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***A research vision***

# Optimization of compact fuzzy controllers used for temperature regulation

# Optimización de controladores difusos compactos utilizados para regulación de temperatura

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**Abstract:** This document shows the optimization of different configurations of a compact fuzzy controller for temperature regulation in a room; such configurations are established considering the analogy with different discrete linear controllers; the model is characterized by several heat transfer components. The results show that the optimization process allows adequate tuning of most of the fuzzy controllers. The initial configuration is relevant for the optimization of the controllers; finally, the best result is obtained with the configuration PID of the compact controller.

**Keywords:** Control, fuzzy system, thermal system.

**Resumen:** Este documento muestra la optimización de diferentes configuraciones de un controlador difuso compacto para la regulación de temperatura en una habitación; tales

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configuraciones se establecen considerando la analogía con diferentes controladores lineales discretos. El modelo se caracteriza por varias componentes de transferencia de calor. Los resultados muestran que el proceso de optimización permite una adecuada sintonía de la mayoría de controladores difusos. Para la optimización de los controladores es de relevancia la configuración inicial, finalmente, el mejor resultado se obtiene con la configuración del controlador compacto de tipo PID.
**Palabras clave:** Control, sistema difuso, sistema térmico.

## 1. Introduction

Fuzzy logic systems allow relating the inputs and outputs of a process through linguistic terms in its description [1]. This feature concedes the use of fuzzy logic in the development of control systems [2]. With this approach, fuzzy logic systems also allow handling nonlinearities as products between variables, saturations and power functions among others, which are presented in nonlinear dynamic systems [3].

This article proposes a fuzzy logic system for the control of a thermal system where the temperature of a house is to be regulated. The model of this plant can be seen in [4], [5], and [6]. This control system seeks to improve the energy efficiency of heating and cooling methods during transient periods of operation. According to [4], these are of great importance for energy management.

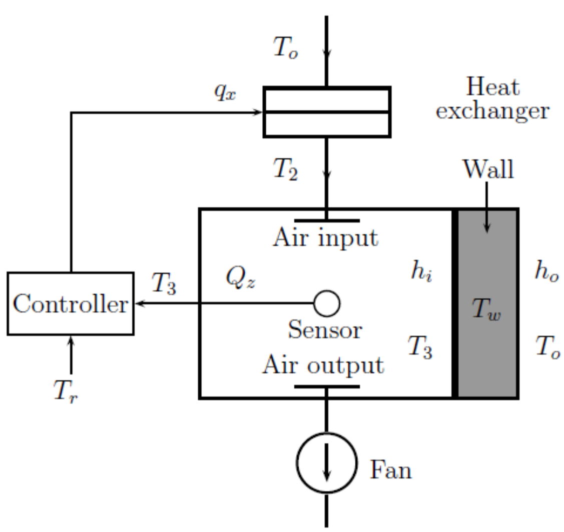
Some works related to the application of fuzzy logic for control of thermal systems can be found in [4] and [7], where a Fuzzy Proportional Integral Sum Derivative (F-PSD) is described. A non-linear proposal is also made based on a fuzzy system of Takagi-Sugeno type where non-linearity is included for the discrete time controller states. Moreover, [6] presents a development for the optimization of an F-PID controller using genetic algorithms. A similar proposal where the particle swarm optimization algorithm is used can be seen in [8]. Finally, [9] carries out the optimization of fuzzy set parameters for a controller.

In this proposal, the controllers are established starting from a discrete time linear controller and then converted into a compact fuzzy form using fuzzy sets. Then, the optimization of the controller parameters is performed with the plant model. The optimization process takes place through the “FMINUNC” function of MATLAB®, which implements the Quasi-Newton method of BFGS (Broyden-Fletcher-Goldfarb-Shanno). With this approach the initial search point is relevant [10].

## 2. System model

Considering the model used in [4], [5] and [6], the system is described by a set of differential equations based on energy balances for the heat exchanger, the air and the wall of the thermal zone. The scheme of the thermal system can be seen in Figure 1.

