasized that it was easier to use the invert and multiply rule. It is understood from the students' statements that they have different expectations from teaching of procedural and conceptual knowledge. Although Mesut stated that he tried to make meaningful teaching, he preferred to say the meaning of the procedures and operations directly by himself. A clinical interview is given below.

> The researcher: You said during teaching that dividing a number by 1/2 is equivalent to multiplying by 2. You said there was an inverse relationship between them. It sounds to me like a doctor giving medicine to a patient without telling the cause.
> Mesut: Yes, I give it as a practical way.

Looking at the conversation above, Mesut said that dividing by ½ and multiplying by 2 had the same meaning. Mesut's approach has been evaluated as teaching as a rule. In the interviews, the preservice teacher stated that his teaching was not meaningful and presented it as a practical way. Therefore, Mesut accepted that his teaching should be classified as teaching as a rule.

Sample cases about teaching "dividing using the common denominator method" as meaningful teaching or teaching as a rule

Faruk noticed that one of the students, Leyla, made a mistake when dividing using the common denominator method. Leyla cancelled 3s in the denominators of the 8/3: 1/3 operation. Faruk asked why she had cancelled the 3s in the denominators. However, Faruk did not receive a meaningful response from the student. In this teaching, the student did not find the information about the meaning of the procedures and operations, and the teacher directly told the reason to the student. He also saw that the student modeled 8/3 and 1/3 in the problem as 3/8 and 1/8. However, instead of talking on this mistake, he drew the correct model to the student. The student's solution to the problem and the model drawn by the preservice teacher in the student's book are given below (see Figure 2). This modeling, which was drawn by Faruk in the student book, was considered as a meaningful teaching of "division by the common denominator method" because Faruk modeled 8/3 and then 1/3 in the student’s notebook. Then, he asked the student how many 1/3 were in 8/3. He mentioned that 8/3 can also be converted to an integer fraction. He emphasized that 8/3 and the integer fraction form "2 and 2/3" were equivalent at this stage.

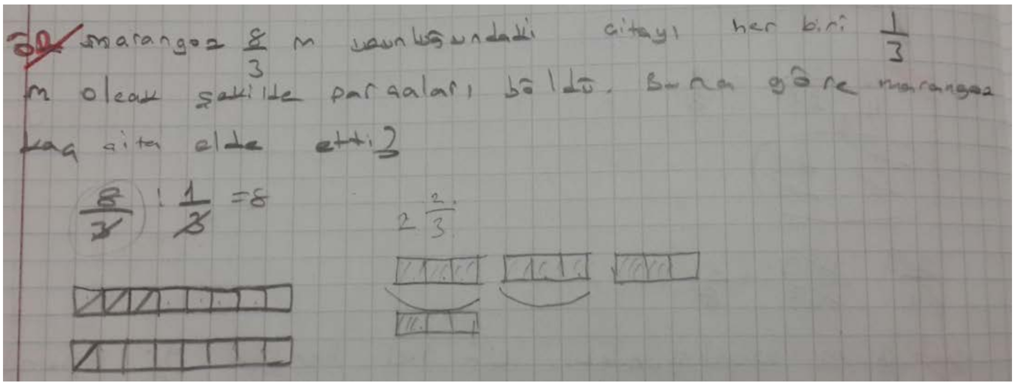

FIGURE 2. *The solution of Leyla's 8/3: 1/3 operation (left) and teacher candidate Faruk's drawing (right)*

However, in the following stages, Faruk turned to "teaching in the form of rules" by dividing the denominator. Below is a part of the clinical interviews with Faruk at the end of the instruction:

> The researcher: Look, this place is very interesting. For example, you say that I teach the logic of mathematics. The student said I memorized the method. He showed you how he did it. I think you learned by memorizing the "denominator equalization" method. Let's watch the relevant part of the video. (The researcher and preservice teacher watching videos).
> The researcher: Yes. He says I memorized it.
> Faruk: Yes.
> The researcher: So, he says I have memorized the method. He did it by memorizing the "denominator equalization method" without knowing its meaning.
> Faruk: Am I not making any further corrections to that conversation? I cannot remember exactly right now.
> The researcher: So, they did not understand the logic of the operation. I want to say that this little explanation for "denominator equalization" is not enough for meaningful teaching. He copied the same thing you did.
> Faruk: As I said, this student was the only problem that I had in the class.

Faruk taught "division by common denominator method" by rule based teaching. In the meantime, one of the students used the phrase "I memorized the method." The researcher and Faruk watched the video recordings. Faruk did not see the student's memorization as a result of his teaching. Faruk suggested that the student had difficulty in understanding as the reason for the student's tendency to memorize.

Sample cases related to the meaningful teaching or rule based teaching for "Equivalent fraction”

In the second stage of her teaching, Feyza focused on the meaning of dividing by using the common denominator method. In order to achieve this, she first thought that the equivalence of fractions should be learned in a meaningful way. Feyza asked to the students the meaning of "equivalent fraction." Feyza did not deem it sufficient with the students' procedural answers. She encouraged them for a more detailed thinking. Feyza stated that she did not behave like this in her previous teaching and included that she directed the students to think more carefully in the second teaching.

> The researcher: I think it is an indicator of improvement that you do not find the operational answers from the students enough and ask for more detailed answers. They modeled 3/4 with the models in front of them. You asked the students to model 6/8. Why did you do that?
> Feyza: I asked the students to cover the whole with small fractions. Students said that there are multiplication in the expansion method. So, they said that they thought that fraction 6/8 was greater than fraction 3/4. At the end of the process of multiplying the numerator and denominator by the same number, the students thought that the value of the fraction

increased. Then, I wanted them to realize that these fractions were equivalent, using the fraction models in front of them. They had 1/8 pieces in their hands. I asked them to model 6/8. Then, I asked them to model 3/4. I wanted them to see that they both represented the same amount.

