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# CONTROLLER DESIGN BASED ON A KINEMATIC ESTIMATOR FOR A 3RRR PLANAR PARALLEL ROBOT DRIVEN BY ELECTRIC MOTORS

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**Abstract.** Parallel robots are used increasingly thanks to several advantages as high rigidity and accuracy, and small link weight. These advantages are achieved by their closed-loop structure of the mechanical system. However, redundant generalized coordinates are often used in dynamic simulation and design of controller for parallel robots. A designed controller normally requires feedbacks of all redundant coordinates and their derivatives. This requirement is hard to achieve in practice, because robots are usually only equipped with sensors for active joints. In addition, some coordinates cannot be easily measured by sensors like encoder. In this paper, a novel method is introduced to estimate three auxiliary generalized coordinates and motion of the moving platform. A kinematic error feedback technique is exploited to ensure the estimated motion converge to the actual motion of the robot. Numerical simulations are performed on a 3RRR parallel robot model with nine generalized coordinates to confirm the reliability and efficiency of the proposed method.

*Keywords*: dynamic modeling, motion control, 3RRR planar parallel robot, kinematic estimation, numerical simulation.

*Classification numbers*: 5.3.2, 5.3.5, 5.3.7.

## 1. INTRODUCTION

Nowadays, parallel robotic manipulators are used widely in industrial applications due to its advantages such as higher accuracy and rigidity, higher loading rates on robot weight than those of serial robots. However, parallel robots have some major disadvantages such as small workspaces and many singularities in the workspace due to their closed loop structure [1, 2]. This kind of robots has attracted numerous researchers. Most of them deal with the dynamic modeling and the controller design. For dynamic modeling of parallel robots, residual generalized coordinates are normally used, and dynamics of parallel robots are described by differential algebraic equations (DAEs). A number of methods for constructing dynamic equations include: Lagrangian equations with multipliers, Newton - Euler equation, Jourdain principle or Kane dynamic equations [3–10].

Similar to serial robots, there are several control laws designed for parallel robots [11–17] such as: proportional–derivative controller (PD) plus gravity compensation, exact linearization, sliding mode control, adaptive control, fuzzy control, neural networks based control, etc. These controllers often require the feedback of all generalized coordinates and their derivatives. This requires the robot to be equipped with a variety of sensors to measure not only the actuated joint variables, but also the auxiliary coordinates as well as the position and velocity of the moving platform. This requirement is not always met and will certainly increase the cost of the robot. To overcome these limitations, this paper proposes the use of the kinematic estimator to determine the dependent generalized coordinates and their derivatives. This approach has been applied in [18] where the connecting links of the robot are modelled by two particles at the ends of the links and the system state is described by only six generalized coordinates. Meanwhile, the connecting links are considered as a rigid body in this paper. Therefore, nine redundant generalized coordinates are used for the robot. The designed kinematic estimator has been integrated with conventional PD and proportional–integral–derivative (PID) controllers. In addition, kinematic error feedback techniques are exploited to ensure that the estimated motion tracks the actual motion of the robot.

The remaining of the paper is structured as follows: Section 2 shows the dynamic model of a parallel robot driven by electric motors. Sections 3.1 and 3.2 present the design of a controller based on a dynamic model with the assumption that all generalized coordinates are measured. Section 3.3 presents the kinematic estimator to determine dependent coordinates. The numerical simulation results are presented in Section 4. Finally, the conclusion in Section 5.

## 2. DYNAMIC MODEL OF PARALLEL ROBOT DRIVEN BY ELECTRIC MOTORS

Let's consider a parallel robot of *n* degrees of freedom driven by *n* electric motors. For parallel robot dynamics, the residual generalized coordinates are often used, as it allows easier in establishing differential equations and is also more convenient to simulate on computers. Defining $\mathbf{q} = [q_1, q_2, ..., q_m]^T = [\boldsymbol{\theta}^T, \mathbf{y}^T]^T = [\boldsymbol{\theta}^T, \boldsymbol{\beta}^T, \mathbf{x}^T]^T, \quad m > n$ is the residual generalized coordinates vector for the robot, in which $\boldsymbol{\theta}$ - active joint variables, $\mathbf{x}$ - coordinates of moving platform and $\beta$ - auxiliary joint variables. The establishment of equations of motion for this system has been presented in numerous references [1, 6, 14, 19, 20]. With Lagrangian multipliers the differential equations of motion for the system are written in the following form:

$$\mathbf{M}_s(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_s(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}_s\dot{\mathbf{q}} + \mathbf{g}_s(\mathbf{q}) = \mathbf{B}_s\mathbf{u} + \boldsymbol{\Phi}_q^T(\mathbf{q})\boldsymbol{\lambda}, \tag{1}$$

$$\phi(\mathbf{q}) = \mathbf{0}, \;\; \boldsymbol{\Phi}(\mathbf{q}) = \partial\phi / \partial\mathbf{q} \tag{2}$$

