Bubble-related ambient noise in the ocean

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An analysis is presented of the mechanisms by which bubbles can generate ambient noise in the ocean and the resulting noise levels are estimated. Bubbles can be extremely efficient amplifiers of water turbulence noise up to 100–200 Hz. At higher frequencies, the Lagrangian spectral intensity of the turbulence is too poor for this mechanism to contribute. Above 1–2 kHz, however, the oscillations by which newly formed bubbles dispose of their initial energy is shown to lead to substantial noise levels. This same process cannot account for the noise in the frequency range intermediate between these two because it would require unrealistically large bubbles, with a diameter of 1 cm or more. A possible mechanism active in this intermediate range, in which relatively large levels of ambient noise are observed, is that of collective oscillations of bubble clouds. In all cases the results obtained by the formal derivations (which are based on an adaptation of Lighthill's theory of aerodynamic noise) are substantiated by simple physical arguments. Other possible noise mechanisms in which bubbles are involved are also briefly considered.

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INTRODUCTION

It is well known that bubbles play an important role in oceanic ambient noise,[1] and several models have been proposed to account for their effects.[1–5] In the present article, we wish to set up a general framework for the description of bubble-generated noise in different frequency ranges. In the range from a few Hz to 100–200 Hz, we consider the amplification of turbulence noise by bubbles along the lines of Ref. 6 (Sec. I). In the region from about 1 to 10 kHz, we examine the hypothesis that noise is produced by freely oscillating individual bubbles (Sec. II). This mechanism is related to that of Ref. 4 (see also Ref. 1), although the mathematical formulation and some significant physical aspects are different. Finally, in the intermediate range from a few hundred Hz to 1 kHz, we study the possibility that the noise is due to collective oscillations of bubble clouds (Sec. III). In this way, one could account for these relatively low frequencies in terms of bubbles having a radius not unrealistically large. Further mechanisms (some real and some unlikely) and a number of early models are briefly considered in Sec. IV.

Bubbles are plentiful in the surface layer of the ocean.[7–9] They are created by biological activity, drop impact such as in sprays, splashes, rain, and, most importantly for our purposes, by wave breaking. It is well known that at wind speeds between 7 and 10 m/s, which are typical of the onset of wave breaking, noise levels undergo a marked increase especially at frequencies of a few hundred Hz and above.[3,10–14] At lower frequencies, this increase is partly masked by the presence of other sources, such as shipping, but is nonetheless documented by the data.[13,14] Here the passive amplification of liquid turbulence noise does not necessarily require wave breaking, but only the occurrence of turbulent patches containing bubbles. Wave breaking would, however, greatly enhance this component of the noise because of increased turbulence levels and bubble numbers.

It should be stressed at the outset that our aim in this article is not to provide ambient noise curves to be compared with data as was done, for example, in Ref. 4. Although this must be the ultimate aim of any theory of ambient noise, we find that too much of the required information is missing at present to attain this goal. Rather, we intend to set up a broad framework for the description of the processes involving bubbles from which indications for future research can be obtained and which, hopefully, will be useful for the quantitative modeling problem once the missing information becomes available. We establish the viability of the proposed mechanisms by rather crude order-of-magnitude estimates. Some attention is also given to a physical understanding of the processes and to estimates of their effectiveness based on simple physical arguments.

I. AMPLIFICATION OF WATER TURBULENCE NOISE

As an acoustic source, turbulence behaves as a quadrupole distribution and is therefore comparatively inefficient.[15–17] If bubbles are present in the turbulent region, however, they respond with volume pulsations to the turbulent pressure fluctuations and the resulting acoustic effect is that of a monopole.[6,17] In the ocean, regions where intense turbulence coexists with a large bubble population are created typically by breaking waves, which appear therefore to be prime potential contributors to ambient noise produced by this mechanism. It should be noted that a breaking wave leaves behind a region of bubbly turbulent liquid that persists for a long time after the passage of the wave.[9] The bubble clouds are observed to penetrate several meters under the surface (up to 6–8 m at wind speeds of 10–12 m/s),[9] and since this must be chiefly due to water turbulence, we can infer that the size of the acoustically active region is of a comparable magnitude. Hence, the volume of bubbly water in which this mechanism can be active extends far beyond the limited crest of white water topping a breaking wave.

For a bubble to respond in the manner described, the pressure fluctuations must be more or less coherent over its surface. This implies that the bubbles capable of amplifying

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the turbulence noise must be engulfed by the turbulent eddies and, therefore, be transported by them. As a consequence, they will respond to the Lagrangian rather than to the higher Eulerian turbulence frequencies. Failure to appreciate this point, as in Ref. 1, may result in a considerable overestimate of the maximum frequency at which this mechanism is effective. Not much is known about the Lagrangian spectrum. Measurements in air suggest that it extends to frequencies about an order of magnitude smaller than the Eulerian one.[18] A maximum frequency of the order of 100–200 Hz appears to be a realistic estimate, and it is interesting to note in this connection that noise data taken in areas with very low shipping tend to present a local minimum in this frequency range.[19] Noise at higher frequencies must be generated by different mechanisms some of which will be considered in the following sections.

In the present treatment of turbulence noise we shall follow the method indicated in Ref. 17, which has the essential advantage of clearly exhibiting the nature of the effective source. The alternative approach of Ref. 6 describes the source in a different way which would be more appropriate for the mechanism studied in Sec. II. The two methods are, however, equivalent and we derive both our results using the first one.

With a view to the breaking wave process, we shall refer to the acoustically active region as to the *turbulent patch* or *spot*. The actual natural process is clearly transient but since its lifetime, measured in tens of seconds, is far greater than the inverse frequencies of interest, we are going to use terminology strictly appropriate for a steady process, and we shall incorporate the transient nature of the event by simple "windowing" in space-time.

A. General theory

We consider the mixture of bubbles and liquid as a single continuum for which mass and momentum equations may be written as

$$\frac{\partial \rho_m}{\partial t} + \nabla\cdot(\rho_m \mathbf{u}) = 0\,, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_m \mathbf{u}) + \nabla\cdot\mathbf{M}_m = -\nabla p + \nabla\cdot\boldsymbol{\tau} + \mathbf{G}\,. \tag{2}$$

Here, $\mathbf{M}_m$ is the average momentum flux in the mixture and ρ_m is the average density given by

$$\rho_m = \beta\rho_G + (1-\beta)\rho\,, \tag{3}$$

where ρ_G and ρ denote the gas and liquid density and β is the gas volume fraction, i.e., the volume occupied by the gas in a unit volume of the mixture. Here, $\mathbf{u}$ denotes the mass-averaged velocity, p the pressure, $\boldsymbol{\tau}$ the total stress (consisting of viscous and Reynolds stresses), and $\mathbf{G}$ the effective body force. Upon elimination of $\rho_m \mathbf{u}$ and subtraction of $c^2\nabla^2\rho_m$, where c is the speed of sound in *pure* water, we find

$$\frac{\partial^2 \rho_m}{\partial t^2} - c^2\nabla^2\rho_m = \frac{\partial^2}{\partial x_i\,\partial x_j} T_{ij} + \nabla\cdot\mathbf{G}\,, \tag{4}$$

where the tensor T_{ij} is given by

$$T_{ij} = M_{ij} + \tau_{ij} + (p' - c^2\rho'_m)\delta_{ij}\,, \tag{5}$$

with primes denoting perturbations of equilibrium values. Equation (4) is similar to that posed by Lighthill as the basis of his well-known theory of aerodynamic noise, and is particularly useful when, as in the present application, one is interested in the acoustic field in the otherwise undisturbed medium far away from the region where $\mathbf{T}$ is appreciably nonzero. In this farfield, there is practically no free gas, $\beta = 0$, and $\rho_m = \rho$, the liquid density. The perturbation ρ' of this quantity can therefore be calculated from the known Green's function for Eq. (4) and is

