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Examining mobile game experiences of prospective primary school teachers and their game designs about teaching math*

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Abstract. In this study, we aimed at introducing to prospective primary school teachers how to present a pattern of numbers through designing a mobile game according to the Theory of Didactical Situations (TDS). Additionally, the games developed by teacher candidates were examined. In our study, the case study method was used, which is one of the qualitative research methods. The participants were 17 prospective primary school teachers (9-females, 8-males). The data were collected through video cameras, audio recorders, and files that include game designs of prospective teachers. In data analysis, the actions concerning mobile gaming experiences of prospective teachers were conducted according to the TDS stages. The games designed by prospective teachers were examined in terms of basic gamification criteria. Findings of the study; it was demonstrated that mobile games designed according to TDS support the prospective *teachers* in many aspects, such as developing a milieu for using gamification in teaching math, internalizing processes of reflecting gamification criteria to the game, and positively influencing the affective characteristics. Moreover, concerning gamification criteria, it was determined that some prospective teachers are successful at gamification of mathematical knowledge; however, most of them have difficulties in game designing, particularly in giving feedbacks about games.

Keywords: Theory of didactical situations, mathematics teaching, mobile game, gamification, prospective teachers

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INTRODUCTION

The idea of integrating emerging current technologies into education and training has required the development of new approaches in mathematics education (Ersoy, 2005; Kiili, 2005; Tall, 1988). One of these approaches is to teach mathematics with games where technological tools are used. Teaching mathematics through gamification is supported, and content development is encouraged on various platforms (Educational Informatics Network [EBA]; Ministry of Education [MEB]; National Council of Teachers of Mathematics [NCTM]).

In the mathematics course curriculum, it is stated that "It should be tried to include math games in the sections deemed appropriate in relation to the content of the unit." and it is requested that the games be used effectively in the teaching process (MEB, 2018a). Why is teaching mathematics with games important? There can be many answers given to this question. In one of these answers, Bishop (1991) underlined the mathematical connections in games and stated that most mathematicians consider games to be equivalent to doing mathematics because they are rules-based behaviors. It can be stated that this understanding has a broad spectrum, including games using technological tools. Additionally, there are many uses of technology in mathematics education.

In mathematics education, technology is generally used in various ways such as teaching numbers, specialization in arithmetic, presenting and presenting mathematical objects, examining many different situations related to an algorithm in a short period of time (Şahin, 2016; NCTM, 2000). Kiili (2005) stated that today, students are in constant interaction with

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technological tools, so using only technological tools in education is not a sustainable approach. He also stated that this problem could be overcome through gamification. At this point, it may be helpful to ask some questions. What is gamification emerging as a new approach in education? What has been done in the studies related to gamification in terms of mathematics teaching? What are the main criteria of gamification?

In mathematics education, technology is often used in various ways, such as teaching numbers, specializing in arithmetic, the concretization of mathematical objects, studying many different situations related to an algorithm in a short time frame (Sahin, 2016; NCTM, 2000). Kiili (2005) stated that today, students are constantly interacting with technological tools, so using only technological tools in education is not a sustainable approach. He also stated that this problem could be solved through gamification. It can be helpful to ask some questions at this point. What is gamification emerging as a new approach in education? What has been done so far in the studies related to gamification in terms of mathematics teaching? What are the basic criteria of gamification?

Deterding, Dixon, Khaled, and Nacke (2011) defined gamification as the use of game design elements in non-game contexts. In this context, many researchers used gamification in teaching different levels of mathematics (Widodo & Rahayu, 2019; Bullón, Encinas, Sánchez & Martínez, 2018; Deniz, 2015; Tüzün, Arkun, Bayirtepe-Yagiz, Kurt & Yermeydan-Uğur, 2008). In these studies, gamification can be used to develop arithmetic specialization and positive attitude towards mathematics learning of primary school students (3rd grade) (Widodo & Rahayu, 2019), to increase academic achievement of middle school students (7th grade) and positively affected their attitudes towards mathematics (Deniz, 2015), to reinforce mathematics subjects in a fun way to university students (Bullón et al., 2018), to provide an effective way of teaching the three dimensional multi-user computer game environment (Quest Atlantis) to participants (1 person 8th grade, 2 people 9th grade, and 1 person 10th grade) (Tüzün et al., 2008).

Another dimension of gamification is about how it is constructed and what its basic criteria are. In this direction, many researchers propose different models and presented different criteria (Akgün, Nuhoğlu, Tüzün, Kaya, & Çınar, 2011; Song & Zhang, 2008; Kiili, 2005; Amory & Seagram, 2003; Prensky, 2001). For instance, Song and Zhang (2008) presented three basic indicators in the educational game design model as an effective learning environment, flow, and motivation. In the experiential game model proposed by Kiili (2005), learning is seen as a cyclical process, and these cycles emerge by gaining experience during the interaction process with the game. Prensky (2001) proposed 6 criteria for gamification, which are: 1. Rules, 2. Objectives and aims, 3. Results and feedback, 4. Conflict, competition, challenge, and opposition, 5. Interaction, 6. Representation or story.

In this study, the term "gamification" refers to the presentation of mathematical knowledge embedded in a game to be taught to the extent of the possibilities provided by a particular technological tool. In this way, learners will be able to do math without being aware of it while playing the game and will be able to intuitively access mathematical knowledge in the context of the game. They will also be able to develop awareness about how mathematical knowledge can be used in a classroom environment through gamification. With this approach, the desirable and attractive aspect of the game and technology on individuals is integrated with the teaching of mathematics, and it is aimed to make mathematics an enjoyable endeavor for all students.

Teaching mathematical knowledge through gamification can be achieved through the production of digital content. This issue has been addressed as education policy in the 2023 Education Vision Document, and steps have been taken to develop digital educational content across various platforms. Moreover, it is seen that a step further is intended to establish an ecosystem for the development of digital content and skills. In this direction, it is planned to expand the development of digital educational content in order to support meaningful and permanent learning, to enable teachers to take an active role in the development process as well as to use these contents and to create a National Digital Content Archive to support all possible teaching situations (MoNE, 2018b).

The idea behind supporting the teaching materials in education and teaching with digital content is that knowledge cannot be passively received but can be actively constructed through cognition (Von Glasersfeld, 1989). In many countries, the constructivist approach was taken as a base in the last quarter of the 20th century (Pon, 2001; Aldridge, Fraser, Taylor & Chen, 2000). Educational curriculum based on the constructivist approach was introduced in Turkey after 2005 (Çiftçi, Sünbül, and Köksal, 2013; Bukova-Güzel & Alkan, 2005). Battista (1994) stated that teachers played a key role in the success of curriculum changes. However, he noted that the beliefs most teachers have about mathematics are incompatible with the ideas underlying the reform attempts. In their observations, Kurtdede Fidan and Duman (2014) determined that the vast majority of teachers did not have enough of the qualifications required by the constructivist approach.

In the constructivist approach, individuals construct information by analyzing existing information in line with their beliefs as a result of their interaction with new ideas or situations they encounter (Airasian & Walsh, 1997). In this respect, knowledge is gained through the interaction process of individuals with any environment that is consciously or unconsciously designed. In this sense, knowledge is generated by the individual in an active manner and to the extent that the individual's cognitive status allows. With the dissemination of such environments in teaching, the individual will begin to learn their own learning action by confronting the objects that form the basis for the creation of knowledge and progress in this regard.

In the mathematics curriculum, it is stated that individuals should be competent in mathematics, science, and technology and that learning action should be formed in the form of learning to learn (MEB, 2018a). In teaching, the teacher is asked not to intervene openly in the teaching process by withdrawing to a certain extent after designing the learning environment appropriately and generally to take on the task of organizing the environment. On the other hand, students are required to acquire and internalize the mathematical knowledge in an environment where the mathematical objects are discussed freely in individual or group interaction with the designed environment and to use it by adapting it to new situations. In that stage, it is thought that prospective teachers experience environments suitable for constructivism through gamification in their learning process and that their own designs for the creation of such environments are worth researching. All of these considerations involve many complex processes. Mathematical theories of education can be used to accurately reflect these processes on the educational environment. Therefore, the arguments of the Theory of Didactical Situations, one of mathematics education theories, are used in this study.

Educational Environment Design with the Theory of Didactical Situations

Theory of Didactical Situations (TDS) offers a game-based learning approach in mathematics teaching (Brousseau, 1997, 2002). One of the main elements of this theory is the concept of the situation (Erdoğan & Özdemir Erdoğan, 2013). Situation is a collection of conditions that, when an individual needs to use mathematical knowledge, must overcome to learn it (Warfield, 2014). In theory, situations that allow students to learn mathematical knowledge based on their own cognitive states are characterized as a-didactic situations. In more detail, a-didactic situations refer to an environment (milieu) in which the mathematical knowledge to be taught is presented through a game and for a while the knowledge to be taught is hidden from the students and the instructional situations (an environment in which teacher interventions are limited) in which the students interact with the target information (their own knowledge) (Warfield, 2014). In a-didactic situations, the design of the environment is considered very important. The environmental design needs to identify objects that can be included in the environment by analyzing changes in the environment to reveal the target knowledge (Erdoğan & Özdemir Erdoğan, 2013; Erdoğan, 2016a). The environment is explained as everything the individual interacts with (Brousseau, 1997). For example, a well-designed game, a problem, the arguments of the opposing group in group work, the mental or concrete materials presented to the environment can be evaluated in this context. Brousseau (1997) stated that to maintain the functionality of a-didactic conditions, and it should include the following stages:

• *Devolution stage*: The stage in which students must take responsibility for the game to achieve meaningful learning without relying on teacher feedback (Ligozat & Schubauer-Leoni, 2010).

