Electromagnetic Scattering by a Cylinder in a Lossy Medium of an Inhomogeneous Elliptically Polarized Plane Wave

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Abstract—In this paper, a rigorous theoretical approach, adopted in order to generalize the Vectorial Cylindrical-Harmonics (VCH) expansion of an inhomogeneous elliptically polarized plane wave, is presented. An application of the VCH expansion to analyze electromagnetic field scattered by an infinite circular cylinder is presented. The results are obtained using the so-called complex-angle formalism reaching a superposition of Vectorial Cylindrical-Harmonics. To validate the method, a Matlab code was implemented. Also, the validity of the methodology was confirmed through some comparisons between the proposed method and the numerical results obtained based on the Finite Element Method (FEM) in the canonical scenario with a single cylinder.

Keywords—electromagnetic scattering, inhomogeneous wave dispersion, vectorial cylindrical-harmonics.

1. Introduction

The problem of solving Maxwell's equations in order to determine the field scattered by an object has been the focus of many researchers for several decades now. All aspects of electromagnetic scattering, approached from a purely geometrical point of view, have already been analyzed. In literature, several works can be found on canonical scattering caused by spherical, spheroidal, conical, cylindrical and ellipsoidal objects [1]–[6]. More complex scenarios have also been considered, such as buried spheres, cylinders and axially symmetric objects [7]–[13]. Ensembles of scattering objects can be found in literature as well [14]–[18]. Furthermore, different types of materials that the scatterer is made of have been studied too [19]–[23].

This article introduces a rigorous method relied upon to represent an elliptically inhomogeneous plane wave polarized as an expansion of Vectorial Cylindrical-Harmonics (VCH). This subject is very interesting because of the general representation of an electromagnetic wave as an inhomogeneous wave. In fact, when an electromagnetic wave propagates in a lossy medium, the wave vector representing it is of a complex nature, as it comprises two components – a phase vector and an attenuation vector. In nature, a wave rarely propagates in a completely lossless medium, especially in the field of biology, where water is the main constituent.

Ivlev was the first author to present a work on an inhomogeneous elliptically polarized plane wave [22], [23], determining the basic structure of propagation and exploring its energy fallout. Subsequently, he proposed the first application of an infinite cylinder. In particular, in these studies, the Adler-Chu-Fano formulation (the phase and attenuation vectors) [24] was used, achieving a result equally elegant as complex. In the current study, the use of the complex-angle [25] formulation showed that the representation of the incident field as a superposition of VCH could be obtained with much less complexity. Moreover, this approach will be generalized and related to scattering caused by a cylinder immersed in a lossy medium.

The paper will also provide numerical comparisons for various developments in cylindrical vector waves. Furthermore, results pertaining to an infinite perfect electric conductor (PEC) cylinder immersed in a lossy medium will be provided. Matlab was used for the implementation of the various formulations, while the relevant model was simulated with COMSOL Multiphysics, a commercial software based on the finite element method (FEM).

The article is structured in the following manner. In Section 2, the formalisms are introduced with the purpose of representing an inhomogeneous wave and of providing the formulas used to proceed from a formalism to another. Subsequently, theoretical aspects are illustrated in order to attain a representation of an inhomogeneous elliptically polarized electric field as a superposition of VCHs. In Section 3, some numerical validations are shown, comparing the numerical results of the formulations implemented in Matlab with the ones obtained with the use of COMSOL Multiphysics. Furthermore, new results for the scattering of an elliptically polarized plane wave at oblique incidence from

a PEC cylinder in a lossy medium are presented. Finally, the conclusions are drawn in Section 6.