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi c}\int d^3y \int ds \frac{\partial^2}{\partial y_i\,\partial y_j} T_{ij}(\mathbf{y},s) \times \frac{\delta(|\mathbf{x}-\mathbf{y}| - c(t-x))}{|\mathbf{x}-\mathbf{y}|}\,, \tag{6}$$

where the effective body force term has been neglected for reasons that will be given below. The origin of the coordinate system is taken to be at the surface somewhere in the turbulent region. Upon integration by parts the double derivative can be transferred to the Green's function and, since this depends on $\mathbf{x}-\mathbf{y}$, it can be turned into a derivative with respect to $\mathbf{x}$ and taken out of the integral. With these steps the integration with respect to s can be carried out to find

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi c}\frac{\partial^2}{\partial x_i\,\partial x_j}\int T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{c}\right)\frac{d^3y}{|\mathbf{x}-\mathbf{y}|}\,.$$

This expression represents the solution to (4) in an unbounded medium. In the present application, the pressure-release condition at the free surface must be accounted for. Since we are concerned here with sound waves in water with wavelengths of tens of meters, we shall ignore the presence of surface waves for simplicity and take the surface as plane. By use of the method of images, we then obtain

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi c}\frac{\partial^2}{\partial x_i\,\partial x_j}\int\left[\frac{1}{|\mathbf{x}-\mathbf{y}|} T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{c}\right) - \frac{1}{|\mathbf{x}-\mathbf{y}_r|} T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}_r|}{c}\right)\right] d^3y\,, \tag{7}$$

where $\mathbf{y}_r$ is the position vector of the source point reflected in the plane free surface,

$$\mathbf{y}_r = \mathbf{y} - 2\hat{\mathbf{z}}z\,. \tag{8}$$

Here we have indicated with $\hat{\mathbf{z}}$ the unit vector normal to the free surface oriented away from the liquid and with z the component of $\mathbf{y}$ in this direction measured from the free surface.

We are interested in the noise at a distance from the surface that is large compared with the size of the surface source region, and we therefore use the approximations

$$|\mathbf{x}-\mathbf{y}|^{-1} \simeq |\mathbf{x}-\mathbf{y}_r|^{-1} \simeq x^{-1}\,,$$

where $x = |\mathbf{x}|$. This approximation is evidently excessively crude to be used in the arguments of the T_{ij}'s. For frequencies up to 150 Hz, say, the wavelength of sound in water is 10 m or greater, which exceeds the dimensions of the turbulent spot radiating sound. This circumstance enables us to set

$$|\mathbf{x}-\mathbf{y}| \simeq x - \mathbf{n}\cdot\mathbf{y}, \quad |\mathbf{x}-\mathbf{y}_r| \simeq x - \mathbf{n}\cdot\mathbf{y}_r\,,$$

where

$$\mathbf{n} = \mathbf{x}/x\,, \tag{9}$$

is the unit vector in the direction of the observer. Furthermore, in the farfield,

$$\frac{\partial^2}{\partial x_i\,\partial x_j}\left(\frac{T_{ij}}{x}\right) = \frac{x_i x_j}{c^2 x^3}\frac{\partial^2 T_{ij}}{\partial t^2} + O(x^{-2})\,,$$

so that (7) becomes

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi c^4 x}\frac{x_i x_j}{x^2}\frac{\partial^2}{\partial t^2}\int\left[T_{ij}\left(\mathbf{y},t-\frac{x-\mathbf{n}\cdot\mathbf{y}}{c}\right) - T_{ij}\left(\mathbf{y},t-\frac{x-\mathbf{n}\cdot\mathbf{y}_r}{c}\right)\right]d^3y\,. \tag{10}$$

Note that here, in principle, the integration region extends over the entire horizontal plane, but only between $-\infty$ and 0 in the third (z) coordinate. The term T_{ij} will, however, vanish outside of the turbulent patch.

This result is accurate in the farfield but no further progress is possible without some simplification of the form (5) of T_{ij}. In a pure fluid, pressure and density perturbations are related by $p' = c^2\rho'$, the last two terms of (5) very nearly cancel, and the dominant effect arises from the velocity fluctuations. If, however, the pressure fluctuations occur in a bubbly mixture, we have

$$p' = c_m^2\rho_m'\,, \tag{11}$$

where c_m is the speed of sound in the mixture approximately given, at the low frequencies of present concern, by[20,21]

$$c_m^2 = p_0/\rho\beta(1-\beta)\,, \tag{12}$$

where p_0 denotes the undisturbed pressure. A remarkable feature of this result is that c_m is much smaller than c as soon as β exceeds a fraction of a percent. For example, with $p_0 = 1$ bar, $\rho = 10^3$ kg/m^3, we find $c_m = 100.5$ m/s and $c_m = 46$ m/s for $\beta = 1\%$ and 5%, respectively. We can therefore write

$$p' - c^2\rho_m' = (1 - c^2/c_m^2)p' \simeq -(c^2/c_m^2)p'\,.$$

Furthermore, in a turbulent field, the first term M_{ij} in the definition (5) of T_{ij} is also of the order of p' and hence can be neglected. Similar arguments can be invoked to justify the neglect of the effective body force $\mathbf{G}$ and stress tensor τ_{ij}.[17] With these approximations, Eq. (10) becomes

$$\rho'(\mathbf{x},t) = -\frac{1}{4\pi c^2 x}\frac{\partial^2}{\partial t^2}\int c_m^{-2}(p'-p_r')\,d^3y\,,$$

where the index r indicates that the argument of p' corresponds to the reflected point $\mathbf{y}_r$ as in the second term of (10). We further assume for simplicity that c_m is approximately uniform in the source region and varies but slowly with time. While these hypotheses are certainly violated to some extent, they should be adequate for the estimates of present concern. With this, the previous expression for ρ' becomes

$$\rho'(\mathbf{x},t) = -\frac{1}{4\pi c^2 c_m^2 x}\frac{\partial^2}{\partial t^2}\int (p'-p_r')\,d^3y\,. \tag{13}$$

It may be noted that (aside from the image effect) this result differs from the one found in standard treatments of turbulence noise[17] by a factor c^2/c_m^2, which explicitly appears in (13). The presence of the bubbles therefore amplifies the turbulence noise intensity by the factor $(c/c_m)^4$, as was pointed out in Ref. 6. The spatial isotropy of the field given by each one of the two terms of (13) suggests that the acoustic source behaves as a monopole of strength

$$q = \frac{1}{c_m^2}\frac{\partial p'}{\partial t}\,. \tag{14}$$

The difference between the real and image sources leads however to radiation with a dipole pattern as will be seen below.