• *Action stage*: The stage in which the student interacts with the milieu and makes sense of it (Warfield, 2014).

• *Formulation stage*: This is the stage in which students express their ideas implicitly developed (e.g., hypotheses) during the action phase (Warfield, 2014).

• *Validation stage*: The stage in which the student explains the validity of the ideas developed during the formulation stage.

• *Institutionalization stage*: The stage in which the class develops (or explains in a way that other people understand) to the mathematical dimension by revising, shaping, and classifying, where necessary, the teacher's ideas in the stages of action, formulation, and validation in sequential or intertwined form (Warfield, 2014).

The other important dimension of the theory is the didactic contract. The didactic contract is defined as a whole of rules (written or unwritten) governing all kinds of relationships implicitly existing between teacher and student (Brousseau, 1988). The existence of the didactic contract can be observed, especially when it is broken (Erdoğan, 2016b; Yavuz, Arslan, and Kepceoğlu, 2011). For instance, in an activity aimed at a mathematical understanding that is likely to arise in the process of solving the problem given to students by a teacher in an environment designed according to constructivist approach, asking the teacher to solve the problem can be considered as an attempt to break the didactic contract. In this case, the teacher may recall the didactic contract or remain uninterested in the question.

When the literature is reviewed, it is seen that there are a limited number of studies in which TDS is used as a theoretical framework for teaching mathematical knowledge in the classroom environment. Samaniego and Barrera (1999) suggested that a-didactic environment could be designed for teaching some problems designed with the help of a TI-92 graphing calculator for 12th-grade students. However, the study did not give any explanation as to how the problem solutions and the application of the course were conducted. 9 in the study of Dikkartın Övez and Akar (2018) gave the students a problem situation and examined the students' ability to access function knowledge in an a-didactic learning environment. The results of the study revealed that groups that interacted strongly with milieu were successful in structuring the concept of function; otherwise, there were difficulties. Similar situations were also observed in the study conducted by Erdoğan, Gök, and Bozkır (2014). In this study, the concept of proportionality in an adidactic environment was studied in the classroom environment for 6th-grade students. A problem situation was developed and presented to the students in accordance with TDS. It was observed that the students progressed in knowledge when interactions with milieu were provided. At this point, researchers face the problem of how to make students' interaction with milieu stronger. An effective way to do this is through gamification.

Similar cases can be observed in other studies. In these studies, it was determined that there were some setbacks in a-didactic situations. One reason these problems occur is an incomplete configuration of the environment design. Gök and Erdoğan (2017) investigated strategies that students intuitively felt a need to use in a non-routine problem-solving process to 6th graders who were not given any training on problem-solving strategies in the a-didactic setting. The study found that many heuristic strategies were used successfully. On the other hand, some strategies were not observed. It is thought that the discussion of which arguments may arise from which unintended strategies can be provided in the environment can be useful for future studies. Baştürk Şahin, Şahin and Tapan Broutin (2017) examined the functionality of the designed environment in the study, where the teaching of prime numbers to 6th-grade students was researched in terms of a student-centered approach in action research they planned in the TDS framework. In this study, it is observed that in some situations it is expected to make inferences from the students about the progress of the information in the environment. On the other hand, there is no discussion about how these implications can be presented without disturbing the a-didactic structure in the environment. In addition, many discussions, such as in verification stage, can be experienced and some stages that require a certain process appear to be less effective than expected. In their study, Arslan, Taşkın, and Kirman Bilgin (2015) investigated the impact of two a-didactic environment designs on the teaching of the field of parallelogram and trapezoid to 7th-grade students on student achievement from an individual study or group study perspective. They found out that the designed environment supports more individual studies of the students. The reason why such environments do not support group work involves a certain degree of uncertainty. Güneş and Tapan Broutin (2017) examined the teaching of Pythagorean relation in an a-didactic setting to 8th-grade students in their research. They suggested that students were not willing to find other solutions after finding one solution. Similar results were found out in different studies (Erumite, Arslan and Erumite, 2012). Arslan, Baran, and Okumuş (2011) examined the processes of finding the center of gravity of the triangle of 8th-grade students in an a-didactic environment. It was indicated that there were obstructions in the process from time to time and that they were dealt with by the guidance of teachers so that the structure of the a-didactic environment could be maintained. Yavuz and Kepçeoğlu (2016) portrayed the in-class actions of mathematics teachers within the framework of TDS. In that study, it was expressed that in-class actions involved many complex processes (design of the didactic environment, teacher's position, didactical time processes, and didactic contract) and that awareness of teachers' actions can be generated through such investigations. In another study, mathematical thinking processes of prospective elementary school mathematics teacher were researched in case of problem-solving in an a-didactic environment (Josephus problem) (Çelik, Güler, Bülbül, & Özmen, 2015). In the designed environment, it was seen that prospective teachers used many mathematical process skills (such as presenting hypotheses, trial, and error, reasoning, proving, and expressing). Even though they presented a model for solving the problem, they were insufficient to explain why this model works in all situations. These inferences obtained from the studies reveal how much attention should be paid to the activity design. It is thought that the problems that may arise with better-designed activities and feedbacks that can be given in appropriate places during the implementation of the activities can be solved. In this context, gamification provides positive indicators for overcoming obstacles in mathematics teaching.

In the design of the environment suitable for the constructivist approach where knowledge does not require teacher intervention through gamification, problems can be handled thanks to many arguments that can be placed in the game during the teaching process. For example, visualization, adding sound effects, coloring, guiding numbers, or texts that can be written into objects can be evaluated in this context. Furthermore, after the game is designed, many pilot applications can be made, and different solutions can be developed for the problems that arise after each pilot application. All this reveals the importance of gamification for mathematics teaching.

Gamification has great potential in the teaching of mathematical knowledge, but it appears to be limited to several studies in the literature and at the academic level (Widodo & Rahayu, 2019; Bullón et al., 2018; Deniz, 2015; Tüzün et al., 2008). This shows that gamification is not adequately used in teaching and cannot be disseminated at an adequate level. This can only be achieved by the effective participation of teachers or future teachers in the dissemination of gamification (MEB, 2018b). Therefore, this study shows how mathematical knowledge can be presented to prospective classroom teachers through gamification in an environment appropriate to the constructivist approach. In this research, prospective teachers need to reach the mathematical knowledge behind the designed game by tackling the limitations and conditions of the game. This approach was followed with the idea that the experience of the future's teachers in the process of playmaking and its aftermath is effective in internalizing the playmaking processes.

In this study, a number pattern was preferred as an object of mathematical knowledge. The reason for this can be explained as patterns form the basis of many subjects in mathematics teaching. In this context, linking patterns with subjects such as algebraic expressions and equations in secondary, school functions in high school and laying the foundations of patterns in primary school (Uyghur-Kabael & Tanışlı, 2010) has been instrumental in addressing this topic in the study.

In this study, the approaches revealed by prospective classroom teachers to experience the situation of teaching through the gamification of a number pattern designed according to TDS are described. The main objective of the study is to teach prospective classroom teachers how to present the number pattern to the students through gamification, rather than teaching the number pattern determined through gamification, and to develop awareness among the prospective teachers about how game-student interaction occurs in this process. Secondly, as a reflection of these acquisitions, the cases of gamification of an information object identified by prospective classroom teachers were examined. In this respect, this study is looking for answers to the following research questions:

- How did prospective classroom teachers interact with the mobile game at different stages of TDS in the process of attaining mathematical knowledge?
- What are the opinions of prospective classroom teachers about the mobile game designed according to TDS?
- How can the games designed by prospective classroom teachers be evaluated in terms of the criteria for gamification?

METHODS

The qualitative research method was used in this study. The model of the study can be specified as the case study. Merriam (2013) described the case study as an in-depth depiction and study of a limited system. In this study, teaching environment design for the presentation of mathematical knowledge with an educational mobile game created using TDS arguments to prospective classroom teachers was examined. The designed application was applied to prospective classroom teachers to internalize the gamification processes. In addition, the games designed by prospective classroom teachers for the teaching of mathematical knowledge determined by them were evaluated in terms of the basic criteria of gamification. The study reflects part of a scientific research project. As part of the project, three games were designed (online tickets, race by numbers, and bacterial colony) and applied to prospective classroom teachers (Classes A and B separately). In this study, the *online ticket* game and the data obtained by the application of this game to prospective classroom teachers in class A were examined.

The aim of the *Online Ticket* game is to design an environment suitable for the constructivist approach in order to teach mathematical knowledge to prospective classroom teachers. Therefore, in this study, the phenomena of the prospective teachers in a constructivist environment, and the implications they place on them are examined within the framework of TDS (Denzin & Lincoln, 2005).