However, in the later stages of teaching, Feyza switched to a rule-based teaching method when teaching equivalent fraction concept. Feyza found that Bahri still did not understand the equivalent fractions. She preferred to give rule-based feedbacks this time instead of asking them to model the given fractions with the materials. When we asked her why she preferred to do so, Feyza stated that her aim in this teaching was not to teach equivalent fractions. She found that the students' notebooks contained unequal parts, all drawings that were not equal to fraction models. Feyza said that they found the result using fraction sets but they might have been sloppy when drawing shapes in their notebooks. During the teaching process, students Seda and Halime made mistakes in drawings while modeling. Figure 3 and Figure 4 illustrate the drawings of these students.

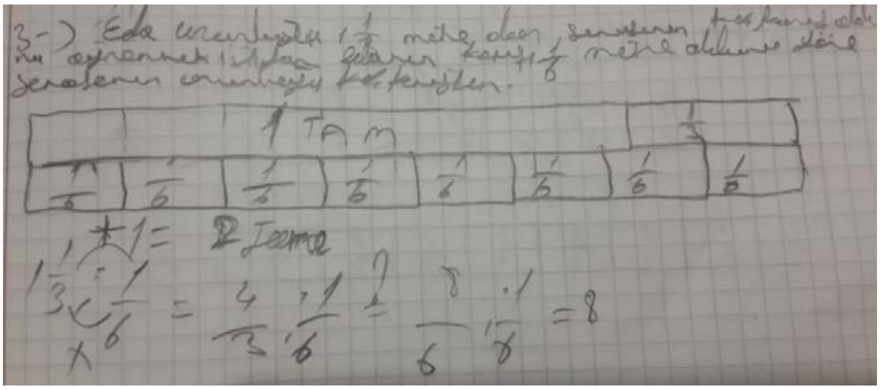

FIGURE 3. *Halime's solution to the problem of dividing "1 and 1/3" into 1/6.*

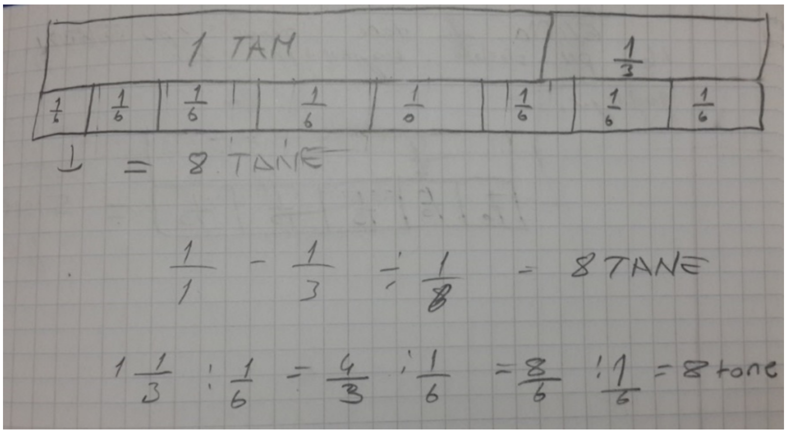

FIGURE 4. *Seda's solution to the problem of dividing "1 and 1/3" into 1/6.*

The situations shown in Figure 3 and Figure 4 were asked during interviews with Feyza. Feyza was asked why she did not interfere with the inaccuracies of the students' drawings. Feyza suggested different reasons:

The researcher: Seda and Halime did not align the drawings that they drew in their notebooks.
Feyza: I asked them to draw in their notebooks to reveal what was done to solve the problem. Because they solved the question with fraction sets, they made their drawings by looking at the model. Perhaps they would have drawn it more easily if they had only tried to model themselves without using a fraction set.

As it can be seen from the conversation above, Feyza did not pay enough attention if students drew their models completely and correctly in this teaching.

Sample cases related to the meaningful teaching or rule based teaching for "Simplification and Extension" operations

None of the preservice teachers conducted an activity for meaningful teaching of the simplification and expansion operations. Çağla did not emphasize that the amount expressed by the fraction did not change while teaching the expansion and simplification operations. On the contrary, she preferred to do rule-based instruction using only procedures. One of the students used the expression "we multiply the fraction by two" while showing the expansion by two. Çağla said that the numerator and denominator were multiplied by 2 while expanding the fraction by 2. However, she did not say that multiplying a fraction by 2 is not an expansion. The student used the word multiplication instead of expansion.

Çağla: When they say multiplication, they meant "expansion". They even write it down under the denominator.
The researcher: But it is also important to use the expression properly. Any student in the classroom may misunderstand that expanding by 2 means multiplying the fraction by 2. You did not make any corrections. You accepted the expression "multiply by 2" instead of expanding by 2.
Çağla: Could be. I did not say the concept properly. I agreed because I understood what they meant.
The researcher: You did not show what extension and simplification meant on the figure.
Çağla: Yes, I did not. How many mistakes I made.

Sample cases related to the meaningful teaching or rule based teaching for compound fraction conversion

Feyza preferred to get the students' ideas instead of defining the combined fraction directly in her second teaching. With these behaviors, she provided students with an opportunity to express their own ideas. When one of the students said he used the rule to convert the integer fraction to the compound fraction, she stated that it can be done without the rule. At this point, we observed that Feyza directed students to the meaning of operations and concepts.

The researcher: Bahri says that he could use the rule to convert the integer fraction into compound fraction. You say we can do it without using a rule.
Feyza: Yes.
The researcher: There is a constant effort to move from rules to meaning. I am very pleased.
Feyza: You know, rules are forgotten, but learning is permanent.