**Figure 1.** Schematic of the thermal system model.




Source: Based on [4].

The equations that relate the variables of the model and represent the dynamics of the system are:

$$\rho_x C_x V_x \frac{dT_2}{dt} = \rho_a f C_a (T_o - T_2) + q_x \quad (1)$$

$$\rho_a C_a V_z \frac{dT_3}{dt} = \rho_a f C_a (T_2 - T_3) + h_i A_i (T_w - T_3) + Q_z \quad (2)$$

$$\rho_w C_w V_w \frac{dT_w}{dt} = h_o A_o (T_o - T_w) - h_i A_i (T_w - T_3) \quad (3)$$

In general, the model parameters are:

- $A$: Wall area $[m^2]$.
- $C$: Specific heat $[KJ/Kg°C]$.
- $f$: Volumetric flow $[m^3/s]$.
- $h$: Convection heat transfer coefficient $[W/m^2K]$.
- $q_x$: Heat exchanger input power $[W]$.
- $Q_z$: Internal thermal load $[W]$.
- $R$: Thermal resistance $[m^2°C/W]$.
- $T$: Temperature $[°C]$.
- $V$: Volume $[m^3]$.
- $\rho$: Density $[kg/m^3]$.

The subscripts for the model variables are:

- $a$: Air.
- $w$: Wall.
- $i$: Interior of the heating zone.
- $o$: Exterior of the heating zone.

• $z$: Heat zone.

## 3. Fuzzy systems

Fuzzy logic systems allow representing an input-output relationship using fuzzy sets and relationships (rules) between these sets. In control systems they allow to establish the different actions to be carried out on the plant [1].

In this regard, different types of fuzzy systems have been proposed such as Mandani, composed of rules of the If-Then form, using linguistic terms represented by fuzzy sets at the input and output. Takagi-Sugeno, which uses rules, composed of fuzzy sets in the antecedent and singleton functions in the output, which corresponds to a particular case of the Mandani model where there is a singleton set (constant value) [11].

The proposal of the fuzzy compact controllers made in this document uses fuzzy sets at the input, at the output the control actions correspond to actuators of constant type. In this way, it can easily establish an initial configuration which is then improved using optimization algorithms.

## 4. Control system

The general scheme of the control system can be seen in Figure 2. Here, it is appreciated that the value of the error is taken to implement the function that is the object of optimization.

For the design of compact fuzzy controllers, linear control strategies are considered as referents:

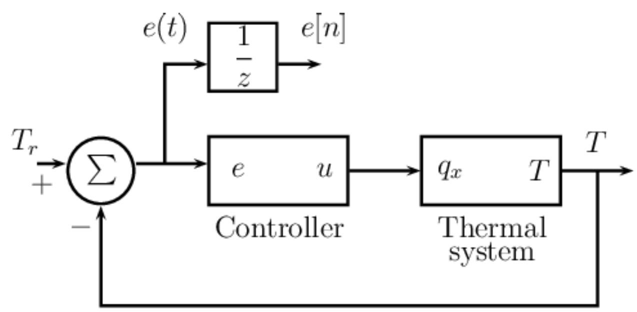
• General controller (G).

• Proportional Integral Control (PI).

• Proportional Derivative Control (PD).

• Proportional Integral Derivative Control (PID).

**Figure 2.** Block diagram of the control system.

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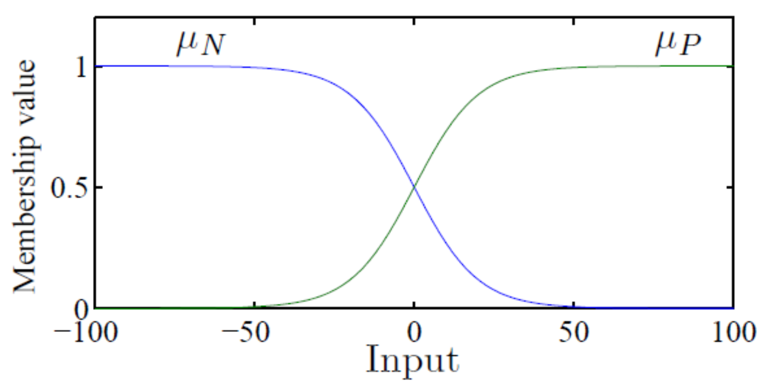
Fuzzy compact controllers are designed using the sigmoidal membership functions shown in Figure 3, which represent positive and negative error values. On the other hand, the control action is associated to an action value (virtual actuator) that is activated directly by the membership value produced at the input.