2. Theoretical Approach

In literature, it is known that an inhomogeneous wave propagating in a lossy medium can be represented through two formalisms. The best one, known as the Adler-Chu-Fano formulation, has a complex propagation vector $k_i = \beta_i + i\alpha_i$ represented by the phase and attenuation vectors, $\beta_i, \alpha_i \in \Re$, respectively. The second one, known as the complex-angle formulation $k_i = k_R + ik_I$ has the propagation vector represented by a superposition of real and imaginary parts which form a complex angle with an axis of the Cartesian reference system $\bar{\vartheta}_i = \vartheta_R + i\vartheta_I$ [25], see Fig. 1. We have used the $\bar{\vartheta}$ symbol to highlight the complex nature of the angle.
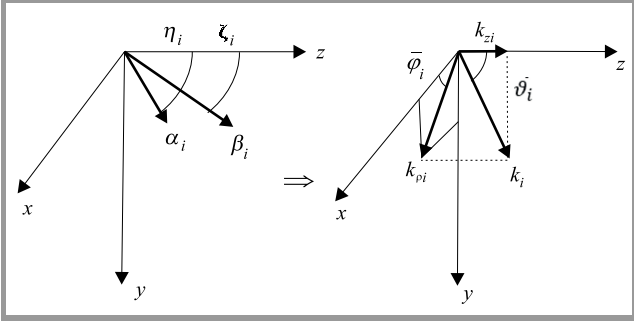

Fig. 1. In the left-hand side figure, the complex wave vector of an inhomogeneous plane wave is presented with the use of phase and attenuation vectors. In the right-hand side picture, the same vector is represented with the use of the complex-angle formalism.

This article demonstrates how to use the complex-angle formalism in order to obtain a simple representation of the field expressed as a superposition of elementary cylindrical waves. Let us consider the following wave, in which the vectors α_i and β_i are forming the angles ζ_i and η_i with axis z, are positioned on the same plane passing through the z-axis and are forming a real angle φ with axis x (see Fig. 1). In this case, the relations between the two formalisms are [25]:

$$\cos \vartheta_R = \frac{k_R \beta \cos \xi + k_I \alpha \cos \eta}{\sqrt{k_R^2 \beta^2 - k_I^2 \alpha^2 + 2(k_R k_I)^2}} \ , \qquad (1)$$

$$\sin \vartheta_R = \frac{k_R \beta \sin \xi + k_I \alpha \sin \eta}{\sqrt{k_R^2 \beta^2 - k_I^2 \alpha^2 + 2(k_R k_I)^2}} \ , \qquad (2)$$

$$\vartheta_I = \frac{1}{2} \text{atanh} \left(\frac{2\beta\alpha}{k^2} \right) \ , \qquad (3)$$

where η and ξ are the angles that the vectors α and β form, respectively, with axis z. Both Eqs. (1) and (2) are needed in order to avoid the indetermination of the value assumed by ϑ_R. For the sake of simplicity, we have worked

on the plane $\varphi = 0$. However, the following considerations can easily be extended to each plane with $\varphi \neq 0$.

It is known that by resolving the scalar Helmholtz equation, the following scalar solution is obtained [26]–[32]:

$$\psi_m = Ae^{im\varphi} Z_m(k_\rho \rho) e^{ik_z z - i\omega t} \ , \qquad (4)$$

having indicated with ρ, ϕ, z the three variables independent of the cylindrical coordinate system, with A – a complex constant, and with k_ρ and k_z the transverse and longitudinal components of the wave vector, respectively, that are defined as:

$$k_\rho^2 + k_z^2 = k^2 \ , \qquad (5)$$

with $k_x = k_\rho \cos \varphi$ and $k_y = k_\rho \sin \varphi$ which are the projections of transversal vector k_ρ on plane $z = 0$. The function $Z_m(k_\rho \rho)$ represents $J_m(k_\rho \rho)$, $Y_m(k_\rho \rho)$, $H_m^{(1)}(k_\rho \rho)$ and $H_m^{(2)}(k_\rho \rho)$, the first, second, third, and fourth Bessel functions, respectively.

At this point, the harmonic vector is defined as follows [27], [30], [32]:

$$\mathbf{M} = \nabla \times (\hat{\mathbf{z}}_0 \psi), \qquad \mathbf{N} = \frac{1}{k} \nabla \times \mathbf{M} \ , \qquad (6)$$

it is always possible to define the electric and the magnetic fields as superpositions of these vectorial functions:

$$\mathbf{E} = \sum_{m=-\infty}^{+\infty} (a_m \mathbf{M}_m + b_m \mathbf{N}_m) \ , \qquad (7)$$

$$\mathbf{H} = \frac{k}{i\omega\mu} \sum_{m=-\infty}^{+\infty} (a_m \mathbf{N}_m + b_n \mathbf{M}_m) \ . \qquad (8)$$