Although Çağla was persistently guided by the researcher in her first teaching, she taught most of the procedures and operations as a rule in her second teaching. Çağla did not dynamically show how the conversion to compound fraction occurs on the model. She did not expect the students to discover that figures should be divided into five parts, instead, she preferred to draw shapes divided directly into five parts. The researcher was asked why she did not change her teaching. Çağla said that it was impossible to change a situation which she had been accustomed to for a long time. However, she stated that she would change her way of teaching over time.

The researcher: Do you need a little bit more time to teach processes in a meaningful and conceptual way?
Çağla: Let's imagine. Human psychology changes by practice. Let's think of a very stingy person. Even if this person suddenly says that I will be generous, he cannot apply it immediately. People change by gaining experience over time. Learning in human psychology is a very interesting thing. I am just learning something. You expect me to transfer it immediately. I cannot implement them immediately, I need a process. My second teaching was better than the first, but it was not enough.

Table 5 presents general information about teaching styles of the six preservice teachers who participated in the study. All six preservice teachers were divided into two groups considering that it would be difficult to prepare lesson plans, group discussions, and clinical interviews.

Table 5. *Themes and categories related to teaching of procedural and conceptual knowledge*

Theme	Category	1.Group						2.Group					
		Çağla		**Faruk**		**Büşra**		**Feyza**		**Mesut**		**Zeliha**	
		1.t	**2.t**	**1.t**	**2.t**	**1.t**	**2.t**	**1.t**	**2.t**	**1.t**	**2.t**	**1.t**	**2.t**
Meaningful Teaching	Meaningful teaching of division concept	-	3	3	1	1	1	4	1	1	-	3	6
	Meaningful teaching of division by invert and multiply rule	1	-	2	-	-	-	-	-	3	-	1	-
	Meaningful teaching of division by common denominator rule	-	2	-	1	-	3	2	1	-	1	-	-
	Meaningful teaching of equivalent fraction concept	-	-	-	-	-	-	1	1	-	-	-	-
	Meaningful teaching of simplification	-	-	-	-	-	-	-	-	-	-	-	-
	Meaningful teaching of the expansion	-	-	-	-	-	-	-	-	-	-	-	-
	Meaningful teaching of compound fraction conversion	-	-	-	2	-	-	-	1	-	-	-	-
	TOTAL	1	5	5	4	1	4	7	4	4	1	4	6
		Çağla		Faruk		Büşra		Feyza		Mesut		Zeliha	
Teaching As A Rule	Rule based teaching of division concept	2	-	2	-	5	-	-	-	2	1	3	-
	Rule based teaching of division by invert and multiply rule	5	2	2	2	-	4	1	-	2	-	2	2
	Rule based teaching of division by common denominator rule	3	1	5	1	-	1	-	-	1	1	2	5
	Rule based teaching of equivalent fraction concept	-	1	1	-	-	1	-	1	-	-	-	1
	Rule based teaching of simplification	1	-	1	2	2	-	-	-	-	-	-	1
	Rule based teaching of the expansion	1	-	-	2	1	1	-	-	-	2	-	1
	Rule based teaching of compound fraction conversion	-	1	-	-	-	-	-	-	-	-	-	1
	TOTAL	12	5	11	7	8	7	1	1	5	4	7	11
		Çağla		Faruk		Büşra		Feyza		Mesut		Zeliha	

*The abbreviation "1.t" in the table refers to the number of behaviors related to the first teaching performed by the preservice teachers. Similarly, the abbreviation "2.t" refers to the number of behaviors related to the second teaching performed by the same preservice teachers.

Accordingly, Çağla, Faruk, and Büşra were placed in the first group, and Feyza, Mesut, and Zeliha were placed in the second group. Each preservice teacher taught twice within the scope of the study. In the table below, the findings related to the first teaching were abbreviated as "1.T", and the findings related to the second teaching were abbreviated as"2.T". For example, there were three behaviors evaluated within the meaningful teaching category of division concept for Zeliha's first teaching. On the other hand, there were six behaviors in the same category in her second teaching (2.T). For this reason, Zeliha's behaviors in the meaningful teaching category of division concept increased in her second teaching.

When the categories of meaningful teaching theme were examined, the first thing that draws attention is that none of the preservice teachers were able to conduct meaningful teaching of the simplification and expansion operations. In addition, meaningful teaching of equivalent fractions and conversion to compound fractions was not handled by almost any teacher. However, the concept of equivalent fractions was necessary to teach the meaning of simplification and expansion. In addition, the same concept was also necessary for the meaningful teaching of division in fractions. When the data related to the meaningful teaching category of equivalent fractions were examined, only Feyza reflected a proper behavior. In the interviews prior to the teaching, Feyza said that the teaching of equivalent fractions is important, and therefore teaching should be started with an instruction on equivalent fractions first. In her teaching, she acted in accordance with this idea. When Table 5 was examined, the numbers of "teaching as a rule" behaviors were quite high compared to the meaningful teaching. If equivalent fractions, simplification and expansion operations had a higher number of meaningful teaching behaviors, then the number of teaching as a rule behaviors could be lower.

Çağla showed only one meaning-based teaching behavior in her first teaching. We observed that this number increased to five in her second teaching. On the other hand, the rule-based teaching behaviors, which was 12 in her first teaching, decreased to five in her second teaching. Hence, we can say that the practices carried out within the scope of the study oriented Çağla to meaning-based teaching. The changes in the number of themes were consistent with what the preservice teacher expressed during the interviews that were conducted after the second teaching. In these interviews, she said that a teaching method without meaning was not correct. She also stated that teachers who are constantly teaching rules based are accustomed to laziness.