The basic result (13) can readily be expressed in the following form:

$$\rho'(\mathbf{x},t) = \frac{1}{2ic^2c_m^2 x}\int_{-\infty}^{\infty}\omega^2\tilde{p}(\mathbf{k},\omega)\exp\left[-i\omega\left(t-\frac{x}{c}\right)\right]d\omega\,, \tag{15}$$

in terms of the Fourier space-time transform of p' defined by

$$\tilde{p}(\mathbf{k},\omega) = \frac{1}{2\pi^2}\int d^2r\int_{-\infty}^{0}dz\int_{-\infty}^{\infty}dt \times p'(\mathbf{x},t)\sin\kappa z\,\exp(i\omega t - i\mathbf{K}\cdot\mathbf{r})\,. \tag{16}$$

In this relation, we have set

$$\mathbf{k} \equiv (k_1,k_2,k_3) \equiv (\mathbf{K},\kappa) \equiv (\omega/c)\mathbf{u} = k(\mathbf{x}/x)\,. \tag{17}$$

From (15) a standard procedure leads to the following expression for the intensity spectrum:

$$\hat{I}(\mathbf{x},\omega) = (\pi\omega^4/2T\rho c c_m^4 x^2)\,|\tilde{p}(\mathbf{k},\omega)|^2\,. \tag{18}$$

The quantity of interest is not so much the intensity spectrum for a single turbulent spot, but rather the ensemble average over many such processes, which we indicate by angle brackets,

$$\langle\hat{I}\rangle(\mathbf{x},\omega) = (\pi\omega^4/2T\rho c c_m^4 x^2)\langle\tilde{p}(\mathbf{k},\omega)\tilde{p}^*(\mathbf{k},\omega)\rangle\,. \tag{19}$$

This quantity can be expressed in a more useful form in terms of the pressure correlation function defined by

$$R(\boldsymbol{\xi},\tau;\mathbf{x},t) = \langle p'(\mathbf{x}+\boldsymbol{\xi},t+\tau)p'(\mathbf{x},t)\rangle\,. \tag{20}$$

We define its (partial) space-time Fourier transform by

$$\tilde{\tilde{R}}(\mathbf{K},\zeta,\omega;\mathbf{r},z,t) = \frac{1}{(2\pi)^{3/2}}\int\int d\xi\,d\eta\int d\tau\,\exp(i\omega\tau - i\mathbf{K}\cdot\boldsymbol{\xi}) \times\langle p'(\mathbf{y}+\boldsymbol{\xi},t+\tau)p'(\mathbf{y},t)\rangle\,, \tag{21}$$

where $\mathbf{y} = (\mathbf{r},z)$ and $\boldsymbol{\xi} = (\xi,\eta,\zeta)$. In terms of $\tilde{\tilde{R}}$ the intensity distribution (19) may be written

$$\langle\hat{I}\rangle(\mathbf{x},\omega) = \frac{\omega^4}{(2\pi)^{3/2}T\rho c c_m^4 x^2}\int_{-\infty}^{0}dz\,\sin\kappa z\int d^2r\int_{-\infty}^{\infty}dt \times\int_{-\infty}^{-z}d\zeta\,\sin\kappa(z+\zeta)\tilde{\tilde{R}}(\mathbf{K},\zeta,\omega;\mathbf{r},z,t)\,. \tag{22}$$

Although the integral over the depth z extends all the way to $-\infty$, the region where $\tilde{\tilde{R}}$ is appreciable only extends at most a few meters under the water surface and it therefore makes sense to introduce an apparent surface source spectral density $\hat{S}(\mathbf{r},\omega;\mathbf{x})$ defined in such a way that the contribution $d\langle\hat{I}\rangle$ to $\langle\hat{I}\rangle$ due to an element dA of the ocean surface placed at $\mathbf{r}$ is given by

$$d\langle \hat{I} \rangle = \hat{S}(\mathbf{r},\omega;\mathbf{x})\cos^2\theta \frac{dA}{x^2}, \tag{23}$$

where θ is the angle that the line going from $\mathbf{r}$, the location of dA, to $\mathbf{x}$ makes with the vertical (Fig. 1). The factor $\cos^2\theta$ is just the dipole directivity factor. On the basis of (22), we are led to the result

$$\hat{S}(\mathbf{r},\omega;\mathbf{x}) = \frac{\omega^4}{(2\pi)^{3/2} T\rho c c_m^4 \cos^2\theta} \int_{-\infty}^{\infty} dt \int_{-\infty}^{0} dz \sin\kappa z \times \int_{-\infty}^{-z} d\zeta \sin\kappa(z+\zeta) \tilde{\tilde{R}}(\mathbf{K},\zeta,\omega;\mathbf{r},z,t). \tag{24}$$

We call this quantity the apparent source density because it exhibits some dependence on the observation point $\mathbf{x}$. This dependence is however weak since, as already remarked, $|\kappa z| \ll 1$ in the region where $\tilde{\tilde{R}}$ is nonzero so that $\sin\kappa z \simeq \kappa z \simeq kx \cos\theta$ to the extent that the angle θ' between the vertical and the line joining the source point $(\mathbf{r},z)$ to $\mathbf{x}$ can be approximated by θ (see Fig. 1). Furthermore, as a function of ζ, $\tilde{\tilde{R}}$ vanishes when ζ exceeds the vertical spatial correlation length of the turbulence, which is much smaller then the depth of the turbulent patch so that $\sin\kappa \times (z+\zeta) \simeq \sin\kappa z$. (Strictly speaking this approximation fails in the immediate neighborhood of the surface where z is comparable to ζ; however, $\kappa\zeta$ is negligible there.) Finally, we also assume as a first approximation the turbulence correlation function to be isotropic. With all these approximations, we may drop the argument $\mathbf{x}$ in (24) to find

$$\hat{S}(\mathbf{r},\omega) = \frac{\omega^6}{(2\pi)^{3/2} T\rho c^3 c_m^4} \int_{-\infty}^{\infty} dt \int_{-\infty}^{0} dz\, z^2 \int_{-\infty}^{0} d\zeta \times \tilde{\tilde{R}}(\mathbf{K},\zeta,\omega;\mathbf{r},z,t). \tag{25}$$

This is the main result of this section. To illustrate the way it can be used, we now give a simple application with a rough numerical estimate. We shall conclude the section with a physical interpretation and a discussion.

B. Illustrative application

Equation (25) gives the average spectral distribution of the sound source consisting of a turbulent water patch containing bubbles. For its quantitative use a considerable amount of information on the surface turbulence caused by breaking waves or other processes is needed that is the object of current research, but not yet available. To elucidate the physical content of our result we shall therefore be content with a simple application based on admittedly crude approximations. It should be stressed that, at the present time, the spirit of these approximations is mostly illustrative and that future research may suggest to proceed in different directions. In any event, it will be seen that the assumptions to be introduced appear to lead to physically plausible results.

