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Revitalizing the moment distribution method: A fast and exact analysis of multi-bay, multi-story frames

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Abstract: *The conventional moment distribution method is revitalized with a new approach which requires only one-cycle of balance and carry-over and no iteration. For a continuous beam, the method begins by assuming the value of the balancing moment, say x, at the first joint. By invoking the moment equilibrium condition successively from the first towards the last joints, balancing and carry-over moments at other joints can be determined in terms of the unknown x without iteration. By means of moment equilibrium of the last joint, the unknown x can be solved exactly, regardless of the number of spans, and the final moment distribution of the whole structure can be easily obtained. Further, for multi-bay, single story frames, the analysis is carried out without the need to separate the analysis into two stages as in conventional MDM, and the final exact moments are found by solving two unknowns only, regardless of the number of bays. As such, the revitalized MDM is particularly advantageous for solving continuous beams and single-story frames with large number of spans or bays. Five examples are given herein to demonstrate the procedures and efficiency of the proposed method.*

Keywords – *One-step moment distribution, plane frames.*

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I. Introduction

The moment distribution method (MDM) was first introduced by Professor Hardy Cross in a paper published in 1932[1]. The essence of the method was clearly expressed in the first paragraph of his paper, "*The essential idea which the writer wishes to present involves no mathematical relations except the simplest arithmetic*". Since its first introduction, the method was widely used by structural engineers for a few decades until seventies, when computational structural analysis started to emerge as the dominant tool for engineering design. Even though the method is no longer routinely used in design office nowadays, the method is still important from the academic and practical perspectives: 1) Unlike energy methods or other advanced matrix approaches, the moment distribution method is a mathematically simple and physically intuitive tool that helps students to develop a 'feel' of how forces and moments are distributed across a frame-type structure and how they are related to relative stiffness and deformation of the members. Its importance in structural engineering curriculum has been thoroughly discussed by Creed [2]; 2) Further, its relevance in practice is no less than in education, as recently highlighted in a technical guidance note published by the Institution of Structural Engineers, UK [3] emphasizing that the method is "*even more vital today than they have ever been if we are to fully understand the output of analysis applications*".

Although the concept of MDM is simple yet versatile, the method requires repeated iteration of balancing and carry-over of joint moments until convergence, thus involving a lot of arithmetic operations and a lengthy process, particularly when multi-bay or multi-story frames are analyzed [4]. In this paper, an alternative, faster and exact approach, which was first proposed and published by Yu in Chinese, back in 1957 [5,6] is revisited. It is a fast-track version of conventional MDM that aims to eliminate its iterative process, thus reducing the amount of arithmetic that the original MDM requires, yet retaining its versatility and strong physical intuition. To further explore the potential of this method, this paper aims to extend this fast and exact approach of MDM to a variety of structures, ranging from a simple continuous beam, a single story frame to more complex, multi-bay or multi-story frames.

II. The moment distribution method: an exact approach without iteration

Taking a continuous beam as an example, the conventional MDM begins by assuming each joint of a structure is fixed. Then, by unlocking and locking each joint in succession, the internal moments at the joints are balanced and carried over repeatedly until the joints have rotated to their final positions and joint equilibrium is approximately satisfied. The final moment at each joint is then determined by summing up the balancing moments and carry-over moments in all previous iterative steps. As such, it is obvious that the conventional MDM is a method of successive approximation that can be carried out until a desired degree of accuracy is achieved. In the exact approach, however, instead of repeatedly balancing and carry-over of moments at each joint until joint equilibrium is satisfied, it begins by assuming the value of the final balancing moment, say x, at the first joint. By invoking the moment equilibrium condition successively from the first towards the last joints, balancing and carry-over moments at other joints can be determined in terms of the unknown x without any iteration. By means of moment equilibrium of the last joint, the value of x can be found exactly and hence the final moment distribution of the whole beam can be easily obtained; details of the method are demonstrated through the following continuous-beam example.

III. Continuous Beams

A five-span continuous beam with span-wise varying stiffness is given in Fig. 1. It is subject to a load combination of superimposed dead Load (DL) and alternate live load (LL) as specified in the Hong Kong Code of Practice 2013[7]. The values of DL and LL are assumed to be 1kN/m and 2kN/m respectively, resulting in a combined load of 1.4DL+1.6LL being 4.6kN/m. The loadings and beam properties are shown in Table 1 and Fig. 1. The distribution factors are calculated and the conventional MDM is carried out as shown in Table 2.