where $\mathbf{M}_s(\mathbf{q}) = \mathbf{M}(\mathbf{q}) + \mathbf{B}\mathbf{J}_m r^2\mathbf{Z}$ is mass matrix, $\mathbf{C}_s(\mathbf{q}, \dot{\mathbf{q}})$ - Coriolis and centrifugal matrix is calculated by Christoffel symbol [4, 7] or using Kronecker product [21], $\mathbf{D}_s = \mathbf{B}(\mathbf{D}_m + \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{K}_e)r^2\mathbf{Z}$ - damping matrix, $\boldsymbol{g}_s(\boldsymbol{q})$ - generalized force due to gravity, $\mathbf{B}_s = \mathbf{B}\mathbf{K}_m\mathbf{R}_a^{-1}r$ - input matrix related to configuration of actuators. The parameters of *n* electric motors are collected in some matrices including: $\mathbf{J}_m = \mathrm{diag}(J_{m,1}, J_{m,2}, ..., J_{m,n})$ -moment of inertia of rotors, $\mathbf{R}_a = diag(R_{a,1}, R_{a,2}, ..., R_{a,n})$ - motor coil resistances, $\mathbf{K}_e = diag(K_{e,1}, K_{e,2}, ..., K_{e,n})$ - back-emf constants, $\mathbf{K}_m = diag(K_{m,1}, K_{m,2}, ..., K_{m,n})$ - torque constants, and $r$ - the ratio of the gearbox. The details of the equations of motion are presented in the articles [11,12,19].

To design a controller based on dynamic models, the differential algebraic equations (1) and (2) will be transformed into active joint coordinates. By using the matrix, $\mathbf{R}(\mathbf{q}) \in \mathbb{R}^{m\times n}$ is defined as follows:

$$\mathbf{R}(\mathbf{q}) = \begin{bmatrix} \mathbf{E} \\ -\mathbf{\Phi}_y^{-1}\mathbf{\Phi}_\theta \end{bmatrix}, \tag{3}$$

where $\mathbf{\Phi}_\theta(\mathbf{q}) = \partial\boldsymbol{\phi} / \partial\boldsymbol{\theta}$, $\mathbf{\Phi}_y(\mathbf{q}) = \partial\boldsymbol{\phi} / \partial\mathbf{y}$ are Jacobian matrices of the constraint equations with respect to (w.r.t) the active and dependent generalized coordinate vector. The dynamic equation of the parallel robot is transferred to the form

$$\mathbf{M}_\theta(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}_\theta(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{D}_\theta\dot{\boldsymbol{\theta}} + \mathbf{g}_\theta(\boldsymbol{\theta}) = \mathbf{R}^T\mathbf{B}_s\mathbf{u} =: \boldsymbol{\tau}_\theta, \tag{4}$$

with

$$\mathbf{M}_\theta(\boldsymbol{\theta}) = \mathbf{R}^T\mathbf{M}_s(\mathbf{q})\mathbf{R} \in \mathbb{R}^{n\times n},\ \mathbf{C}_\theta(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = \mathbf{R}^T\left[\mathbf{M}_s(\mathbf{q})\dot{\mathbf{R}} + \mathbf{C}_s(\mathbf{q},\dot{\mathbf{q}})\mathbf{R}\right] \in \mathbb{R}^{n\times n}$$

$$\mathbf{D}_\theta = \mathbf{R}^T\mathbf{D}_s\mathbf{R},\ \mathbf{g}_\theta(\boldsymbol{\theta}) = \mathbf{R}^T\mathbf{g}_s(\mathbf{q}),\ \boldsymbol{g}_\theta \in \mathbb{R}^n.$$

In the equation (4) the following properties are still guaranteed: $\mathbf{M}_\theta(\boldsymbol{\theta})$ is a symmetric and positive and $\mathbf{N}_\theta = \dot{\mathbf{M}}_\theta(\boldsymbol{\theta}) - 2\mathbf{C}_\theta(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ is a skew-symmetric matrix. These properties are very important for control design shown in the following section. Dynamic model (4) is base for control design.

Noting that in case of modeling the connecting links by two masses at two ends the auxiliary joint variables $\beta$ do not appear in the above equations. This case has been solved in the work [18]. In this paper, the connecting links are modeled by rigid bodies, so that the auxiliary joint variables are needed to describe motion of these links. Therefore, the number of generalized coordinates increase, and the problem becomes more complex than the problem solved in the previous work [18].

## 3. CONTROLLER DESIGN WITH A KINEMATIC ESTIMATOR

The objective of the control problem is to find the law of the motor voltage so that the motion of the moving platform tracks the given trajectory defined by $\mathbf{x}_d(t)$. A control law can be designed in actuated joint space such that $\mathrm{e}_\theta = \boldsymbol{\theta} - \boldsymbol{\theta}_d \to 0$ or in operational space such that $\mathbf{e}_x = \mathbf{x} - \mathbf{x}_d \to 0$. In this section, the methods of control design in active joint space are presented. This approach requires to solve inverse kinematics to find $\boldsymbol{\theta}_d(t)$ from $\mathbf{x}_d(t)$. The basis for the design of controller in joint space is equation (4).

### 3.1. Position control by PD controller plus gravity compensation

For position control problem, we can apply the law PD + gravity compensation as follows:

$$\mathbf{u} = (\mathbf{R}^T\mathbf{B}_s)^{-1}\left(-\mathbf{K}_p\mathbf{e}_\theta - \mathbf{K}_d\dot{\mathbf{e}}_\theta + \mathbf{g}_s(\boldsymbol{\theta})\right) \tag{5}$$

where $\mathbf{e}_\theta = \boldsymbol{\theta} - \boldsymbol{\theta}_d$.