**Figure 3.** Fuzzy sets used for each antecedent (input).

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### 4.1. General controller

The discrete linear controller considered consists of a second order system of the form:

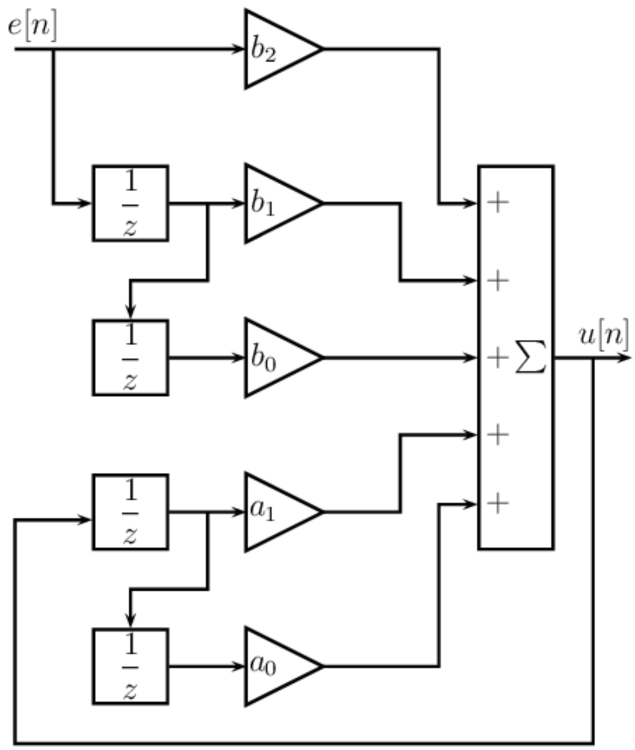
$$\frac{U(z)}{E(z)} = \frac{b_2+b_1z^{-1}+b_0z^{-2}}{1+a_1z^{-1}+a_0z^{-2}} \quad (4)$$

The respective equation in discrete time is:

$$u[n] = -a_1 u[n-1] - a_0 u[n-2] + b_2 e[n] + b_1 e[n-1] + b_0 e[n-2] \quad (5)$$

The representation of this controller using a block diagram can be seen in Figure 4.

**Figure 4.** Linear controller in discrete time.

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On the other hand, the structure of the compact fuzzy controller can be seen in Figure 5. In this diagram, it is to be noted that the inputs from the memory elements enter the fuzzy system where the non-linear relationships of the system states are defined. It is also noteworthy that the structure used with the memory elements is similar for the two systems considered (linear and fuzzy).

Each of the discrete controller variables is modeled using the fuzzy sets shown in Figure 3. Graphically, the set of all rules can be seen in Figure 6.

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**Figure 5.** Structure of the fuzzy controller.