Faruk's number of meaning-based teaching behaviors decreased from five in his first teaching to four in his second teaching. His number of rules-based teaching decreased from 11 to seven. The decrease in the number of rules-based instruction was a positive development. The decrease in the number of meaningful teaching behaviors was thought to be due to the use of activities that were not appropriate for the students' levels in his second teaching. In the activity he used during the second teaching, Faruk wrote two fractions one by one and put a big line between them. Taking the share of the fraction above, he asked to the students how the value of two fractions changed by dividing. He pointed out the share of the fraction above and said that he changed it. He asked how this change affects the division of the two fractions. For example, he told that if the denominator of the upper fraction increases, the value of the fraction would decrease. Moreover, he added that if the numerator of the second fraction increases, the value of the fraction would decrease. Finally, he told that if the denominator of the lower fraction increases, the value of the fraction would decrease. The students could not fully understand what Faruk said. Faruk realized this but spent more time instead of leaving this activity. In the clinical interviews at the end of the teaching, He stated that he had overused this activity unnecessarily. He therefore admitted that his second teaching had failed.

When the number of meaning-based instruction of Büşra was examined, the number meaning-based teaching behaviors increased from one in her first teaching to four in her second teaching. The number of rules-based teaching behaviors decreased from eight to seven. When evaluated together, the practice carried out within the scope of the study directed Büşra to meaningful teaching. At the beginning of the study, Büşra said that mathematical operations should be taught on the basis of meaning. During the interviews, which were conducted at the end of her second teaching, she said that she did not know how to provide meaning-based teaching, although she said that meaningful teaching was important. Büşra stated that during this study process, she learned how meaning-based instruction should be. Besides, she said that she initially thought the operations can only be learned by memorization. However, she later stated that she saw that operations and concepts could be taught in relation with this activity. Although Feyza conducted meaning-based instruction seven times in her first teaching, this number decreased to four in her second teaching. Rule-based teaching behaviors were observed as one in both teaching. Feyza believed that meaningful teaching was important at every stage of teaching. Although her number of themes for meaningful teaching had fallen, this decline should not be regarded as negative because she preferred to use less activity and questions in her second teaching and devoted more time to their meanings based on the experience gained from the first teaching. The decrease in Feyza's meaningful teaching theme was thought to be due to the fact that she used less activity than she used in her first teaching.

Mesut's number of themes related to meaningful teaching decreased from four to one. The number of rules-based instruction was five in his first teaching and it was four in his second teaching. At every stage of this study, Mesut considered meaningful teaching as an unnecessary effort. He expressed this view in an interview as follows, "I think it is best to memorize the rules. If students do not memorize, they cannot do exactly what they know. Until now, information has always been taught by memorization in the education system. We cannot change it. My teachers did not say anything about learning subjects by making sense of them. So, there is no need to waste time. You just need to memorize." He also stated that it was very difficult for him to prepare a lesson plan. He said that if a ready plan was given to him, he could apply it very well. Mesut did not pay attention to the plan he prepared for his second teaching. He wanted to use the activities prepared by the remaining five preservice teachers by revising their teaching plans. However, this request was not accepted by the researcher and he was asked to prepare his own original plan. Mesut participated in the study voluntarily. However, he said that he considered meaningful teaching ineffective. Therefore, it was unnecessary to prepare a lesson plan based on meaningful teaching for him. For these reasons, Mesut's second teaching was not good eanough and was inefficient. Zeliha's meaning-based instruction has increased from four to six. At this point, there was progress in Zeliha. In the interviews, she said that she found meaningful teaching quite important. On the other hand, the number of rules-based instruction increased from seven to 11. Zeliha was asked why she tended to follow rule-based teaching, especially in the final stages of her second teaching. Zeliha said that she emphasized the meaning of operations in her first teaching, and therefore she wanted to practice on the rules in her second teaching.

Within the scope of the study, the researcher directed the preservice teachers to conduct self-evaluation both during the pre-teaching lesson plan preparation stages and at the end of the clinical interviews. At all stages, the content of the lesson plans that preservice teachers prepared and their teaching with real students had been evaluated in terms of whether these plans and their teaching were meaning based or not. This process, which took place in two cycles, increased the awareness of all preservice teachers and developed them in many ways such as preparing lesson plans, planning instruction, thinking on their own teaching, and comparing their own teaching with those of their friends. As a result of the detailed analysis of their teaching, some preservice teachers, especially those who believed in meaningful teaching and were open to development, developed faster. The practical skills of preservice teachers were difficult to change even in such an environment where comprehensive feedbacks, group discussions, preparing and thinking together, thinking on their own teaching, and the opportunity to compare with their friends opportunities were provided.

DISCUSSION and CONCLUSIONS

In this study, preservice teachers were expected to teach the concepts and operations related to division in fractions based on an approach that pays attention to meaning. It is stated that the preservice teachers' lack of content knowledge may have prevent them for providing meaningful teaching of operations and concepts. When the studies related to the division by fractions are examined, it is understood that the operations and concepts related to division by fractions are one of the least understood subjects by the preservice teachers (Li, 2008; Sinincrop, Mick & Kolb, 2002). In the clinical interviews conducted with preservice teachers within the scope of this study, we observed that almost all of them did not know the meaning of operations and concepts and could not explain why the invert and multiply rule is used when dividing fractions. In a study conducted by Yeşildere (2008), preservice teachers with inadequate content knowledge could not properly teach related concepts and had difficulty in detecting student errors. In addition, as in Mesut's case, it should be taken into consideration that beliefs against the constructivist approach may prevent their developments. Similarly, Gökkurt, Şahin, and Soylu (2012) emphasized that having a good mathematical content knowledge does not guarantee that the subject can be taught well. In another study conducted by Gökkurt, Şahin, Soylu, and Soylu (2013), it was found that even if the preservice teachers identified the student errors, they were not able to correct these errors by applying appropriate pedagogical methods. Throughout the study, the researcher made suggestions to the preservice teachers to include the meanings of the procedures, operations, and concepts in their plans. When the findings of the study are evaluated together with this role of the researcher, it was difficult to guide them even if they knew the meaning of the procedures, operations, and concepts. Hence, knowing a subject alone does not guarantee that it can be taught well (Kahan, Cooper & Bethea, 2003). These difficulties are thought to be due to the lack of experience of preservice teachers on not knowing how to demonstrate positive pedagogical behaviors.