The stability of the closed system is proven by selecting the following Lyapunov function

$$V = \frac{1}{2}\dot{\boldsymbol{\theta}}^T \mathbf{M}_\theta(\mathbf{q})\dot{\boldsymbol{\theta}} + \frac{1}{2}\mathbf{e}_\theta^T \mathbf{K}_p \mathbf{e}_\theta . \tag{6}$$

Differentiating of $V$ with respect to (w.r.t.) time, under consideration of equation (4) and skew-symmetric property of matrix $[\dot{\mathbf{M}}_\theta - 2\mathbf{C}_\theta]$, one obtains

$$\begin{aligned} \dot{V} &= \dot{\boldsymbol{\theta}}^T \mathbf{M}_\theta(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \tfrac{1}{2}\dot{\boldsymbol{\theta}}^T \dot{\mathbf{M}}_\theta(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} + \dot{\boldsymbol{e}}_\theta^T \mathbf{K}_p \boldsymbol{e}_\theta \\ &= -\dot{\boldsymbol{\theta}}^T \left(\mathbf{D}_\theta(\boldsymbol{\theta}) + \mathbf{K}_d\right)\dot{\boldsymbol{\theta}} \le 0 \end{aligned} \tag{7}$$

**3.2. PID controller with inverse dynamics**

For tracking control, the method based on inverse dynamics, exact feedback linearization, or computed torque control can be applied. The control law is chosen as follows:

$$\mathbf{u} = (\mathbf{R}^T \mathbf{B}_s)^{-1} \left[\mathbf{M}_\theta(\boldsymbol{\theta})\mathbf{v} + \mathbf{C}_\theta(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{b}_\theta(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\right] \tag{8}$$

where

$$\mathbf{v} = \ddot{\boldsymbol{\theta}}_d - \mathbf{K}_D \dot{\mathbf{e}}_\theta - \mathbf{K}_P \mathbf{e}_\theta - \mathbf{K}_I \int_0^t \mathbf{e}_\theta(\tau) d\tau . \tag{9}$$

Substituting (8) into (4) yields

$$\mathbf{M}_\theta(\mathbf{q})(\ddot{\boldsymbol{\theta}} - \mathbf{v}) = \mathbf{0} . \tag{10}$$

Due to matrix $\mathbf{M}_\theta(\mathbf{q})$ is positive definite, from equation (10), one obtains

$$\ddot{\boldsymbol{\theta}} - \mathbf{v} = \mathbf{0} . \tag{11}$$

Combining with equation (9), one obtains

$$\ddot{\boldsymbol{\theta}} - \ddot{\boldsymbol{\theta}}_d + \mathbf{K}_D \dot{\mathbf{e}}_\theta + \mathbf{K}_P \mathbf{e}_\theta + \mathbf{K}_I \int_0^t \mathbf{e}_\theta(\tau) d\tau = \mathbf{0} \tag{12}$$

or

$$\ddot{\mathbf{e}}_\theta + \mathbf{K}_D \dot{\mathbf{e}}_\theta + \mathbf{K}_P \mathbf{e}_\theta + \mathbf{K}_I \int_0^t \mathbf{e}_\theta(\tau) d\tau = \mathbf{0} . \tag{13}$$

By differentiating (13) w.r.t. time one gets

$$\dddot{\mathbf{e}}_\theta + \mathbf{K}_D \ddot{\mathbf{e}}_\theta + \mathbf{K}_P \dot{\mathbf{e}}_\theta + \mathbf{K}_I \mathbf{e}_\theta = \mathbf{0} . \tag{14}$$

If the positive definite matrices $\mathbf{K}_D, \mathbf{K}_P, \mathbf{K}_I$ are selected as the diagonal one, from (14) we obtain the third order linear differential equations as follows

$$\dddot{e}_{\theta i} + k_{Di}\ddot{e}_{\theta i} + k_{Pi}\dot{e}_{\theta i} + k_{Ii} e_{\theta i} = 0, \; i = 1,2, \ldots \tag{15}$$

Characteristic equation (15) has the form

$$\lambda_i^3 + k_{Di}\lambda_i^2 + k_{Pi}\lambda_i + k_{Ii} = 0 \text{, i = 1, 2, 3} \tag{16}$$

According to Hurwitz criterion, the conditions for the characteristic equation (16) have negative real parts as follows:

$$k_{Di} > 0, \quad k_{Pi} > 0, \quad k_{Ii} > 0, \qquad k_{Di}k_{Pi} - k_{Ii} > 0, \quad i = 1,2,3 \tag{17}$$

Thus, if we choose the coefficients $k_{Di}, k_{Pi}, k_{Ii}$ satisfy the condition (17), the solution of the system (15) will converge asymptotically to zero. This leads to $\boldsymbol{\theta}(t) \to \boldsymbol{\theta}_d(t)$ and $\mathbf{x} \to \mathbf{x}_d$.

### 3.3. Design of a kinematic estimator based on constraint equations

Control laws (5) and (8) require not only active joint variables $\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}$ but also auxiliary variables $\mathbf{y}, \dot{\mathbf{y}}$. In order to get these variables for feedback, it is required to equip more robots with more sensors in addition to the active encoder variable encoders. To avoid this cost, the paper proposes to estimate these variables from kinematic constraint equations. This section presents the construction of an estimator that meets the above requirements.