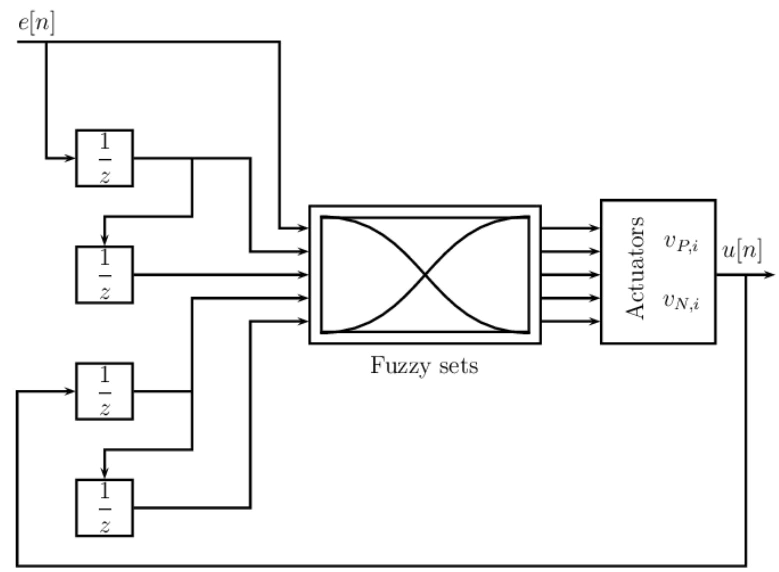


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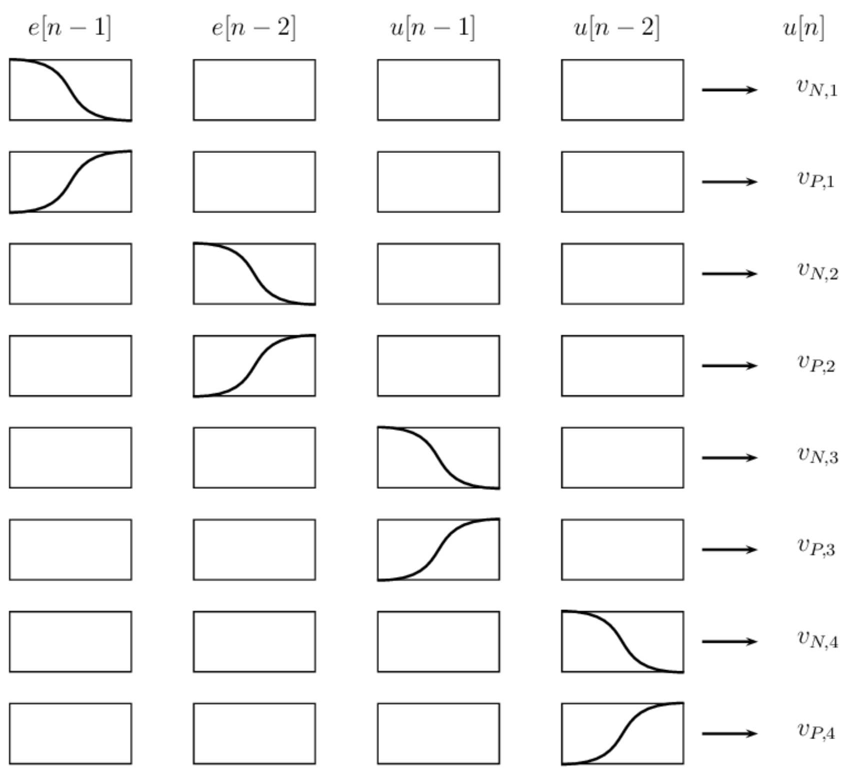
**Figure 6.** Graphical representation of the fuzzy system rules.

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In this way, the respective rules for each input are:

- *If* $e[n-1]$ *is* $\mu_{N,1}$ *then* $u[n]$ *is* $v_{N,1}$.
- *If* $e[n-1]$ *is* $\mu_{P,1}$ *then* $u[n]$ *is* $v_{P,1}$.
- *If* $e[n-2]$ *is* $\mu_{N,2}$ *then* $u[n]$ *is* $v_{N,2}$.
- *If* $e[n-2]$ *is* $\mu_{P,2}$ *then* $u[n]$ *is* $v_{P,2}$.
- *If* $u[n-1]$ *is* $\mu_{N,3}$ *then* $u[n]$ *is* $v_{N,3}$.
- *If* $u[n-1]$ *is* $\mu_{P,3}$ *then* $u[n]$ *is* $v_{P,3}$.
- *If* $u[n-2]$ *is* $\mu_{N,4}$ *then* $u[n]$ *is* $v_{N,4}$.
- *If* $u[n-2]$ *is* $\mu_{P,4}$ *then* $u[n]$ *is* $v_{P,4}$.

