

## Estimating Model Life Tables using both Abridged Life Table and Brass Relational Two- parameter Logit System in Twenty Six States in Sudan From the Population Census Data (2008)

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### Abstract

The objective of the study is to enhance methodology in the area of adult mortality and to attempt to correct defective data in Sudan and the twenty-six States to estimate adult mortality and life table using Brass relational two-parameter logit system, also to estimate the probability of dying and expectation of life at birth using adjusted data in the twenty-six States in Sudan through Brass relational two-parameter logit system. A comparison was made between the adjusted life table for Sudan and the twenty-six States with the original Abridge model life table in order to test the reliability of the method used. The data was gathered from Population Census Data 2008. Values for  $e_0$  calculated by Brass Logit System when compared with those calculated using the abridge life table, were reasonable and realistic, the abridge life table  $e_0$  might be affected by the defective data. Life expectancy  $e_0$  for the whole country found to be 59 years using abridge life table, where Life expectancy calculated using Brass method found to be 54 years, in Gezira, Eastern Equatoria, Nahr El Nile, Northern and Sinnar states Life expectancy  $e_0$  equal to 74, 72, 71, 75 and 72 years respectively, where  $e_0$  values calculated by Brass Logit System for the same group of states were 58, 56, 58, 59 and 57 respectively. In Northern Bahr El Ghazal, Unity and Warrap, Bahr El Ghazal  $e_0$  values rise from 30, 30, 44, and 46 to 44, 55, 54 and 51 respectively after using Brass Logit System. In Sudan  $\square$ , indicates low mortality relative to the standard, Blue Nile, Northern Bahr El Ghazal, West Darfur have positive values of  $\square$  indicates high mortality relative to the standard. North Kordfan state  $\square = -2.545524$  out of the reasonable range. In Sudan and Blue Nile high value of  $\square$  indicates a low infant and child mortality and a high adult mortality relative to the standard. The majority of states recorded a value of  $\square$  higher than the upper limit of the reasonable range, except Al Gadarief reported indicates a high infant and child mortality and low adult mortality.

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تقدير نماذج جداول الحياة باستخدام طريقتي جدول الحياة المختصر ونموذج برأس العلائقي اللوجستي ذو المعلمتين في ست وعشرين ولاية من ولايات السودان  
من بيانات التعداد السكاني الخامس 2008م

### المستخلص

تهدف هذه الدراسة للإضافة المنهجية في مجال وفيات البالغين في محاولة لتصحيح البيانات المعيبة في السودان وولاياته الست وعشرين لتقدير وفيات البالغين وجدول الحياة، وذلك باستخدام نموذج برأس اللوجستي ذو المعلمتين بالإضافة إلى تقدير ومقارنة قيم احتمال الوفاة وتوقع الحياة عند الميلاد من جدول الحياة المختصر وجدول الحياة العلائقي باستخدام نموذج برأس اللوجستي ذو المعلمتين لاختبار صلاحية النموذج. وقد استخدمت الدراسة لهذا الغرض بيانات التعداد السكاني الخامس 2008.

بمقارنة قيم توقع الحياة عند الميلاد المحسوبة من النموذج العلائقي بتلك التي تم حسابها باستخدام جدول الحياة المختصر، وجدت قيم النموذج العلائقي أكثر واقعية من تلك التي تم حسابها باستخدام جدول الحياة المختصر والتي ربما تأثرت بالبيانات المعيبة. العلائقي توقع الحياة عند الميلاد لكل السودان وجد أنه يساوي 59 عاماً من النموذج العلائقي بينما بلغ 54 عاماً باستخدام جدول الحياة المختصر في ولايات الجزيرة، شرق الاستوائية، نهر النيل، الشمالية و سنار، وجد توقع الحياة عند الميلاد 74، 72، 71، 75 و 72 عاماً على الترتيب، باستخدام النموذج العلائقي وجد توقع الحياة عند الميلاد لنفس مجموعة الولايات بنفس الترتيب 58، 56، 58، 56 و 57 عاماً. في شمال بحر الغزال، الوحدة، و واراب زادت قيم توقع الحياة عند الميلاد التي تم حسابها باستخدام جدول الحياة المختصر من 30، 30، 44 و 46 إلى 44، 55، 54، و 51 بنفس الترتيب عند استخدام النموذج العلائقي.

وبالنسبة لكل السودان فإن قيمة معلمة النموذج  $\alpha$  تشير إلى معدلات منخفضة للوفيات قياساً بالمعيار، على مستوى ولايات النيل الأزرق، شمال بحر الغزال، غرب دارفور فإن قيمة معلمة النموذج  $\beta$  الموجبة تشير إلى ارتفاع معدلات الوفيات قياساً بالمعيار. في حين أنّ قيمة معلمة النموذج  $\gamma$  لشمال كردفان بلغت 2.545524- وهو ما يعد خارج المدى المعقول. ميل المنحنى  $\alpha\beta\gamma$  يشير إلى انخفاض وفيات الأطفال والرضع وارتفاع وفيات البالغين قياساً بالمعيار في معظم الولايات سجلت قيم ل  $\alpha\beta\gamma$  أعلى من الحد الأعلى للمدى المعقول، باستثناء ما سجل لولاية القضايف الذي يُشير لارتفاع وفيات الأطفال والرضع وانخفاض وفيات البالغين قياساً بالمعيار.  $\alpha$