As a first step, we assume the possibility of factoring R in the form

$$R(\boldsymbol{\xi},\tau;\mathbf{x},t) = Q(\boldsymbol{\xi},\tau)\, W(\mathbf{x},t). \tag{26}$$

The physical picture underlying this assumption is the following. If the turbulence were steady and uniform the dimensionless function W would equal 1 for any $\mathbf{x}$ and t and Q would be the pressure correlation function appropriate for this steady, uniform case. It may be tentatively supposed that a local, slowly transient event could be approximately described by a function Q giving the structure of the turbulence multiplied by a function W giving the relative intensity and space-time location of this structure. This assumption is most likely very crude but it represents the simplest possible extension of the steady homogeneous case and is useful for purposes of estimation. It follows from these considerations that W is a dimensionless function of order 1 on some compact set of space-time and zero outside. Define the quantity

$$L^3(\mathbf{r},t) = \int_{-\infty}^{0} z^2 W(\mathbf{r},z,t)\,dz, \tag{27}$$

which, from the meaning of W, will be of the order of the cube of the vertical size of the acoustically active turbulent spot. It may be expected that the length L defined here grows, reaches a maximum size, and then decreases as the turbulent spot is formed and then disintegrates and therefore we write

$$\int_{-\infty}^{\infty} L^3(\mathbf{r},t)\,dt = L_0^3\Theta, \tag{28}$$

where L_0, Θ are average values of the vertical extent and lifetime of the spot, which we take to be independent of the position $\mathbf{r}$ on the surface. With these definitions and approximations, (25) may be written

$$\hat{S} = (L_0^3\Theta\omega^6/2\pi T\rho c^3 c_m^4)\tilde{Q}(\mathbf{K},\omega), \tag{29}$$

where

$$\tilde{Q}(\mathbf{K},\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{0} d\zeta \int\int d\xi\, d\eta \int_{-\infty}^{\infty} d\tau \times \exp(i\omega\tau - i\mathbf{K}\cdot\boldsymbol{\xi})\,Q(\boldsymbol{\xi},\tau). \tag{30}$$

In order to proceed further we shall estimate Q on the basis of results available from the theory of homogeneous turbulence. In this theory it can be shown that[22]

$$\langle p'(\mathbf{x},t)p'(\mathbf{x}',t)\rangle = \frac{\rho^2}{6r}\int_r^{\infty} y(y-r)^3 \frac{\partial^4\langle u_i u_j u_k'' u_l''\rangle}{\partial y_i \partial y_j \partial y_k \partial y_l}\,dy, \tag{31}$$

where $r = |\mathbf{x}-\mathbf{x}'|$ and $\mathbf{u}$ denotes the velocity at $\mathbf{y}$ while $\mathbf{u}''$ is the velocity at $\mathbf{x}'-\mathbf{x}+\mathbf{y}$. According to an approximation due to Batchelor,[22,23] the quadruple velocity correlation can be expressed in terms of double correlations. This approximation is not entirely satisfactory since it leads to a non-strictly positive expression for the turbulent energy spectrum. Nevertheless, we shall use it here in view of its relative simplicity and of the fact that the error introduced appears

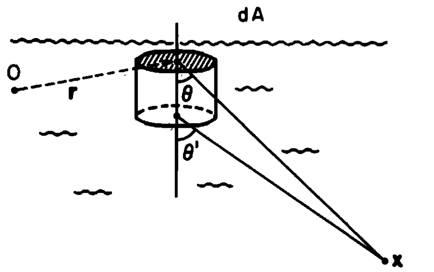

FIG. 1. Definition of the surface source density according to Eqs. (28) and (29).

to be not too large and should not exceed that arising from our other assumptions. With Batchelor's approximation (31) becomes[22]

$$\langle p'(\mathbf{x}+\boldsymbol{\xi},t)p'(\mathbf{x},t)\rangle = 2(\rho\,\overline{u'^2})^2 \int_{|\xi|}^{\infty} \left(\mu - \frac{|\xi|^2}{\mu}\right)\left(\frac{df}{d\mu}\right)^2 d\mu\,, \tag{32}$$

where

$$f(|\xi|) = \langle \mathbf{u}(\mathbf{x}+\boldsymbol{\xi},t)u(\mathbf{x},t)\rangle / \overline{u'^2}\,, \tag{33}$$

and $\overline{u'^2}$ denotes the mean-square fluctuating component of the turbulent velocity. An expression for this quantity is available for the case of low Reynolds number turbulence and is[22,24]

$$f = \exp(-|\xi|^2/2l^2)\,. \tag{34}$$

For the high Reynolds number case an empirical expression is[22]

$$f = \exp(-|\xi|/l)\,. \tag{35}$$

For the present purposes we can make an approximation based on the fact that $|\mathbf{K}|l \ll 1$; i.e., the turbulence correlation length l is small compared with the wavelength of sound. With this, we show in the Appendix that

$$\widetilde{Q} = C(\rho\,\overline{u'^2})^2 \int_{-\infty}^{\infty} l^3 e^{i\omega\tau}\, d\tau\,, \tag{36}$$

where C is a numerical constant with the values

$$C = (8\pi^{1/2})^{-1}, \quad C = (10\pi)^{-1},$$

for the cases of Eqs. (34) and (35), respectively. More generally, if we merely assume

$$f = f(|\xi|/l) \tag{37}$$

we have

$$C = \frac{2}{15\pi} \int_0^{\infty} u^4 \left(\frac{df}{du}\right)^2 du. \tag{38}$$

Since f must decay rather rapidly when its argument increases beyond 1, one may expect $C = O(10^{-1})$.

The generality of (36) is particularly useful here in view of the uncertainty with which the turbulence spectrum is known. This problem is compounded by the fact that the expressions (31) to (35) are strictly applicable only at equal times, i.e., for $\tau = 0$. We get around this difficulty by using (36) allowing l to depend on τ. This procedure implies only that the functional form (37) holds for any τ, which is a weaker assumption than requiring the validity of a specific expression such as (34) or (35). We shall also use $C = (10\pi)^{-1}$, which is appropriate for the large Reynolds number case and appears to be a reasonable estimate of the order of magnitude of C in general.

It may be expected that l has a maximum, l_0 say, for $\tau = 0$ and decreases for positive and negative values of τ. A functional form possessing these characteristics that we may use for purposes of illustration is

$$l = l_0 \exp[-(\tau/\tau_0)^2], \tag{39}$$

with l_0, τ_0 constants. With this one finds from (36)

$$\widetilde{Q} = [1/10(3\pi)^{1/2}](\rho\,\overline{u'^2})^2 l_0^3 \tau_0 \exp(-\tfrac{1}{12}\omega^2\tau_0^2). \tag{40}$$

As another example we may use

$$l = l_0/[1 + (\tau/\tau_0)^2], \tag{41}$$

to find

$$\widetilde{Q} = \tfrac{1}{80}(\rho\,\overline{u'^2})^2 l_0^3 \tau_0 (3 + 3\omega\tau_0 + \omega^2\tau_0^2) e^{-\omega\tau_0}. \tag{42}$$

With these results we have

$$\hat{S} = \frac{(\rho\,\overline{u'^2})^2 L_0^3 l_0^3 \Theta \tau_0 \omega^6}{20\pi(3\pi)^{1/2} T\rho c^3 c_m^4} \exp\left(-\frac{1}{12}\omega^2\tau_0^2\right), \tag{43}$$

for l given by (39), while

$$\hat{S} = \frac{(\rho\,\overline{u'^2})^2 L_0^3 l_0^3 \Theta \tau_0 \omega^6}{160\pi T\rho c^3 c_m^4} e^{-\omega\tau_0}(3 + 3\omega\tau_0 + \omega^2\tau_0^2), \tag{44}$$

for l given by (41). If we write

$$\hat{S} = \left[(\rho\,\overline{u'^2})^2 L_0^3 l_0^3 \Theta / T\rho c^3 c_m^4 \tau_0^5\right] F_j(\omega\tau_0), \quad j = 1,2, \tag{45}$$

we find

$$F_1(Z) = [Z^6/20\pi(3\pi)^{1/2}]\exp(-Z^2/12), \tag{46}$$

$$F_2(Z) = (Z^6/160\pi)(3 + 3Z + Z^2)e^{-Z}, \tag{47}$$

for the two cases, respectively. In Fig. 2, we show graphs of these two functions. Both have maxima for ω equal a few times τ_0^{-1}, after which they rapidly decrease.