The constraint equations at the position level are rewritten as follows

$$\boldsymbol{\phi}(\mathbf{q}) \equiv \boldsymbol{\phi}(\boldsymbol{\theta}, \mathbf{y}) = 0. \tag{18}$$

By differentiating w.r.t time one gets the constraint equations at the velocity

$$\dot{\boldsymbol{\phi}}(\mathbf{q}) = \boldsymbol{\Phi}_q(\mathbf{q})\dot{\mathbf{q}} = \boldsymbol{\Phi}_\theta(\boldsymbol{\theta}, \mathbf{y})\dot{\boldsymbol{\theta}} + \boldsymbol{\Phi}_y(\boldsymbol{\theta}, \mathbf{y})\dot{\mathbf{y}} \tag{19}$$

Assuming that the robot does not cross singular region, it means $\det \boldsymbol{\Phi}_y(\boldsymbol{\theta}, \mathbf{y}) \neq 0$, from (19) one gets

$$\dot{\mathbf{y}} = -\boldsymbol{\Phi}_y^{-1}(\mathbf{q})\boldsymbol{\Phi}_\theta(\mathbf{q})\dot{\boldsymbol{\theta}} = -\boldsymbol{\Phi}_y^{-1}(\boldsymbol{\theta}, \mathbf{y})\boldsymbol{\Phi}_\theta(\boldsymbol{\theta}, \mathbf{y})\dot{\boldsymbol{\theta}} \tag{20}$$

By integrating equation (20) with the initial condition $\boldsymbol{y}(0)$ one gets the value of variables $\boldsymbol{y}(t)$. The values $\boldsymbol{y}(t)$ obtained after integrating the equation (20) may no longer satisfy the constraint equation (18) due to the cumulative errors in the calculation. Due to the error of the constraint equation, it is possible to lead to position and velocity drift in the numerical simulation. To eliminate this drift, kinematic error feedback techniques is proposed in this study. The idea of the method is that instead of solving $\dot{\mathbf{y}}$ from equation (19), we use the equation

$$\dot{\boldsymbol{\phi}}(\mathbf{q}) = -\mathbf{K}\boldsymbol{\phi}(\mathbf{q}), \tag{21}$$

where **K** is a positive matrix chosen by the designer, $\mathbf{K} > 0$.

So equation (19) is modified as

$$\dot{\phi}(\mathbf{q}) = \boldsymbol{\Phi}_q(\mathbf{q})\dot{\mathbf{q}} = \boldsymbol{\Phi}_\theta(\boldsymbol{\theta}, \mathbf{y})\dot{\mathbf{q}} + \boldsymbol{\Phi}_y(\boldsymbol{\theta}, \mathbf{y})\dot{\mathbf{y}} = -\mathbf{K}\phi(\mathbf{q}) \tag{22}$$

and one gets

$$\dot{\hat{\mathbf{y}}} = -\boldsymbol{\Phi}_y^{-1}(\boldsymbol{\theta}, \hat{\mathbf{y}})\left[\boldsymbol{\Phi}_\theta(\boldsymbol{\theta}, \hat{\mathbf{y}})\dot{\boldsymbol{\theta}} + \mathbf{K}\phi(\boldsymbol{\theta}, \hat{\mathbf{y}})\right]. \tag{23}$$

In equation (23) the notations $\dot{\hat{\mathbf{y}}}$ and $\hat{\mathbf{y}}$ have been used to distinguish the variables in the generalized coordinates of the robot are $\dot{\mathbf{y}}$ and $\mathbf{y}$. Note that, equation (21) is equivalent to $\dot{\mathbf{e}} + \mathbf{K}\mathbf{e} = \mathbf{0}$, so that with positive diagonal matrix $\mathbf{K}$, its solution has the form $e_i(t) = e_i(0)\exp(-k_{ii}t)$. Obviously, these solutions will converge to zero, it means the error of the constraint equation is guaranteed to converge to zero. Thus, the constraints are still guaranteed without being broken.

From the theoretical basis mentioned above, we have a parallel robot control scheme using the above kinematic estimation as depicted in Figure 1.