Let us consider a simple inhomogeneous plane wave, using the Fourier series formulas and mathematical identities of the exponential functions [27], [30], [33], [34]. The following is obtained:

$$e^{i(k_x x + k_y y)} = e^{ik_\rho \rho \cos(\varphi - \varphi_i)} \qquad (9)$$

$$= \sum_{m=-\infty}^{+\infty} J_m(k_\rho \rho) e^{im(\varphi - \varphi_i)} e^{im\frac{\pi}{2}} \ . \qquad (10)$$

Now, multiplying both members of Eq. (9) by $e^{ik_z z}$ and considering Eq. (4), the following is reached:

$$e^{i\mathbf{k} \cdot \mathbf{r}} = \sum_{m=-\infty}^{+\infty} \psi_m e^{-im\varphi_i} e^{im\frac{\pi}{2}} \ . \qquad (11)$$

Multiplying both members of Eq. (11) by $\nabla \times \mathbf{z}_0$, and considering Eq. (6), the following result is achieved:

$$\nabla \times \mathbf{z}_0 e^{i\mathbf{k} \cdot \mathbf{r}} = \sum_{m=-\infty}^{+\infty} \mathbf{M}_m(k\mathbf{r}) e^{-im\varphi_i} e^{im\frac{\pi}{2}} \ , \qquad (12)$$

where the first member, using the following identities $\nabla \times \mathbf{z}_0 e^{i\mathbf{k} \cdot \mathbf{r}} = i\mathbf{k} \times \mathbf{z}_0 e^{i\mathbf{k} \cdot \mathbf{r}} = -ik_\rho \mathbf{h}_0(\bar{\vartheta}_i, \varphi_i) e^{i\mathbf{k} \cdot \mathbf{r}}$ can be written as follows:

$$\mathbf{h}_0(\bar{\vartheta}_i, \varphi_i) e^{i\mathbf{k} \cdot \mathbf{r}} = \frac{i}{k_\rho} \sum_{m=-\infty}^{+\infty} i^m \mathbf{M}_m(k\mathbf{r}) e^{-im\varphi_i} \ , \qquad (13)$$

with $\mathbf{h}_0$ the unit vector contained in (x, y) plane. Now, considering the further mathematical identities $\nabla \times \mathbf{h}_0(\bar{\vartheta}_i, \varphi_i)e^{i\mathbf{k}\cdot\mathbf{r}} = i\mathbf{k} \times \mathbf{h}_0(\bar{\vartheta}_i, \varphi_i)e^{i\mathbf{k}\cdot\mathbf{r}} = ik\mathbf{v}_0(\bar{\vartheta}_i, \varphi_i)e^{i\mathbf{k}\cdot\mathbf{r}}$, and taking into account the curl of both members, we obtain:

$$\mathbf{v}_0(\bar{\vartheta}_i, \varphi_i)e^{i\mathbf{k}\cdot\mathbf{r}} = \frac{1}{k_\rho} \sum_{m=-\infty}^{+\infty} i^m \mathbf{N}_m(k\mathbf{r})e^{-im\varphi_i} \ , \qquad (14)$$

having indicated with $\mathbf{v}_0$ the vertical unit vector with respect to the plane (x, y). For an exhaustive exposition, the formulations for the vertical and horizontal components on a Cartesian reference system are reported:

$$\mathbf{v}_{i0} = \cos \bar{\vartheta}_i \cos \varphi_i \mathbf{x}_0 + \cos \bar{\vartheta}_i \sin \varphi_i \mathbf{y}_0 - \sin \bar{\vartheta}_i \mathbf{z}_0 \ , \qquad (15)$$

$$\mathbf{h}_{i0} = -\sin \varphi_i \mathbf{x}_0 + \cos \varphi_i \mathbf{y}_0 \ . \qquad (16)$$