When the findings of the preservice teachers' teaching are examined, Çağla and Büşra's number of meaningful-based instruction increased and their number of rule-based instruction decreased. Although there has been an increase in the number of behaviors related to Zeliha's meaningful teaching, there has been some increases in her rule-based teaching behaviours. The increase in Zeliha's rule-based teachings stems from her rule-based teaching of equivalent fractions and expansion and simplification in her second teaching. Equivalent fractions and expansion and simplification are essential to teach the meaning of division in fractions. Therefore, it is very important to teach these subjects based on meaning. In addition to this, it is very important to perform division by natural numbers and to teach meaningful multiplication operations in fractions. Some preservice teachers tried to make sense of the invert and multiply process without knowing the meaning of multiplication in fractions, but they were not successful. Ma (1999) stated that in order to teach division by fractions, division by natural numbers should be understood. In the same study, the interpretation of the inverse multiplication is explained by the addition, multiplication and division of fractions. Although, in this current study, the researcher emphasized the meanings of concepts and operations during the whole process, the rule-based instruction was still encountered in their second teaching. In this case, it was not enough to provide preservice teachers with information about the content knowledge in order to make meaningful teaching. It is thought that as the preservice teachers have more teaching experience, their rule-based teaching should decrease over time.

When Faruk's themes related to teaching of procedures, operations, and concept were examined, there was no significant change in his meaningful teaching behaviors, but there was some decrease in his rule-based teaching behaviors. During the interviews with Faruk, he said that his second teaching could be better, but he emphasized unnecessarily too much of an activity that he thought it was not suitable for the students' levels. Davis (1997) stated that in order to ensure effective mathematics teaching and active participation of students, the activities in which students are centered should be used. Indeed, the activity used by Faruk was not suitable to both students' levels, and it was not a student-centered activity. For this reason, he did not acquire the achievement that he aimed in his second teaching.

Since the beginning of the study, Feyza had shown an attitude against rule-based teaching. In accordance with this view, Feyza showed very little rule-based behavior in both of her teachings. There was a slight decrease in the number of meaning-based instruction in her second teaching. In the interviews with Feyza, she stated that her first teaching did not go as she planned, and that her lesson plan was quite intense. She explained her plan to prepare less activities in her second teaching and will think more about them. When the Feyza's second teaching was examined, she actually used less activities than her first teaching. In these activities, Feyza directed the students to talk and comment on the subject. Therefore, the decline in Feyza's meaningful teaching themes cannot be seen as failure.

When Mesut's frequencies of themes were examined, his number of meaning-based instruction was decreased. In general, Mesut preferred a rule-based teaching. Mesut stated that he was in favor of conducting rules-based teaching, he saw meaningful teaching as a waste of time. Moreoever, he thought that the students who applied the rules well were successful in the exams. He also stated that he could not fully understand the meanings of concepts and operations. Therefore, he failed in meaningful teaching. Similarly, Lo and Luo (2012) emphasized that preservice teachers cannot make meaningful teaching on fractions unless they learn division by fractions. In this study, the researcher focused on the meaning of the concepts of division in all processes, but despite all efforts, Mesut preferred applying a rule-based teaching, although he knew the meanings of some operations and concepts.

Mesut said that it was difficult for him to prepare a lesson plan in all the meetings. He said he was searching for an already prepared plan from the internet, and it was unnecessary to make sense of dividing the two fractions through using shapes. He also stated that he had difficulty in making sense of the division operation. Mesut wanted to use only the rules without including figures in his plan for his first teaching. We think that this approach was the reason for the decrease in his meaningful teaching. In addition, it should be taken into consideration that the instruction on dividing the two fractions in the second teaching was more difficult than the first teaching. Borko et al. (1992) observed a preservice teacher who failed to teach division in fractions based on meaning. At the end of the study, they concluded that some negative beliefs of the preservice teachers about the teaching could prevent their development. When the findings of the preservice teachers for the meaningful teaching were considered, five of the six preservice teachers who participated in the study made progress in general. At the end of the study, almost all of the preservice teachers have reached to a certain level to make explanations about the meaning of the inverse multiply rule. Because the meaning of inverse multiplication requires the interpretation of many related concepts together, it is thought that there would be no progress if such an application was not made and only teaching was given. Although the courses taken at the university focus on these concepts and operations, it should be remembered that knowing a subject and its meaning does not guarantee that it will be taught well (Gökkurt, Şahin and Soylu, 2012). In the Li and Kulm (2008) study, 46 preservice teachers were asked to explain the meaning of the inverse multiplication process. However, none of them were able to provide a reasonable and acceptable answer.

Ma (1999) observed that Chinese teachers emphasized the meanings while teaching the division operation in fractions. Ma (1999) emphasized that the teaching tradition of each country is different. According to Ma (1999), teachers in the United States mostly teach in the form of rules, while teachers in China emphasize concepts more. All of the preservice teachers who participated in this current study said that they had not previously taken a course for meaningful teaching of the procedures. In addition, they said that they always went through rule-based teaching in their primary, secondary, and high school education. For example, Çağla said in one statement: We did not know the meaning of the words process and concept. We mostly went through an education about the processes. What we call concept are actually operational information. We learned the meaning of the concepts during this study. We have just learned the relationships between processes and concepts. It can be said that meaningful teaching should become a tradition.