## 1. Introduction:

No method that gives complete satisfactory results in estimating adult mortality. What direct methods for measuring mortality are available? Survey data on deaths that occurred in the last year and registration data are often available, but in developing countries these data are rather poor. Although many surveys have been undertaken, none have given satisfactory results, since they are affected by problems of both reference period and omissions. Civil registration is also generally deficient, except in a few developing countries such as Chile. Is it possible to attempt to correct omissions in civil registration by comparing the estimation of childhood mortality with direct estimation obtained from registration or from survey data, and to use this comparison to estimate the overall incompleteness of the effective data? It can be seen, therefore, that estimation of adult mortality presents a very difficult problem. (*Brass, W.1975*)

One approach would be to select a model life table with a childhood mortality level, such a model life table could easily be selected from one of the available sets of reference tables; i.e., the United Nations Model Life Tables, the Regional Model Life tables, and those based on the logit system. However, whichever of the model systems is used, this approach is not satisfactory because the relationship between childhood mortality and adult mortality is not very strong. Populations with the same mortality level for the adult population may have very different levels of childhood mortality. For example Turkey has a relatively low adult mortality in comparison with its childhood mortality, (*Brass,1975*).

It is significant that two seemingly opposed designations have been applied to the same thing; "Mortality Table." or the "Life Table." Quite in accord with this dual character of the life table, its applications may be broadly classed in two categories--applications relating primarily to mortality and death rates, and applications relating primarily to survivals, (*Deevey,1988*).

Many problems in population study must be answered. Incidentally, it should be remarked that life tables, as generally constructed, represent a fixed mortality of a particular calendar year or period. Such a table tells us what would be the number -of survivors to age 10, 20, etc., if the mortality at each age remained constant as of the calendar year or period for which it is constructed. On this occasion we are concerned with applications to general demographic problems. In the first category, applications relating more particularly to mortality, we have, first of all, the direct use of the life table as a gauge or measure of the mortality in a given population or group of persons. Standardized death rates, on the other hand, have the disadvantage that they depend' on an arbitrarily selected standard population. The life table is free from this arbitrary feature, and, of course, with its several Columns, exhibiting, for a "cohort" or "generation" traced from birth through life, the number of survivors, the number of deaths, the death rate, and the expectation of life at each' age, such a table gives much more detailed information than a general death rate, whether crude or standardized. (*Deevey.S.E, J,1988*).

The expectation of life (or life expectancy), is defined as the number of years which an individual at a given age could expect to live at present mortality levels, if mortality at each age remains constant in the future(Wikipedia, the free encyclopedia), can also defined as life expectancy at birth the number of years a newborn infant would live if prevailing patterns of age-specific mortality rates at the time of birth were to stay the same throughout the child's life|(Un Common Database, Human Development,99(2000)).The entry includes total population as well as the male and female components. Life expectancy at birth is also a

measure of overall quality of life in a country and summarizes the mortality at all ages. It can also be thought of as indicating the potential return on investment in human capital and is necessary for the calculation of various actuarial measures. (Shepard, Jon; Robert W. Greene ,2003)

Life tables can be extended to include other information in addition to mortality, for instance health information to calculate health expectancy. Health expectancies, of which disability-free life expectancy (DFLE) and Healthy Life Years (HLY) are the best-known examples, are the remaining number of years a person can expect to live in a specific health state, such as free of disability. Two types of life tables are used to divide the life expectancy into life spent in various states: 1) multi-state life tables (also known as increment-decrement life tables) based on transition rates in and out of the different states and to death, and 2) prevalence-based life tables (also known as the Sullivan method) based on external information on the proportion in each state. Life tables can also be extended to show life expectancies in different labor force states or marital status states. Life tables are also used extensively in biology and epidemiology. The concept is also of importance in product life cycle management. (Deevey.S.E, J,1988).

The Brass relational system of model relational life tables is derived from a mathematical relationship rather than empirical life tables. This is very useful because it has been found by Brass(1971) that if one takes two life tables, and take logits of their  $l_x$ s, then the relationship between the two sets of the logits is remarkably linear, that is drawing a graph of one against the other. This important discovery is essentially an empirical rather than a theoretical finding through Brass (1975) gives some justification. Sometimes it does not give exactly a straight line; especially in the extreme age rang.( Colin Newell,1988-1989) .

The study attempt to answer these questions what is the situational status of adult mortality in Sudan? Is it possible to mathematically derive an adjusted model life table to estimate the level of mortality and expectation of life at birth in Sudan? If possible, what will be the precision of such derivation in practical application?

This paper aim to enhance methodology in the area of adult mortality and to attempt to correct defective data in Sudan twenty-six Stats to estimate adult mortality and life table using Brass relational two- parameter logit system, also to estimate the probability of dying and expectation of life at birth using adjusted data in twenty-six States in Sudan through Brass relational two-parameter logit system . A comparison was made between the adjusted life table for Sudan and twenty-six Stats with the real Abridge model life table in order to test the reliability of the method.

## 2. Material and Methods:

The data was gathered from Population Census Data 2008.For Sudan and Twenty-Six State.

### 2.1 Abridged life tables:

A **life table** (also called a **mortality table**) is a table which shows, for each age, what the probability is that a person of that age will die before his next birthday. From this starting point, a number of statistics can be derived and thus also included in the table: (Wikipedia, the free encyclopedia)

- The probability of surviving any particular year of age
- Remaining life expectancy for people at different ages

${}_nq_x$ : The probability of dying between exact age  $x$  and  $x+n$ .

$${}_nq_x = \frac{{}_nM_x}{1 + n(1 - a_x) \cdot {}_nM_x}$$

${}_na_x$ : The proportion of the interval lived by those who die, but sometimes its defined as the average number of years lived by those who die. Thus the normal 5 years intervals  ${}_na_x$  may be 0.5 or 2.5.

$$q_0 = \frac{m_0}{1 + 0.7 m_0}$$

$${}_4q_1 = \frac{4({}_4m_1)}{1 + 2.7({}_4m_1)}$$

$${}_{\infty}q_{95} = 1.0$$

For Age Specific Death Rate  $m_0$  the moving average of five age groups (1-4), (5-9), (10-14), (15-19) and (20-24), is used to obtain the value for age 0.