### 4.2. PI Controller

In this case, the controller has the form:

$$C(z) = k_p + \frac{K_i}{1-z^{-1}} = \frac{(K_p+K_i)-K_p z^{-1}}{1-z^{-1}} \quad (6)$$

In general terms it can be expressed as:

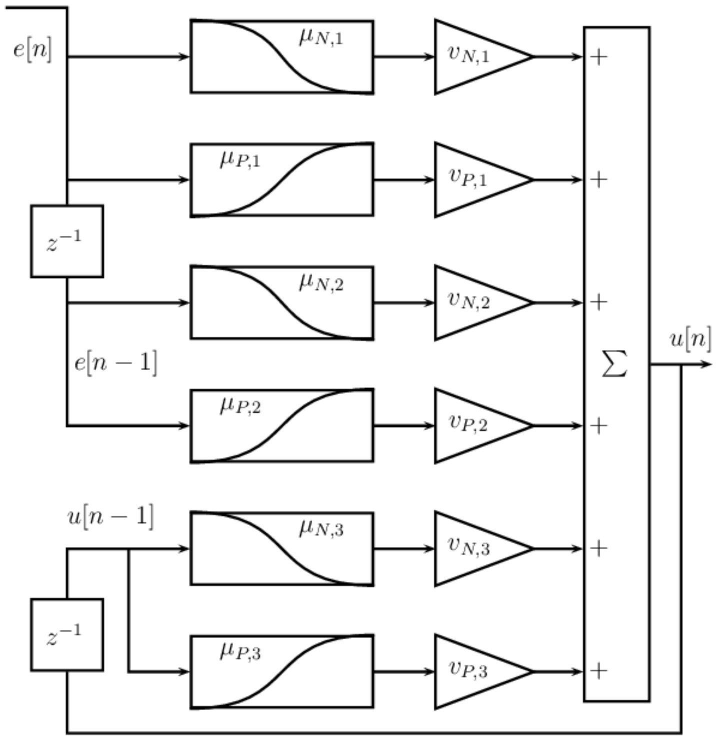
$$C(z) = \frac{b_0-b_1 z^{-1}}{1-z^{-1}} \quad (7)$$

Thus, the equation in discrete time of the PI controller corresponds to:

$$u[n] = u[n-1] + b_0 e[n] - b_1 e[n-1] \quad (8)$$

With reference to the difference equation for this controller, it is proposed the implementation in Figure 7.

**Figure 7.** PI compact fuzzy system.

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### 4.3. PD controller

The PD controller can be represented as follows:

$$C(z) = K_p + K_d(1 - z^{-1}) = (K_p + K_d) - K_d z^{-1} \quad (9)$$

A general expression for this would be:

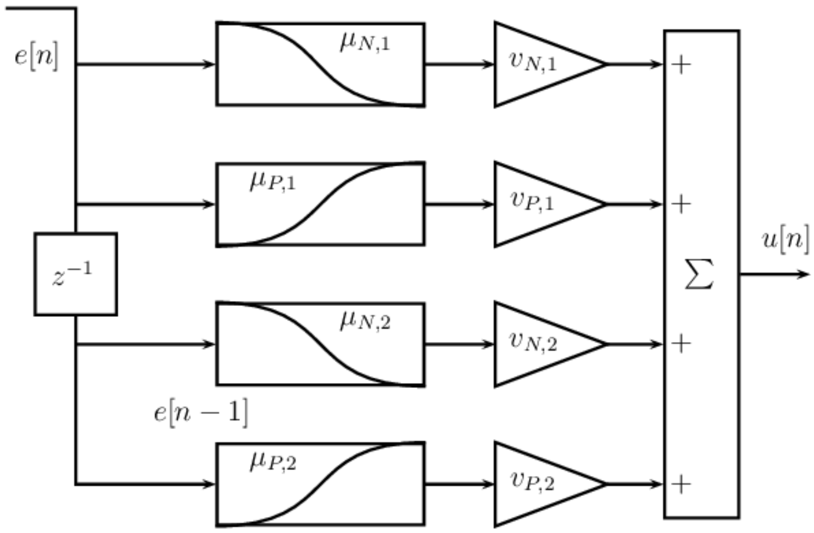
$$C(z) = b_0 - b_1 z^{-1} \quad (10)$$

The corresponding equation in discrete time is:

$$u[n] = b_0 e[n] - b_1 e[n-1] \quad (11)$$

Using the respective fuzzy sets for negative and positive values is obtaining the scheme in Figure 8.

