Achievable Error-Rate Analysis of OFDM Communication Systems Incorporating MRC Diversity Technique over Correlated Nakagami-*m* Fading Channels

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***Abstract*—For the purpose of more effectively investigating the bit error-rate (BER) and symbol error-rate (SER) performances achieved by orthogonal frequency division multiplexing (OFDM) communication systems employing maximal-ratio combining (MRC) receiver architecture, concise closed-form asymptotic BER and SER formulas are derived over correlated Nakagami-*m* fading channels. By utilizing the proposed simple asymptotic BER and SER expressions, explicit insights into the achievable error-rate performance (i.e., modulation gain and diversity order) can be also obtained for the various modulation schemes and channel conditions, particularly in the high signal-to-noise ratio (SNR) regime. To be specific, we derive the exact expressions of the modulation gain obtained from the *L*-branch MRC OFDM systems with binary signals and *M*-ary quadrature amplitude modulation (*M*-QAM) schemes, and also show that the full diversity of order *mL* can be asymptotically achieved even in correlated Nakagami-*m* fading environments.**

***Index Terms*—Correlated Nakagami-*m* Fading Channels; Error-Rate Analysis; MRC Diversity; OFDM.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been recognized as an effective technique for reliable communication systems in wireless environments at high date rate due to its robustness against frequency-selective fading, low-cost implementation using fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) techniques, and flexibility in subcarrier allocation [1, 2], where the fundamental concept of OFDM is based on splitting the total transmission bandwidth into a number of orthogonal subcarriers to transmit data symbols using the subcarriers in parallel. Thus, OFDM has been adopted for several standards (e.g., IEEE 802.11a/g/n IEEE 802.16, IEEE 802.20, etc.) and various applications (e.g., Digital Audio Broadcasting, Digital Terrestrial Video Broadcasting, etc.). On the other hand, various diversity reception methods have been also considered as promising techniques that can dramatically improve the performance of wireless communication systems by mitigating the detrimental effects of the multipath fading in wireless mobile channels [2]. Among the widely used diversity combining techniques (i.e., maximal-ratio combining (MRC), equal-gain combining (EGC), switch and stay combining (SSC), selection combining (SC), etc.), the MRC diversity technique is well known as an optimum reception scheme, since it always shows better error-rate performance than either EGC or SC, which is from the fact that the output signal-to-noise ratio (SNR) can be maximized by MRC diversity [2]. Therefore, it has been of great interest to evaluate the performance achieved by OFDM communication systems incorporating various diversity techniques in different fading channels [2-10].

Nakagami-*m* fading distribution has been widely adopted owing to its capability to model a wider class of fading conditions and to fit well into the empirical multipath fading data [2]. Thus, the authors in [8] derived efficient but complicated closed-form formulas of bit error-rate (BER) and symbol error-rate (SER) for OFDM systems employing MRC diversity with binary signals (e.g., binary phase-shift keying (BPSK) and binary frequency-shift keying (BFSK)) and *M*-ary quadrature amplitude modulation (QAM) signals over spatially correlated Nakagami-*m* fading, which can be considered as being more practical in wireless systems. In particular, they used the moment generating function (MGF) approach, since it is a powerful tool for simplifying the theoretical analysis by leading to simple expression to the average BER and/or SER for a variety of wireless communication systems over fading channels [2]. On the basis of the existing complicated BER/SER formulas derived in [8], which are expressed in terms of hypergeometric functions, hence, our aim in this paper is to further simplify the formulas by judiciously exploiting the high SNR approximation technique and thus to gain more physical insights into the asymptotic error-rate performance achieved by OFDM communication system with MRC diversity reception even in correlated Nakagami-*m* fading environments. In addition, the closed-form expressions of the asymptotically achievable modulation gain and diversity order, which can be readily formulated from the derived concise asymptotic BER/SER formulas, enable us to predict more effectively the achievable error-rate performance of OFDM communication system incorporating MRC diversity technique in various system configurations and channel conditions, without time-consuming computer simulations.