The moving average is a type of finite impulse response filter used to analyze a set of data points by creating a series of averages of different subsets of the full data set. Given a series of numbers and a fixed subset size, the moving average can be obtained by first taking the average of the first subset. The fixed subset size is then shifted forward, creating a new subset of numbers, which is averaged. This process is repeated over the entire data series. Thus, a moving average is not a single number, but it is a set of numbers, each of which is the average of the corresponding subset of a larger set of data points. A moving average may also use unequal weights for each data value in the subset to emphasize particular values in the subset.

${}_nM_x$ : Age Specific Death Rate (ASDR).

$${}_nM_x = \frac{\text{Deaths during year of person aged } x}{\text{Population aged } x \text{ at mid - year}}$$

${}_np_x$ : The probability of surviving between exact age  $x$  and  $x+n$ . It is just a component of  ${}_nq_x$ . Thus:

$${}_np_x = 1 - {}_nq_x \quad \text{or} \quad {}_np_x + {}_nq_x = 1$$

Often  ${}_np_x$  like  ${}_nq_x$  is expressed per 1000 or 10000 in order to avoid lots of zeros.

$l_x$ : This is the number of persons alive at exact age  $x$ .

$l_0$  is an arbitrary number called the radix. Usually it will be around number such as 1 or 1000 or 100000.

$$l_x = l_{x-n} \cdot {}_np_{x-n}$$

${}_nd_x$ : This is the number of persons dying between age  $x$  and  $x+n$

$${}_nd_x = l_x - l_{x+n}$$

${}_nL_x$ : This is the number of persons years lived between age  $x$  and  $x+n$ .

$${}_nL_x = \frac{n}{2}(l_x + l_{x+n})$$

$$L_0 = 0.3 l_0 + 0.7 l_1$$

$${}_4L_1 = 1.3 l_1 + 2.7 l_5$$

$${}_{\infty}L_{95} = l_{95} \log_{10} l_{95}$$

$T_x$  : This is the total number of persons lived at exact age x. It is thus simply the  ${}_nL_x$  column cumulated from the bottom.

$e_x$  : This is the expectation life at age x, or the average number of years a person aged x has to live.

$$e_x = \frac{T_x}{l_x}$$

## 2.2 Brass relational two- parameter logit system life tables:

The aim is to discover which one of the finite family of life table that can be generated by varying  $\alpha$  and  $\beta$  the raw data are most like. The raw data may be a complete life table, or they may be just a handful of values.

The logits of the observed  $l_x$ s are fitted plotted on a graph against the logits of standard life table. Then a straight line is fitted to the points. The standard values are plotted on the X-axis, the observed values on the Y-axis

### 2.2.1 Fitting a Logit model life table

### 2.2.2 The linear regression model

In the simple equation of a straight line  $\alpha$  is the intercept – the point at which the line crosses the y-axis, and  $\beta$  is the slope or gradient of the line:

Altering  $\alpha$  will affect the level of mortality, while altering  $\beta$  will affect the relationship between child mortality and adult mortality

As stated by Brass (1975), the reasonable range for  $\alpha$  is (-1.5 to + 0.8). A high (i.e. positive) value indicates high mortality relative to the standard, while a low (i.e. negative) value indicates low mortality relative to the standard. For  $\beta$  a reasonable range is roughly (0.6 to 1.4), a low value indicates a high infant and child mortality and low adult mortality, where a high value indicates a low infant and child mortality and a high adult mortality relative to the standard.

The intercept and the slope of the line,  $\alpha$  and  $\beta$  are calculated using the **Group Average Method**, this simply involves taking the average of the childhood mortality points and the average of the adulthood mortality points. Thus having started out of the **three child** and **five adult** mortality estimates. The fitted logits are then computed by putting  $\alpha$  and  $\beta$  and the standard logits on the straight line equation. Lastly anti logits are taken to produced asset of *fitted  $l_x$ s* – the fitted of life table.

**The Logit transformation. The formula is**

$$\text{Logit}(p) = 0.5 \log_e \left( \frac{1-p}{p} \right)$$

**P= A set of  $l_x$  values are just proportion if the radix is 1.0**

### Remark:

It is possible to estimate  $q_x$  from  $M_{xS}$  using in the formula  $a_x$  ( the fraction of a year lived ). The values that  $a_0, a_1, \dots$  take vary from country to country and according to the level of mortality .The  $a_{xS}$  used here were calculated from California registration data 1970 (Chaing( no date 76-8)).

$$A_0= 0.09 \quad a_1=0.43 \quad a_2=0.45 \quad a_3=0.47 \quad a_4=0.49 \quad a_5=0.50$$