**Figure 8.** Compact PD fuzzy system.

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### 4.4. PID Controller

In this case, the PID controller is described as:

$$C(z) = K_p + \frac{K_i}{1-z^{-1}} + K_d(1 - z^{-1}) \tag{12}$$

After doing the operations the obtained result is:

$$C(z) = \frac{(K_p+K_i+K_d)-(K_p+2K_d)z^{-1}+K_d z^{-1}}{1-z^{-1}} \tag{13}$$

In general, it can be written as:

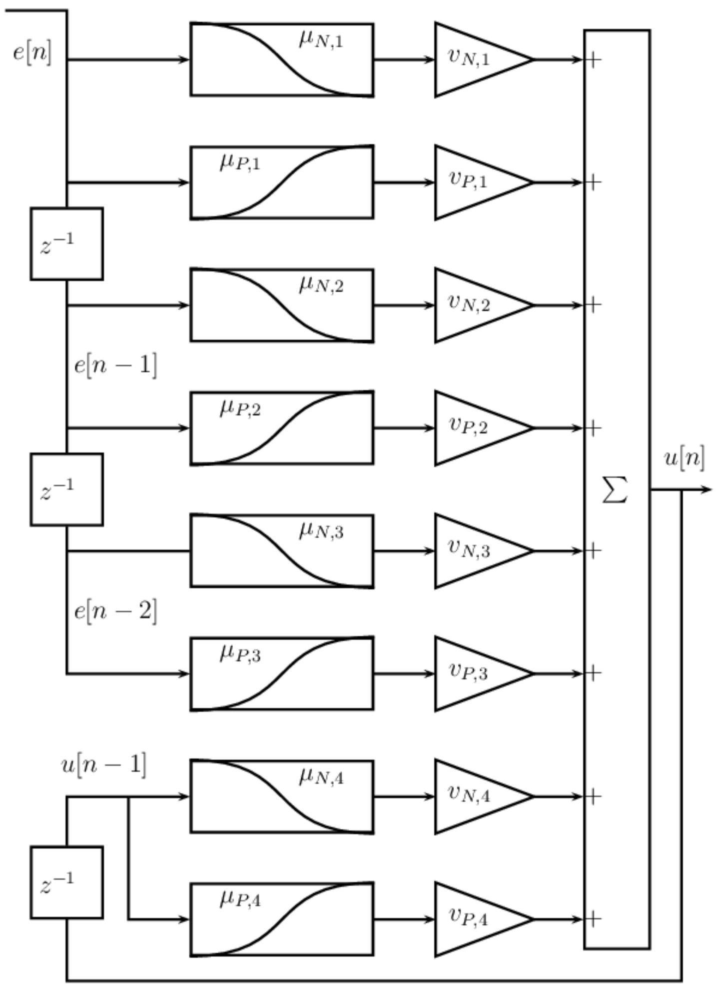
$$C(z) = \frac{b_0-b_1 z^{-1}+b_2 z^{-1}}{1-z^{-1}} \tag{14}$$

The respective difference equation for this controller is:

$$u[n] = u[n-1] + b_0 e[n] - b_1 e[n-1] + b_2 e[n-2] \tag{15}$$

Thus, the respective compact fuzzy PID controller is shown in Figure 9.

**Figure 9.** PID compact fuzzy system.

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## 5. Optimization process

To implement the optimization process, a function is used to establish the dynamics of the plant with the control system, where the parameters of the controller $X$ are taken as input and the response of the control system $M(X)$ as output. The result of this function is used to calculate the performance index to be optimized $J(X)$. In this way, the optimization process is carried out using the function that implements the respective performance index.