TDS allows designing environments suitable for the constructivist approach (Artigue, 1994; Laborde, 2007). However, it was observed that prospective classroom teachers had theoretical knowledge in this context but could not reflect it into practice. Therefore, the basic elements of TDS, its stages, the roles of students and teachers in these stages, the problems used by the theory, and the environment design were explained in 4 hours. In addition, in order to internalize these processes by prospective teacher, *race to 20* (Brousseau, 1997), *farmers' wheat harvest problem* (Erdoğan et al., 2014), *Caesar and the prisoners problem* (Gök & Erdoğan, 2017) were applied to each application in the classroom setting with 2 lesson hours allocated. The application was made by one of the researchers, and after each application, prospective teachers were informed about the processes of TDS. After all these processes were completed, the *Online Ticket* game was implemented.

Research Participants

The research participants were 17 (9 female and 8 male) prospective classroom teachers studying at a state university in the Eastern Anatolia region. Participants were selected by the purposive sampling method. It is aimed to obtain rich data on the research topic in purposive sampling (Yıldırım and Şimşek, 2016). In this respect, the participants are the prospective classroom teachers who wish to take part in the game process voluntarily as part of the requirements of the Teaching Mathematics I course.

Voluntary participants from the Classroom Teaching Department Class A were divided into two groups, and the application was carried out. Other students remained in the position to monitor the practice process. One of the groups was formed as 9 and the other as 8. The game was implemented in the context of a competition in which groups played against the computer. It started with any of the groups (e.g., Group 1), and one participant from that group played the game against the computer on the smartboard. Then a student from the other group (Group 2) similarly played the game against the computer on the smartboard. The winner of the game earned 1 point on behalf of their group. (A separate scoring from the scoring in the game). Participants were given odd numbers AK1, AK3,..., AK9 in Group 1 and even numbers AK2, AK4,..., AK8 in Group 2. The application was conducted by one of the researchers, and this person was coded as a teacher (T) in the findings.

Online Ticket Game and Scenario

FIGURE 1. *Introduction and levels of the game*

This is a game involving the player's challenge against the computer. The context of this game is based on the purchase of tickets for a friendly match between Turkey and Brazil ahead of the 2018 World Cup by fans via a website (artificial intelligence or an algorithm controls the operations of Brazil fans.). However, the distribution of the tickets for this football match is not sold at random. They are sold according to the rules described below.

The seat indicated by the press (B) in each stand is reserved for the press, so it is not for sale.

2. The stands of the stadium are named A, B, and C and have a capacity of 24, 35, 48 fans, respectively, except for the press box. Seats in the stands are numbered from 1 onwards.

3. The sale of tickets in any stands (e.g., Stand A) is carried out from small numbers to large.

4. Ticket purchase is made in order (A Turkish after a Brazilian, then vice versa, etc.), and the number of fans is changed (the fans of both teams have at least 1 maximum 2 tickets for the first at least 1 to 4 tickets for the second, and at least 1 to 6 tickets for the third time, etc.). In other words, the number of tickets that the fans can buy alternately is determined to be at least 1 and at most 2 more than the maximum number of tickets that can be received in the previous purchase.

5. The first ticket in the stands can be bought by the Turkish or Brazilians.

6. In sections A, B, and C, the fan who purchased the last ticket (i.e., the person with the largest number of seats) wins 2 prize tickets to the 2018 World Cup.

How do you think it is possible to always win 2 prize tickets for each stand in this friendly match?

The aim of the game is to determine the numbers that win 2 award tickets for the 2018 World Cup by purchasing the last ticket in each stand and to develop a suitable strategy considering the common characteristics of these numbers.

Feedback Provided By the Mobile Game for the Acquisition of Target Information in the Game

In order for the target information to be revealed in the game, it is necessary to give positive or negative feedback to prospective classroom teachers in the process of playing the *online ticket* game. Accordingly, the following feedbacks are planned to be given in the *online ticket* game.

- Total number of seats
- Displaying each move in a different color
- Win-lose situations
- Monitoring the total number of seats reached at the end of each move,
- Earning points when the seat numbers in the winning algorithm are purchased,
- Seats are numbered.

Game Design and Coding

The game was written by one of the researchers. With Android, game design and coding operations are performed as a result of applying certain process steps, respectively. The game design began with the design of the screens to be created in the game and the creation of it with charcoal. After that, many features of each element in the design scene, such as style and position, are indicated. The coding part was carried out using Java language in the Android Studio environment version 2.1.

Below is the starting position of the game process for the Stand A (24 seats) in the online ticket game (Figure 2). In this image, the *A section* in the top row shows the selected Stand A, and the *score* explains the points, the level, and the final point when played correctly. The *new game* shows the button that can be used when you want to play again, with the I⊲ button switching to the previous game and the ⊳I button switching to the next game. Below the arrows to the left and right of the visual are given in turn the number of seats that the player and computer can choose. Two seats (seats 1 and 2) are marked in blue on the playing area. If the player chooses any seat, it is marked in red. Then it will be artificial intelligence's turn (Brazil fans), and the choices of artificial intelligence will be marked in yellow. Selections are made without skipping from small to large. Seats that have not yet been elected are marked in grey. The number of seats that can be selected in each move is indicated on the left and right of the screen, and information about these selections is given at the bottom of the screen.

FIGURE 2. *First game for stand a after piloting*

Game Application and Data Collection

The pilot study was carried out by applying the designed game to 2 prospective classroom teachers. The piloting lasted 57 minutes. As a result of it, it was decided to make changes to the visual design of the game and some technical issues. Easy, medium and difficult levels were designed for Stands A, B, and C in the game. These levels were not changed, but a button (forward and back button) was added to switch between different games. It was decided to shorten the wait time between turn selections of players and computers. During the selection of seats, it was decided to add a sound effect. In addition, a button was added to create an option for the player or computer to start the game first. It was understood that this button had the potential to play an important role in confirming or rejecting certain hypotheses that may be given by game testing. In addition, the "selection over" button was added in order that comments can be made about the game progress at any moment of the game. In this way, players' strategies can be questioned through the game.

The main application lasted 70 minutes. This was carried out on a smartboard to make it easier to implement and discuss the game in a classroom environment instead of a mobile phone. Data from the main application were collected through the recording of the event with a video camera and a voice recorder. These data were transferred to the computer by means of repeated listening of the recordings. These tools were used consciously. Through a cameraman, the camera recorded data on situations such as the teacher in the environment, the movement, the game process being played on the board. On the other hand, it is not possible for the camera to fully monitor in-group conversations and in-group dynamics. In this context, the discussions and the group dynamics were recorded by putting the sound recorders on the tables of the groups. Thus, data loss was minimized.

After the main application, the participants were given a period of 1 week and asked to design a game for the teaching of mathematical knowledge to be determined by themselves through gamification. They were informed that they could use TDS during the gamification process, but not only limited to it. Gamification involves many processes, such as the game's scenario, coding, and implementation. Since the coding part requires expertise and permission is required for the application part, it was requested to present the design dimension for coding and the possible approaches for implementation in presenting them during the gamification process.

A Game against Computer and Some Possible Solutions

FIGURE 3. *An example game*

In this game, the seats in Stand B (35 seats) are offered for sale. In this direction, seats are purchased from the smallest numbered seat to the largest numbered. The purchase process was started by Turkish fans first (at least 1 and at most 2 seats can be selected), and they purchased 1 seat (Seat Number 1). Then it was time for Brazilian fans (at least 1 and at most 2 seats can be selected), and they purchased 1 seat (Seat Number 2). Then it was the turn of the Turkish fans (at least 1 and at most 4 seats can be selected), and 1 seat was chosen (seat number 3). Then it was up to the Brazilian fans (at least 1 and 4 seats can be selected), and they had 1 seat (Seat Number 4). Later, Turkish fans (at least 1 and at most 6 seats can be selected) selected 4 seats (5-8 seats). Next, Brazilian fans (at least 1 and at most 6 seats can be selected) had 1 seat (Seat Number 9). Turkish fans (at least 1 and at most 8 seats can be selected) chose 6 seats (10-15 seats). Brazilian fans purchased 5 seats (16-20 seats) (at least 1 and at most 8 seats can be selected). Turkish fans (at least 1 and at most 10 seats can be selected) selected 4 seats (21-24 seats). Brazilian fans (at least 1 and at most 10 seats can be selected) had 9 seats (25-33 seats). Finally, Turkish fans (at least 1 and at most 12 seats can be selected) chose 2 seats (seats 34 and 35). It is seen that one of the Turkish fans won the game. The game process described above is given in the findings section as T: 1 → B: 2 → T: 3 → B: 4 → T: 5-8 → B:9 → T: 10-15 → B: 16-20 → T: 21-24 → B: 25-33 → T: 24-25. (T: purchases by Turkish fans and B: purchases by Brazilian fans). In the game, some winning strategies can be expressed as follows:

- Selecting the largest numbered seat in each row,
- Selecting seats 3, 8, 15, 24 and 35 in order,
- Choosing the seats with the preceding number of each square number,
- Choosing the seats by recognizing the additive pattern, i.e. seat 3, then (3+ “5” = 8) seat 8, then (8+ “7” = 15) seat 15, (15+ “9” = 24) seat 24, and the last one (24+ “11” = 35) seat 35.