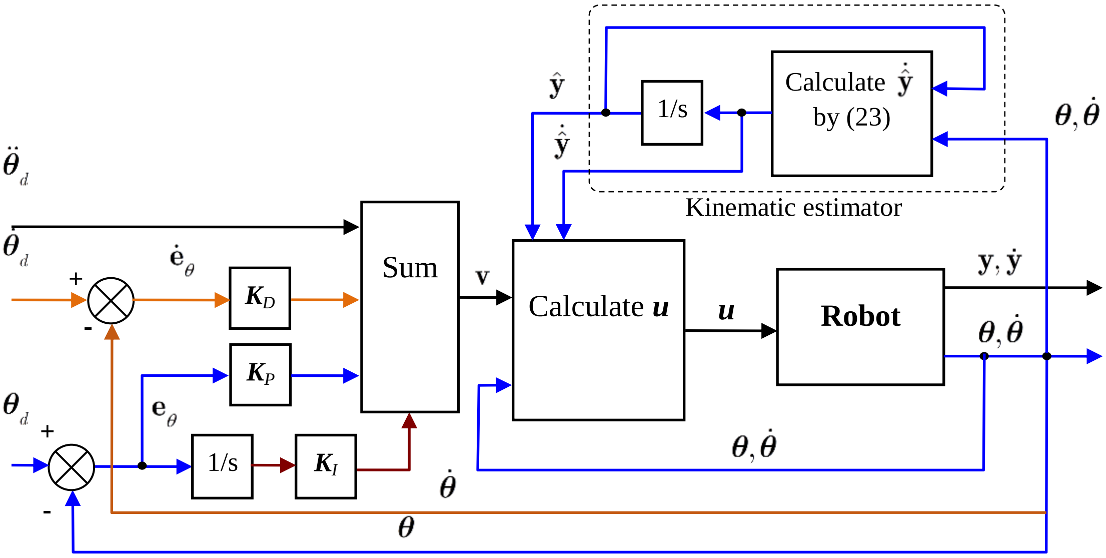


*Figure 1.* The control diagram in joint space with a kinematic estimator.

## 4. SIMULATION RESULTS AND DISSCUSION

Control laws and kinematic estimator presented in the previous section will be applied to a 3RRR planar parallel robot moving in the horizontal plane. This robot has a fixed and a moving platform are equilateral triangle $O_1O_2O_3$ with side $L_0$ and $B_1B_2B_3$ with side $a$, and the same three legs are composed of two corresponding lengths, $O_iA_i = l_1$, $A_iB_i = l_2$. Active joints are driven by DC electric motors through gearbox transmission (Figure 2.).

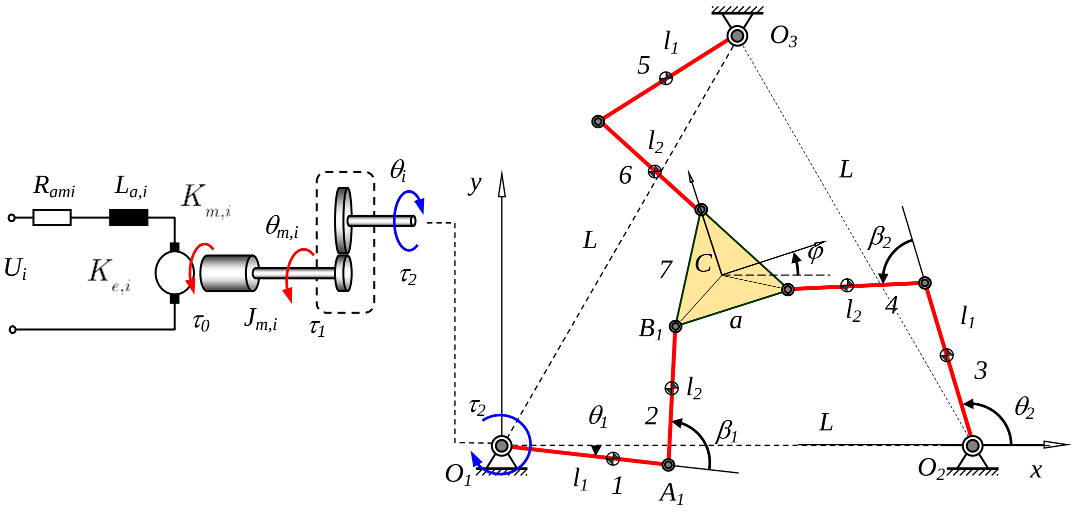


*Figure 2.* Model of 3RRR parallel planar robot and DC motor with gearbox.

**4.1. The dynamic model of a 3RRR parallel robot**

For this model, the following vectors have been used:

$$\boldsymbol{q} = [\boldsymbol{\theta}^T, \boldsymbol{y}^T]^T = [\boldsymbol{\theta}^T, \boldsymbol{\beta}^T, \boldsymbol{x}^T]^T = [\theta_1, \theta_2, \theta_3, \beta_1, \beta_2, \beta_3, x_C, y_C, \varphi]^T .$$

The equation of motion is derived with the kinetic and potential energy:

$$T(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\sum_{k=1}^{7}(m_k v_{Ck}^2 + J_{Ck}\omega_k^2), \qquad \Pi(\boldsymbol{q}) = 0 . \tag{24}$$

The constraint equations of the manipulator are given as follows

$$\boldsymbol{\phi}(\boldsymbol{q}) \equiv \boldsymbol{\phi}(\boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{x}) = \boldsymbol{0}:$$

$$x_{O1} + l_1\cos\theta_1 + l_2\cos(\theta_1 + \beta_1) = x_C + b\cos(\varphi + \alpha_1)$$
$$y_{O1} + l_1\sin\theta_1 + l_2\sin(\theta_1 + \beta_1) = y_C + b\sin(\varphi + \alpha_1)$$

$$x_{O2} + l_1\cos\theta_2 + l_2\cos(\theta_2 + \beta_2) = x_C + b\cos(\varphi + \alpha_2)$$
$$y_{O2} + l_1\sin\theta_2 + l_2\sin(\theta_2 + \beta_2) = y_C + b\sin(\varphi + \alpha_2)$$

$$x_{O3} + l_1\cos\theta_3 + l_2\cos(\theta_3 + \beta_3) = x_C + b\cos(\varphi + \alpha_3)$$
$$y_{O3} + l_1\sin\theta_3 + l_2\sin(\theta_3 + \beta_3) = y_C + b\sin(\varphi + \alpha_3)$$