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To implement the performance function, the vector $X$ corresponds to the set of parameters of the controller, $n$ the discrete time variable and $N$ the total number of data used, in this way, the performance function corresponding to the mean square error, which is calculated as:

$$J = \frac{1}{N}\sum_{n=1}^{N} \left(y_{ref}(n) - y_{out}(n, X)\right)^2 \quad (16)$$

The process of implementation uses the "FMINUNC" function of MATLAB®, which employs the BFGS Quasi-Newton (Broyden-Fletcher-Goldfarb-Shanno) algorithm, which is a method where a successive matrix approach of Hessiana is performed [10].

## 6. Results

This section presents the results obtained when optimizing the compact fuzzy controllers where the optimization variables correspond to the parameters of the membership functions $\mu_{N,i}$, $\mu_{P,i}$ and the respective actuators $v_{N,i}$, $v_{P,i}$. The parameters of the thermal system are taken in accordance with those presented in [4] having as initial temperature $35°\text{C}$ and the desired $25°\text{C}$. Table 1 shows the values of the objective function for the controllers before and after optimization.

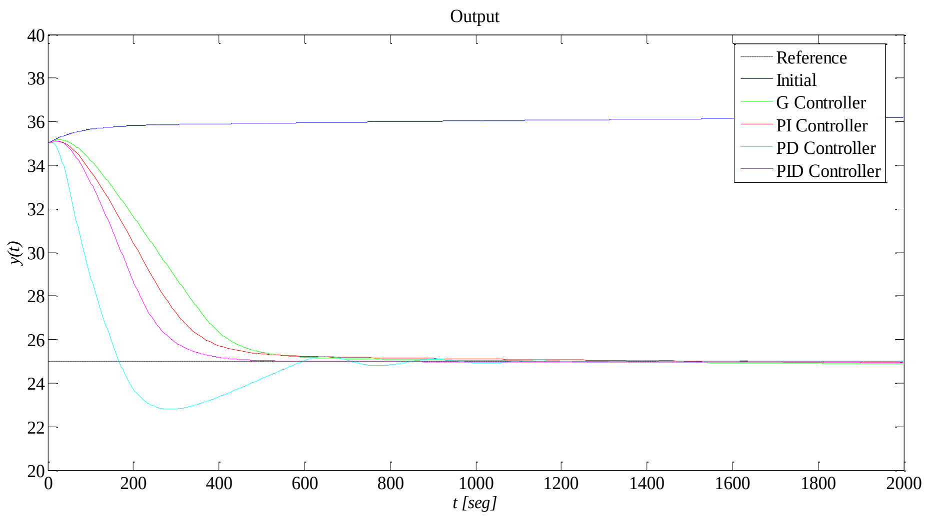
**Table 1.** Values of the objective function.

| System | G | PI | PD | PID |
|---|---|---|---|---|
| Before | 126.0260 | 126.0380 | 126.0260 | 126.0260 |
| After | 4.0952 | 3.3863 | 1.5980 | 2.7933 |

Source: own.

The results of the optimization process are presented in Figure 10, where it is see the response of the system without optimizing and the output of the system with each of the controllers after being optimized.

**Figure 10.** Results of the optimization process.

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As can be seen in Table 1 and in Figure 10, the fuzzy controller without optimization has a high error value. After the optimization process it is observed that the proposed controllers allow the system to reach the reference value. The best performance value is the PD control; however, the reference is passed before stabilizing. Considering this may be a not desire behavior, then the best performance would be the PID controller.

## 7. Conclusions

It is noticeable that it is possible to perform the optimization of different compact fuzzy controllers applied to the temperature regulation of a thermal system.

The initial search point for the optimization process considered allows for proper tuning of the controllers.

Through the optimization process, the improvement in the performance of compact fuzzy controllers is observed. The optimization strategy used allows a quick adjustment of the controller parameters.

In a future work, it is expected to develop more models of compact fuzzy controllers that can be used for adaptive control processes.

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