**Childhood mortality  $l_2, l_3, l_5$**

**Adulthood mortality**  $l_{45}, l_{50}, l_{55}, l_{60}, l_{65}$

**Childhood point**  $(x_c, y_c)$

**Adulthood point**  $(x_a, y_a)$

Having fitted the line, the values of  $\alpha$  and  $\beta$  can be calculated. For  $\beta$  using the group average points such as:

$$\beta = \frac{y_a - y_b}{x_a - x_b}$$

For  $\alpha$  using the relation:

$$\alpha = y_b - \beta x_b$$

If the Logit  $l_x$  is denoted by  $Y(x)$  and the , Logit of the standard as  $Y_s(x)$  as has become conventional in the literature .Having now calculated,  $\alpha$  and  $\beta$ , it is possible to calculate the logits of the fitted model through the equation:

$$Y_{fit}(x) = \alpha + \beta Y_s(x)$$

Then the fitted  $l_x$ s are computed by taking the anti-logits of using the  $Y_{fit}(x)$  equation

$$Fitted l_x = \frac{1}{1 + e^{2Y_{fit}(x)}}$$

### 3. Results

#### 3.1 Life Expectancy Estimation for Sudan, Abridged life table case:

Life expectancy values presented in Table (1.1) were resulting from applying the abridged

life table presses, as shown in material and methods section. Such as  $e_x = \frac{T_x}{l_x}$ .

Not that  $m_0$  in age 0 is resulting from using the moving average method on the five age groups (1-4), (5-9), (10-14), (15-19) and (20-24), thus  $m_0$  for Sudan is equal to 0.00702 as presented in Table (1.1) (this process also applied to the 26 states data (see tables in appendix).



Table (1.1) Life Expectancy Estimation for Sudan

Age	$P_x$	$D_x$	$nm_x$	$nq_x$	$nP_x$	$L_x$	$nd_x$	$nL_x$	$T_x$	$e_x$
0	1056307	210340	0.00702	0.00698	0.99302	100000	698.209	99511.3	5913988	59
1-4	4741387	140574	0.02965	0.1098	0.8902	99301.8	10903.7	367767	5814476	59
5-9	5735808	53021	0.00924	0.04518	0.95482	88398.1	3993.41	432007	5446709	62
10-	4955546	22618	0.00456	0.02256	0.97744	84404.7	1904.46	417262	5014702	59
15-	4066624	23514	0.00578	0.0285	0.9715	82500.2	2351.17	406623	4597440	56
20-	3396486	21452	0.00632	0.03109	0.96891	80149.1	2491.74	394516	4190816	52
25-	2983456	15688	0.00526	0.02595	0.97405	77657.3	2015.25	383249	3796300	49
30-	2397095	14137	0.0059	0.02906	0.97094	75642.1	2198.11	372715	3413052	45
35-	2236681	10848	0.00485	0.02396	0.97604	73444	1759.7	362821	3040337	41
40-	1711234	11569	0.00676	0.03324	0.96676	71684.3	2382.88	352464	2677516	37
45-	1264822	8530	0.00674	0.03316	0.96684	69301.4	2298.11	340762	2325052	34
50-	1065568	11044	0.01036	0.05051	0.94949	67003.3	3384.56	326555	1984290	30
55-	619436	6491	0.01048	0.05106	0.94894	63618.7	3248.18	309973	1657735	26
60-	675818	11768	0.01741	0.08343	0.91657	60370.6	5036.89	289261	1347762	22
65-	388365	7907	0.02036	0.09687	0.90313	55333.7	5360.07	263268	1058501	19
70-	408061	11882	0.02912	0.13571	0.86429	49973.6	6782.01	232913	795233	16
75-	193783	7697	0.03972	0.18066	0.81934	43191.6	7802.96	196451	562320	13
80-	175634	9265	0.05275	0.23303	0.76697	35388.6	8246.52	156327	365869	10
85-	62931	5223	0.083	0.34367	0.65633	27142.1	9327.95	112391	209542	8
90-	38905	4004	0.10292	0.40928	0.59072	17814.2	7291.01	70843.4	97151.3	5
95 +	31014	3303	0.1065	1	0	10523.2	10523.2	26307.9	26307.9	3
Total	38204960	610874								

Source: Population Census Data 2008

### 3.2 Fitting Model Life Table Sudan 2008

**Remark:** It is possible to estimate  $q_x$  from  $M_{xS}$  using in the formula  $a_x$  (the fraction of a year lived). The values that  $a_0, a_1, \dots$  take vary from country to country and according to the level of mortality. The  $a_x$ s used here were calculated from California registration data 1970 (Chaing (no date 76-8)). See Table (4.2) below. (this process also applied to the 26 states data (see tables in appendix).

Table (1.2) Fitting Model Life Table Sudan 2008

Age	$l_x$	$P$	Logit $P$	African Standard Logit $P$
2	92550	0.925503	-0.54712	-0.8053
3	89801	0.898014	-0.47237	-0.7253
5	86869	0.86869	-0.41028	-0.6514
45	69301	0.693014	-0.17681	-0.1073
50	67003	0.670033	-0.15381	-0.0212
55	63619	0.636187	-0.12135	0.0832
60	60371	0.603706	-0.0914	0.21
65	55334	0.553337	-0.0465	0.3746



Source: Population Census Data 2008

### 3.3 Life expectancy estimation for Sudan using Brass Relational Two- Parameter Logit System

The intercept and the slope of the line,  $\alpha$  and  $\beta$  were calculated using the Group Average Method, which is simply involves taking the average of the childhood mortality points and the average of the adulthood mortality points. The intercept  $\alpha = -0.36918$ , the slope of the line  $\beta = 2.328949$

The fitted logits are then computed by putting  $\alpha$  and  $\beta$  and the standard logits on the straight line equation. Lastly anti logits are taken to produced a set of fitted  $L_x$ s. As presented in Table ( 1.3 ) (this process also applied to the 26 states data (see tables in appendix).