Data Analysis

The analysis of the research data was conducted by direct quotations at different stages of TDS, with a descriptive analysis method that aims to reflect the stages as they occur. In this context, interactions between participants in different stages of TDS, behaviors expected from participants, and the progress in the knowledge in the environment are planned to be made as given in Table 1.

Table 1. *The roles of individuals, interaction and change in knowledge at different stages of TDS*

Stages	Interaction	Teacher	Participants	Change in Knowledge
Devolution	T → Px	Active	Passive	Understanding the game
Action	G ↔ Px	Guide	Active	Informal strategies
Formulation	T ← Px ↖ Py	Guide	Active	Formulate hypotheses by generalizing informal strategies
Validation	G ↔ Px ↘ ↕ Py	Guide	Active	Convincing, proving or refuting the opponent
Institutionalization	T → Px	Active	Passive	Presenting formal information

T: Teacher, Px, and Py: Participants, x and y=1, 2,…, 17; Game: G

As can be seen in Table 1, the teacher takes an active role in the devolution stage, explaining how to conduct the activity (forming groups, the nature of their feedback, scoring, etc.), the purpose and rules of the game. At this stage, the teacher must place situations in which the game will be understood by the participants. During the action stage, prospective classroom teachers interact with the game and develop some informal models that they think are winning strategies. After these models are discussed within the group, they are generalized and formulated as hypotheses of the groups. In the validation stage, these hypotheses are confirmed in the context of the game in large class discussions. In this process, knowledge of the class is constantly revised with confirmed hypotheses. In this way, progress is made in the context of the game towards the target information. In the stages between devolution and institutionalization, prospective classroom teachers are active, and the teacher has an organizing role. Finally, in the institutionalization stage, the knowledge that the prospective teachers reach in the context of the game is explained by the teacher in the formal sense.

Another dimension of the study are the files prepared by prospective classroom teachers for gamification, and they were evaluated in the context of the basic criteria given by Prensky (2001). These rules include 1. Rules, 2. Objectives and aims, 3. Results and feedback, 4. Conflict, competition, challenge, and opposition, 5. Interaction, 6. Representation or story. Descriptive analysis of the files prepared by prospective teachers was done in terms of the criteria stated. In the findings, if the relevant criterion was fully observed, it was marked as yes, while other cases were marked as no. The online ticket game is analyzed in the context of these criteria in Table 2 to give an idea of how the analyzes performed.

Table 2. *Analysis of online ticket game in terms of gamification criteria*

Criteria	Game Process	Yes / No
Rules	Online ticket game and scenario (See Method)	Yes
Objectives	Online ticket game and scenario (See Method)	Yes
Feedback	Feedback provided by the mobile game for the acquisition of target information in the game (See Method)	Yes
Challenge	A game against the computer and some possible solutions (See Method)	Yes
Interaction	See Table 1	Yes
Story	Online ticket game and scenario (See Method)	Yes

Table 2 shows that the online ticketing game meets all the criteria in the context of gamification. This game was applied to the prospective classroom teachers trying to raise awareness of them about all the processes of gamification.

Regarding the validity and reliability of the research, the game was designed by a mathematics educator who specialized in TDS and its fundamental arguments. Limitations and conditions were determined within the scope of a-didactic situations within the framework of TDS concerning the emergence of target knowledge in the design process. The integration of these into mobile gaming and how knowledge can be presented implicitly in the a-didactic environment was planned with the researcher who designs software. In this planning, the limitations of the program in which the game was written were also taken into consideration. As a result, a game called *online ticket* was developed between the mathematical dimension of the game and the software dimensions under the cooperation of two researchers. On the other hand, during the design process, the prototype version of the game was applied at different times to two researchers, a specialist teacher, and a middle school student. After each trial, the game was revised again and strengthened in terms of both the emergence of target information and the mechanics of the game. It can be suggested that revealing the processes in the research clearly increases the reliability of the research. In the study, the processes of online ticket game in the context of TDS were controlled by two researchers who are experts in mathematics education. In addition, the files prepared by prospective teachers about the game were analyzed independently by two researchers in the context of the basic criteria of gamification. For the reliability of these analyses, the Consensus/(Consensus + disagreement) x 100 formula proposed by Miles and Huberman(1994) was utilized. The value calculated in this study was found to be 74% and considered reliable for the study.

RESULTS

The findings of the study are given in two sub-sections. The first involves prospective classroom teacher’s mobile gaming experiences designed according to TDS. The second consists of the analysis of the games designed by prospective classroom teachers in the context of gamification.

A Mobile Game Experience of Prospective Classroom Teachers

In a student-centered environment, the interaction between game, teacher, and prospective teachers at different stages of TDS is given below.

Devolution Stage

The teacher indicated to prospective classroom teachers that they would play a game on the smartboard. First, he divided the class into two groups and then made the following statements:

> T: … we have prepared a game. In this game, there will be a friendly match before the World Cup. In this friendly match ... the fans ... They buy tickets through an online system. It is in real life too...
> AK2: Yes, yes!

> T:… Of course, the rules are a little different… but the concept is the same, guys… The aim of this game is briefly to buy the last ticket. Of course, you are Turkish fans... artificial intelligence is Brazilian... What will you do?… You will try to purchase the seat with the largest number in the end… The rules of the game are: As you see here, this is Stand A; of course, there will be Stand B and Stand C, and this is Level 1, guys. We can learn the level here. [In the box, it says Level: 1, Step: 1, and Score: 0] Stand A is always filled with 24 fans, there is a B here. B represents the press box… You cannot choose B... But you can choose the following. [24 remaining seats]. Of course, how is the selecting process? In turn, you choose, and computer does. Let's look at this. Turkey is selected, and there are 1-2 tickets, right?... Brazil will be active after you select, and they will purchase 1-2 tickets... We put a button like this. Brazil can start the game first too… [First computer started the game] Now the computer has chosen [seat 1 selected], you can choose the seats numbered 2 and 3. For example, let me play the game so that you can see. Thus, you get a better grasp of the game. I chose 2… then there is a button saying that the selection is over. You have to click on it, so the computer knows that your turn is over now. Now everything is changed. Four seats were opened. First, 2 seats are opened, then 4 seats, then 6 seats, then 8 seats, and it will increase like this… and will continue in this way.

As can be seen from these explanations, the game is designed to allow groups to play the game against artificial intelligence. This game was designed as mobile but implemented via a smartboard through an intermediate program. The teacher stated the context, rules, and purpose of the game. Then the teacher played the two games randomly on the smartboard. He won one of these, while he lost the other. These games were played without any strategy and the rules of the game were tried to be explained. The teacher explained some of the variables that changed on the screen of the smartboard (points at the top of the screen, the texts changing on the right, left and bottom of the screen) after each move made during the game playing process and explained how the prospective teachers would benefit from the game environment. It can be stated that presenting the game using the touch screen feature of the smartboard facilitates the adaptation of prospective teachers to the game. In this way, the direction of the source of information was devolved from the teacher to the game. Therefore, prospective teachers will be able to use the changed objects effectively on the game screen in the next stage (action stage). Recognizing, analyzing, and drawing conclusions about the changes of objects in the environment in the gameplay has an important role in the emergence of strategies that will win the game. These strategies initially appear informally and can be developed during the game process. As a result, in the development of these strategies, changing objects in the game process give the participants positive or negative feedback. It is of great importance for the game to give positive or negative feedback to the players, to maintain the dynamic structure in the environment and to provide progress and development in knowledge.

When the devolution stage is examined in terms of gamification, it can be stated that prospective teachers are an effective way to present the rules and objectives of the game. It is also seen that at this stage, it offers a specific approach to prospective teachers on how to design arguments that students can get feedback from the game object and how to present them to the environment.

Another important aspect of the game can be explained as the quick start strategies and adaptation process. It can be indicated that the design of the game is easy, that it contains information that guides the player (for example, coloring), and that it contains descriptions (text at the bottom of the screen) in this direction. It can also be suggested that the game is presented with a daily life context in which online tickets are purchased for a football match, allowing it to be easily understood by prospective teachers. In other words, the presentation of the story or scenario in the process of playmaking by giving the context of daily life accelerates the adaptation process.

As the teacher moved from one move to another during the gameplay, he explained the changes on the screen after clicking on the "selection over" button. This button acts as a tool that gives the player enough thinking in the game as he moves from one turn to another. New moves cannot be made without the player pressing this button. This allows the player to progress through the game at his own pace. The interaction during the devolution stage is given in Figure 4.

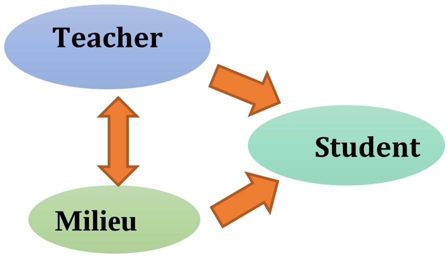

FIGURE 4. *Interaction during the devolution stage*

At this stage, in the process of explaining the points that are not understood about the instructions of the game, behaviors contrary to the didactic contract between the teacher and the prospective teachers may occur. An example of these is presented below.