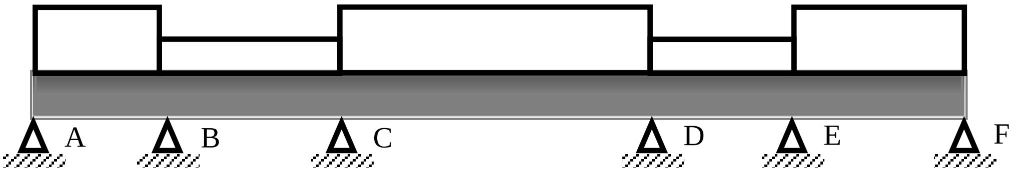

Fig. 1 – Five span continuous beam

Table 1: Properties and loading on continuous beam

	Span AB	Span BC	Span CD	Span DE	Span EF
Length (m)	3	5	6	3	5
Section (mm)	300x500	300x600	300x800	300x500	300x600
Loading (kN/m)	4.6	1	4.6	1	4.6
Relative Stiffness (I/L) (mm^3)	$1.04x10^6$	$1.08x10^6$	$2.13x10^6$	$1.04x10^6$	$1.08x10^6$

Table 2: Analysis of continuous beam by conventional MDM

Joint	A	B		C		D		E		F
DF		0.42	0.58	0.34	0.66	0.67	0.33	0.56	0.44	
FEM	0	5.175	-2.083	2.083	-13.800	13.800	-0.750	0.750	-14.375	0
BAL		-1.299	-1.793	3.984	7.733	-8.744	-4.307	7.630	5.995	
C.O.			1.992	-0.897	-4.372	3.867	3.815	-2.154		
BAL		-0.837	-1.155	1.791	3.478	-5.147	-2.535	1.206	0.948	
C.O.			0.896	-0.578	-2.574	1.739	0.603	-1.268		
BAL		-0.376	-0.520	1.073	2.080	-1.569	-0.773	0.710	0.558	
C.O.			0.536	-0.260	-0.785	1.040	0.355	-0.387		
BAL		-0.225	-0.311	0.355	0.690	-0.934	-0.460	0.217	0.170	
C.O.			0.178	-0.156	-0.468	0.345	0.186	-0.229		
BAL		-0.075	-0.103	0.212	0.411	-0.356	-0.175	0.128	0.101	
C.O.			0.106	-0.052	-0.178	0.205	0.064	-0.088		
BAL		-0.045	-0.061	0.078	0.152	-0.180	-0.089	0.049	0.039	
		2.318	-2.318	7.633	-7.633	4.066	-4.066	6.564	-6.564	

The beam is then re-analyzed by the one-step approach and the procedures are given in Table 3. It can be observed that the distribution factors, carry-over factor and fixed end moment remain the same as those in MDM. The one-step approach is carried out from left to right as follows:

1. Balance joint B by an unknown moment x. The moment is distributed on each side of joint B in proportional to the stiffness ratio.
2. It results in final moment M_{BA} = (5.175+0.42x)
3. Equilibrium at joint B requires that the final moment M_{BC} = (-5.175-0.42x).
4. Subtract the fixed end moment and balance moment from the final moment M_{BC} (from step 3) in order to get the carry-over moment M_{BC} = (-3.092-x).
5. Determine the balance moment M_{CB} = (-6.184-2x) at C which is twice the carry-over moment M_{BC} (from step 4).
6. Determine the carry-over moment M_{CB} = 0.29x which is half the balance moment M_{BC}. Hence summing up the carry-over, FEM and the balance moment gives the final moment M_{CB} =(-4.101-1.71x)
7. Step 3 to step 6 are applied repeatedly for joint C up to joint E (the second last joint).
8. Determine the balance moment M_{EF} = (-151.34-23.182x) from the balance moment M_{ED} based on the distribution factors (0.56:0.44) at joint E. The distribution is thus completed.