### 3.4 Calculation of $e_0$ from Abridge life table and Brass Logit System for Sudan and by states

Values for  $e_0$  calculated from the Brass Logit System when compared with those calculated using abridge life table, tend to be more reasonable and realistic, the abridge life table  $e_0$  might be affected by the defective data. Table ( 2 ).

Table ( 1.3 ) life expectancy estimation for Sudan using Brass Relational Two- Parameter Logit System

Age	$Y_{sx}$	$Y_{fitt\ x}$	Proportio n	Fitted $l_x$	$nd_x$	$nL_x$	$T_x$	$e_x$
0			1	100000	457	99679.7	5401985.0	54
1	-0.9972	-2.69161	0.9954254	99543	653	99215.8	5302305.2	53
2	-0.8053	-2.24468	0.9888917	98889	493	98642.4	5203089.4	53
3	-0.7253	-2.05836	0.9839568	98396	351	98219.9	5104446.9	52
4	-0.682	-1.95752	0.9804423	98044	293	97897.8	5006227.0	51
5	-0.6514	-1.88626	0.9775139	97751	1312	485476.	4908329.2	50
10	-0.5498	-1.64963	0.9643919	96439	636	480606.	4422852.7	46
15	-0.5131	-1.56416	0.9580330	95803	1231	475939.	3942246.4	41
20	-0.4551	-1.42908	0.9457240	94572	2008	467841.	3466307.2	37
25	-0.3829	-1.26093	0.9256425	92564	2491	456593.	2998465.5	32
30	-0.315	-1.1028	0.9007302	90073	3075	442677.	2541872.3	28
35	-0.2496	-0.95048	0.8699786	86998	4013	424955.	2099195.1	24
40	-0.1817	-0.79235	0.8298453	82985	5463	401266.	1674239.1	20
45	-0.1073	-0.61907	0.7752191	77522	7738	368263.	1272973.0	16
50	-0.0212	-0.41855	0.6978364	69784	11103	321160.	904709.14	13
55	0.0832	-0.17541	0.5868072	58681	14647	256787.	583548.22	10
60	0.21	0.119902	0.4403410	44034	17265	177007.	326761.17	7
65	0.3746	0.503247	0.2676871	26769	14545	97481.2	149754.13	6
70	0.5818	0.985805	0.1222380	12224	8570	39693.6	52272.838	4
75	0.8611	1.63628	0.0365365	3654	3018	10722.6	12579.198	3
80	1.2433	2.526405	0.0063540	635	583	1719.11	1856.5458	3
85	1.781	3.778681	0.0005223	52	51	134.013	137.43228	3
90	2.5634	5.600851	1.36666E-	1	1	3.43311	3.4187521	3
95+	3.709	8.268895	6.58375E-	0	0	-	-0.0143626	-2

