Examples of air-entraining flows

H. N. Oğuz, A. Prosperetti, and A. M. Lezzi[a)]
Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Maryland 21218

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Four examples of air-entraining flows at the free surface of a liquid are briefly considered: (a) the transient impact of a jet, (b) the application of an excess pressure, (c) two counter-rotating vortices below the surface, and (d) a disturbance on a vortex sheet.

Despite their widespread occurrence (breaking waves, hydraulic jumps, impacting drops, splashes, jets, and others) and importance (underwater noise generation, water treatment, gas–liquid reactors, metal and glass industry) surprisingly little is known about air-entraining flows. It appears that several different mechanisms are at work depending on the conditions, but they are rather poorly understood.[1–3] In the first important paper on air entrainment by jets falling into a pool of the same liquid, Lin and Donnelly[1] showed that, for a laminar jet, bubbles originate from the breakup of an air film that separates the jet liquid from the pool liquid down to a depth of several diameters below the free surface.[4] In the case of a turbulent jet, on the other hand, air entrainment is intermittent and more intimately related to unsteady processes.

In this Letter, we shall consider several examples of transient flows giving rise to air entrainment. While these examples illustrate some physical aspects of the phenomenon, they also present enough of a fluid dynamical interest to warrant consideration in their own right.

In all the cases to be described, the flow is incompressible, inviscid, and potential, possibly with embedded regions of localized vorticity. Surface tension and gravity effects are described by means of the nondimensional Weber and Froude numbers defined by $\mathrm{We} = \rho U^2 L/\sigma$, $\mathrm{Fr} = U^2/gL$, respectively. Here, ρ and σ are the liquid density and interfacial tension, g is the acceleration of gravity, and U and L are suitable velocity and length scales. The time scale is given by L/U. The numerical method used is essentially that described in Ref. 5 and is based on a boundary-integral approach.

The first example is that of the transient impact of a vertical cylindrical jet on a plane liquid surface. The jet's impact velocity and radius R are taken as the reference velocity and length scales. At the initial instant the jet is just in contact with the plane free surface of the receiving liquid. We show the free surface configuration at successive instants of time in Fig. 1 for $\mathrm{We} = 20.60$ and $\mathrm{Fr} = 68.03$. With an impact velocity of 1 m/sec and the physical properties of water, the corresponding jet radius would be 1.5 mm. No air entrainment is expected for these parameter values in steady conditions. At first, the jet penetrates into the receiving liquid surrounded by a cylindrical shroud of air. This shroud then collapses due mainly to the action of gravity, thus entrapping air. The calculation was stopped just before the occurrence of contact between the facing liquid surfaces.

In the parameter range of Fig. 1, in which the kinetic energy of the jet is relatively large, we find that the thickness of the air shroud is close to the jet's radius, with little dependence on We and Fr. Furthermore, the bottom of the air shroud advances into the receiving liquid with an almost constant velocity essentially independent of We and Fr and very nearly equal to half of the jet's velocity. A simple argument can be given to explain this feature. When the impact velocity is smaller, however, the mechanics is different. The air film is now much less thick with very pronounced waves on the jet's surface and a much smaller amount of air entrained.

A common transient process giving rise to air entrainment—as evidenced by high-speed movies[1,6]—can be described as follows. Occasionally a disturbance occurs on the surface of the falling jet and is convected downward. When it reaches the surface of the pool, it imparts a local impulse that moves the free surface downward much in the same way as the cavity formed by an impacting object. When this depression is sufficiently deep, it cannot fill before it closes off near the free surface. Thus, in a manner analogous to that found in the previous case, one or more bubbles are entrained.

The discussion of the last example considered in this Letter will make clear that the numerical simulation of this process is very complex. Therefore we look here at a related, but much simpler, situation. We consider a liquid of infinite depth unbounded on one side and bounded by a vertical wall on the other. At time zero, a constant excess pressure $\Delta p \equiv \rho U^2$ begins to act on the liquid surface over a portion of width L adjacent to the wall. Figure 2 (in which the physical system is rotated clockwise by 90°) shows a series of snapshots of the free surface for the case $\mathrm{We} = 6.62$, $\mathrm{Fr} = 50$. For water, these numbers correspond to $L = 1$ mm, $U = 0.7$ m/sec. A consideration of the Bernoulli integral suggests that the applied Δp will eventually be overpowered by the hydrostatic head so that the deepening of the cavity will stop. However, the present situation presents obvious similarities with the previous one and a similar sequence of events can therefore be expected.