Considering the moment equilibrium at joint E, i.e. M_{ED} + M_{EF} =0, leads to x = -6.866. By substitution, all the bending moment can be found and shown in the last row of Table 3. It is now clear that, by comparing Tables 2 and 3, the conventional MDM's iterative process is no longer necessary and the procedure is shortened with much less arithmetic operations. The MDM's basic concept and physical intuition are still retained in the present approach and the final moments are obtained exactly. Most importantly, regardless of the number of spans, the present approach only requires the solution of the single unknown balance moment x.

Table 3: One-step moment distribution for a 5-span continuous beam.

Joint	A	B		C		D		E		F
DF		0.42	0.58	0.34	0.66	0.67	0.33	0.56	0.44	
FEM	0	5.175	-2.083	2.083	-13.800	13.800	-0.750	0.750	-14.375	0
BAL		0.42x	0.58x	-6.184-2x	-12-3.882x	59.802+11.184x	29.455+5.509x	-192.641-29.504x	-151.34-23.182x	
C.O.			-3.092-x	0.29x	29.901+5.592x	-6-1.941x	-96.307-14.752x	14.728+2.755x		
Final Moment	0	5.157+0.42x	-5.175-0.42x	-4.101-1.71x	4.101+1.71x	67.602+9.243x	-67.602-9.243x	-177.136-26.749x	-165.715-23.182x	
	0	2.29	-2.29	7.64	-7.64	4.14	-4.14	6.52	-6.52	0

IV. A box culvert under symmetrical loads

A box culvert is subject to a point load acting on its top slab, see Fig. 2(a). It is supported by a uniform bearing pressure of 5 kN/m at the base. Lateral soil loads on both sides are assumed as point loads acting at one-third of the depth from the base. Because of symmetry, the frame is not subject to sway deformation. Fixed end moments, distribution and carry-over factors are calculated as per conventional MDM. Symmetry is not taken into account in the analysis. The procedure for a close-box frame commences with two unknown balancing moments x and y at joints A and D respectively. The one-step procedure then proceeds to joints B and C, as demonstrated in Table 4:

1. Balance joint A and D by unknown moments x and y respectively. These two moments are distributed to M_{AB} , M_{AD}, M_{DA} and M_{DC}. They are then carried over to M_{BA} , M_{DA}, M_{AD} and M_{CD}.
2. Equilibrium of joint A gives carry-over moment M_{AB} = (7.78 –x – 0.235y). Similarly, equilibrium of joint D gives carry-over moment M_{DC} = (–2.23–0.235x–y).
3. Determine the balance moments M_{BA} and M_{CD} at B and C respectively, which are twice the carry-over moment M_{AB} and M_{DC} (from step 2).
4. Determine the balance moment M_{BC} from the balance moment M_{BA} (step 3) based on the distribution factors at joint B. Same principle applies to joint C.
5. Carry over the balance moment M_{BC} in step 4 to obtain the carry-over moment M_{CB}. Same applies to the balance moment M_{CB} from step 4.

To solve for the unknown moments x and y, moment equilibrium is enforced at B and C, that is:

M_{BA}+M_{BC} = 0 gives:

→25.56-1.735x-0.47y+9.6025-1.983x-1.304y = 0

and

M_{CB}+M_{CD} = 0 gives

→-11.13-0.47x-1.735y+7.385-1.305x-1.982y = 0

On solving these two equations: x = 12.871 and y = - 7.154
The final moments thus obtained are shown in Fig.2(b).

Table 4: One-step moment distribution for a box culvert under symmetrical loads

	D	A		B		C		D
	DA	AD	AB	BA	BC	CB	CD	DC
DF	0.47	0.47	0.53	0.53	0.47	0.47	0.53	0.53
FEM	-4.44	2.22	-10	+10	-2.22	4.44	-6.67	6.67
BAL	0.47y	0.47x	0.53x	15.56 -2x -0.47y	13.8 -1.774x -0.417y	-3.955-0.418x-1.774y	-4.46-0.47x-2y	0.53y
CO	0.235x	0.235y	7.78 -x -0.235y	0.265x	-1.9775-0.209x-0.887y	6.9-0.887x-0.208y	0.265y	-2.23-0.235x -y
FINAL Moment	-4.44 +0.235x +0.47y	2.22 +0.47x +0.235y	-2.22 -0.47x-0.235y	25.56-1.735x-0.47y	9.6025-1.983x-1.304y	7.385-1.305x-1.982y	-11.13-0.47x-1.735y	4.44-0.235x-0.47y
Moments (kNm)	-4.778	6.588	-6.588	6.59	-6.59	4.767	-4.767	4.778