Ultimately, we can affirm that any obliquely polarized elliptical field, with respect to the surface of a cylinder, can be represented as a linear combination of two components, one vertical and one horizontal, each multiplied by its polarization coefficient (E_{vi} and E_{hi}, respectively):

$$\mathbf{E}(\mathbf{r}) = \left[E_{vi}\mathbf{v}_0(\bar{\vartheta}_i, \varphi_i) + E_{hi}\mathbf{h}_0(\bar{\vartheta}_i, \varphi_i) \right] e^{i\mathbf{k}\cdot\mathbf{r}}$$
$$= \frac{1}{k_\rho} \sum_{m=-\infty}^{+\infty} i^m \left[iE_{hi}\mathbf{M}_m(k\mathbf{r}) - E_{vi}\mathbf{N}_m(k\mathbf{r}) \right] e^{-im\varphi_i} \qquad (17)$$

Imposing the following definitions [32]:

$$a_m = \frac{E_{hi}}{k_\rho} i^{m+1} e^{-im\varphi_i} \ , \qquad (18)$$

$$b_m = -\frac{E_{vi}}{k_\rho} i^m e^{-im\varphi_i} \ , \qquad (19)$$

$$\mathbf{k}_i = k \left(\sin \bar{\vartheta}_i \cos \varphi_i \mathbf{x}_0 + \sin \bar{\vartheta}_i \sin \varphi_i \mathbf{y}_0 + \cos \bar{\vartheta}_i \mathbf{z}_0 \right) \ , \qquad (20)$$

$$\mathbf{M}_m = \mathbf{m}_m e^{im\varphi} e^{ik_z z - i\omega t} \ , \qquad (21)$$

$$\mathbf{N}_m = \mathbf{n}_m e^{im\varphi} e^{ik_z z - i\omega t} \ , \qquad (22)$$

with

$$\mathbf{m}_m = im\frac{Z_m(k_\rho \rho)}{\rho}\boldsymbol{\rho}_0 - k_\rho \frac{\partial Z_m(k_\rho \rho)}{\partial \rho}\boldsymbol{\varphi}_0 \ , \qquad (23)$$

$$\mathbf{n}_m = i\frac{k_z k_\rho}{k}\frac{\partial Z_m(k_\rho \rho)}{\partial \rho}\boldsymbol{\rho}_0 - \frac{mk_z}{k}\frac{Z_m(k_\rho \rho)}{\rho}\boldsymbol{\varphi}_0$$
$$+ \frac{k_\rho^2}{k} Z_m(k_\rho \rho)\mathbf{z}_0 \ , \qquad (24)$$

the electric field can be written in the following elegant way:

$$\mathbf{E}_i(k\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \left[a_m \mathbf{M}_m(k\mathbf{r}) + b_m \mathbf{N}_m(k\mathbf{r}) \right] \ . \qquad (25)$$

We can extend what has been done so far to a more generic case, where the phase vector, attenuation vector, and the z axis are not positioned on the same plane. In this case, angle $\bar{\varphi}_i$ is complex as well. Following the same logical

reasoning as adopted in the previous case and exploiting the mathematical identities [33], the following is obtained:

$$\mathbf{E}_i(k\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \left[\bar{a}_m \mathbf{M}_m(k^*\mathbf{r}) + \bar{b}_m \mathbf{N}_m(k^*\mathbf{r}) \right] \ , \qquad (26)$$

with

$$\bar{a}_m = \frac{E_{hi}}{k_\rho}(-i)^{m-1} e^{-im\bar{\varphi}_i} \ , \qquad (27)$$

$$\bar{b}_m = -\frac{E_{vi}}{k_\rho}(-i)^m e^{-im\bar{\varphi}_i} \ , \qquad (28)$$