T: There was something you didn't understand?
AK3: Well, sir, you've done it right now, but what if we do it wrong?
T: And if we do wrong, the computer wins?
AK1: How did you do it, right?
T: You will figure it out.

In such situations, the teacher may act to ignore the question or to remind the didactic contract. In this study, due to the constructivist approach, the teacher reminded the didactic contract by stating that the task of introducing the game was for teacher, but the task of finding the winning strategy was for students. It was determined that the devolution stage offers opportunities for prospective teachers in terms of how the rules, purpose, and scenario of the game can be transferred to the learners in terms of gamification. This stage took 10 minutes. It was also observed that prospective teachers largely understood the game. Then the teacher opened a new game and asked one of the group members to play the game on the smartboard. In this way, transition to action stage took place.

Action, Formulation, and Validation Stages

In this study, the stages of action, formulation, and validation are intertwined and given together since it is not possible to cut clear the boundaries of these stages. First, the action stage is the stage where prospective teachers interact with the game and make some inferences about the game from win-loss situations at the end of the game. The teacher initiated the action stage with the following words.

> T: Yes, it's your turn now. I start a new game. We are competing against the computer. And here is what you need to keep in mind: is it always possible to win in this game?.... You think about that too, okay? Let's start here, one at a time. Here you are. And We are also competing against the computer between groups. We are scoring that, too. The winner receives 1 point, while the loser receives no points.

When these explanations are examined from the point of view of gamification, it is understood that the interaction between the game and prospective teachers begins at the action stage. At the beginning of this stage, it is predicted that the interaction will be mostly between the game and prospective teachers. Prospective teachers will notice the feedback mechanisms in the game throughout this stage. Although these are explained by the teacher during the devolution stage, there will be no feedback unless these are rediscovered by prospective teachers during the action stage. In addition, the teacher's suggestion to direct prospective teachers to investigate as to whether there is always a winning strategy in the game suggests that the game in the environment involves the challenge for the players. The game difficulty level was structured as easy, medium, and difficult. There were also three stages (Stands A, B, and C) at each level (see Method section). Designing the game from easy to difficult can also be described as a challenge to a certain extent. The first games were played at the easy level and the next games at the medium and difficult levels. The game processes of prospective teachers on the smartboard are given below in Table 3.

Table 3. *Game processes of prospective teachers in the action, formulation, and validation stages*

Number of Game	Player	Game Process	Win/Lose
1	AK5	B:1 →T:2→B:3→T:4→B:5-8→ T:9-12→ B:13-18→T: 19-24	1
2	AK2	B:1 →T:2→B:3→T:4→B:5-8→T: 9→B:10-14→T:15→B:16→T:17-24	1
3	AK1	B:1-2 →T:3→B:4-5→T:6-7→B:8-11→T: 12-13→B:14-19→T:20-23→B:24-30→T:31-35	1
4	AK4	T:1-2→B:3→T:4-7→B:8-10→T: 11-12→B:13→T:14-21→B:22-26→T:27-35	1
5	AK3	T:1→B:2-3→T:4-5→B:6-8→T: 9-11→B:12-17→T:18-25→B:26-30→T:31→B:32-34→T:35-46→B:47-48	0
6	AK6	T:1-2→B:3-4→T:5-6→B:7-9→T: 10-14→B:15-16→T:17-24→B:25-27→T:28→B:29-32→T:33→B:34-36→T:37-48	1
7	AK9	B:1→T:2→B:3→T:4-7→B: 8→T:9→B:10-15→T:16→B:17-24	0
8	AK10	T:1→B:2→T:3→B:4→T: 5-10→B:11-12→T:13-15→B:16-21→T:22→B:23-25→T:26→B:27-35→T:36-48	1
9	AK11	B:1-2→T:3-4→B:5→T:6→B: 7→T:8-9→B:10-15 →T:16-21→B:22-24→T:25→B:26-35	0
10	AK8	T:1→B:2→T:3→B:4-6→T: 7→B:8-11→T:12-15→B:16-17→T:18-24	1
11	AK13	B:1-2→T:3-4→B:5→T:6→B: 7→T:8-9→B:10-15 →T:16-21→B:22-24→T:25→B:26-35	0
12	AK12	B:1→T:2→B:3→T:4→B: 5-8→T:9→B:10-15 →T:16→B:17-24	0
13	AK15	B:1→T:2-3→B:4-5→T:6-7→B: 8→T:9→B:10-15 →T:16-23→B:24→T:25→B:26-35	0
14	AK4	B:1-2→T:3-4→B:5→T:6-9→B: 10-15→T:16→B:17-24 →T:25→B:26-35	0
15	AK4	K_4 T:1→B:2-3→T:4→B: 5→T:6-9→B:10-15 →T:16→B:17-24→T:25→B:26-35	
16	AK7	B:1-2→T:3-4→B:5→T:6→B: 7-10→T:11-14→B:15 →T:16→B:17-24→T:25→B:26-35	0
17	AK1	T:1→B:2-3→T:4-7→B: 8-10→T:11-13→B:14-15 →T:16-23→B:24→T:25-31→B:32-35	0
18	AK8	T:1→B:2→T:3→B: 4-7→T:8-9→B:10-15 →T:16-17→B:18-24→T:25-34→B:35	0
19	AK11	T:1→B:2-3→T:4-7→B: 8-10→T:11→B:13-15 →T:16-20→B:21-24→T:25→B:26-35	0
20	AK11	T:1-2→B:3-4→T:5→B: 6-8→T:9-12→B:13-15 →T:16-21→B:22-24→T:25-27→B:28-35	0
21	AK2	T:1→B:2-3→T:4-6→B: 7→T:8-9→B:10-15 →T:16-18→B:19-24→T:25→B:26-35	0
22	AK17	T:1→B:2→T:3-4→B: 5→T:6-11→B:12-15 →T:16→B:17-24	0
23	AK16	B:1-2→T:3→B: 4→T:5-8→B:9-12 →T:13-15→B:16-21→T:22-24	1
24	AK16	B:1-2→T:3→B: 4-5→T:6-8→B:9-11 →T:12-15→B:16-18→T:19-24→B:25→T:26-35	1
25	AK16	T:1→B: 2-3→T:4-7→B:8-10 →T:11-15→B:16-19→T:20-24→B:25-27→T:28-35	1

Win: 1, Lose: 0, Brazil or Computer: B, Player or Turkey: T

Table 3 shows that 25 games were played in the stages of action, formulation, and validation. For example, the 18th game was played by student coded AK8. Since this game process started with T, the first player started to play (if it started with B, it would be understood that the computer started the game first) and since there were 35 seats, it was understood that the selection was made in Stand B (See Method). The game process is as follows: In move 1, the player chose seat 1, and computer seat 2, in move 2, the player selected seat 3, and computer seats from 4 to 7, and in move 3, players selected seats 8 and 9, and computer selected seats 10 to 15, in move 4, player selected seats 16 and 17, and computer selected seats from 18 to 24, in move 5, player chose seats from 25 to 34 and computer chose seat 35. The computer (or Brazilian fans) won the game because the last seat was chosen by the computer. The other games can also be interpreted in a similar way. In this table, it is seen that prospective teachers won the game because of the easy level of the first games, but mostly artificial intelligence (computer) won because the games were played at medium and difficult levels. The fact that the game contains three different levels and the difficulty experienced by prospective teachers to win the game as the levels progress shows that the challenge criterion is sufficient in terms of gamification. The following dialogue is interesting about the existence of a winning strategy in the game that the computer constantly wins the game (from game 11 to game 22).

> T… Look, you were always winning at level 1. What happened? (Meaning that they could not win the game in the next levels) If the computer always wins, is there a strategy that always wins?
> AK8: There is.
> AK6: Yes, teacher. There is, actually.
> T: Think about that strategy now, slowly.

When this dialogue is evaluated in terms of gamification, the computer's continuous win indicates the existence of a winning strategy in the game. This supports the idea that more careful monitoring of the computer's moves and feedback can be provided from here. The recognition of the feedback provided by the game led to the emergence of winning strategies in the game.

The teacher then asked the groups to come up with hypotheses that they now think are winning strategies. In these games, the teacher only assumed an organizing role in the environment, while prospective teachers interacted with the game and proposed hypotheses (winning strategies) in the game. In Table 4, the hypotheses suggested by prospective teachers in the game process are given.

Prospective teachers presented a total of 10 hypotheses in Table 4, which they considered to be the winning strategy in the game during the game process. Some of these hypotheses were approved by reasoning (hypothesis 2), and some (hypotheses 3, 6, 8, and 9) were rejected by replay. Some hypotheses are neither approved nor rejected (1, 4, 5, and 7. hypotheses). It can be stated that these hypotheses carry uncertainty or need to be confirmed. The hypotheses given in the first games are often ambiguous, but the hypotheses given towards the end of the game are either confirmed or rejected. While ambiguities do not give clues to the emergence of the winning strategy, hypotheses that have been approved or rejected are of great importance because they give positive or negative feedback in terms of the emergence of the winning strategy. From hypotheses approved or rejected in the context of gamification, prospective teachers gained a lot of feedback. This feedback made progress in the knowledge in the environment and always played a big role in the emergence of the winning strategy. In recent games involving approved or rejected hypotheses, it can now be said that the requirements of the validation stage were met. It can be argued that hypotheses 2 and 6 (one approved and the other rejected) and the fact that the computer won the game continuously for a period of time played a role in the emergence of the winning strategy in this game process. Following the game process carefully, AK16 found one of the winning strategies and tested this strategy in 3 different games at a difficult level to get it approved. In the dialogue below, the winning strategy and the classroom knowledge of this strategy are given.