Ball (1990), Chiu (2009), Chen et al. (2011) stated that it is necessary to guide preservice teachers to solve verbal problems by drawing. Hence, their understanding of teaching concepts will develop. In parallel with these views, the researcher directed the preservice teachers, who were participants of this study, to draw figures while solving the division questions in fractions. However, some preservice teachers did rule-based teaching, although they understood division operation using figures. Some preservice teachers tried to teach the concepts by drawing shapes. However, they told the student step by step how these shapes will be. The student did exactly what he was told without interpreting. Ball (1990), Chiu (2009), Chen et al. (2011) suggested teaching verbal problems by drawing. However, it has been found that this suggestion may not be beneficial for everyone. The teachers' behaviors are important in drawing activities.

Some preconcepts need to be understood for meaningful teaching of division in fractions. These concepts are equivalent fractions, simplification and expansion, and conversion to compound fractions. When the preservice teachers' teachings were examined, only one of them taught the concept of equivalent fractions in her both teachings based on meaning. In all interviews conducted by the researcher, the importance of these concepts was emphasized. However, all preservice teachers taught the simplification and extension procedures on the basis of rules. Two of the preservice teachers had shown compound fraction conversion through meaning-based teaching using figures. Hence, despite all these efforts, it was very difficult to direct preservice teachers to meaningful teaching of concepts since they taught the necessary preliminary knowledge more on using rule-based teaching. Therefore, they had difficulty in making meaningful teaching on the rule-based information.

One of the most striking findings of the study is about the relationship between conceptual and procedural knowledge. In the interviews conducted with the preservice teachers, it was seen that the division questions can be formed with different operations, especially in fractions with an integer result. It has been found that these division questions can be easily calculated using addition, subtraction, and even multiplication. The connections among division, addition, subtraction, and multiplication concepts in fractions contribute to procedural fluency by increasing the ability to form relationships between operations. These results are similar to those reported by Armstrong and Bezuk (1995). Accordingly, in order to teach the division of fractions meaningfully, it is necessary to know all concepts related to division by natural numbers and fractions.

Researchers such as Borko et al. (1992) and Ma (1999) stated that divisions with fractions have different interpretations. They also stated that the concept of division is already difficult to learn in itself, and that the division together with fraction makes learning more difficult. In the beginning, the preservice teachers had difficulty in understanding the operations and concepts about division in fractions. However, during the study process, they both learned their meaning and gained experience in teaching. For example, in interviews with Zeliha, she said that she has made progress in teaching practice through this study. She explained this with the following words: "At the beginning of the study, we said that we cannot do such teaching. We did not know how. But now I believe we can teach mathematics in a meaningful way. Because I learned it myself during the study process. There were times when I was really surprised, when I learned the meaning of things. In this process I learned many things."

In the light of the data obtained from the study, content knowledge has an important place in teacher training. However, there is a difference between having superficial and in-depth mathematics content knowledge. Preservice teachers should know all the reasons of the procedural and conceptual knowledge they have learned. Otherwise, they have difficulty in making justifications when teaching. For this reason, we suggest that faculty members in teacher training institutions should focus not only on increasing preservice teachers' knowledge but also should focus on developing their analysis and synthesis skills in mathematics. Therefore, the in-depth knowledge of the subject is not enough alone.

Although preservice teachers know the operations and concepts that he/she will teach, they sometimes fail because of their negative attitudes and prejudices. Sometimes they could not teach properly because they did not have enough pedagogical skills. Based on our findings, preservice teachers should know the meaning of operations and concepts, believe that they can teach these meanings to the students, and should have the ability to apply pedagogical methods to transfer these meanings. The preservice teachers who knew the meaning of operations and concepts at the beginning of the study and advocated to a constructivist and student-centered teaching, which supports a meaning-based instruction, can behave in contradiction with these ideas during their actual teaching. We found that some preservice teachers did not prefer to teach the content knowledge they know and did not act in accordance with their attitudes and opinions. It should be remembered that there may be contradictions between the knowledge, opinions, and practices of these preservice teachers. In order to eliminate these contradictions, we recommended that preservice teachers should be given more feedback. During this process, attention should be paid to giving more feedback by the instructors. In the following studies, it may be useful to reveal the contradictions between the knowledge, attitude, instructional views, beliefs, and actual teaching of the teachers and preservice teachers.

REFERENCES

Abrosse, R., Clement, L., Philipp, R., & Chauvot, J. (2004). Assessing Preservice elementary school teachers' beliefs about mathematics and mathematics learning: Rationale and Development of a Constructed-Response-Format Belief Survey. *School Science and Mathematics Journal, 104*(2), 56–69.