C. Estimate

We now consider the order of magnitude of the source level produced by the proposed mechanism and show that it is indeed comparable to the observed one. For this purpose, we use Eq. (45) for $\hat{S}$ with the function $F = 10$, which is the right order of magnitude around the maximum as shown in Fig. 2. For the vertical size of the turbulent region we take $L_0 \simeq 1$ m. We note that this is substantially less than the depth reached by bubbles even in moderate winds,[9] and is selected here as a plausible size of the region where the turbulence induced by breaking waves is intense. For the lifetime of the turbulent event (i.e., the breaker), we take

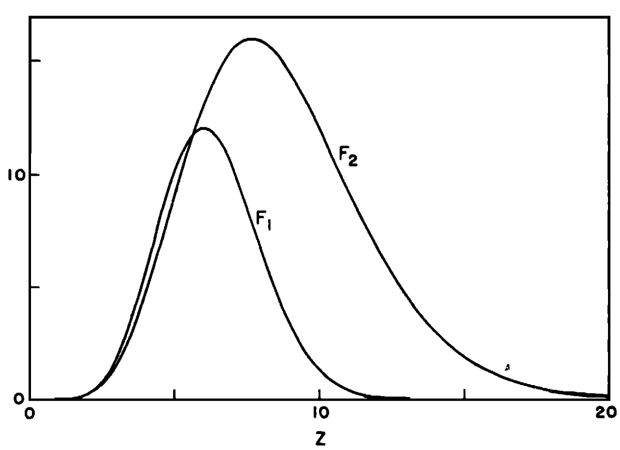

FIG. 2. The functions $F_1(Z)$ and $F_2(Z)$ defined in Eqs. (51) and (52). These are the normalized spectral shapes of the amplified turbulence noise according to the two functional forms (44) and (46) for the dependence of the turbulent correlation length on the delay time τ.

$\Theta = 100$ s, and for the averaging time $T = 1$ s. A reasonable estimate for the turbulent velocity fluctuations may be $(\overline{u'^2})^{1/2} = 0.15$ m/s, for the spatial correlation length $l_0 = 0.1$ m, and for the correlation time $\tau_0 = (10\ \text{Hz})^{-1} = 0.1$ s. Although the air volume fraction β at the top of a breaking wave can reach 20%,[25] the biggest bubbles do not penetrate deeply and are very quickly removed by buoyancy and we shall assume $\beta = 5\%$ as an indicative value of the average volume fraction in the active turbulent spot. Finally, we take $\rho = 10^3$ kg/m^3 and $c = 1500$ m/s. Our choice $F \sim 10$ implies that our estimate is appropriate for the frequency region around a few times τ_0, say 50 Hz. With the previous numerical values we find from (45) $S \simeq 3.39 \times 10^{-12}$ W s/m^2 *re*:1 μPa^2/Hz, which corresponds to

$$\hat{S} \simeq 67.2 \text{ dB}. \tag{48}$$

Experimental values for surface source levels are not known precisely and the same data show a large variability, up to 5–10 dB, even for similar conditions of wind speed and wave height.[26] Wilson[27] has published estimates based on various data that we shall use here fully aware of their very tentative nature. These estimates for the source level at 50-Hz range from 55.4 dB for a wind speed of 7.5 m/s to 72.0 dB for a wind speed of 20 m/s. Before comparing these numbers with (48), it must be stressed that our result refers to a single turbulent event, while Wilson's are average values. We can convert our result to his by multiplying it by the percentage w of the ocean surface in which turbulent events take place at any given time. This can be estimated from Monahan's data on the oceanic whitecap coverage[28] which, according to the empirical formula $w = 3.84 \times 10^{-4} U^{3.41}$ (U in m/s), would range from 0.37% to 10% for the wind speed values indicated above. The extent of the region of high turbulence due to a whitecap is not confined however to the white water patch, since the breaking process leaves behind a considerable amount of turbulence in which a large number of smaller bubbles remains entrained. It therefore appears reasonable to increase the previous estimate of the fraction of water surface containing turbulent patches by a factor 5–10. Even with this correction our estimate (48) is in line with Wilson's. This example is clearly very crude and our sole purpose in presenting it was to show that, with reasonable values of the quantities involved, noise levels consistent with observation may be generated.

In the absence of bubbles, the result (45) for the source level should be multiplied by $(c_m/c)^4$ as already mentioned after Eq. (13). With the numerical values used for this example $c_m/c \simeq 0.031$, which leads to a reduction by about 60 dB of the estimate (48). This conclusion confirms the very inefficient character of low-velocity turbulence as an acoustic source. To compensate for this reduction, the thickness of the turbulent layer should be increased by two orders of magnitude, even with the unrealistic assumption of a uniformly high-turbulence level throughout it. These considerations imply that, as soon as a few bubbles are entrained, their effect on ambient noise is quickly dominant over that due to turbulence in water without bubbles.

D. Physical interpretation

The result (45) previously obtained for the surface source density can be interpreted in physical terms as follows. If, upon expansion or contraction of the bubbles, the gas volume per unit volume changes by $\Delta\beta$, the associated variation of liquid mass per unit volume is $-\rho\Delta\beta$. The apparent rate of liquid mass creation or destruction per unit volume is therefore

$$q = \rho \frac{\partial \beta}{\partial t}, \tag{49}$$

and this quantity may then be considered as an effective monopole source intensity. Due to the smallness of the gas density and to the near incompressibility of the liquid, $\rho\Delta\beta$ also equals very nearly the change in the mixture density (3) so that from (49) the form (14) previously given is recovered. As is well known from the theory of monopole radiation, the pressure field in the liquid responds to $\partial q/\partial t$. Furthermore, the free surface introduces an image effect leading to an effective dipole emission. Therefore, the monopole radiation field should be multiplied by a factor $kL \cos\theta$, where L is of the order of the vertical size of the source. With these considerations we may write directly the following relation for the radiated pressure field:

$$p' \simeq \frac{kL \cos\theta}{c_m^2 x} \frac{\partial^2}{\partial t^2} \int p'\, dV. \tag{50}$$

The instantaneous acoustic intensity is $p'^2/\rho c$. If the acoustic emissions have a typical duration Θ and are observed for a time T, an intensity of the order Θ/T is recorded on the average. (This factor is greater than 1 when $\Theta > T$ because then more than one acoustic emission is probable during the observation time.) The observed intensity is thus, upon ensemble averaging,

$$\langle I \rangle \simeq \frac{\Theta}{T\rho c} \left(\frac{kL}{c_m^2 x} \cos\theta \right)^2 \times \left\langle \left(\frac{\partial^2}{\partial t^2} \int p'(\mathbf{y},t) d^3y \right) \left(\frac{\partial^2}{\partial t^2} \int p'(\mathbf{y}',t) d^3y' \right) \right\rangle. \tag{51}$$