Table 4. *The hypothesis of prospective teachers*

No.	Game	Player	Hypotheses Presented and Verification Process of Some	Win/Lose	Approval/ Rejection
1	1	AK5	… I did not quite get it, but mostly I tried to go double-double.	1	?
2	10	AK8	… I improvised at first... and then I started counting. Now I, well, always left the last seat to myself... I had to give the following numbers (16-23) to the opponent in order to take it (pointing to 24). That's why I chose 15. It had to choose one of these. From 16 to 23, no matter which one it chose, 24 was all mine.	1	Approval
3	7	AK9	I counted 10 backward from the last seat, but it was not enough. I made it to 13.	0	Rejection
4	12	AK12	Teacher, before selecting 1-10, I had 1-8 beforehand. If I chose 10, well, (changed the decision) If I chose 18, I think there was a chance to come.	0	?
5	11	AK13	T: So, how could you win the game? AK13: Well, it is like this. When I am at 8, 10 more seats are opened for it. I had to select, depending on this. If it was 10 for me, I needed to choose 12. So, I made two mistakes. Ö: What do you need to select to win, for example? AK13: Like 23 and 22.	0	?
6	14 and 15	AK4	AK4: (presents hypothesis) If we start from 1, the square of numbers, 1's square is 1. 2's square is 4. 3's square is 9. Then, we will select 16 here and reach 25 (played and lost). But I think we need to start from 1... But we did not choose 1. Ö: What is your strategy? (game restarted) AK4: Square numbers... (lost game) It is never possible to win (laughs).	0	Rejection
7	18	AK8	AK4: According to moves, the number of tickets is according to the number of seats; in fact, we should count backward, not forward. AK8: … Wait a minute; then, we have to give 25 to it… (his group members asked him to choose 25) I told you. I swear, I told you. After all, we need to leave 25 to it to win the game. Actually, if it was 24, I would choose 24. I would leave 25 to it.	0	?
8	19 and 20	AK11	AK11: I selected the wrong one. If I chose 21, teacher, it was okay. Ö: Let's see the hypothesis to see if it is true. Because your friend actually proposed a hypothesis. He says, if I choose 21, I win… (played again but could not win)	0	Rejection
9	22	AK17	AK17: Could I choose this one? I could have won if I had chosen this. Ö: Which one? AK17: 14. (played but lost)	0	Rejection
10	23, 24, and 25	AK16	AK16: ... when we get the last seats (the largest numbered seat in each row), the computer has to get the other seats, and we get the last seat.	1	Approval

AK16: Sir, my strategy was this. Anyway, when we got the latest seats (he changed his word), we always had to get the latest seats. When we get the last seats (the largest numbered seat in each row), the computer has to get the other seats, and the last seat is left

for us. For example, there is a pattern here. 3 to 8 (referring to seats 3, 8, 15, 24, respectively). It increases two at a time. There are 7 numbers between 8 and 15. And it goes up like 5, 7, 9.
T: Then let's see. Let's try the strategy you found (opening the Stand B). I wonder if we can win this time. Let's see…
AK2: Selecting, selecting.
AK4: I think it is true… Yes (won again, applauding)
T: Once again, it showed the accuracy of the strategy. Now, what have we done here, guys? The game was starting the computer. Now is the opposite. Does the strategy still work? Let's see (AK16 started first this time)… Our friend won again. Do you think this strategy is always right?
AK16: I think we practiced it three times. I think this strategy is always right in this game.
AK2: We found the formula, teacher, the formula.
AK4: n^2-1
T: n^2-1. How so?
AK4: Our friend did it. We did something from there. The square of the two is 4, 4-1=3, so the square of the three is 9, 9-1=8, the square of the four is 16, 16-1=15, and the square of the five is 25, 25-1=24. It goes like this.
AK2: That is right.

In the dialogue, the prospective teacher with the code AK16 appears to approve the hypothesis presented by reasoning on the game through trial and error. In this way, it is understood that prospective teacher stated the winning number as follows: {3, 8, 15, 24, 35,…}. He also noticed that there was a sum pattern among these numbers. He argued that if the winning number sequence is specified as {3, 8, 15, 24, 35, respectively}, there are as many differences between the two consecutive terms of this sequence as the single numbers that can be specified as {5, 7, 9,…}. AK4, a member of the same group who later joined the dialogue, presented a different approach by giving algebraic expression of the winning numbers. Thus, three different ways of winning strategies were explored. It was determined that the action, formulation, and validation stages of the game involved the challenge of recognizing, providing feedback from the arguments in the game, and strong interaction between the game and the player in these processes. In this study, the intertwined action, formulation, and validation stages lasted a total of 50 minutes. The interaction of the class at these stages is given in Figure 5.

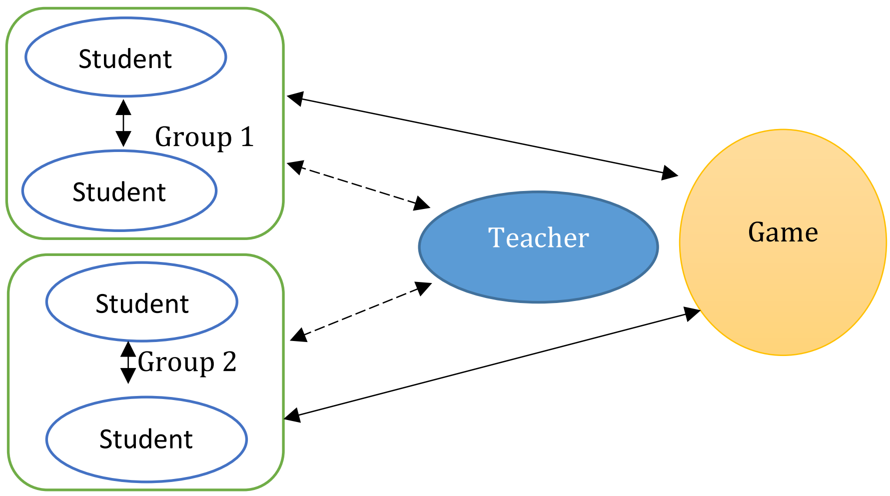

FIGURE 5. *Interaction in the action, formulation, and validation stages*

Institutionalization Stage

At this stage, the solutions found by prospective teachers in the context of the game were expressed by the teacher. Then the mathematical concept behind this game and how this concept is presented by embedding it into the game is explained. The introduction of mathematical knowledge in gamification by embedding it under the requirements of a-didactic environment design rather than directly sharing it offers an effective way of fulfilling the challenge criterion of gamification. Accordingly, the teacher made the following explanations:

> T: You have found it, there was more than one solution in this game. AK16 found one, and AK4 found the other. Congratulations. Yes, 3, 8, 15, 24, 35, 48 were the winning numbers. They have a formula. One minus of the squared numbers, guys... You selected the square numbers.
> AK2: Yes, when we get one minus, the result is obvious.
> AK4: We needed to choose one minus.
> T: You could do this, actually look here there is Stand B (representing the press). You never paid attention to it. If you select 1 in the press, you see, if you select 1 in the press, but you cannot choose. OK. It chooses 2, 3, 4, but 4 is 3 here.
> AK4: Yes. There you go.
> T: 5, 6, 7, 8, 9, but 9 here is 8.
> AK4: Just like we found, it is 8.
> AK8: Oh, yes.
> T: What is it now? It chooses 10, 11, 12, 13, 14, 15, 16, but 16 is 15 here. We can go on like this. In fact, when you include the press here, it means the square numbers. So square numbers are the solution. As our friend AK16 said, it could be solved through the pattern too. There are as many as 3, 8, 15, 24, 35, 48 winning numbers and an increasing sequence of odd numbers between them. Of course, this series starts at 5. It is like 5, 7, 9, 11, and 13.

As can be seen here, if the press seat 1 is selected in the game, and the seats in the game are reordered in ascending order, a position such as this is achieved,: 1 seat (press seat) in row 1 from top to bottom, 4 seats in row 1 and 2, 9 seats in row 1, 2 and 3 together, 16 seats in rows 1 to 4, and 25 seats in rows from 1 to 5. When it is arranged in this way, it will be obvious that there is a relationship between the number of rows and the seats. This relationship reveals a single strategy that allows the game to be reached in different ways in line with limitations in the context (such as selecting at least 1 and at most 2 in the first move, and at least 1 and at most 4 in the second move).

Finally, it is seen that prospective teachers who reach the number pattern underlying the game as target information in three different ways in the context of the game have very positive impressions about the activity and thus enjoy teaching mathematics. In this respect, some of the prospective teachers' opinions about the mobile game designed are given below.