$$\mathbf{k}_i = k^* \left(\sin \bar{\vartheta}_i \cos \bar{\varphi}_i \mathbf{x}_0 + \sin \bar{\vartheta}_i \sin \bar{\varphi}_i \mathbf{y}_0 + \cos \bar{\vartheta}_i \mathbf{z}_0 \right) \ , \qquad (29)$$
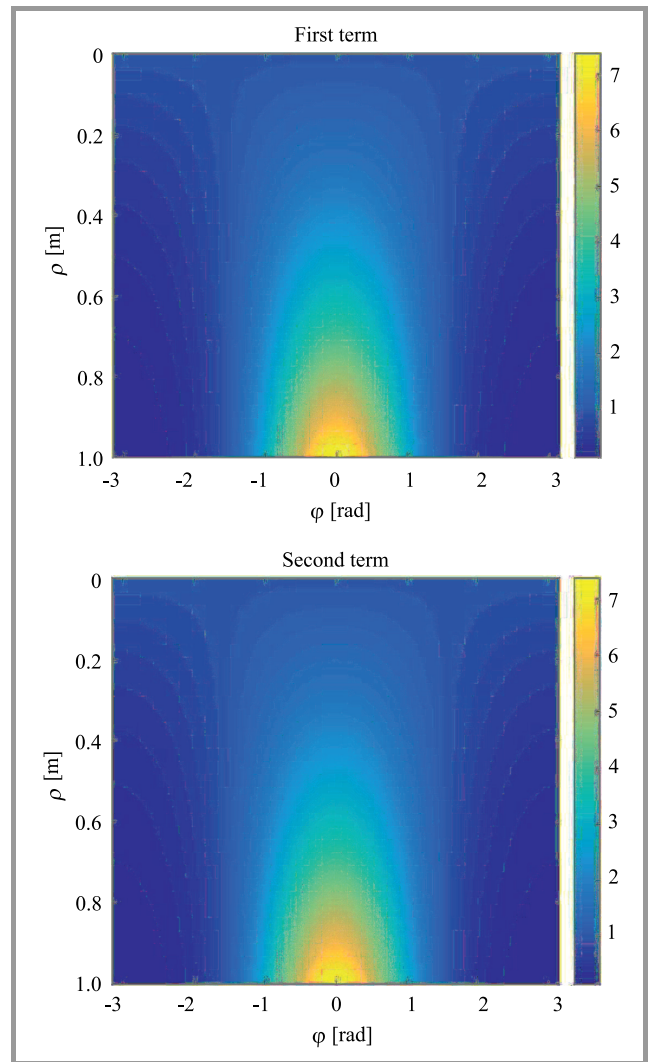
and with k^* the complex conjugate of the wave number k.

Fig. 2. Comparison of the absolute value of the inhomogeneous electric field as a function of φ and ρ in the case of $\varphi \in R$, see Eq. (25). The upper figure shows the results implementing the inhomogeneous plane wave – see the first member of Eq. (25); in the lower figure, results implementing the superposition of the VCHs are shown – see the second member of Eq. (25). (For color pictures visit www.nit.eu/publications/journal-jtit)
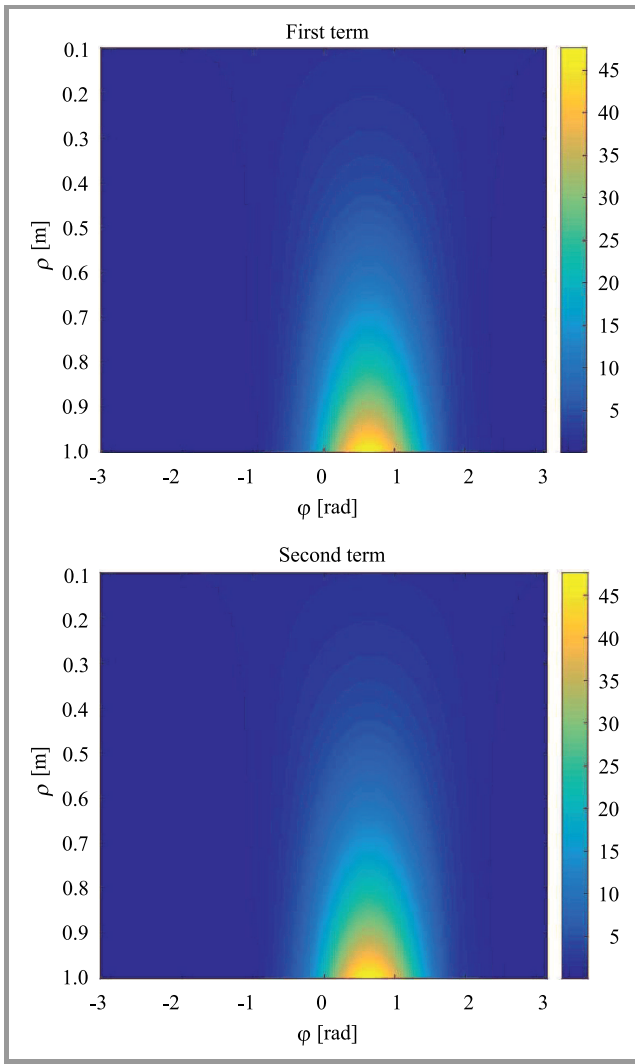
Fig. 3. Comparison of the absolute value of the inhomogeneous electric field as a function of φ and ρ in the case of $\varphi \in I$ – see Eq. (26). The upper figure shows the results implementing the inhomogeneous plane wave– see the first member of Eq. (26), while the lower figure shows the results implementing the superposition of VCHs – see the second member of Eq. (26).
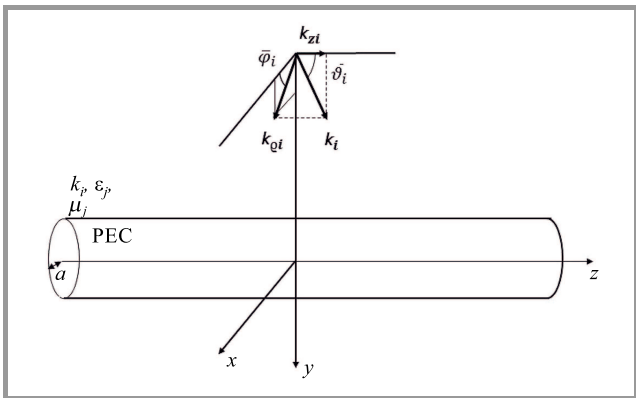