This problem is somewhat analogous to the steady separated flow past a plate normal to the incoming stream which, in two dimensions and in the absence of gravity and surface tension, admits of an exact closed-form solution. A numerical treatment of the axisymmetric case has been given by Brennen.[7] The analytic solution shows that, in contrast with the previous axisymmetric situation, the

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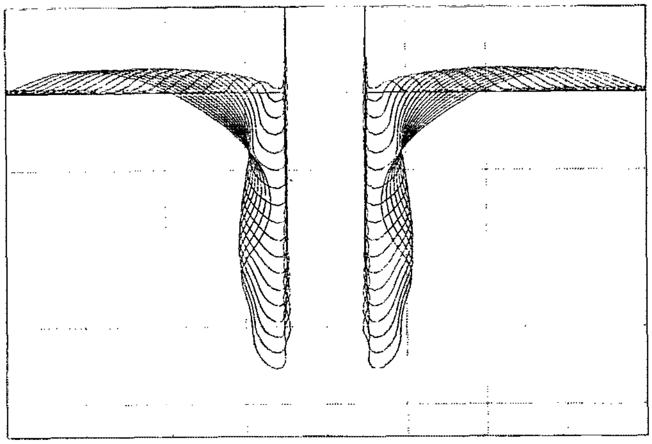

FIG. 1. Successive configurations of the free surface produced by a 1.5 mm radius water jet impacting at 1 m/sec (We = 21, Fr = 68). The traces are approximately equally spaced in time (within 12%) and separated by 0.855 dimensionless time units; the first trace is for the dimensionless time $Ut/R = 0.726$ and the last one for 15.3. The grid spacing is equal to the jet's diameter $2R$.

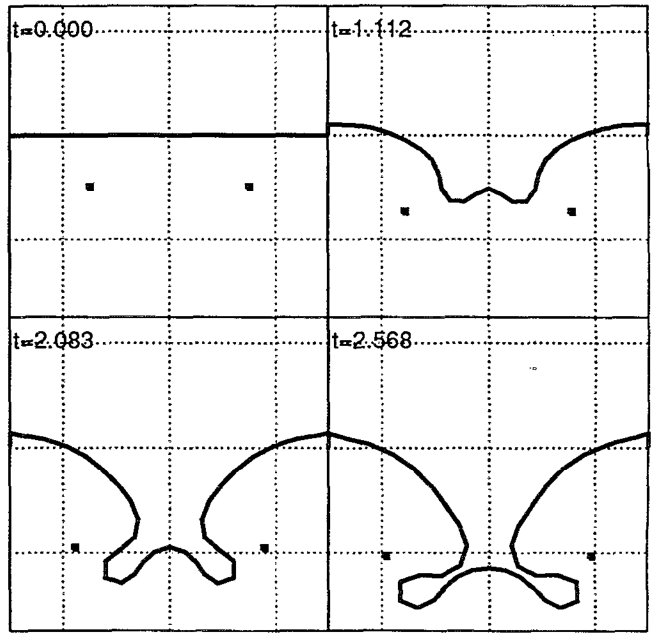

FIG. 3. Successive surface deformation produced by a pair of counter-rotating vortices (left vortex clockwise) for We = 687, Fr = 2.04, $d/L = 3$. The grid spacing is twice the initial depth of submergence L. With $L = 5$ cm, for water, this case corresponds to $\Gamma/2\pi L = 1$ m/sec.

thickness of the separation zone increases without bound away from the plate. As a consequence of this outward motion, which is also clearly present in our numerical results, the gravity-driven inward closing of the cavity is retarded so that, when it is finally completed, the cavity depth is larger than before and its aspect ratio strikingly different.

A body of experimental evidence exists connecting air entrainment to the presence of vortices in the neighborhood of a free surface.[8] We shall formulate a hypothesis as to the origin of these vortices in connection with the last problem to be described below. Here, we look at a simple example that illustrates the manner in which air may be entrained in these circumstances. At time zero, two counter-rotating line vortices (the left one rotating clockwise) are impulsively started on a line parallel to a free surface. According to Helmholtz's theorem, their subsequent motion evolves according to the regular part of the velocity field evaluated at their location. The dimensional circulation Γ is held constant. The initial separation is d and the initial depth of submergence L. The latter quantity is taken as length scale and the velocity scale is $U = \Gamma/2\pi L$. This and related problems have been the object of several recent studies[9–12] none of which, however, is concerned with air entrainment.

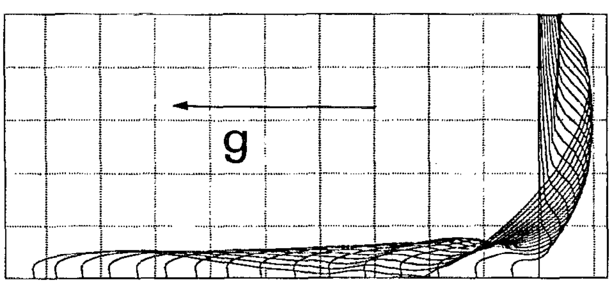

FIG. 2. Successive configurations of the free surface produced by an overpressure of 0.49 KPa acting over a 1 mm distance from a plane wall (We = 6.6, Fr = 50). (The picture is rotated clockwise by 90°; gravity points to the left.) The first trace is for $\sqrt{\Delta p/\rho t/L} = 2.72$, and the last three for 30.337, 32.127, and 33.726; all others are approximately equally spaced and separated by 2.12 dimensionless time units (within 14%). The spacing of the grid shown is $4L$.