The mass matrix $\boldsymbol{M}_s(\boldsymbol{q})$ is obtained as

$$m_{11} = J_{c1} + J_{c2} + \frac{1}{4}m_1 l_1^2 + m_2 l_1^2 + r^2 J_r ,\ m_{12} = \frac{1}{2}m_2 l_1 l_2\cos(\theta_1 - \beta_1) ,m_{21} = m_{12},$$

$$m_{22} = \frac{1}{4}m_2 l_2^2 ,\ m_{33} = J_{c_1} + \frac{1}{4}m_1 l_1^2 + m_2 l_1^2 + r^2 J_r ,\ \ m_{34} = \frac{1}{2}m_2 l_1 l_2\cos(\theta_2 - \beta_2) ,$$

$$m_{43} = m_{34},\ \ m_{44} = J_{c_2} + \frac{1}{4}m_2 l_2^2 \ ,\ \ m_{55} = J_{c_1} + \frac{1}{4}m_1 l_1^2 + m_2 l_1^2 + r^2 J_r ,$$

$$m_{56} = \frac{1}{2}m_2 l_1 l_2\cos(\theta_3 - \beta_3),\ \ m_{65} = m_{56},\ \ m_{66} = J_{c_1} + \frac{1}{4}m_2 l_2^2 ,$$

$$m_{77} = m_3 ,\ \ \ m_{88} = m_3 ,\ \ \ m_{99} = J_{c_3} .$$

the other elements are zero.

The Coriolis and centrifugal matrix $\boldsymbol{C}_s(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is given with

$$c_{21} = -\frac{1}{2}m_2 l_1 l_2\sin(\theta_1 - \beta_1)\dot{\theta}_1 , \qquad c_{12} = \frac{1}{2}m_2 l_1 l_2\sin(\theta_1 - \beta_1)\dot{\beta}_1 ,$$

$$c_{43} = -\frac{1}{2}m_2 l_1 l_2\sin(\theta_2 - \beta_2)\dot{\theta}_2 , \qquad c_{34} = \frac{1}{2}m_2 l_1 l_2\sin(\theta_2 - \beta_2)\dot{\beta}_2 ,$$

$$c_{65} = -\frac{1}{2}m_2 l_1 l_2\sin(\theta_3 - \beta_3)\dot{\theta}_3 , \qquad c_{56} = \frac{1}{2}m_2 l_1 l_2\sin(\theta_3 - \beta_3)\dot{\beta}_3 ,$$

the other elements are zero.

The damping matrix $\boldsymbol{D}_s$ is given with

$$\boldsymbol{D}_s = R_a^{-1} r^2 K_m K_e diag([1,0,1,0,1,0,0,0,0]) .$$

The control input matrix $\boldsymbol{B}_s$ is given with zero elements except for:

$$B_s(1,1) = B_s(3,2) = B_s(5,3) = R_a^{-1} r K_m .$$

Some simulations with a 3-DOF planar parallel manipulator which moves in the horizontal plane driven by 3 actuators are implemented. The numerical simulations are carried out in Matlab. Here the Baumgarte's technique is applied in the simulation [22]. The model of the robot is shown in Fig. 1. The mechanical parameters of the robot are given as follows [12, 14, 23].

Base: $L$ = 1.2 m,

Two links of the legs:

$l_{i,1} = 0.581$ m, $m_{i,1} = 2.072$ kg, $J_{C1} = 0.13$ kg.m$^2$,

$l_{i,2} = 0.620$ m, $m_{i,2} = 0.750$ kg, $J_{C2} = 0.03$ kg.m$^2$,

Platform: $a = 0.2$ m, $m_7 = 0.978$ kg, $J_{C7} = 0.007$ kg.m$^2$, $\alpha_i = [\frac{7}{6}\pi, -\frac{1}{6}\pi, \frac{1}{2}\pi]$.

Gear trans.: $r$ = 10 (transmission ratio)

DC motor : $J_m$ = 0.01 kg.m$^2$, $K_m$ = 3.00 Nm/A, $K_e$ = 0.10 Vs/rad, $R_a$ = 3.00 Ohm.

The coefficient in the kinematic estimator is chosen **K** = 100. During numerical simulation we need to solve algebraic differential equations. These equations will be implemented in MATLAB to simulate the response of the system corresponding to the configuration $\Delta_8(-1,-1,-1)$ of the robot [24].