If the spectrum of the turbulence has a cutoff at the maximum frequency $\omega_{\max}$, the spectral density $\langle \hat{I} \rangle$ will be related to $\langle I \rangle$ by $\langle \hat{I} \rangle \simeq \langle I \rangle / \omega_{\max}$, approximately. Furthermore, in the frequency domain, $\partial/\partial t \to i\omega$. Upon setting $k = \omega/c$ Eq. (51) leads therefore to

$$\langle \hat{I} \rangle \simeq \Theta \omega^6 L^2 \cos^2\theta / T\rho c^3 c_m^4 x^2 \omega_{\max} \times \left\langle \int d^3y \int d^3\xi\, p'(\mathbf{y},t) p'(\mathbf{y}+\boldsymbol{\xi},t) \right\rangle. \tag{52}$$

As a function of $\boldsymbol{\xi}$ the integral vanishes at distances greater than a typical length of order l. For distances less than l, it has a magnitude of the order $(\rho \overline{u'^2})^2$. As a function of $\mathbf{y}$, it vanishes outside a region of horizontal extent S and vertical dimension L. We thus have

$$\langle \hat{I} \rangle \sim \frac{\Theta \omega^6 L^3 l^3 (\rho \overline{u'^2})^2}{T\rho c^3 c_m^4 \omega_{\max}} \frac{S \cos^2\theta}{x^2}. \tag{53}$$

Since, as already remarked, ω_{max} is of the order of τ_0^{-1}, this result coincides in order of magnitude with (45).

In addition to providing some insight into the physics underlying the previous result, this argument is useful in showing that the numerous assumptions made in its derivation were not unreasonable. Therefore, we may have some confidence that future refinements will not totally subvert our main conclusions.

E. Discussion

The theory developed above ascribes the wind-dependent part of the low-frequency ambient noise spectrum to the passive response of bubbles excited by turbulent pressure fluctuations occurring in the water in their vicinity. In considering the frequency range in which this mechanism may be significant, it should be kept in mind that, as already noted, the bubbles are entrained by the turbulent structures. This results in a considerable reduction of the maximum frequency at which the present mechanism may be expected to be effective since the spectrum of the excitation will be poor in high-frequency components.

Our result (45) for the source spectral density exhibits a very strong dependence on many of the quantities appearing in it. The turbulence correlation time enters raised to the fifth power, the turbulent velocity and the speed of sound in the bubbly liquid raised to the fourth power, and the depth of the bubbly layer and turbulence correlation distance raised to the third power. The dependence of our result on wind speed and other environmental parameters is through these quantities that are at present so poorly known that they can only be very roughly estimated. Considerable more research is needed to express these dependencies explicitly and we hope that our work may be useful to guide this research, in addition to predicting actual spectra when the missing information becomes available.

In view of all the underlying uncertainties, the numerical estimate that we have given in (48), although plausible, must be taken as an indicative value rather than a firm quantitative prediction. As is clear from (45), the frequency dependence of the noise generated by the proposed mechanism is contained in the function $F(\omega\tau_0)$ shown in Fig. 2. A measurement of the noise spectrum produced by breaking events can therefore either disprove or give at least a broad support to our results, leading also to an experimental value for the frequency-independent part of (45).

Another important question is the relationship of this noise generation mode with the presence of breaking waves. Bubbles are present near the ocean surface also in the absence of breaking waves[7,9] and turbulent spots can therefore radiate as postulated. However, breaking waves result in a far greater number of bubbles and turbulent intensity. The noise output is therefore expected to increase substantially in the presence of breaking waves.

II. NOISE IN THE kHz RANGE: SINGLE-BUBBLE OSCILLATIONS

The breaking of waves in the spilling mode gives rise to a large number of bubbles and, as discussed by Longuet-Higgins and Turner,[25] the gas volume fraction near the surface can exceed 20%. Not much is known about the mechanism of formation of these bubbles. Some recent results obtained by high-speed cinematography seem to imply that air is entrained along the line where the water mass rolling down the face of the wave meets the relatively smooth liquid surface ahead of it.[29] This formation process is therefore quite different from that which occurs when a drop hits a liquid surface, where the bubble is generated by the pinching off of the crater produced on the surface.[30,31] If the two processes are indeed so widely different, the procedure of Ref. 4 in which noise produced by drop-impact bubbles is used to estimate the effect of breaking-wave bubbles may be questionable.

Whatever the mechanism of production, it is to be expected that the bubbles have some initial mechanical energy since, at the instant at which a closed cavity is formed, the liquid particles on its surface will have some nonzero velocity. Freshly entrained bubbles are also occasionally seen to split and again one expects the resulting fragments not to be initially spherical and in equilibrium.

This residual initial energy can only be disposed of by executing damped free oscillations in the course of which acoustic waves are emitted at the natural frequency of the bubble which is approximately given by[32,33]

$$\omega_0 = (1/R)(3\kappa p_0/\rho)^{1/2}. \tag{54}$$

Here R is the equilibrium radius of the bubble, κ the polytropic index that ranges between 1 and the ratio of the specific heats (1.4 for air), and surface tension effects have been disregarded for simplicity. Equation (54) is rather accurately valid also when the bubble oscillates nonspherically, provided that this motion is accompanied by volume changes.[34]

A. Theory

To set up a theoretical description of noise emission by freely oscillating bubbles we start again from (10) and note that the developments carried out in the previous application of this result essentially amount to the approximation

$$T_{ij} = -c^2\rho'_m\delta_{ij}. \tag{55}$$

Now, however, we cannot use Eq. (11) to express ρ'_m in terms of p' because, in the frequency range of present concern, the bubbly mixture is strongly dispersive. Rather, we calculate ρ'_m from the definition (3) of ρ_m neglecting the gas density and the liquid compressibility to find

$$\rho'_m \simeq -\rho\beta', \tag{56}$$

where β' is the fluctuation in the gas volume fraction. Upon substitution of (55) and (56), Eq. (10) becomes

$$p'(\mathbf{x},t) = \frac{\rho}{4\pi c^2 x}\frac{\partial^2}{\partial t^2}\int\left[\beta'\left(\mathbf{y},t-\frac{x-\mathbf{n}\cdot\mathbf{y}}{c}\right) - \beta'\left(\mathbf{y},t-\frac{x-\mathbf{n}\cdot\mathbf{y}_r}{c}\right)\right]d^3y. \tag{57}$$

This relation can equally well be obtained using the expression (49) for the monopole source strength in (13) or, in a somewhat different way, following the steps set forth in Ref. 7.