An example of the behavior of this system is shown in Fig. 3 for We = 687, Fr = 2.04, $d/L = 3$. With $L = 5$ cm, for water, these parameters correspond to $U = 1$ m/sec. In this case, the vortex strength is large enough that air is sucked below the free surface and under the vortices. If the vortex strength is reduced by a factor of 2, the vortices are found to always remain below the free surface. In this case, a surface depression forms initially, but it then fills up from the bottom only generating surface waves. Evidently, in the parameter space We, Fr, and d/L, a threshold exists separating entraining from nonentraining cases.

It is very easy to show by putting a little dye in a jet that flow separation occurs near the point of impact with the free surface of the receiving liquid even when the velocity is lower than the air-entrainment threshold (Fig. 4). In an inviscid framework, therefore, the appropriate model involves a vortex sheet that separates the jet liquid from the pool liquid. In our final example, we want to investigate, albeit in a very qualitative way, the possible role of this structure on air entrainment. As in the second example, the two-dimensional version of this problem is considered. The jet velocity is taken as the reference velocity and the jet thickness is infinite.

A configuration in which, upon entering the receiving liquid, the jet retains its velocity and plane shape and is

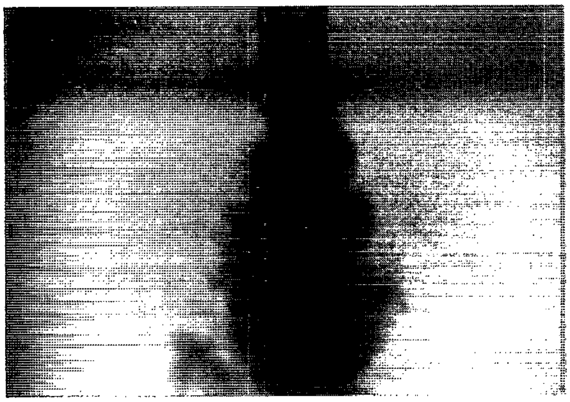

FIG. 4. A 3 mm diameter water jet colored with food dye entering a water pool with an approximate velocity of 1.6 m/sec. The jet velocity is too low to cause air entrainment. Note the flow separation just below the free surface of the pool.

separated by a plane vortex sheet from the pool of liquid at rest and with a plane free surface, is a steady solution of the mathematical model. We introduce an initial perturbation of the jet surface, $x = \delta y^2 \exp[-a(y-b)^2]$ (where δ, a, b are constants and x and y are horizontal and vertical coordinates, the latter measured upward from the undisturbed free surface of the receiving liquid), and let it evolve in time according to the proper dynamics including convection, surface tension, and gravity. The process to be described happens very fast, and therefore surface tension is far more important than gravity. For this reason, we choose as fundamental length a quantity of the order of the curvature of the disturbance, $L = 1/|\delta|a$. In the examples shown below, we take $\delta/L = -0.1$, $b/L = 1$, $aL^2 = 10$, We = 10, Fr = 10. We have also carried out simulations with a positive value of δ finding similar results.

A basic indeterminacy for the modeling of this process arises because the angle with which the free surface of the pool connects with the vortex sheet depends in a complicated way on surface tension and viscosity, and therefore cannot be determined within the framework described. In

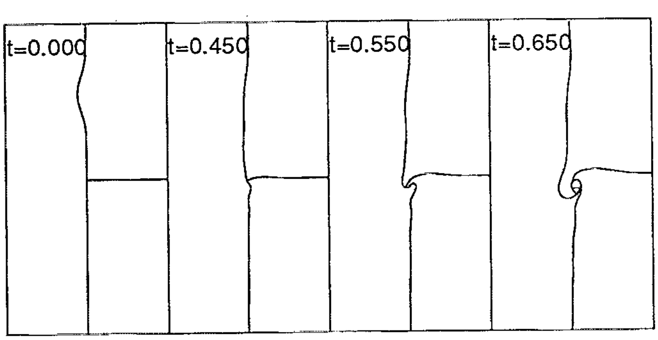

FIG. 5. A disturbance on a jet (first frame) is convected into a liquid pool. The continuation of the jet's boundary below the horizontal free surface is a vortex sheet. The angle with which the free surface meets the vortex sheet is fixed at $\pi/2$. Note the rolling up of the vortex sheet. The horizontal and vertical scales are equal. The size of each frame is 2×4 dimensionless units.

the case shown here, we prescribe this angle to be $\pi/2$. The result strongly suggests the possibility of air entrainment. A weaker disruption of the free surface occurs, however, if the angle is allowed to vary.

An interpretation of the process of air entrainment in this situation might be the following. In normal conditions, the vortex sheet separating the jet liquid from the incoming liquid will become unstable at a certain depth below the free surface. When a perturbation disturbs it, however, the instability is triggered much closer to the free surface, the sheet rolls up giving rise to a strong vortex, and air is entrained as in Fig. 3.

Each one of the flows considered in this note is interesting and we plan to present the result of their systematic investigation in future publications. Our purpose here was to bring together several inter-related and interacting aspects of them, all of which have a bearing on air entrainment by unsteady processes.

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[a)]Present address: Dipartimento di Ingengneria Meccanica, Facoltà di Ingegneria, Università di Brescia, Via Valotti 9, 25060 Brescia, Italy.

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