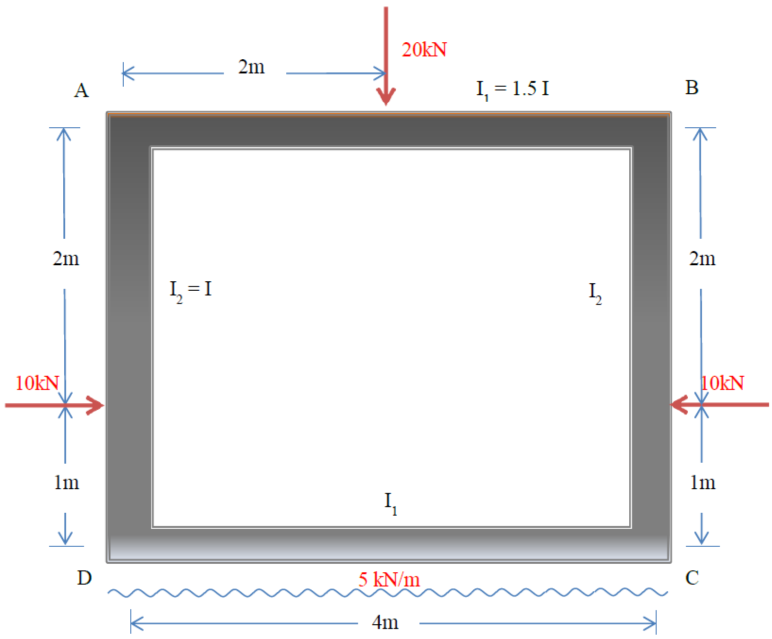

Fig. 2(a) A box culvert subject to symmetrical load

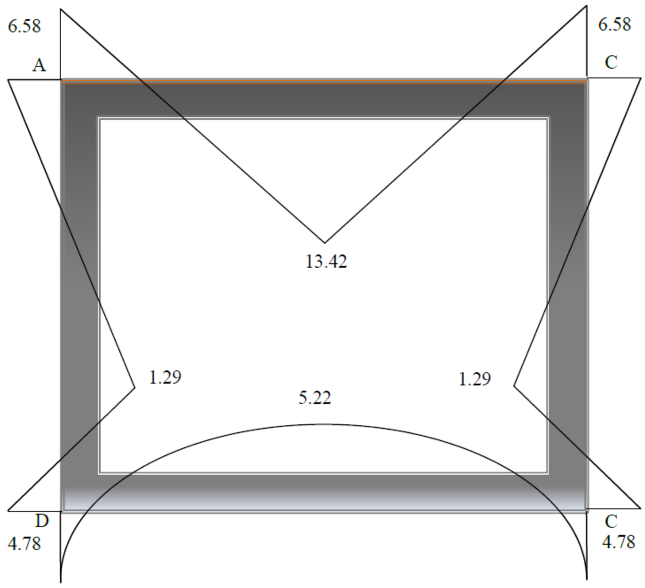

Fig. 2(b) bending moment diagram (all values in kNm)

V. An unsymmetrical single-bay, single story frame

In conventional MDM for frames with side sway, the analysis is separated into two stages: the first stage assumes that the frame is horizontally propped without sway and the resulting horizontal propping force is determined. In the second stage, the frame is assumed to sway under an arbitrary horizontal load and resulting moments are scaled to achieve horizontal equilibrium. Moments from both stages are then summed to give the final distribution.