II. System and Channel Models

Let us consider an OFDM system, where $S(k)$ is the k-th OFDM data block to be transmitted with N subcarriers. Assuming that the channel experiences quasi-static frequency-selective fading, after the IFFT processing to modulate the input signal, the time-domain OFDM symbol can be given as $s(n)=1/\sqrt{N}\sum_{k=0}^{N-1}S(k)\exp(j2\pi k/N)$, $n=0,...,N-1$. After the IFFT modulation procedure, suitable cyclic prefix (i.e., guard interval) is inserted to the time-domain OFDM block, to overcome the problem of inter-block interference (IBI), which will be removed before the demodulation procedure at the receiver. Then, the signal is up-converted before the transmission into the channel and down-converted at the receiver side. After the down-conversion, the cyclic prefix is removed and the demodulation is carried out through the FFT, the output of which is represented as:

$$R_i(k)=1/\sqrt{N}\sum_{n=0}^{N-1}r_i(n)\exp(j2\pi k/N)=H_i(k)S(k)+N(k),$$

for $k=0,...,N-1$, where the subscript i stands for the i-th diversity branch of the OFDM receiver with L-branch MRC, $r_i(n)$ is the received signal, $N(k)$ is an additive complex Gaussian noise with zero mean, and $H_i(k)$ is the frequency-domain channel impulse response, given as $H_i(k)=1/\sqrt{N}\sum_{n=0}^{N-1}h_i(n)\exp(-j2\pi k/N)$ with the Nakagami-*m* distributed random variable $h_i(n)$ [2-4].

When γ_i is the instantaneous signal-to-noise ratio (SNR) of *i*-th receiver branch, the overall SNR at the output of MRC diversity can be expressed as $\gamma_T=\frac{E_S}{\sigma^2}\sum_{i=1}^{L}|H_i|^2=\sum_{i=1}^{L}\gamma_i$, with the average symbol energy, E_S, and the variance of the zero-mean complex Gaussian noise, σ^2 [8].

In practice, multiple receive antennas can be closely spaced, and thus the received signals can be correlated. In this case, under the assumption of equal correlation between any two receive antennas, the correlation coefficient is given by [2]

$$\rho=\frac{E\left\{h_i(k)h_j(k)\right\}}{\sqrt{E\left\{h_i(k)h_i^*(k)\right\}E\left\{h_j(k)h_j^*(k)\right\}}}, \tag{1}$$

where $E\{\cdot\}$ denotes the expectation with respect to all the random variables within the braces.

Then, for the overall SNR γ_T at the receiver, the corresponding probability density function (PDF) and moment generating function (MGF) are given, respectively, as [2, 8]

$$f(\gamma_T)=\frac{\left(\frac{\gamma_T m}{\bar{\gamma}_T}\right)^{Lm-1}\exp\left(-\frac{\gamma_T m}{\bar{\gamma}_T(1-\rho)}\right)}{\left(\frac{\bar{\gamma}_T}{m}\right)(1-\rho)^{(L-1)m}(1-\rho+L\rho)^m\Gamma(Lm)} \times {}_1F_1\left(m,Lm;\frac{Lm\rho\gamma_T}{\bar{\gamma}_T(1-\rho)(1-\rho+L\rho)}\right),\ \gamma_T\geq 0, \tag{2}$$

$$\mathcal{M}(s)=\left(1-\frac{\bar{\gamma}_T(1-\rho+L\rho)s}{m}\right)^{-m}\left(1-\frac{\bar{\gamma}_T(1-\rho)s}{m}\right)^{-m(L-1)}, \tag{3}$$

where $\mathcal{M}(s)=\int_0^\infty\exp(s\gamma_T)f(\gamma_T)d\gamma_T$ denotes the MGF of a random variable γ_T, m is the Nakagami fading parameter (cf. $m\geq 1/2$), $\bar{\gamma}_T$ is the average overall effective SNR at the output of MRC, and ${}_1F_1(a,b;x)$ is the confluent hypergeometric function [11, Eq. (07.20.02.0001.01) and Eq. (07.20.07.0001.01)]. Furthermore, by exploiting the high SNR approximation technique to Equation (3) with $\bar{\gamma}_T\to\infty$, we obtain the following asymptotic form of MGF as

$$\mathcal{M}^\infty(s)=\left(-\frac{\bar{\gamma}_T(1-\rho+L\rho)s}{m}\right)^{-m}\left(-\frac{\bar{\gamma}_T(1-\rho)s}{m}\right)^{-m(L-1)}. \tag{4}$$