### 4.2. PD controller plus gravity compensation with kinematic estimator

In this simulation, the moving platform is controlled from initial coordinate $\boldsymbol{x}_0$ = [0.36; 0.38; 0.2]$^T$ to desired coordinate $\boldsymbol{x}_d$ = [0.45; 0.51; 0.0]$^T$. The parameters of PD controller plus gravity compensation are chosen as:

$$\boldsymbol{K}_P = 950\,\mathrm{diag}(1,1,1); \qquad \boldsymbol{K}_D = 120\,\mathrm{diag}(1,1,1).$$

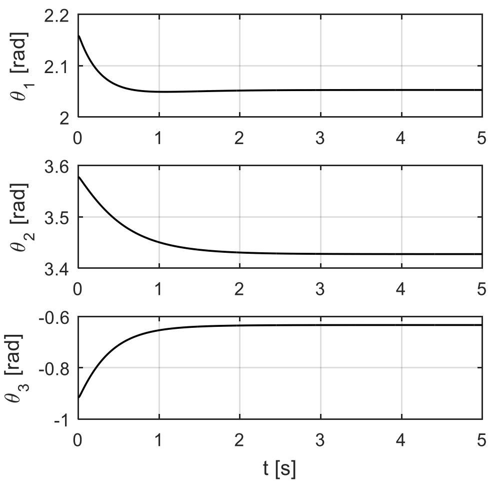


*Figure 3.* Actuated joint variables vs time.

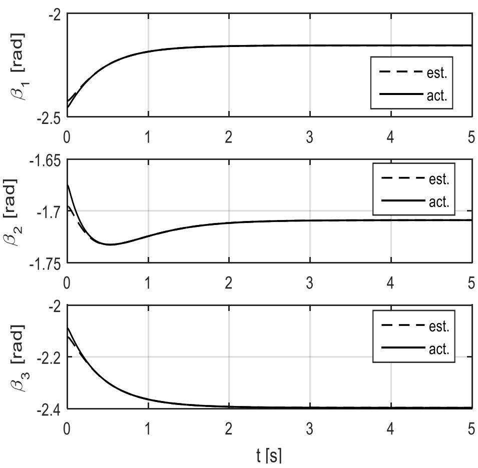


*Figure 4.* Auxiliary joint variables and its estimated value vs time.

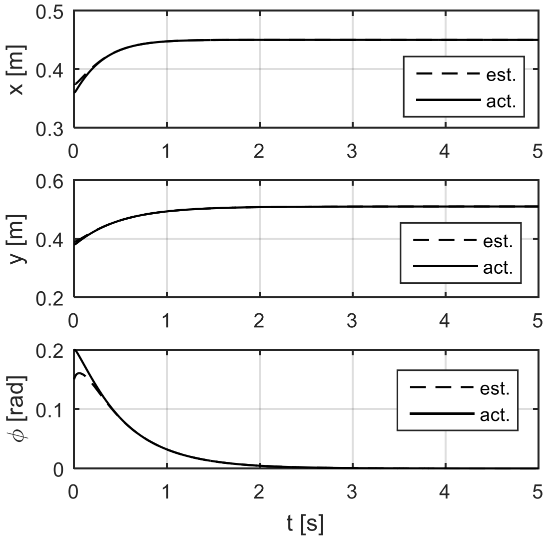


*Figure 5*. Platform position and its estimated value vs time.

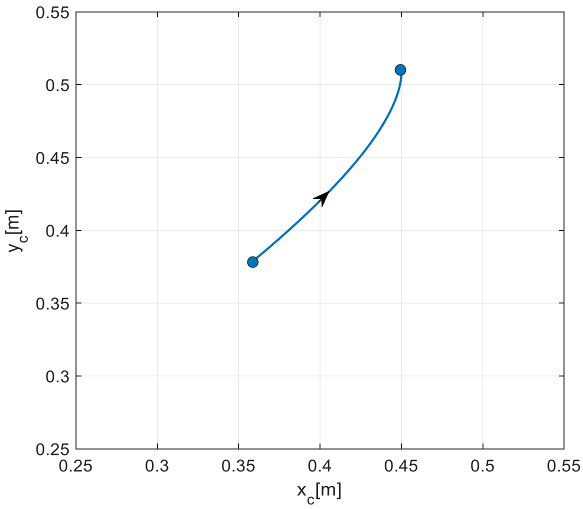


*Figure 6.* Trajectory of moving platform – position control.

The time history of active joint variables, auxiliary variables, the position of the moving platform as well as the trajectory of the center of the moving platform are shown in Figs. 3-6. Figs. 3-5 indicate that the moving platform reaches its desired position after about 2.5 s. The results show that the estimated values converge to their actual values after about 0.5 s (Figs. 4, 5, 11).

### 4.3. PID controller with kinematic estimator based inverse dynamics

In these simulations, the center of the platform will be moved along a circular trajectory, while its orientation is constant, $\varphi = 0$ [rad]. The trajectory has a center at $(x_C, y_C)$ = (0.30, 0.40) [m] and radius $R$ = 0.12 [m].

The parameters of PID controllers are chosen as

$$\mathbf{K}_P = 2850\,\mathrm{diag}(1,1,1);\ \ \mathbf{K}_I = 100\,\mathrm{diag}(1,1,1);\ \ \mathbf{K}_D = 250\,\mathrm{diag}(1,1,1).$$

Simulation results are shown in Figs. 7-10 and 12. The actuated joint variables track the desired motion after about 0.5 s (Fig. 7). The estimated values of auxiliary joint variables as well as the position of the moving platform converge to their actual values after about 0.4 s (Fig. 8-9). The differences between the estimated values and their actual values converge fast to zero (Fig. 10). Fig. 12 shows that the center of the moving platform tracks the given trajectory.