In the one-step approach, both stages are carried out at the same time. For the simple frame with an inclined leg shown in Fig. 3(a), fixed-end moments due to the vertical point load on the beam and a horizontal sway of x units are calculated, as given in Table 5. All members are of the same EI. For a unit horizontal side-sway, it can be shown that the fixed end moments generated in member AB, BC and CD would be in the ratio of √2/2: 1: 1 or 70.7: 100: 100 (see inset in Fig. 3a). The process starts by assuming a balance moment of 4y at joint C. The procedure is then carried out as follows:

1. Balance joint C by an unknown moment 4y. The moment is distributed on each side of joint C in proportional to the stiffness ratio. It results in final moment M_{CD} = (-100x+2y).
2. Equilibrium at joint C requires that the final moment M_{CB} = (100x-2y).
3. Determine the carry-over moment M_{CB} = (-75-4y)
4. Determine the balance moment M_{BC} = (-150-84), which is twice the carry-over moment M_{CB} (from step 4).
5. Based on the stiffness ratio at joint B, the balance moments M_{BA} = (-106.41-5.6752y).
6. Determine the carry-over moment M_{BC} and M_{AB} = (-53.205-2.836y), which are half the balance moment M_{CB} and M_{BA} respectively.
7. Determine the carry-over moment M_{AB} = (-53.205-2.836y).

The final moments are shown in Table 5.

By considering conventional equilibrium conditions, we can solve for x and y as follows:
Equilibrium at joint B gives:
$M_{BA}+M_{BC} = 0$
$\rightarrow -225+100x-7y-106.41-70.7x-5.6752y = 0$ (1)
Horizontal equilibrium of the frame requires:
$H_A = H_D$
$\rightarrow 365.385-541.4x+3.4872y = 0$ (2)
Solving (1) and (2) gives:
x= 0.5198 and y = -24.945
The final moments thus obtained are shown in Fig. 3(b).

Table 5: One-step moment distribution for a portal frame with an inclined leg

	AB	BA	BC	CB	CD	DC
DF		0.415	0585	0.5	0.5	
FEM	-70.7x	-70.7x	100x-75	100x+75	-100x	-100x
BAL		-106.41-5.6752y	-150-8y	2y	2y	
CO	-53.205-2.836y		y	-75-4y		y
FINAL Moment	-53.205-70.7x-2.8376y	-106.41 -70.7x-5.6751y	-225 +100x-7y	100x-2y	-100x+2y	-100x+y
Moments (kNm)	-19.1684	-1.58987	1.58987	101.8656	-101.8656	-76.9207

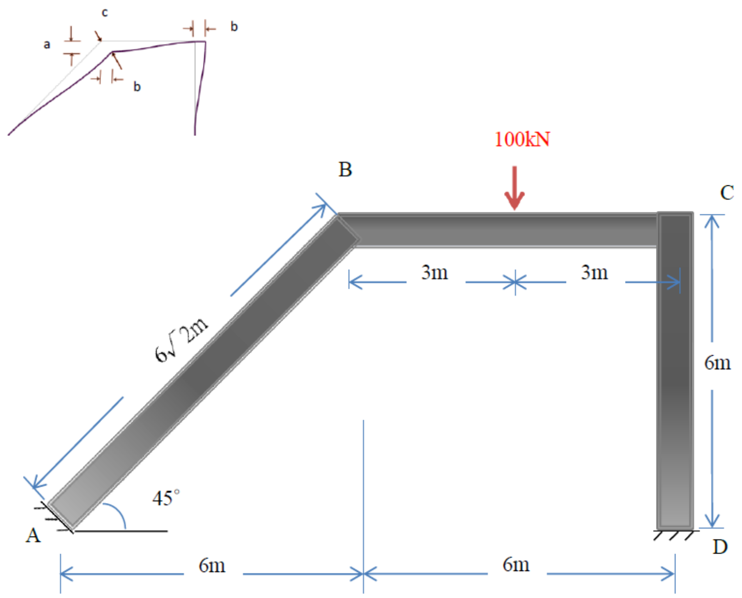

Fig 3(a) An inclined leg portal frame (inset: for a unit horizontal sway b, a:b:c = 1:1:√2/2)