In (57), just as in (10), the zero-pressure boundary condition at the ocean's surface has been satisfied by the image method. As before this procedure ignores the wavy nature of the ocean surface. This approximation is justified because the dimensions of the region where the bubbles are generated (the leading edge of the mass of water rolling down the face of the wave) is expected to be small compared with the characteristic length of the waves. If we now take the position of the reflected source point $\mathbf{y}_r$ to be as in Eq. (8), we introduce a more serious error since this implies taking the ocean surface to be plane. At a frequency of a few kHz the wavelength of sound in water is of the order of 1 m, which is comparable to the size of the breaking wave. Ignoring the presence of the surface wave has the effect of neglecting the scattering phenomena associated with the propagation of the sound from the breaking wave, where it is generated, to the ocean depth. The net result will be an underestimation of the acoustic intensity at points where more than one ray may contribute, and an overestimation in "shadow" zones which see the acoustic source at a nearly grazing angle. These errors will probably be mitigated to some extent by the time averaging implicit in all data acquisition procedures employed so far. However, if comparison with experiments carried out for single breaking events were attempted, one could expect to find some differences caused by this approximation. It is difficult to assess the magnitude of this error, but it is hard to believe that it would affect the order of magnitude of the result. An *a posteriori* way to verify this expectation would be to use the source intensity to be derived with a propagation model containing scattering losses. For the present, we are obliged to proceed in this way by the great complexity that accounting for the surface waves would introduce in the problem. Therefore, retracing the steps leading from (13) to the intensity (18) we find

$$\hat{I}(\mathbf{x},\omega) = (\pi\rho\omega^4/2Tcx^2)|\tilde{\beta}(\mathbf{k},\omega)|^2, \quad (58)$$

where, as in (16),

$$\tilde{\beta}(\mathbf{k},\omega) = \frac{1}{2\pi^2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{0} dz \int d^2r \sin\kappa z \times \exp(i\omega t - i\mathbf{K}\cdot\mathbf{r})\beta(\mathbf{x},t). \quad (59)$$

The analysis can be continued in the same way as before by introducing a correlation function for β and an average source intensity as in (25). In this case, however, a slightly more mechanistic approach is possible, which we now describe.

B. Simple model and estimate

According to the mechanism envisaged here the bubbles are formed in the violently mixing region at the front of the mass of water spilling down the face of the wave and execute oscillations that dissipate their initial energy. Bubbles oscillating in the kHz range have typical Q factors of the order of 100. The entire emission from these bubbles is therefore expected to last a fraction of a second. We therefore neglect the effect of buoyancy.

Let us suppose that, in the time interval $d\tau$ centered around $\tau(>0)$ there are $n(\mathbf{x},\tau)d^3x\, d\tau$ bubbles created in the volume element d^3x around $\mathbf{x}$. Suppose also, again for simplicity, that these bubbles start out with the same initial volume v_0 and volumetric rate of change $\dot{v}_0$. At time $t>\tau$, the volume of a bubble created at time τ is then $v(t-\tau)$ while, for $t<\tau$, $v=0$ since the bubble does not yet exist.

With these ideas, we may express the gas volume fraction surrounding the point $\mathbf{x}$ at time t in the form

$$\beta(\mathbf{x},t) = \int_0^t n(\mathbf{x},t)v(t-\tau)d\tau. \quad (60)$$

It is readily shown that the time Fourier transform of this relation is

$$\hat{\beta}(\mathbf{x},\omega) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} \beta(\mathbf{x},t)e^{i\omega t}\,dt = 2\pi\hat{n}(\mathbf{x},\omega)\hat{v}(\omega), \quad (61)$$

if the condition $\beta=0$ for $t<0$ is used. The spatial part of the transform (59) leads then to

$$\tilde{\beta}(\mathbf{k},\omega) = \frac{\hat{v}(\omega)}{\pi}\int d^2r\int_{-\infty}^{0} dz \times \sin\kappa z\ \hat{n}(\mathbf{x},\omega)\exp(-i\mathbf{K}\cdot\mathbf{r}). \quad (62)$$

For the bubble generation rate $n(\mathbf{x},t)$ we take a very simple "box" model illustrated in Fig. 3. Bubbles are generated at a constant rate N_0 in a region at the front of the breaking wave. This region has length L in the direction parallel to the wave front, depth δ, and width l normal to the wave front. The region also moves forward with the speed U of the wave but, since the associated frequency, U/l, is so much smaller than the kHz frequencies of present concern, we can ignore this motion. Hence, we write

$$n(\mathbf{x},t) = N_0\begin{cases} 0\leqslant x\leqslant l, & -L/2\leqslant y\leqslant L/2 \\ -\delta\leqslant z\leqslant 0, & 0\leqslant t\leqslant\Theta, \end{cases} \quad (63)$$

while $n=0$ otherwise. Here, Θ is the lifetime of the wave. With this specification, the calculation indicated in (61) is readily carried out with the result

$$\tilde{\beta} = \frac{N_0\hat{v}(\omega)}{(2\pi)^{1/2}}\frac{\sin(k_2L/2)}{k_2/2}\frac{1-\exp(-ilk_1)}{ik_1} \times\frac{\cos\kappa\delta-1}{\kappa}\frac{\exp(i\omega\Theta)-1}{i\omega}, \quad (64)$$

where, as before, $\mathbf{K}=(k_1,k_2)$. Using the fact that $lk\ll1$, $\kappa\delta\ll1$, and taking the observation point $\mathbf{x}$ on the plane of symmetry (x,z) so that $k_2=0$ [cf. (17)], we find

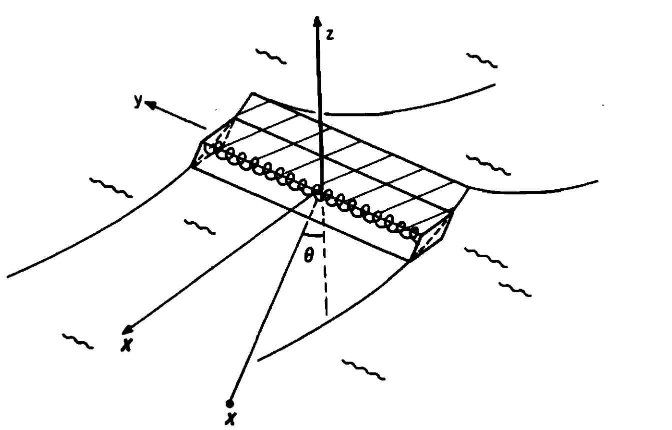

FIG. 3. Conceptual model for air entrainment at the "toe" of a wave breaking in the spilling mode.

$$\tilde{\beta} \simeq \frac{N_0 \hat{v}(\omega)}{2(2\pi)^{1/2}} \frac{lL\delta^2}{c} \cos\theta\,[1-\exp(i\omega\Theta)], \qquad (65)$$

where we have used the fact that $\kappa/\omega = \cos\theta/c$. Here, θ is the angle that the line connecting the observation point **x** and the origin makes with the vertical (Fig. 3).

To calculate $\hat{v}(\omega)$, we express the instantaneous bubble radius in the form $R[1+X(t)]$ and, for small amplitude oscillations, we write

$$v'(t) = \tfrac{4}{3}\pi R^3[(1+X)^3 - 1] \simeq 4\pi R^3 X. \qquad (66)$$

Furthermore, for damped oscillations,

$$X(t) = X_0 e^{-bt}\cos(\omega_0 t + \phi), \quad t>0, \qquad (67)$$

where X_0, ϕ are the initial amplitude and phase and b is the damping constant at the natural frequency ω_0 (Ref. 33), while $X(t)=0$ for $t<0$. We then find

$$\hat{v}(\omega) = \frac{4\pi}{(2\pi)^{1/2}} R^3 X_0 \frac{(i\omega - b)\cos\phi + \omega_0 \sin\phi}{(i\omega - b)^2 + \omega_0^2}, \qquad (68)$$

from which

$$|\hat{v}|^2 = 4\pi R^6 X_0^2 [\omega^2 + b^2 + \omega_0^2 + (b^2 - \omega_0^2 + \omega^2)\cos 2\phi - 2\omega_0 b \sin 2\phi]/[(\omega_0^2 - \omega^2 + b^2) + 4\omega^2 b^2]\,. \qquad (69)$$