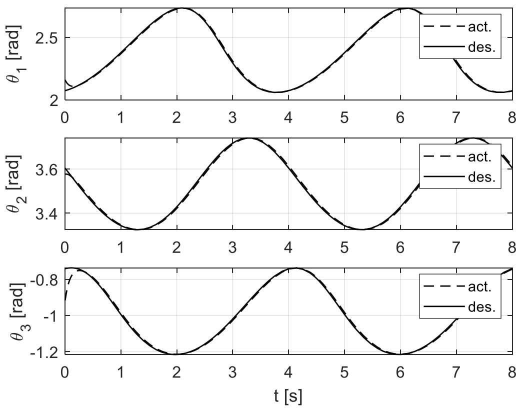


*Figure 7.* Actuated joint variables and its desired value vs time.

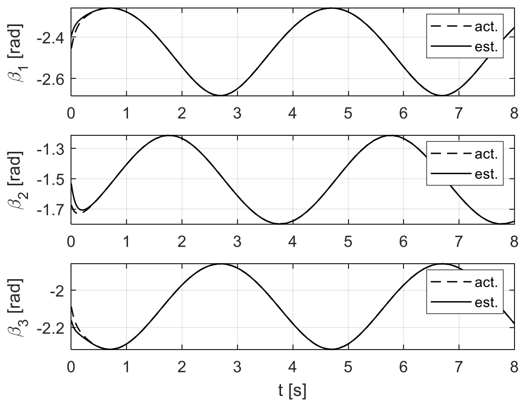


*Figure 8.* Auxiliary joint variables and its estimated value vs time.

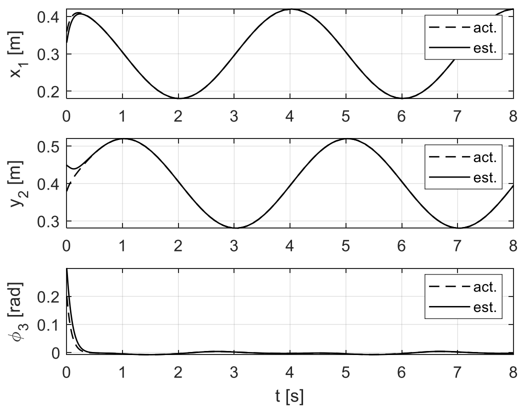


*Figure 9.* Platform position and its estimated value vs time.

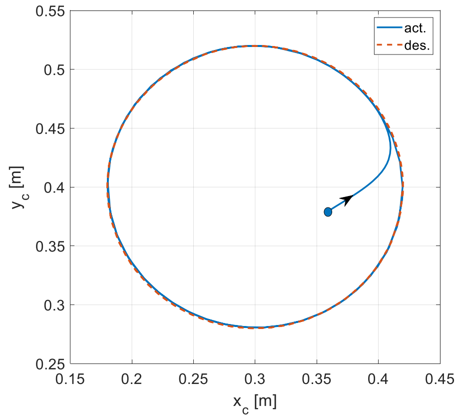


*Figure 10.* Trajectory of moving platform – tracking control.

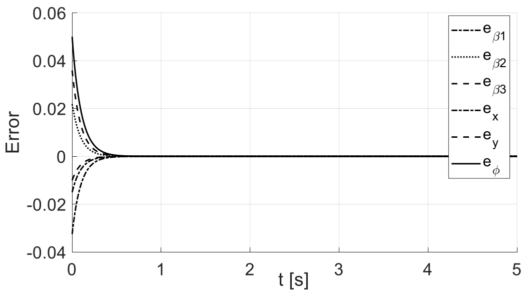


*Figure 11.* Differences between estimated and actual values (Position control).

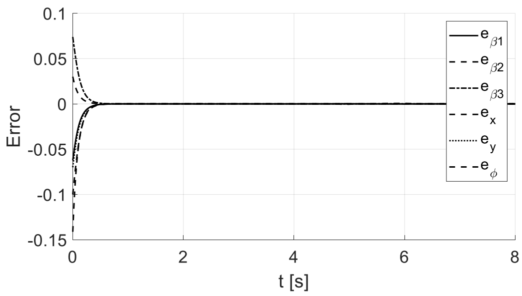


*Figure 12.* Difference between estimated and actual values (Tracking control).

*Remarks*: In the both cases of position control and trajectory control, the estimated variables $\hat{\boldsymbol{y}}$ track to their true values $\boldsymbol{y}$. The position error when using the estimated data in the controller is nearly zero. This proves that kinematic estimator has generated feedback signals for the controller without using sensors to measure the motion of the moving platform as well as the motion of the connecting rods.

## 4. CONCLUSION

This paper has successfully proposed and implemented the approach for estimating generalized coordinates that are depended on kinematic constraint equations. The motion of the moving platform and the connecting links are successfully estimated for feedback controllers. Here the kinematic error feedback technique is included to ensure reducing the effect of cumulative errors during the integration process. The designed estimator is also successfully integrated with the conventional PD and PID controllers. The numerical simulation results with the controller in the joint space show the effectiveness of the proposed method. With this approach, we are able to completely create the feedback signals needed for the controller without the need to equip more sensors for the 3RRR planar parallel robot even though the dynamic model of the system is nonlinear and complicated. Therefore, this approach can also be applied for the spatial parallel robot. The problem of combining the kinematic estimator with the modern controller in the operational space will be studied in the future.

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