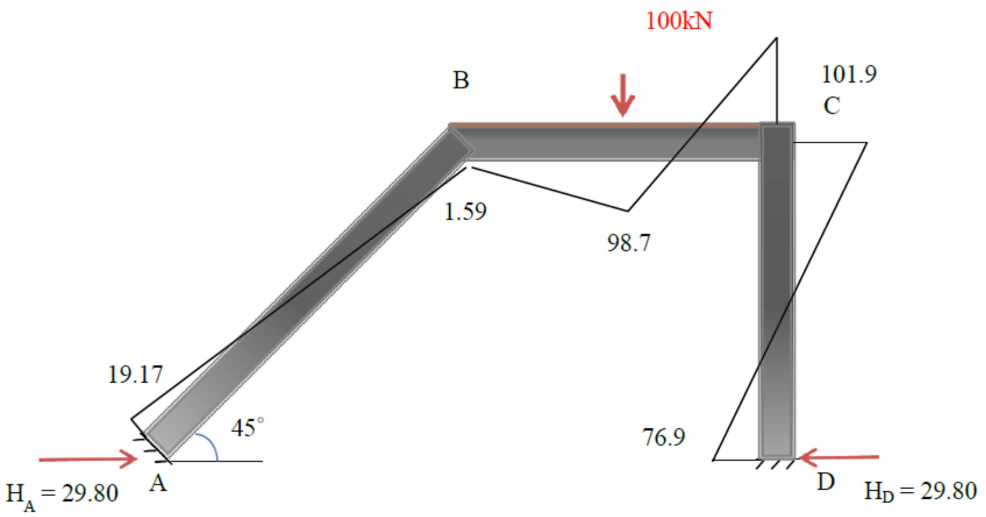

Fig. 3(b) bending moment diagram (all moments in kNm)

VI. A double-bay, single story frame

The present method is further applied to a double-bay portal frame as shown in Fig.4(a). It is subject to vertical and horizontal loads. Assuming a side-sway of y units, the fixed end moments are determined as per the conventional approach. The process starts by applying a balance moment of x at joint B. The procedure is then carried out as in the previous frame example. The final moments in terms of x and y are shown in Table 6.

Further, equilibrium of joint D requires:
$M_{DC}+M_{DE} = 0$
$\rightarrow 38x-2030y+2033.26 = 0$
Horizontal equilibrium requires the sum of column shears and horizontal load to vanish:
$(M_{AB}+M_{BA})/9+(M_{CF}+M_{FC})/9+(M_{DE}+M_{ED})/6+100 = 0$
$\rightarrow 5.283x-321.111y+350 = 0$
Solving these two equations gives:
x = 38.9773 and y = 1.7312

The final bending moment diagram is shown in Fig. 4(b). It is noteworthy that, for multi-bay portal frames, there are only two unknowns to be determined, irrespective of the number of bays involved.