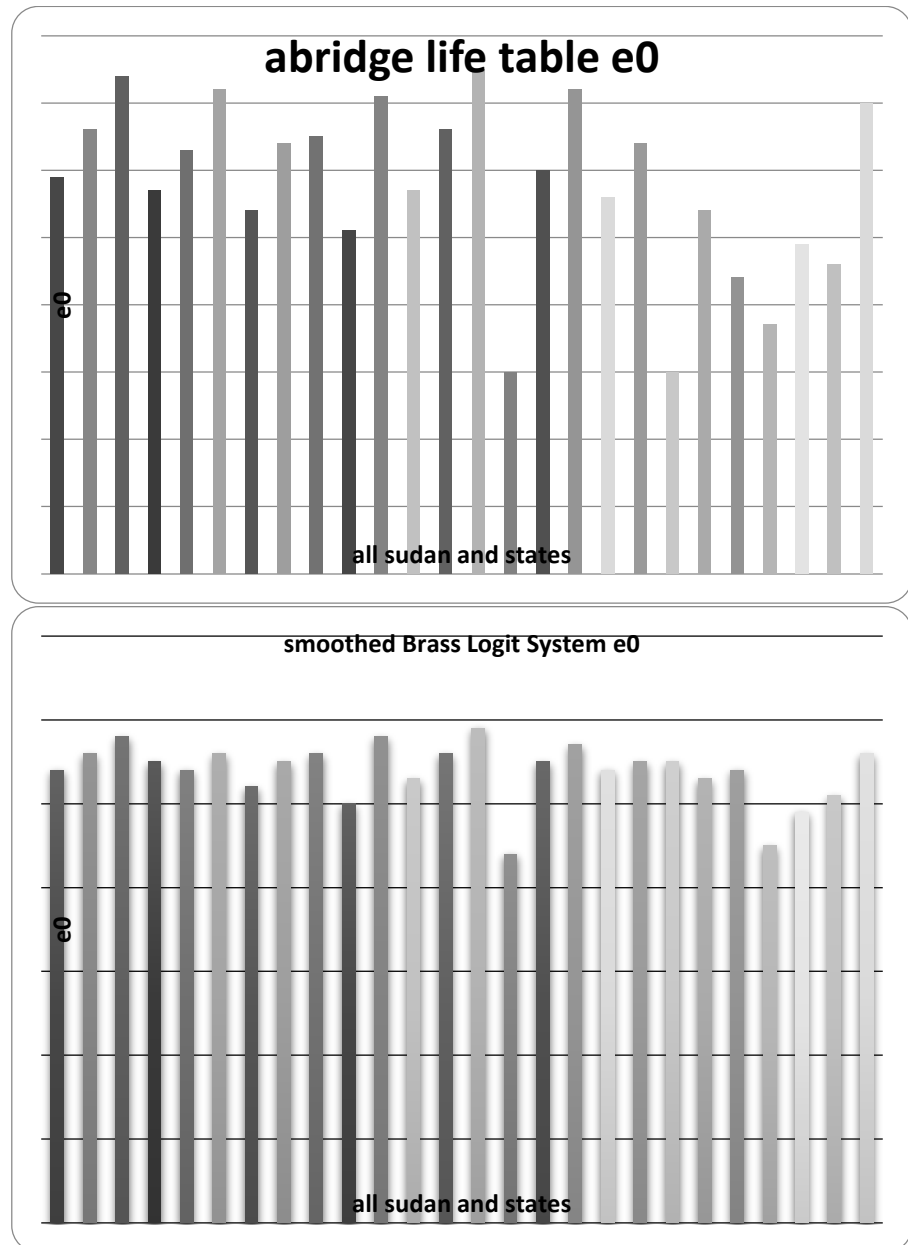
Source: Population Census Data 2008

Table (2)  $e_0$  calculated from Abridge life table and Brass Logit System for Sudan and by states

	abridge life table $e_0$	smoothed Brass Logit
All Sudan	59	54
<b>States</b>		
Al Gadariief	66	56
Al Gezira	74	58
Blue Nile	57	55
Central Equatoria	63	54
Eastern Equatoria	72	56
Jonglei	54	52
Kassala	64	55
Khartoum	65	56
Lakes	51	50
Nahr El Nile	71	58
North Darfur	57	53
North Kordfan	66	56
Northern	75	59
Northern Bahr El Ghazal	30	44
Red Sea	60	55
Sinnar	72	57
South Darfur	56	54
South Kordfan	64	55
Unity	30	55
Upper Nile	54	53
Warrap	44	54
West Darfur	37	45
Western Equatoria	49	49
Western Bahr El Ghazal	46	51
White Nile	70	56

Source: Population Census Data 2008

The effect of using Brass Logit System can clearly be observed in Figure (1) below.  
**Figure (1 ) Data presented in table (4.4)**



### 3.4 Brass Logit System $\lambda$ and $\mu$ values

As shown in Table (3) All the states were within the reasonable rang mentioned by (Brass, 1970) except North Kordfan state which showed a negative low  $\lambda = -2.545524$ , which indicates a low mortality relative to the standard. For  $\mu$ , all values were out of the reasonable range, except Al Gadariief state 0.10786 and Unity state 1.335809 found to be within the reasonable range.

**Table (3) Brass Logit System  $\lambda$  and  $\mu$  values for Sudan and by states**

	$\lambda$	$\mu$
Sudan	-0.36918	2.328949
<b>States</b>		
Al Gadariief	-0.52974	0.10786
Al Gezira	-0.61065	2.523281
Blue Nile	0.44441	3.068183
Central Equatoria	-0.39029	2.267814
Eastern Equatoria	-0.53843	2.841479
Jonglei	-0.24487	2.15285
Kassala	-0.446	2.429902
Khartoum	-0.48547	2.248307
Lakes	-0.17959	1.946571
Nahr El Nile	-0.5526	2.051597
North Darfur	-0.31738	2.133868
North Kordfan	-2.545524	2.545524
Northern	-0.59112	1.739115
Northern Bahr El Ghazal	0.058407	1.857654
Red Sea	-0.4396	2.616472
Sinnar	-0.5648	2.679618
South Darfur	-0.34407	2.527042
South Kordfan	-0.40399	2.094
Unity	-0.36918	1.335809
Upper Nile	-0.3241	2.543292
Warrap	-0.36918	2.104799
West Darfur	0.023636	1.637274
Western Equatoria	-0.14164	1.584106
Western Bahr El Ghazal	-0.22459	2.285264
White Nile	-0.51382	2.382648

Source: Population Census Data 2008

## 4. Discussion:

### 4.1 Life Expectancy Estimation for Sudan, Abridged life table case:

The  $e_0$  for the whole country found to be 59, when compared with World Bank estimation for Sudan Life expectancy at birth 54.21 years at 2010. Which is near to the value of  $e_0 = 54$  calculated using Brass method.

### 4.2 life expectancy estimation for Sudan using Brass Relational Two- Parameter Logit System

The intercept and the slope of the line,  $\alpha$  and  $\beta$  were calculated using the Group Average Method, which is simply involves taking the average of the childhood mortality points and the average of the adulthood mortality points. The intercept  $\alpha = -0.36918$ , the slope of the line  $\beta = 2.328949$ . A Negative low  $\alpha$ , indicates low mortality relative to the standard. High value of  $\beta$  indicates a low infant and child mortality and a high adult mortality relative to the standard.

### 4.3 Calculation of $e_0$ from Abridge life table and Brass Logit System by states.

Values for  $e_0$  calculated from the Brass Logit System when compared with those calculated using abridge life table, looks to be more reasonable and realistic, the abridge life table  $e_0$  might be affected by the defective data. Table (2) below showed that  $e_0$  calculated from the original data using abridge life table in Gezira, Eastern Equatoria, Nahr El Nile, Northern and Sinnar stats equal to 74, 72, 71, 75 and 72 years respectively, where  $e_0$  values calculated by Brass Logit System for the same group of states were 58, 56, 58, 59 and 57 respectively. A clear drop down and similarity in  $e_0$  values can be observed among the states. In Northern Bahr El Ghazal, Unity and Warrap, Bahr El Ghazal  $e_0$  values rise from 30, 30, 44, and 46 to 44, 55, 54 and 51 respectively after using Brass Logit System.