III. CONVENTIONAL EXACT BER/SER EXPRESSIONS

By adopting the well-known MGF approach, the exact BER expression for OFDM incorporating MRC diversity with binary signaling in arbitrary fading can be given as [2]

$$\bar{P}_b=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\mathcal{M}\left(\frac{-\alpha}{\sin^2\phi}\right)d\phi, \tag{5}$$

where $\alpha=1$ for BPSK, $\alpha=1/2$ for BFSK. Then, from Equation (3) and Equation (5), the exact average BER formula can be formulated, after some straightforward mathematical manipulations, as [8]

$$\bar{P}_b=\frac{1}{2\pi}\mathcal{M}(-\alpha)\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(mL+\frac{1}{2}\right)}{\Gamma(mL+1)} \times F_1\left(\frac{1}{2},m,m(L-1);mL+1;\frac{1}{1+\frac{\bar{\gamma}_T(1-\rho+L\rho)\alpha}{m}},\frac{1}{1+\frac{\bar{\gamma}_T(1-\rho)\alpha}{m}}\right). \tag{6}$$

In addition, the exact SER formula for coherent square *M*-QAM modulations in arbitrary fading channels is given by [2]

$$\bar{P}_s=\frac{4\eta}{\pi}\int_0^{\frac{\pi}{2}}\mathcal{M}\left(\frac{-\beta}{\sin^2\phi}\right)d\phi-\frac{4\eta^2}{\pi}\int_0^{\frac{\pi}{4}}\mathcal{M}\left(\frac{-\beta}{\sin^2\phi}\right)d\phi, \tag{7}$$

where $\beta=1.5/(M-1)$ and $\eta=1-1/\sqrt{M}$. Then, by inserting Equation (3) into Equation (7), and further manipulating it, the exact closed-form SER expression can be obtained as [8]

$$\bar{P}_s = \frac{2\eta}{\pi}\mathcal{M}(-\beta)\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(mL+\frac{1}{2}\right)}{\Gamma(mL+1)} \times F_1\left(\frac{1}{2}, m, m(L-1); mL+1; \frac{1}{1+\frac{\bar{\gamma}_T(1-\rho+L\rho)\alpha}{m}}, \frac{1}{1+\frac{\bar{\gamma}_T(1-\rho)\alpha}{m}}\right) - \frac{4\eta^2}{\pi}\left(\frac{\bar{\gamma}_T(1-\rho+L\rho)\beta}{m}\right)^{-m} \times\left(\frac{\bar{\gamma}_T(1-\rho)\beta}{m}\right)^{-m(L-1)} 2^{-\left(mL+\frac{1}{2}\right)} F_1\left(mL+\frac{1}{2}, m, m(L-1); mL+1; \frac{-m}{2\bar{\gamma}_T(1-\rho+L\rho)\beta}, \frac{-m}{2\bar{\gamma}_T(1-\rho)\beta}\right). \quad (8)$$

Finally, with respect to all N subcarriers, the total average error-rate of the OFDM system with L-branch MRC can be expressed as

$$\bar{P}_{Total} = 1-(1-\bar{P})^N \approx N\bar{P}. \quad (9)$$

Here, we note that owing to the highly complicated formulas (i.e., Equation (6) and Equation (8)), which involve the Appell hypergeometric function $F_1(a,b_1,b_2;c;x_1,x_2)$ [11, Eq. (07.36.02.0001.01) and Eq. (07.36.07.0001.01)], it is quite hard to gain any physical insights into the error-rate performance obtained from the considered several modulation schemes.