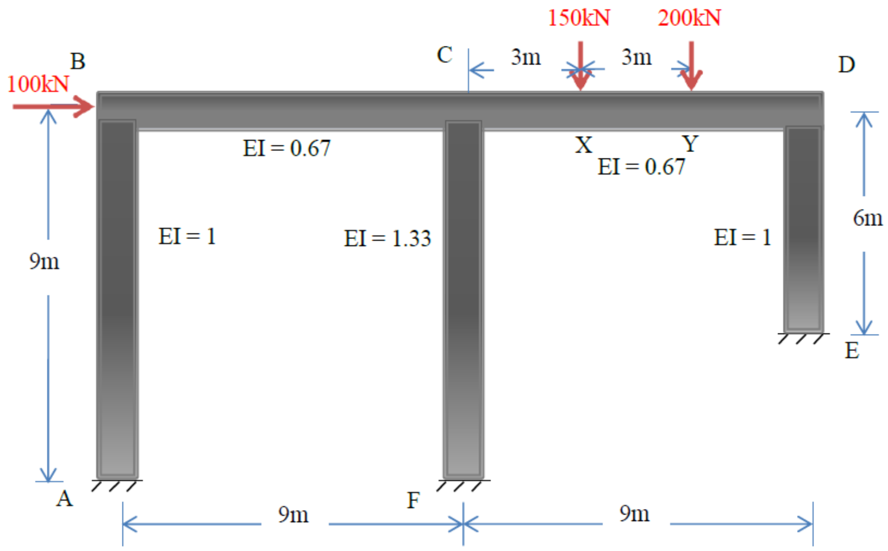

Fig. 4(a): A double-bay portal frame

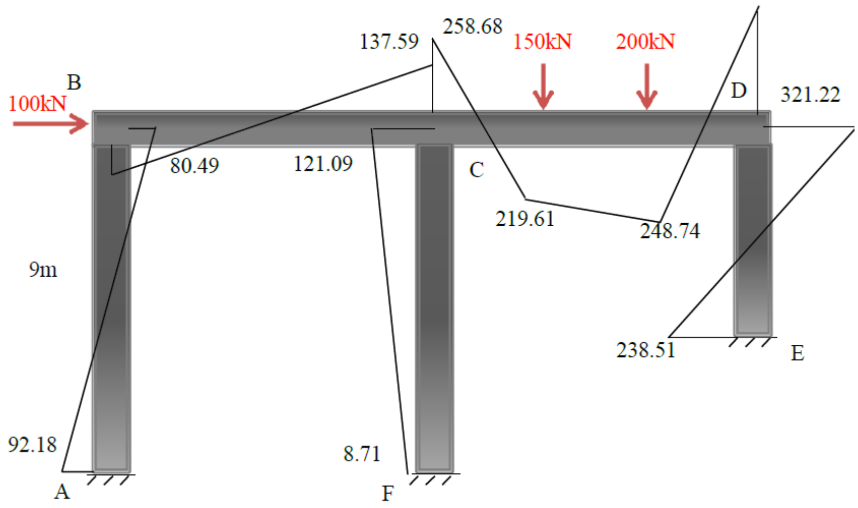

Fig. 4(b) bending moment diagram (all moments in kNm)

Table 6: One-step moment distribution for a double-bay portal frame

	AB	BA	BC	CB	CF	CD	DC	DE	ED	FC
DF	1	0.6	0.4	0.25	0.5	0.25	0.4	0.6	1	1
FEM	-60y	-60y			-80y	-333.3	366.66	-90y	-90y	-80y
BAL		0.6x	0.4x	120y-2x	240y-4x	120y-2x	666.6+15.6x-800y	1000+23.4x-1200y		
CO	0.3x		60y-x	0.2x		333.3+7.8x-400y	60y-x		500+11.7x-600y	120y-2x
FINAL Moment	-60y +0.3x	-60y +0.6x	60y-0.6x	120y-1.8x	160y-4x	-280y +5.8x	1033.26+14.6x-740y	1000+23.4x-1290y	500+11.7x-690y	40y-2x
Moments (kNm)	-92.18	-80.49	80.49	137.59	121.09	-258.68	321.22	-321.22	-238.51	-8.71

VII. A two-story rectangular frame

In this example, the method is applied to a two-storey frame to demonstrate its versatility. The frame is subject to two horizontal loads as shown in Fig.5. All members are of the same EI. The fixed end moment due to the first and second floor inter-story drift are assumed to be x and y units respectively. In addition, two balancing moments 3p and 3q are assumed at joints B and E respectively, see Table 7. The one-step approach is then carried out as follows:

1. Half of the balance moment M_{EB} (= 0.5q) is carried over to joint B.
2. It results in final moment M_{BE} = (p+0.5q).
3. Equilibrium at joint B requires that final moment M_{BC} = (-x-2p-0.5q).
4. Subtracting fixed end moment and balance moment from final moment gives the carry-over moment M_{BC} = (-x-y-3p-0.5q)
5. Determine the carry-over moment M_{CB} = 0.5p.
6. Determine the balance moment M_{CB} = (-2x-2y-6p-q) which is twice the carry-over moment M_{BC}. Hence find the total moment M_{CB} can be found.
7. Based on the stiffness ratio at joint C, the balance moments M_{CD} and M_{CB} are the same.
8. Carry-over moment M_{CD} is half of the balance moment M_{CB}. Summing the FEM, balance moment and carry-over moment gives the total moment M_{CD}.
9. Repeating the previous steps for joints E and D would give total moments M_{DC} and M_{DE}.