Akın, A. & Kabael, T. (2016). Bir matematik eğitimi araştırmasına dayalı öğretim deneyi deneyimi. E*ğitimde Nitel Araştırmalar Dergisi [Journal of Qualitative Study in Education], 4*(3), 7-27.
Armstrong, B. E.,& Bezuk, N. (1995). Multiplication and division of fractions: The search for meaning. In J. Sowder & B. P. Schappelle (Eds.). *Providing a foundation for teaching mathematics in the middle grades,* 85–120. New York: SUNY.
Australian National Curriculum Board (2009). T*he shape of the Australian Curriculum. National Curriculum Board.* [Çevrim-içi: http://www.acara.edu.au/verve/_resources/The_Shape_of_the_Australian_Curriculum_May_2009_file.pdf, Erişim tarihi: 15.12.2015.]
Aytekin, C. & Toluk Uçar, Z. (2014). Ortaokul öğrencilerinin kesirlerde tahmin becerilerinin incelenmesi. *İlköğretim Online, 13*(2), 546-563.
Aytekin, C. (2012). *İlköğretim ikinci kademe öğrencilerinin kesirlerde tahmin becerilerinin incelenmesi.* Yayımlanmamış Yüksek Lisans Tezi. Abant İzzet Baysal Üniversitesi Eğitim Bilimleri Enstitüsü, Bolu.
Ball, D. L. (1990). Preservice elementary and secondary teachers' understanding of division. *Journal for Study in Mathematics Education, 21*(2), 132-144.
Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Study in Mathematics Education, 23*(3), 194-222.
Byrnes, J. P. & Wasik, B. A. (1991). Role of conceptual knowledge in mathematical procedural learning. *Developmental Psychology, 27*, 777-786.
Carpenter, T. P. (1986). Conceptual Knowledge as a Foundation for Procedural Knowledge. In J. Hiebert (Ed.), *Conceptual and Procedural Knowledge: The Case of Mathematics,* 113-132. Hillsdale, NJ: Erlbaum.
Chen, L., Van Dooren, W., Chen, Q., & Verschaffel, L. (2011). An investigation on Chinese teachers' realistic problem posing and problem solving ability and beliefs. *International Journal of Science and Mathematics Education, 9*(4), 919–948.
Chiu, M. (2009). Approaches to the teaching of creative and non-creative mathematical problems. *International Journal of Science and Mathematics Education, 7*(1), 55–79.
Cobb, P. & Steffe, L.P. (1983). The constructivist Researcher as Teacher and Model Builder. *Journal for Study in Mathematics Education, 14*(2), 83-94.
Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Study in Mathematics Education, 28*(3), 355–376.
Dubinsky, E.,& Harel, G. (1992). The Nature of the Process Conception of Function. In G.Harel, & Dubinsky, E. (Ed.), *The concept of function: Aspects of epistemology and pedagogy,* (Vol. 25). Washington: Mathematical Association of America.
Entwistle, N.,& Tait, H. (1990). Approaches to learning, evaluations of teaching, and preferences for contrasting academic environments. *Higher Education, 19*(2), 169-194.
Glencoe Math (2013). *Grade 6-8.* McGraw Hill Education. Bothell, Washington.
Glencoe Math (2014). *Grade 6-8.* McGraw Hill Education. Bothell, Washington.
Gray, E.,& Tall, D. (1993). Success and Failure in Mathematics: The Flexible Meaning of Symbols as Process and Concept. *Mathematics Teaching,14*(2), 6-10.
Gray, E.,& Tall, D. (1994). Duality, ambiguity, and flexibility: a "proceptual" view of simple arithmetic. *Journal of Study in Mathematics Education, 25*(2), 116-140.
Haapasalo, L. (1993). Systematic constructivism in mathematical concept building. In p. Kupari & l. Haapasalo (Eds.), *Constructivist and curricular issues in the school mathematics education. Mathematics education study in Finland*, Yearbook 1992-1993. 9-22. University of Jyväskylä: Institute for Educational Study.
Haapasalo, L.,& Kadijevich, D. (2000). Two types of mathematical knowledge and their relation. *Journal für mathematik didaktik, 21*(2), 139-157.
Hiebert, J.,& Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics, 1–27.* Hillsdale, NJ: Erlbaum.
Hiebert, J.,& Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics,* 199–223. Hillsdale, NJ: Erlbaum.
Hofer, B. K.,& Pintrich, P. R. (1997). The development of epistemological theories: Beliefs about knowledge and knowing and their relation to learning. *Review of Educational Study, 67*(1), 88-140.
Hunting, R.P (1983). Emerging methodologies for understanding internal processes governing children's mathematical behaviour. *The Australian Journal of Education, 27*(1), 45-61.

Järvelä, J.,& Haapasalo, L. (2005). *Three Types of Orientations by Learning Basic Routines in ICT.* Paper presented at the Learning and Instruction on Multiple Context and Settings III. Proceedings of the Fifth Joensuu Symposium on Learning and Instruction, Joensuu.

Kadijevich, D.,& Haapasalo, L. (2001). Linking procedural and conceptual mathematical knowledge through CAL. *Journal of Computer Assisted Learning, 17*(2), 156-165.

Kaput, J. (1982). *Differential effects of the symbol systems of arithmetic and geometry on the interpretation of algebraic symbols.* Paper presented at the Annual Meeting of the American Educational Study Association, NY.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics.* Washington, DC: National Academy Press.

Kline, M. (1980). *Mathematics: The Loss of Certainty.* New York: Oxford University Press.

Lauritzen, Pal (2012). *Conceptual and Procedural Knowledge of Mathematical Functions.* Publications of the University of Eastern Finland.

Lesh, R., Kelly, A., (2000) Multitiered Teaching Experiments. In A. Kelly, R. Lesh (Eds.), *Study Design in Mathematics and Science Education. 197-230.* Lawrence Erlbaum Associates, Mahwah, New Jersey.

Li, Y. (2008). What do students need to learn about division of fractions? *Mathematics Teaching in the Middle School, 13,* 546–552.

Li, Y.,& Kulm, G. (2008). Knowledge and confidence of Preservice mathematics teachers: The case of fraction division. ZDM–*The International Journal on Mathematics Education, 40,* 833–843.

Lo, J.J. ve Luo, F. (2012). Preservice elementary teachers' knowledge of fraction division. *J Math Teacher Educ, 15,* 481–500.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates.

MacGregor, D. (2013). *Developing Mathematical Proficiency. Eps literacy and intervention.* [Çevrim-içi: http://eps.schoolspecialty.com/downloads/study_papers/series/AcademyMATH_study.pdf, Erişim tarihi: 15.12.2015]

Macievejewski, W., Mgombelo, J. ve Savard, A. (2011). *Meaningful Procedural Kknowledge in Mathematics Learning.* CMESG/GCEDM Proceedings. [Çevrim-içi:http://www.math.ubc.ca/~wes/writing/meaningfulproceeduralknowledgeinmathematicslearning.pdf, Erişim Tarihi: 15.12.2015]

Marton, F.,& Säljö, R. (1976). On qualitative differences in learning: I. Outcome and process. *British Journal of Educational Psychology, 46*(1), 4-11.