### 4.4 Brass Logit System $\alpha$ and $\beta$ values

As stated by Brass (1975), the reasonable range for  $\alpha$  is (-1.5 to + 0.8). A high (i.e. positive) value indicates high mortality relative to the standard, while a low (i.e. negative) value indicates low mortality relative to the standard. For  $\beta$  a reasonable range is roughly (0.6 to 1.4), a low value indicates a high infant and child mortality and low adult mortality, where a high value indicates a low infant and child mortality and a high adult mortality relative to the standard.

As shown in table (3) All states within the reasonable range for  $\alpha$ , the majority of the state have negative value of  $\alpha$  within range (-0.14164 - 0.61065) which indicates that low mortality relative to the standard. Blue Nile, Northern Bahr El Ghazal, West Darfur have positive values of  $\alpha$  (0.44441, 0.058407, 0.023636) indicates high mortality relative to the standard. Except North Kordfan state  $\alpha = -2.545524$  out of the reasonable range.

For the slope of the line  $\beta$ , table (4.5) showed **3.06818** for Blue Nile, as the highest positive value, where Al Gadariief reported the lowest positive value **0.10786**

The majority of states recorded a value of  $\beta$  higher than the upper limit of the reasonable range for  $\beta$  which is (0.6 to 1.4) (high value indicates a low infant and child mortality and a high adult mortality relative to the standard.) only for Al Gadariief and Unity states  $\beta = 0.10786$  and 1.335809 respectively.

### References

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Table (4.1) Life Expectancy Estimation for Sudan and States (e<sub>x</sub>)

Age	Sudan	Al Gadarief	Al Gezira	Blue Nile	Central Equatoria	Eastern Equatoria	Jonglei	Kassala	Khartoum	Lakes
0	59	66	74	57	63	72	54	64	65	51
1-4	59	65	73	57	62	71	54	63	65	51
5-9	62	69	73	63	65	74	60	65	65	57
10-14	59	67	69	60	63	72	59	63	61	58
15-19	56	63	65	56	59	68	57	59	56	57
20-24	52	58	60	53	55	65	55	55	52	55
25-29	49	55	56	50	52	61	53	51	47	55
30-34	45	50	51	46	48	57	50	47	43	52
35-39	41	46	47	42	44	53	48	43	39	52
40-44	37	42	42	38	40	48	44	38	34	48
45-49	34	37	38	33	37	44	41	34	30	44
50-54	30	33	33	29	33	40	37	30	26	40
55-59	26	29	29	25	29	36	34	27	23	37
60-64	22	25	25	22	25	31	30	23	19	33
65-69	19	21	22	18	21	27	26	19	16	29
70-74	16	17	18	15	18	23	23	16	13	24
75-79	13	14	15	13	15	19	19	13	10	19
80-84	10	12	12	10	13	15	14	10	8	16
85-89	8	10	8	8	12	10	10	7	5	11
90-94	5	6	6	5	7	8	6	5	4	8
95 +	3	3	3	3	3	3	3	3	3	3

Source: Population Census Data 2008



Table (4.1) Life Expectancy Estimation for Sudan and States (e<sub>x</sub>)

Source: Population Census Data 2008

Age	Nahr El Nile	North Darfur	North Kordfan	Northern State	Northern Bahr El Ghazal	Red Sea	Sinnar	South Darfur	South Kordfan	Unity State
0	71	57	66	75	30	60	72	56	64	30
1-4	70	57	65	75	30	60	71	55	63	30
5-9	69	59	66	72	41	63	72	59	63	34
10-14	66	56	63	68	45	60	68	57	60	34
15-19	61	51	58	63	46	57	64	53	56	36
20-24	56	48	54	58	48	53	60	49	52	43
25-29	52	44	50	54	51	49	55	46	48	47
30-34	47	41	45	49	51	44	51	42	44	48
35-39	43	38	42	45	51	40	47	39	40	49
40-44	38	34	37	40	49	36	43	35	36	48
45-49	34	31	33	36	47	32	38	31	32	46
50-54	29	27	29	31	43	28	34	28	29	42
55-59	25	24	26	27	39	24	30	25	25	39
60-64	21	20	22	23	34	20	26	22	22	35
65-69	17	18	18	20	30	17	23	19	19	30
70-74	14	15	15	16	26	15	19	16	17	26
75-79	11	12	12	13	22	12	15	13	14	23
80-84	9	10	10	10	17	11	11	10	11	18
85-89	6	7	8	8	13	8	8	7	8	13
90-94	4	6	5	5	8	6	6	5	6	8
95 +	3	3	3	3	3	3	3	3	3	3

Source: Population Census Data 2008

Table (4.1) Life Expectancy Estimation for States (ex)

Age	Upper Nile	Warrap	West Darfur	Western Equatoria	Western Bahr El Ghazal	White Nile
0	54	44	37	75	46	70
1-4	53	44	36	75	45	69
5-9	62	52	40	72	56	69
10-14	63	53	39	68	55	65
15-19	60	53	35	63	53	61
20-24	57	53	32	58	52	57
25-29	55	53	30	54	49	53
30-34	51	51	28	49	46	49
35-39	47	49	26	45	41	45
40-44	44	45	23	40	38	40
45-49	40	42	21	36	34	36
50-54	35	38	19	31	30	32
55-59	31	35	17	27	26	28
60-64	27	31	15	23	22	24
65-69	22	27	13	20	19	21
70-74	18	23	12	16	14	17
75-79	15	18	10	13	15	14
80-84	10	14	9	10	11	10
85-89	10	11	8	8	6	8
90-94	7	7	5	5	4	6
95 +	3	3	3	3	3	3