> T: Did you like the game? Let's get your feedback in a nutshell.
> AK1: I liked it very much. It was very nice.
> AK2: it was really good. Actually, we initially went like 4, 9, 16, 25.

This stage lasted 10 minutes. At this stage, it was revealed that mathematical knowledge was hidden within the game by a specific and purposeful conversion and that this approach provided an effective way for the challenge criterion in gamification. In addition, approaches to how to take the information embedded in the game out of context can also be observed at this stage. In this stage, the target information obtained in the context of the game is presented by giving a mathematical dimension, so it can be stated that there is an interaction between the teacher and the prospective teachers. This interaction is given in Figure 6.

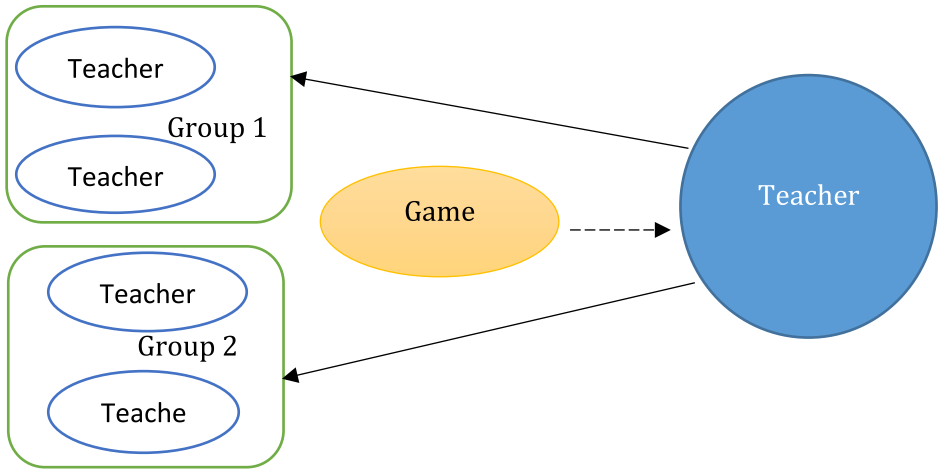

FIGURE 6. *Interaction in the institutionalization stage*

Games Designed by Teacher Candidates

In this section, the games designed by prospective classroom teachers to teach a mathematical knowledge determined by them are examined in terms of the gamification criteria specified by Prensky (2001). In this respect, the results of these designs are given in Table 5.

Table 5. *Evaluation of games developed by prospective teachers in terms of gamification*

Participant	Mathematical Knowledge	Rules	Objective	Feedback	Challenge	Interaction	Story
AK1	Long division	✓	✓	-	✓	✓	✓
AK2	Four operations	✓	✓	-	✓	-	✓
AK3	Odd-even number	✓	✓	-	✓	✓	✓
AK4	Addition	✓	✓	-	✓	✓	✓
AK5	Multiplication	✓	✓	-	✓	✓	✓
AK6	Multiplication	✓	✓	-	✓	✓	✓
AK7	Multiplication	✓	✓	-	-	-	✓
AK8	Exponential numbers	✓	✓	-	✓	✓	✓
AK9	Arithmetic mean	-	-	-	✓	✓	✓
AK10	Number pattern	✓	✓	-	✓	✓	✓
AK11	Square and cubes of a number	-	-	-	✓	✓	✓
AK12	Number pattern	✓	✓	✓	✓	✓	✓
AK13	Finding numbers with prime factors	✓	✓	✓	✓	✓	✓
AK14	Prime multipliers	✓	✓	✓	✓	✓	✓
AK15	Number pattern	✓	✓	✓	✓	✓	✓
AK16	Prime numbers	✓	✓	-	✓	✓	✓
AK17	Writing numbers with the forces of 2	✓	✓	-	-	-	✓

As can be seen in Table 5, it is understood that classroom teacher candidates often create successful designs, except for the feedback criterion in the gamification of the mathematical information they identified. On the other hand, the lack of feedback in the gamification process is a lack that cannot be ignored. Failure to receive feedback from the game constitutes an obstacle to maintaining the dynamic structure in the environment. The reason is that the player who interacts with the game draws meaning from the environment through the feedback he receives from the game, he reaches new information in this direction and leads towards the target information requested to be present in the game. The lack of feedback during the gamification process may result in players not being able to access the mathematical knowledge they are asked to find in the game. In Figure 7, some sections of the game designs developed by AK4 and AK13 are given.

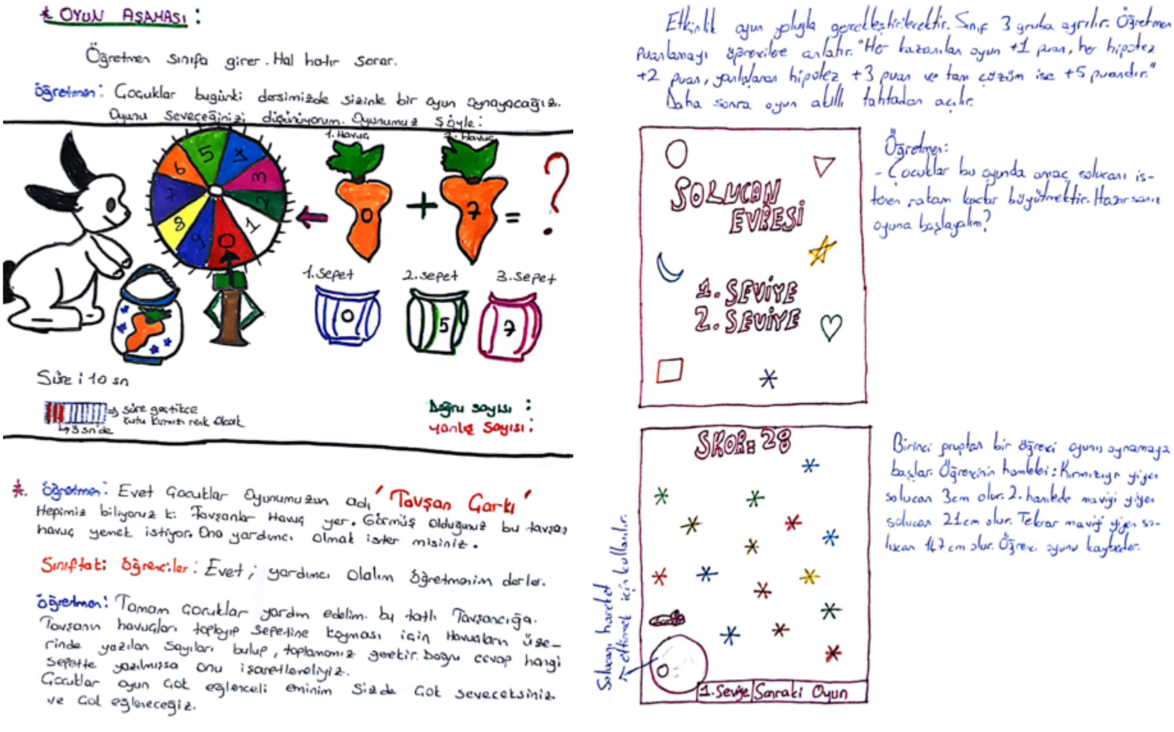

FIGURE 7. *Parts of some game designs (AK4 on the left, and AK13 on the right)*

In Figure 7, prospective teacher AK4 presents the teaching of the addition process with a game called the "*rabbit wheel*." In the details of this game, it is stated that the number of the first addend is marked on the wheel, and the other is randomly assigned by artificial intelligence, and options are created accordingly. It was stated that the game was designed by the prospective teacher in order to reinforce the addition process. However, it is seen that the game does not include any feedback mechanism other than providing an option for finding a total for a student who does not know the addition process. Other criteria of gamification are met in this game. In the second game, it is seen that prospective teacher AK13 assigns a number value to some symbols in the game, which he named as the worm phase. In the game a worm is moved on the screen and grows when it reaches these icons. The aim of the game at the beginning of the game is to grow the worm by the number written as a score. For this reason, it is necessary to discover how symbols grow worms during trial and error processes. When the score is exceeded, the game is lost. It is understood that the worm given here with a length of 1 unit grows by a prime number multiplier after each symbol it reaches (For example, it is understood that some symbols have scores such as stars: 2, month: 3, heart: 5 in Figure 7). This game was designed at a sufficient level in terms of gamification criteria. In addition, the fact that a positive number given in the game is constructed by prime numbers (fundamental theorem of arithmetic) is examined. Based on these explanations, it can be stated that prospective classroom teachers gained some awareness about the processes of gamification of mathematical knowledge. It was also determined that some prospective teachers successfully designed the mathematical knowledge determined according to the relevant criteria, while most of them realized gamification lacking feedback dimension.

DISCUSSION and CONCLUSIONS

In this study, the experiences of prospective classroom teachers of the process of gamification during the application of an educational mobile game developed by the designation and gamification of a number pattern within the framework of TDS were examined. In the study, the game designs of prospective teachers about the gamification of mathematical knowledge were also examined in terms of the criteria of gamification. In the games designed by prospective teachers, it can be stated that gamification is generally successful except for the feedback criterion. In failing to meet the feedback criteria, it can be stated that deficiencies in the field knowledge and design of the teaching process (Shulman, 1986), as well as related situations with the transformation of knowledge in a related way to a teaching purpose (Chevallard & Bosch, 2014), are effective.