IV. Concise Closed-form Asymptotic BER/SER Formulas

To further simplify the complicated exact BER and SER expressions in Equation (6) and Equation (8), respectively, and obtain more comprehensible insights into the achievable error-rate performance, we apply the high SNR approximation technique with $\bar{\gamma}_T \to \infty$ to the existing formulas, and thus derive the concise closed-form asymptotic BER and SER formulas of OFDM employing MRC for binary signals and square M-QAM modulations, respectively, as

$$\bar{P}_b^{\infty} = \frac{1}{2\pi}\mathcal{M}^{\infty}(-\alpha)\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(mL+\frac{1}{2}\right)}{\Gamma(mL+1)} = \frac{\left(\frac{m}{\alpha}\right)^{mL}\Gamma\left(mL+\frac{1}{2}\right)}{2\sqrt{\pi}\Gamma(mL+1)(1-\rho+L\rho)^m(1-\rho)^{m(L-1)}}\left(\frac{1}{\bar{\gamma}_T}\right)^{mL}, \quad (10)$$

$$\bar{P}_s^{\infty} = \frac{2\eta}{\pi}\mathcal{M}^{\infty}(-\beta)\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(mL+\frac{1}{2}\right)}{\Gamma(mL+1)} - \frac{4\eta^2}{\pi}\left(\frac{m}{\beta}\right)^{mL} \times\left(\frac{1}{\bar{\gamma}_T(1-\rho+L\rho)}\right)^m\left(\frac{1}{\bar{\gamma}_T(1-\rho)}\right)^{m(L-1)} 2^{-\left(mL+\frac{1}{2}\right)} = \frac{\left(\frac{m}{\beta}\right)^{mL}}{(1-\rho+L\rho)^m(1-\rho)^{m(L-1)}} \times\left(\frac{2\eta}{\sqrt{\pi}}\frac{\Gamma\left(mL+\frac{1}{2}\right)}{\Gamma(mL+1)} - \frac{4\eta^2}{\pi}2^{-\left(mL+\frac{1}{2}\right)}\right)\left(\frac{1}{\bar{\gamma}_T}\right)^{mL}, \quad (11)$$

where the identity $F_1(a,b_1,b_2;c;0,0)=1$ is exploited [11, 12]. Then, it is obvious that Equation (10) and Equation (11) are much simpler than Equation (6) and Equation (8), respectively, since the derived formulas are expressed only in terms of elementary functions, whereas the existing expressions include complicated hypergeometric functions. Combining Equation (10) or Equation (11) with Equation (9), we can finally obtain the concise closed-form asymptotic expression for the total average error-rate of the OFDM system with N-point IFFT/FFT and L-branch MRC as $\bar{P}_{Total}^{\infty} = 1-(1-\bar{P}^{\infty})^N \approx N\bar{P}^{\infty}$.

As indicated in [2], the asymptotic error-rate can be expressed as $\bar{P}^{\infty} = (G_c^{\infty}\cdot\bar{\gamma})^{-G_d^{\infty}}$ at high SNR, where G_c^{∞} and G_d^{∞} denote the modulation gain and asymptotic diversity order, respectively. Thus, from Equation (10), Equation (11), and Equation (12), we can obtain the corresponding modulation gain and asymptotic diversity order achieved by the considered OFDM system incorporating MRC diversity technique with binary signals and square M-QAM modulations, respectively, as

$$G_{c,binary}^{\infty} = \left[\frac{N\left(\frac{m}{\alpha}\right)^{mL}\Gamma\left(mL+\frac{1}{2}\right)}{2\sqrt{\pi}\Gamma(mL+1)(1-\rho+L\rho)^m(1-\rho)^{m(L-1)}}\right]^{-\frac{1}{mL}}, \quad (12)$$
$$G_{d,binary}^{\infty} = mL,$$

$$G_{c,M-QAM}^{\infty} = \left[\frac{N\left(\frac{m}{\beta}\right)^{mL}}{(1-\rho+L\rho)^m(1-\rho)^{m(L-1)}} \times\left(\frac{2\eta}{\sqrt{\pi}}\frac{\Gamma\left(mL+\frac{1}{2}\right)}{\Gamma(mL+1)} - \frac{4\eta^2}{\pi}2^{-\left(mL+\frac{1}{2}\right)}\right)\right]^{-\frac{1}{mL}}, \quad (13)$$
$$G_{d,M-QAM}^{\infty} = mL,$$

where it is apparent that the achievable modulation gain is affected by the Nakagami fading parameter m, the channel correlation parameter ρ, the subcarrier number of OFDM (i.e., N), the number of MRC branches (i.e., L), and the modulation type (i.e., α , β , and η), while the full-diversity order of mL is achieved for the all considered modulations