Math Connects (2012). *Grade 6-8.* McGraw Hill Education.

MEB (2009). *6-8.Sınıflar Öğretim Programı.* Ankara

MEB (2013). *Ortaokul matematik dersi öğretim programı.* Ankara

Milgram, R. James (2005). *The mathematics preservice teachers need to know. Stanford University Department of Mathematics* [Çevrim-içi: http://math.stanford.edu/pub/papers/milgram/FIE-book.pdf, Erişim Tarihi: 15.12.2015]

NCTM (2013). *Proficiency teaching children mathematics.* [Çevrim-içi: http://cliu21cng.wikispaces.com/file/view/Making%20Shifts%20Toward%20Proficiency.pdf/462127488/Making%20Shifts%20Toward%20Proficiency.pdf, Erişim Tarihi: 15.12.2015]

Nesher, P. (1986). Are mathematical understanding and algorithmic performance related. *For the Learning of Mathematics, 6*(3), 2-9.

Pajares, M. F. (1992). Teacher's beliefs and educational study: Cleaning up a messy construct. *Review of Educational Study, 62*(3), 307-332.

Picker, S. H. & Berry, J.S. (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics, 43*(1), 65–94.

PISA (2012). *Ulusal ön raporu. Yenilik ve eğitim teknolojileri genel müdürlüğü.* [Çevrim-içi:http://yegitek.meb.gov.tr/meb_iys_dosyalar/2013_12/13053601_pisa2012_ulusal_n_raporu.pdf, Erişim Tarihi: 15.12.2015]

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Study in Mathematics Education, 28,* 550–576.

Resnick, L.,& Omanson, S. F. (1987). Learning to understand arithmetic. In G. R. (Ed.), *Advances in Instructional Psychology, 3.* Hillsdale, NJ: Erlbaum.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Study in Mathematics Education, 20*(4), 338–355.

Schoenfeld, A. H. (2007). Method. In F. K. Lester (Ed.), *Second handbook of study on mathematics teaching and learning,* 69–110. Greenwich, CT: information Age Publishing.

Sfard, A. (1991). On the dual nature of mathematical conceptions: theoretical reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22,* 1-36.
Sinincrop, R., Mick, H. W., & Kolb, J. R. (2002). Interpretations of fraction division. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions,*153–161. Reston: NCTM.
Steffe, L. P. (1991). The constructivist teaching experiment: Illustrations and implications. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education,* 177-194. Boston, MA: Kluwer Academic Press.
Steffe, L.P. & Thompson, P. (2000). Teaching experiment methodology: Underlying principles and essential elements. In A. E. Kelly ve R.A. Lesh (Eds.), *Handbook of Study design in mathematics and science education* (pp. 267-306), Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.
Steffe, L.P. (1984). The teaching experiment methodology in a constructivist study program. In M. Zweng. (ed.). *Proceeding of the fourth International Congress on Mathematical Education,* 469-471.
Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*(2), 105–127.
Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the study. In D. A. Grouws (Ed.), *Handbook of Study on Mathematics Teaching and Learning,* 127–146. New York: Macmillan.
Tirosh, D. (2000). Enhancing pre-service teachers' knowledge of children's conceptions: the case of division of fractions. *Journal for Study in Mathematics Education, 31*(1), 5-25.
Toluk Uçar, Z., Pişkin, M., Akdoğan ,E. N., ve Taşçı,D. (2010). İlköğretim öğrencilerinin matematik, matematik öğretmenleri ve matematikçiler hakkındaki inançları. *Eğitim ve Bilim, 35*(155), 131-144.
Utley, J. ve Redmond, A. (2008). *Preservice elementary teacher attitudes towards and knowledge of the division of fractions.* Paper presented at the annual meeting of the Study Council on Mathematics Learning, Oklahoma City, OK.
Vergnaud, G. (1990). Epistemology and psychology of mathematics education. In P. K. Nesher, J. (Ed.), *Mathematics and cognition: a study synthesis by the international group for the psychology of mathematics education.* Cambridge: Cambridge University Press, 14-30.
Zucker, B. (1984). *The relation between understanding and algorithmic knowledge in decimals.* Unpublished Master Thesis. University of Haifa.
Gökkurt, B., Şahin Ö. & Soylu, Y. (2012). Matematik Öğretmenlerinin Matematiksel Alan Bilgileri ile Pedagojik Alan Bilgileri Arasındaki ilişkinin incelenmesi. *The Journal of Academic Social Science Studies, 5*(8), 997-1012.
Gökkurt, B., Şahin, Ö., Soylu, Y. & Soylu, C. (2013). Examining Pre-Service Teachers' Pedagogical Content Knowledge on Fractions in Terms of Students' Errors. *International Online Journal of Educational Sciences, 5*(3), 719-735.
Işıksal, M. (2006). *İlköğretim matematik öğretmen adaylarının kesirlerde çarpma ve bölmeye ilişkin alan ve pedagojik içerik bilgileri üzerine bir çalışma.* (Yayımlanmamış doktora tezi). Orta Doğu Teknik Üniversitesi, Ankara.
Kahan, J., Cooper, D., & Bethea, K. (2003). The role of mathematics teachers' content knowledge in their teaching: a framework for study applied to a study of student teachers. *Journal of Mathematics Teacher Education, 6*, 223-252.
Yeşildere, S. (2008, Ağustos). *İlköğretim matematik öğretmen adaylarının sayı örüntüleri ile ilgili pedagojik alan bilgilerinin incelenmesi.* VIII Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi. Bolu: Abant İzzet Baysal Üniversitesi.