At different stages of TDS, the interaction between teacher candidates and teachers can be said to be conducted in parallel with the interaction predicted in theory throughout the game process. Balacheff and Kaput (1997) described a critical feature of codable systems as the enormous complexity of didactic interactions can be managed by focusing on one activity or discovery at a time. In this context, it can be stated that the mobile game designed provides a suitable atmosphere for prospective classroom teachers in terms of internalizing the criteria of gamification during the process of teaching mathematical knowledge.

The devolution stage is of critical importance in this study because at this stage, the teacher tried to teach the students how to play the game by introducing the rules of the game, on the other hand he explained how to make use of the educational mobile game, which would enable the teachers to reach the target information and give them positive or negative feedback during the game process. Also, at this stage, the purpose of the game was explained by the teacher. Therefore, during this stage, the rules, scenario (story), and the purpose of the game were successfully devolved to prospective classroom teachers.

Firstly, in the introduction of the game, the fact that adding some parameters to the game screen (the seats to be selected are blue, the number of seats can be selected at the bottom of the screen, the player who chooses the order of selection on the right and left of the screen) and the game is played with the touch screen in the form of mobile game instead of playing with the keyboard or mouse revealed that prospective teachers were able to adapt to the game faster and focus their energies on the strategy that won the game. At this stage, when the teacher made a lot of explanations about the understanding of the mobile game and showed them in a practical way by playing two games randomly, this accelerated this process. On the other hand, when technology is not properly integrated into education, it may present some difficulties (Noe, 2010). Examples of this include a very complicated game menu and a lack of knowledge of how to start the game. However, the initial strategies of the game designed in this study consisted of a simple phenomenon such as touching only one of the lit blue seats, eventually reaching the largest numbered seat, and there is no such obstacle thanks to the guiding commands on the game screen.

Secondly, the game should be well designed in such a way as to provide positive or negative feedback to prospective teachers. Brousseau (1997) stated that there might be some obstruction in the playing process. Accordingly, he emphasized that it takes a certain time for students to discover the number 14 after reaching 17 of the winning numbers (dividend numbers that are divided by 3 and have 2 as remainder), and that it is not very easy in the game where he introduced the stages of TDS in the form of the game "*Race to 20*" designed for finding the Euclidean division. Sun and Tapan Broutin (2017) stated in their study that in an activity designed according to TDS, the devolution stage and the action stage were intertwined because some students did not know what to do in the action phase. By being aware of such risks, different parameters (seats numbering, different colors of players' choices on the screen, seat ranking, and points gained in the game) were added to the game by evaluating the benefits provided by the technology in this study. The explanation of these parameters by the teacher in the context of the game has a positive effect on the functioning of the process in making it easier to move to other stages of TDS and not having any turning back in the stages.

On the other hand, during the devolution stage, prospective teacher AK1 made an attempt to break the didactic contract. Prospective teachers were previously informed that they would seek solutions to problem situations within the framework of the constructivist approach in the activities within the scope of this course. It is seen that the teacher intervened with the sentence, "You will find it," referring to this didactic contract. It is thought that prospective teacher's exposure to traditional methods during school years and in many lessons and the fact that he has not fully internalized the constructivist approach has been effective in this kind of attempt. A student-centered approach, in theory, has been introduced in the teaching programs in Turkey since the 2005 curriculum (MEB, 2005, 2013, 2018a). However, it may not be true to suggest that this happens literally in practice in terms of teaching mathematics. Similar studies within the framework of TDS have also seen attempts by students to break the didactic contract (Sun & Tapan Broutin, 2017). The main reason for these attempts can be explained by the dominance of the teaching environments in which those in the teaching position are active.

The action, formulation, and validation stages of TDS are intertwined in this study. The structure of the designed game and the need for prospective teachers to convince their opponents using this context are thought to elicit such a situation. As a matter of fact, after several games were played during the action stage, it was observed that the stages were intertwined because random strategies were tested by trial and error in the context of the game, and they wanted to discover the winning strategy in a short way. Similar results are reviewed in the literature that different stages are intertwined in the activities designed according to TDS (Arslan et al., 2011; Baştürk Şahin et al., 2017; Dikkartın Övez & Akar, 2018).

A total of 25 games were played during the action, formulation, and validation stages in which the teacher candidates were active, and 10 hypotheses were presented. It appears that hypotheses were given more frequently in recent games and that prospective teachers wanted to play the game again to confirm their hypotheses. In this process, it was observed that the perception that the hypothesis is false if prospective teachers lose the game and that it is true if they win prevails in the classroom. The most important indicator of this is that AK16, the first to reveal the winning strategy in the game, shows the accuracy of its strategy by winning in three different games in a pragmatic (trial and error) manner. The perception that the hypothesis is correct in the position of winning and the hypothesis is wrong in the position of losing shows that the designed game is functional in the sense of positive or negative feedback. It can also be stated that the teacher's intervention is minimized.

The way in which seats are sorted by AK16 of the winning strategy in the game (wins reaching the last seat in each row) and number pattern was found to be (3, 8, 15, 24, 35,...). However, it is unclear how this was discovered by the prospective teacher. On the other hand, given that artificial intelligence (computer) is the winner in the game process from game 11 to game 22, it is possible that prospective teacher who watches these games carefully may have discovered the winning strategy from the choices the computer makes in its moves. After the winning strategy is revealed in the game, it is seen that a group of students from the Group 2 revised a hypothesis they had given earlier and obtained the winning strategy in the game in a different way. In their initial hypotheses, these students stated the winning strategy as square numbers without considering the press seat. This strategy was later revised again as one minus of square numbers without calculating the press seat. It was determined that this strategy was introduced by several prospective teachers in Group 2 in in-group discussions. In this sense, it can be stated that in-group discussions involve a process of interaction beyond the situations observed in the classroom.

In this study, prospective classroom teachers were able to greatly internalize the criteria of gamification to provide challenge, interaction, and feedback in the intertwined stages (action, formulation, and validation). The design of the game from easy to difficult and always hiding the winning strategy naturally gave rise to the challenge. This approach kept alive the motivation for the game to be played continuously. The fact that the game contains parameters for the emergence of goal acquisition and that prospective teachers recognize them during the game process and provide feedback from the game has ensured the continuous progression of the knowledge in the environment. It is undeniable that the intense interaction during the active stages of the students contributes to the smooth functioning of these processes.

The strategies obtained in the context of the game were explained by removing them from the context of the game by prospective teachers who reach the target number pattern in the context of the game in three different ways during the institutionalization stage. Accordingly, when the press seat is taken into account, and the seats are renumbered, it is stated that the winning strategy in the game is square numbers (n^2, $n>1$, and n is a natural number). In this sense, multiple solutions for the problem of the game, designed within the framework of TDS, can be presented. In activities prepared within the framework of TDS, it is indicated that students are reluctant to explore the other solution after finding one solution for the problem situation (Sun & Tapan Broutin, 2017; Erumit et al., 2012). This study is distinguished from others by the emergence of multiple solutions. It is thought that presenting the problem situation with a mobile game as a result of the integration of technology into education through gamification was instrumental in the emergence of this difference.

It was found out that that prospective teachers expressed positive views about the activity and thus enjoyed the teaching of mathematics in that way. In mathematics teaching, developing positive attitudes towards mathematics teaching can be possible by taking advantage of the attractive features of gamification and technology. The activities applied within the framework of TDS in the literature (Baştürk Şahin et al., 2017; Shepherd, 2016) specified that using technology in education can provide individuals with experiences they will enjoy (Sahin, 2016).

In this study, the benefits of the experience of prospective classroom teachers in presenting mathematical knowledge with an educational mobile game developed within the framework of TDS in terms of teaching mathematics can be listed as follows:

- Participants have an experience of teaching in an atmosphere that they enjoy because knowledge is taught through gamification and raises awareness about how to design these environments,
- The mobile game provides an example of the integration of technology into education as it provides material for future teachers to use in the classroom,
- Prospective teachers have largely understood the basic elements of an approach to the presentation of mathematical knowledge through an educational game in line with the criteria of gamification.
- Limited teacher interventions and the student-centered approaches in which students access knowledge through a designed environment are effective in understanding the designs for their use in the classroom environment.

All of these benefits reveal the interactive aspect of technology. In other words, it shows that the relationships between learners and mathematical knowledge, and learners and teachers are evolving around a new common denominator (technology) (Balacheff & Kaput, 1997). In this respect, the results of the study showed that prospective classroom teachers could learn environmental design from mathematical concepts related to the presentation of a number pattern in the classroom environment with an educational mobile game designed within the framework of TDS. It is predicted that teaching mathematical concepts through mobile games will offer prospective teachers a different teaching experience and guide them on how to design such environments when they become teachers in the future. For researchers, it may be suggested to examine the contributions of longitudinal studies in which all the processes (design, coding, and application) of gamification for mathematics teaching within the scope of TDS to the mathematics education and to investigate the detailed description of these processes.

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