V. Numerical Results

In this section, we evaluate the error-rate performance of OFDM systems incorporating *L*-branch MRC diversity technique with binary signals (i.e., BPSK and BFSK) and square *M*-QAM modulations, considering the spatial correlation in Nakagami-*m* fading channels. Specifically, we demonstrate the comparisons between the BER/SER curves from the existing exact error-rate formulas (i.e., Equation (6) and Equation (8)) and those from our derived concise closed-form asymptotic formulas (i.e., Equation (10) and Equation (11)) for several modulation schemes over various fading environments.

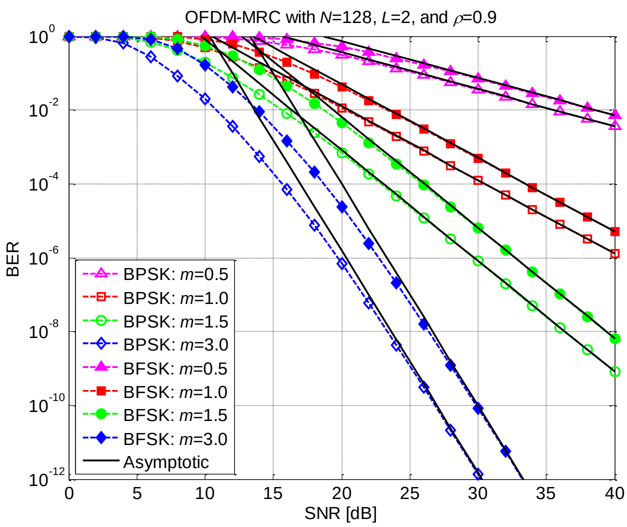

Figure 1: BER vs. SNR of OFDM-MRC with BPSK/BFSK, *N*=128, *L*=2, ρ=0.9, *m*={0.5, 1.0, 1.5, 3.0} in correlated Nakagami-*m* fading channels

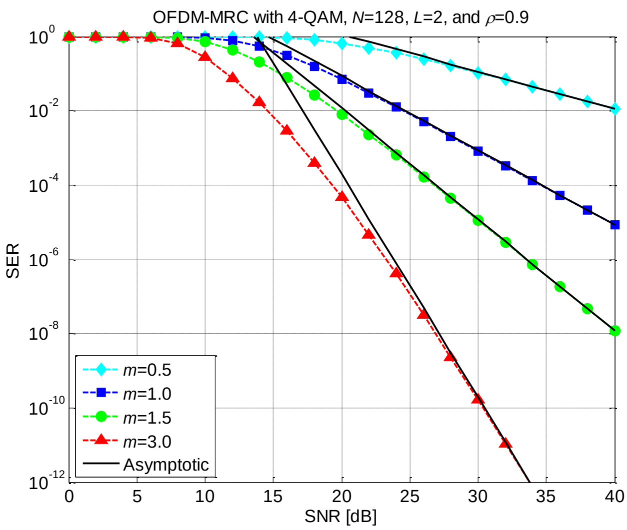

Figure 2: SER vs. SNR of OFDM-MRC with 4-QAM, *N*=128, *L*=2, ρ=0.9, *m*={0.5, 1.0, 1.5, 3.0} in correlated Nakagami-*m* fading channels

Figure 1 and Figure 2 illustrate the average BER/SER performances versus the average SNR $\bar{\gamma}_T$ for $N = 128$, $L = 2$, and $\rho = 0.9$ with BPSK/BFSK and 4-QAM modulations over various Nakagami-*m* fading conditions (i.e., $m = 0.5$, 1.0 (Rayleigh fading), 1.5, 3.0). From the figures, we can clearly observe that for the considered fading scenarios, the lines from the derived asymptotic BER/SER formulas show an excellent match with the curves from the exact but complicated BER/SER expressions at high SNR, which evidently demonstrates that the proposed analytical asymptotic results are very useful to effectively and accurately assess the achievable error-rate behavior of the considered OFDM systems with MRC diversity.