Considering the moment equilibrium at joints C and D, that is:

$M_{CB}+M_{CD} = 0$

$M_{DC}+M_{DE} = 0$

Thus giving:

$5x+4y+12p+5q= 0$ (3)

and

$5x+4y+5p+12q= 0$ (4)

Horizontal equilibrium of the second storey beam requires:

$(M_{CB}+M_{BC})/L + (M_{DE}+M_{ED})/L + P_2 = 0$

$6x+2y+9p+9q = P_2 L$ (5)

Horizontal equilibrium of the first storey beam requires:

$(M_{AB}+M_{BA})/L + (M_{EF}+M_{FE})/L + P_1 + P_2= 0$

$4x + 1.5p + 1.5q = -(P_1 + P_2) L$ (6)

Assuming P_1 = 10, P_2 = 10 and L = 5, by solving (3) to (6):

x = -40, y = -35, p = 20, q = 20

The final bending moment diagram is shown in Fig. 5(b). It is worthy of note that, in case of multi-story frames, the number of unknowns involved would increase in proportional to the number of stories and the number of bays.

VIII. Conclusion

The present approach inherits the physical intuition of the original MDM whilst enhancing its computing efficiency. No complicated mathematics or computers are required in the analysis. It is demonstrated in the examples that the revitalized MDM is particularly advantageous for solving continuous beams and single-story frames with large number of spans or bays. The present approach allows academics to revitalize the classical moment distribution method as a learning tool for structural engineering students. In practice, the method still serves as an effective complementary tool for engineers to check against output from commercial software. Examples presented herein enabled academic and engineers to further develop its potential in practice and in education.

Table 7: One-step moment distribution for a two-story frame

	A	B			C		D		E			F
	AB	BA	BE	BC	CB	CD	DC	DE	ED	EB	EF	FE
DF		1/3	1/3	1/3	1/2	1/2	1/2	1/2	1/3	1/3	1/3	
FEM	x	x		y	y			y	y		x	x
BAL		p	p	p	-2x-2y-6p-q	-2x-2y-6p-q	-2x-2y-p-6q	-2x-2y-p-6q	q	q	q	
CO	0.5p		0.5q	-x-y-3p-0.5q	0.5p	-x-y-0.5p-3q	-x-y-3p-0.5q	0.5q	-x-y-0.5p-3q	0.5p		0.5q
FINAL Moment	x+0.5 p	x+p	p+0.5q	-x-2p-0.5q	-2x-y-5.5p-q	-3x-3y-6.5p-4q	-3x-3y-4p-6.5q	-2x-y-p-5.5q	-x-0.5p-2q	0.5p+q	x+q	x+0.5q
Moment (kNm)	-30	-20	30	-10	-15	15	15	-15	-10	30	-20	-30

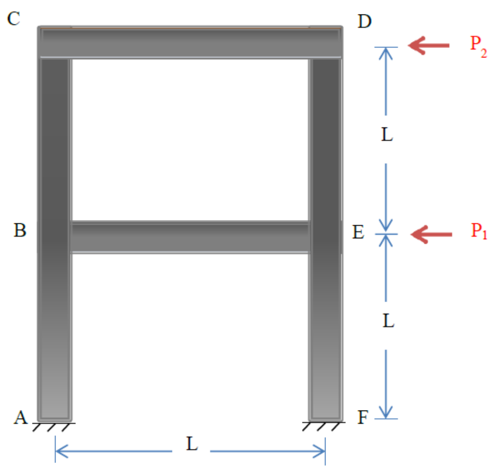

Fig. 5(a) A two-story frame with uniform EI subject to horizontal loads

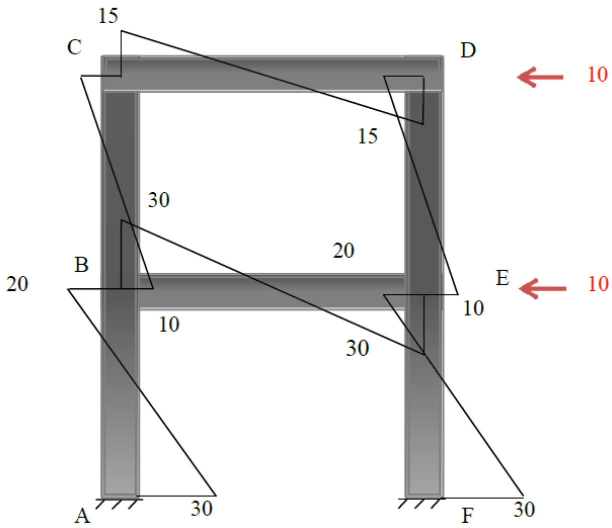

Fig. 5(b) Bending moment diagram (all moments in kNm)

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