Fig. 4. Statement of the validation problem. A complex wave vector of an inhomogeneous plane wave (with ϑ and $\varphi \in I$) is impinging on an infinite circular PEC cylinder with radius a and with its longitudinal axis parallel to axis z.

3. Validation and Numerical Results

Validation was performed by comparing the determined formulation and the canonical case of electromagnetic scattering. In particular, an infinite PEC cylinder has been considered and Eqs. (25) and (26) have been taken into consideration. Equation (25) has been validated considering the inhomogeneous plane wave with the following parameters $k = 2 - i2$ 1/m, $\rho = [0, 1]$ m, $\vartheta = \pi/2$, $\varphi = [-\pi, \pi]$. The results representing the inhomogeneous plane wave and its representation in the form of a VCH superposition were found to be perfectly compatible (see Fig. 2). Subsequently, the most general case of an inhomogeneous plane wave with φ complex was validated. In particular, the following fictitious parameters of $k = 2 - i2$ 1/m, $\rho = [0.1, 1]$ m, $\vartheta = \pi/2$, $\varphi = [-\pi + i, \pi + i]$ were considered. Once again, the comparison between the inhomogeneous plane wave and its representation in the form of a VCH superposition showed that they are perfectly stackable (see Fig. 3).

After the validation, the actual method to study the scattered electric field by an infinite PEC cylinder immersed in a lossy medium was implemented (see Fig. 4). For this purpose, the following representation of the electric scattered field was considered [32], [35]:

$$\mathbf{E}_{sc}(k^{*}\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \left[c_m \mathbf{M}_m^{(3)}(k^{*}\mathbf{r}) + d_m \mathbf{N}_m^{(3)}(k^{*}\mathbf{r}) \right] \ , \quad (30)$$

having indicated with the apex $^{(3)}$ that we are considering the third Bessel function in the VCHs, and where:

$$c_m = -a_m \frac{\dot{J}_m(k_\rho^{*}a)}{\dot{H}_m^{(1)}(k_\rho^{*}a)} \ , \quad (31)$$

$$d_m = -b_m \frac{J_m(k_\rho^{*}a)}{H_m^{(1)}(k_\rho^{*}a)} \ , \quad (32)$$

are the scattering coefficients [32], and where a is the cylinder radius.