Source: Population Census Data 2008

Table (3) Brass Logit System  $\alpha$  and  $\beta$  values for Sudan and by states

Source: Population Census Data 2008

State	$y_c$	$y_a$	$x_c$	$x_a$	$\alpha$	$\beta$
<b>Sudan</b>	-0.476591129	-	-0.7273333333		-0.36918	2.328949
<b>Al Gadariief</b>	- 0.488623646	- 0.20834	-0.7273333333	0.10786	-0.52974	0.10786
<b>Al Gezira</b>	- 0.669483651	- 0.33849	-0.7273333333	0.10786	-0.61065	2.523281
<b>Blue Nile</b>	- 0.3856826	- 0.11347	- 0.11347	0.10786	0.44441	3.068183
<b>Central Equatoria</b>	- 0.513965141	- 0.14568	-0.7273333333	0.10786	-0.39029	2.267814
<b>Eastern Equatoria</b>	- 0.525872781	- 0.23194	-0.7273333333	0.10786	-0.53843	2.841479
<b>Jonglei</b>	- 0.400609604	- 0.01266	-0.7273333333	0.10786	-0.24487	2.15285
<b>Kassala</b>	- 0.527630164	- 0.18392	-0.7273333333	0.10786	-0.446	2.429902
<b>Khartoum</b>	- 0.614440219	- 0.24296	-0.7273333333	0.10786	-0.48547	2.248307
<b>Lakes</b>	- 0.398687611	0.030371	-0.7273333333	0.10786	-0.17959	1.946571
<b>Nahr El Nile</b>	- 0.738410613	-0.33132	-0.7273333333	0.10786	-0.5526	2.051597
<b>North Darfur</b>	- 0.478623801	-0.08722	-0.7273333333	0.10786	-0.31738	2.133868
<b>North Kordfan</b>	- 0.54390597	-0.2158	-0.7273333333	0.10786	-2.545524	2.545524
<b>Northern</b>	- 0.883780823	-	-0.7273333333	0.10786	-0.59112	1.739115
<b>Northern Bahr El</b>	-0.19082269	0.258773	-0.7273333333	0.10786	0.058407	1.857654
<b>Red Sea</b>	-0.476591129	-0.15739	-0.7273333333	0.10786	-0.4396	2.616472
<b>Sinnar</b>	-0.587461063	-0.27578	-0.7273333333	0.10786	-0.5648	2.679618
<b>South Darfur</b>	-0.402006334	-0.0715	-0.7273333333	0.10786	-0.34407	2.527042
<b>South Kordfan</b>	-0.576984116	-0.17813	-0.7273333333	0.10786	-0.40399	2.094
<b>Unity</b>	-0.332234856	0.292999	-0.7273333333	0.10786	-0.36918	1.335809
<b>Upper Nile</b>	-0.378171428	-0.04978	-0.7273333333	0.10786	-0.3241	2.543292
<b>Warrap</b>	-0.308260765	0.088544	-0.7273333333	0.10786	-0.36918	2.104799
<b>West Darfur</b>	-0.309880141	0.200232	-0.7273333333	0.10786	0.023636	1.637274
<b>Western</b>	-0.498007188	0.029226	-0.7273333333	0.10786	-0.14164	1.584106
<b>Western Bahr El</b>	-0.34357221	0.021897	-0.7273333333	0.10786	-0.22459	2.285264
<b>White Nile</b>	-0.607357126	-0.25683	-0.7273333333	0.10786	-0.51382	2.382648

Source: Population Census Data 2008

Table (4.1) life expectancy estimation for Sudan and States ( $e_x$ ) using Brass Relational Two- Parameter Logit System

Age	Sudan	Al Gadarief	Al Gezira	Blue Nile	Central Equatoria	Eastern Equatoria	Jonglei	Kassala	Khartoum	Lakes
0	54	56	58	55	54	56	52	55	56	50
1	53	55	57	54	54	55	51	54	55	50
2	53	54	56	53	53	55	51	54	55	50
3	52	53	55	52	52	54	50	53	54	49
4	51	52	54	51	52	53	49	52	53	49
5	50	51	53	50	51	52	49	51	52	48
10	46	47	49	46	46	47	44	47	48	44
15	41	42	44	41	42	42	40	42	43	39
20	37	37	39	36	37	38	35	37	38	35
25	32	33	35	31	33	33	31	33	34	31
30	28	28	30	27	29	29	27	29	30	28
35	24	24	26	23	25	24	24	25	26	24
40	20	19	22	19	21	20	20	20	22	20
45	16	15	18	15	17	16	16	17	18	17
50	13	12	14	11	13	12	13	13	14	13
55	10	9	10	8	10	9	10	10	11	11
60	7	6	7	6	8	7	8	7	8	8
65	6	5	5	4	6	5	6	5	6	6
70	4	4	4	4	4	4	4	4	4	5
75	3	3	3	3	3	3	4	3	4	4
80	3	3	3	3	3	3	3	3	3	3
85	3	3	3	3	3	3	3	3	3	3
90	3	2	2	2	3	2	3	3	3	3
95+	-2	-4	-3	-4	-2	-4	-2	-2	-2	-1

Source: Population Census Data 2008