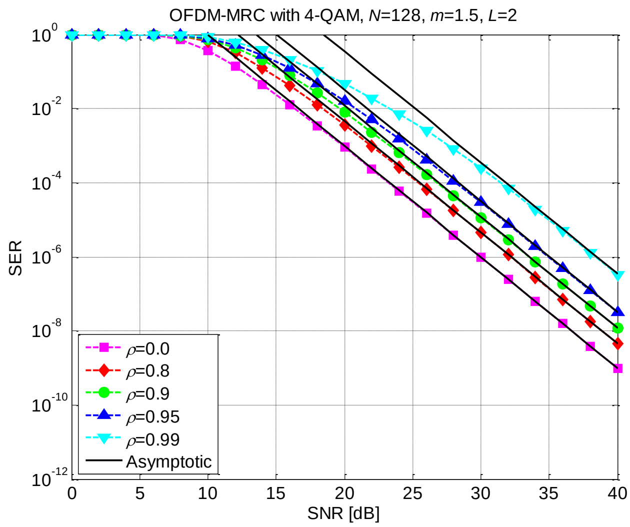

Figure 3: SER vs. SNR of OFDM-MRC with 4-QAM, *N*=128, *L*=2, *m*=1.5, ρ={0.0, 0.8, 0.9, 0.95, 0.99} in correlated Nakagami-*m* fading channels

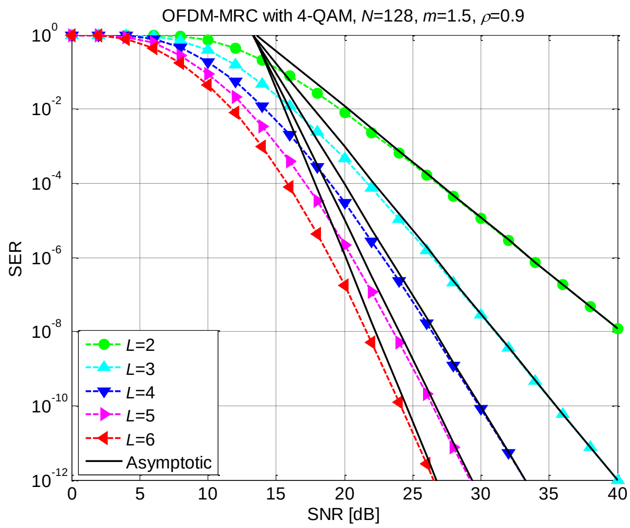

Figure 4: SER vs. SNR of OFDM-MRC with 4-QAM, *N*=128, *m*=1.5, ρ=0.9, *L*={2, 3, 4, 5, 6} in correlated Nakagami-*m* fading channels

In Figure 3 and Figure 4, we investigate the error-rate performance of the OFDM system employing MRC diversity with $(N = 128,\ L = 2,\ m = 1.5,\ \rho = \{0.0, 0.8, 0.9, 0.95, 0.99\})$ and $(N = 128,\ m = 1.5,\ \rho = 0.9,\ L = \{2, 3, 4, 5, 6\})$ for 4-QAM modulation, respectively. To be specific, Figure 3 depicts that the increase of the correlation coefficient leads to the degradation of the error-rate performance, and the derived asymptotic error-rate lines still show an excellent agreement with the exact error-rate curves at high SNR for various values of ρ . In addition, Figure 4 shows that the error-rate performance is improved with the number of MRC diversity branches as well as the asymptotic diversity order, which can be also directly observed as the magnitude of the slope of BER/SER against SNR, in the high SNR region, on a log-log scale, tightly converges to mL .

VI. CONCLUSION

In this paper, utilizing the high SNR approximation technique, the concise and efficient closed-form asymptotic BER/SER formulas have been derived by exploiting the conventional exact but complicated BER/SER expressions for the OFDM communication systems employing MRC diversity reception with various modulation schemes over spatially correlated Nakagami-*m* fading channels. Furthermore, the corresponding modulation gain and diversity order achieved by the considered system have been also derived, which are useful to effectively evaluate the achievable error-rate behavior. Some analytical results have obviously verified the accuracy and efficacy of our derived closed-form asymptotic formulas under various system configurations and channel environments.

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