To validate this last scenario, the results obtained by verifying Eq. (31) in Matlab and COMSOL were compared. The results were computed based on the following input values: frequency 300 MHz, $\varphi_i = 0$ rad, $E_{0_h} = 1$ V/m, $E_{0_v} = 0$ V/m, $\varepsilon = 1 + 1i$ (relative dielectric constant), $\mu = 1$ (relative magnetic permeability), cylinder radius 12.5 cm, $\xi = \pi/8$ rad, and $\eta = \pi/6$ rad. The scattered electric field was calculated along the line $x = -25$ cm, $y = [-25 \div 25]$ cm, and $z = 0$ cm. Figure 5 shows the results achieved with two different numerical methods, with a perfect match between them.

4. Conclusions

In this paper, a rigorous method applied in order to expand an elliptically polarized plane wave that is inhomogeneous in terms of vectorial cylinder harmonics is presented. The solution has been achieved using the complex-angle formalism focusing on the problem of determining the expansion
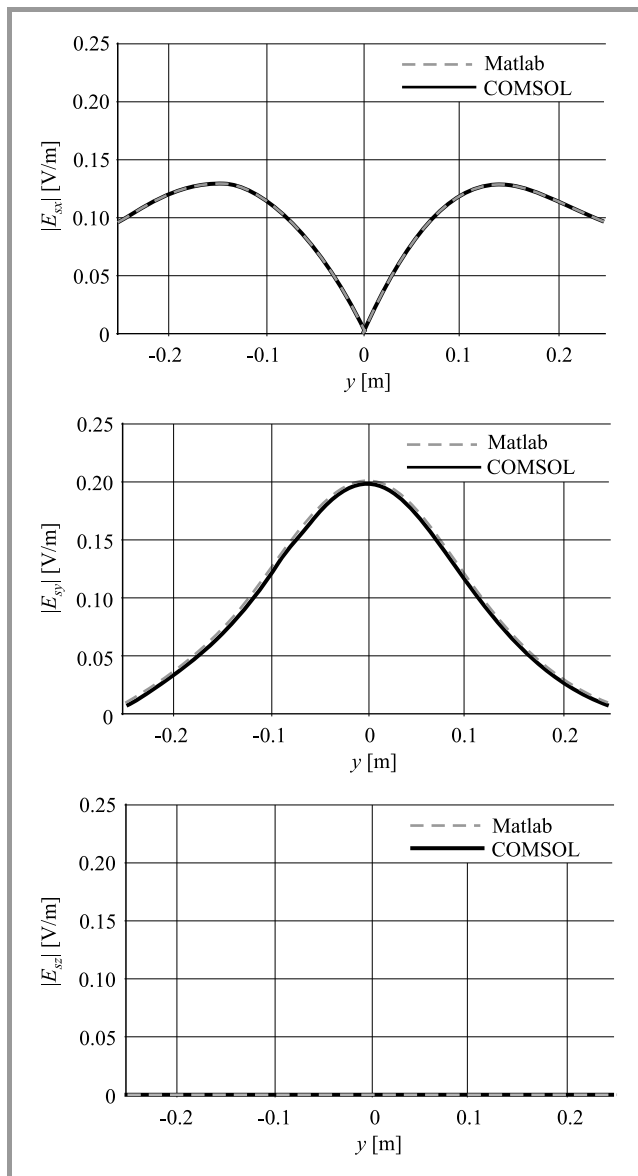
Fig. 5. Comparison of numerical results obtained with Matlab code (red, dashed line) and COMSOL simulations (blue, solid line) for the three components of the scattered electric field along a line of the following coordinates: $x = -25$, $y = [-25 \div 25]$, $z = 0$ cm.

coefficients. In this way, an elegant and light formalism was obtained. To validate the procedure, some numerical results have been presented. Furthermore, comparisons with simulations performed in the COMSOL environment have been performed. In particular, the case of scattering caused by a perfectly conductive electric cylinder with a circular section and of infinite length was considered to compare the results, and perfect accordance was reached in all scenarios. Thanks to the minimal invasiveness of the formalism, the cylindrical harmonics defined for the complex angle enjoy all the same properties as the simple cylindrical harmonics. Therefore, they are elegantly applicable to more complex cases, such as an ensemble of cylinders or cylinders buried in lossy media.

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