This expression shows that, in general, $|\hat{v}|^2 \sim \omega^{-2}$ for large ω. This weak rate of decrease arises because $X(t)$, as given by (67), is discontinuous at $t=0$. In reality, even if (67) is applicable for $t>0$, the amplitude $X(t)$ would gradually, if rapidly, build up from the value 0 to X_0 over a nonzero time interval $-\bar{t} < t \leqslant 0$, and the asymptotic behavior of $|\hat{v}|^2$ for large ω would be dictated by the detailed nature of this buildup. In practice, the exact asymptotic behavior is irrelevant because the corresponding frequencies are so large that other mechanisms intervene to account for the corresponding ambient noise. However, since $|\hat{v}|^2$ must be multiplied by ω^4 in the intensity expression (58), to avoid an unphysical increase of the intensity with frequency, we try to obtain as fast as possible a decrease of $|\hat{v}|^2$ for large ω. This we do by demanding continuity of (67) at $t=0$, which requires $\phi = \pi/2$ so that

$$|\hat{v}|^2 = 8\pi R^6 X_0^2 \omega_0^2/[(\omega_0^2 + b^2 - \omega^2)^2 + 4\omega^2 b^2]\,, \qquad (70)$$

and $|\hat{v}|^2 \sim \omega^{-4}$ for large ω. If more detailed information on the buildup of the amplitude X from 0 to X_0 were available, one would expect a behavior similar to that described by this equation up to a frequency of the order of $\bar{t}^{-1}$, followed by an even more rapid decline.

Upon use of the results derived for n, $\hat{v}$, and β in the intensity equation (58), and division by the dipole factor $Ll\cos^2\theta/x^2$, we find the following expression for the source density:

$$\hat{S} = \frac{9}{8\pi^3}\frac{\rho}{Tc^3}\beta_0^2 X_0^2 Ll\delta^4 \times \frac{\omega^4\omega_0^2}{(\omega_0^2 - \omega^2 + b^2)^2 + 4\omega^2 b^2}\frac{\sin^2(\tfrac{1}{2}\omega\Theta)}{\Theta^2}, \qquad (71)$$

where

$$\beta_0 = \tfrac{4}{3}\pi R^3 N_0 \Theta, \qquad (72)$$

is the average air volume entrained in the wave per unit volume of the "entraining region" defined in (63). It will be seen in the physical interpretation of these results given below that the term $\sin^2(\tfrac{1}{2}\omega\Theta)$ appears as a consequence of the assumption that all bubbles are created with the same initial conditions. This is clearly rather artificial, and we therefore replace this term by its average, 1/2, which does not affect the order of magnitude of our results. If we furthermore let

$$V = Ll\delta, \qquad (73)$$

be the volume of the entraining region, we finally obtain

$$\hat{S} = \frac{9}{16\pi^3}\frac{\rho\beta_0^2 X_0^2 V\delta^3}{Tc^3\Theta^2}\frac{\omega^4\omega_0^2}{(\omega_0^2 - \omega^2 + b^2)^2 + 4\omega^2 b^2}. \qquad (74)$$

Since $b \ll \omega_0$,[33] as a function of ω this expression exhibits a maximum very close to ω_0, where it approximately takes the value

$$\hat{S}(\omega_0) = \frac{9}{64\pi^3}\frac{\rho\beta_0^2 X_0^2 V\delta^3\omega_0^4}{Tc^3 b^2\Theta^2}, \qquad (75)$$

while it asymptotes to the much smaller value $(4b^2/\omega_0^2)\hat{S}(\omega_0)$ for $\omega \to \infty$. We show in Fig. 4 a graph of the function

$$G(\omega) = \frac{\omega^4}{\omega_0^4}\left[\left(1 - \frac{\omega^2}{\omega_0^2} + \frac{b^2}{\omega_0^2}\right)^2 + 4\frac{\omega^2}{\omega_0^2}\frac{b^2}{\omega_0^2}\right]^{-1}, \qquad (76)$$

for bubbles having a radius of 5, 1, and 0.1 mm. The corresponding values of $\omega_0/2\pi$ are 0.6525, 3.238, and 31.52 kHz and those of b are 0.02528, 0.2708, and 7.224 kHz. The maximum at ω_0 is sharp because of the smallness of b with respect to ω_0. However, this feature is only due to the use of bubbles of a single size and the maximum could be considerably broadened (and lowered) by taking a spectrum of bubble sizes R, allowing β_0 and X_0 to depend on R, and averaging (71) over R. Here we are only interested in orders of magnitude and (75) is sufficient for this purpose.

Reasonable values for the dimensions of the "entraining box" are $L = 2$ m, $l = 0.1$ m, $\delta = 0.1$ m. For β_0, on the basis of Ref. 25, we take the conservative estimate 10%. (This estimate is larger than the one used in the previous section because that model referred to a much thicker layer of fluid.) As before, we take $T = 1$ s, $c = 1500$ m/s, $\rho = 10^3$ kg/m^3, and we also assume an initial oscillation amplitude $X_0 = 0.1$. Using the values of ω_0 and b previously quoted we find, for $R = 5$ mm and $R = 1$ mm, $\hat{S}(\omega_0) = 4.75\times 10^{-10}$ W s/m^2 and 2.51×10^{-9} W s/m^2, i.e.,

$$\hat{S} = 88.7 \text{ dB}, \quad \hat{S} = 95.9 \text{ dB}.$$

The estimates of Ref. 27 for the source intensity at a frequency of 500 Hz range between 56.2 dB at a wind speed of 7.5 m/s and 72.3 dB for 30 m/s. At 1000 Hz, the corresponding values are 56.2 and 70.6 dB. Our estimates are thus seen to be 20–30 dB higher. However, substantial reductions can be expected due to the averaging over bubble radii already mentioned. Furthermore, the result (75), which applies to the source intensity of an "active" (i.e., entraining) surface patch, must be multiplied by the percentage of the ocean surface which is in such an "active" condition at any given time. The empirical correlation previously quoted gives 0.37% at 7.5 m/s and 42% at 30 m/s. In view of these considerations, we are therefore led to believe that the mechanism considered can give rise to emissions of an intensity adequate to match the data.

C. Physical interpretation

Inserting back into (75) the dipole factor $Ll\cos^2\theta/x^2$, the definition of β_0 (72), and the factor $\sin^2\omega\Theta/2$, we find an expression for the intensity which we rewrite in the following way to facilitate its interpretation:

$$\hat{I}(\omega_0) = \frac{\rho N_0^2 L^2 l^2 \delta^4 R^6 X_0^2 \Theta^2 \omega_0^6}{8\pi T c^3 b^2}\left(\frac{\sin\frac{1}{2}\omega_0\Theta}{\frac{1}{2}\omega_0\Theta}\right)^2 \frac{\cos^2\theta}{x^2}. \quad (77)$$

This result can be justified in simple terms by the use of some basic physical considerations.

We begin by noting that, according to the previous model, bubbles created at time Θ, when the breaking wave is about to die out, radiate with a phase difference $\omega_0\Theta$ from bubbles created near $t = 0$. This phase difference builds up linearly with time as the spilling front advances down the slope of the wave. The situation is very similar to that encountered in the Fraunhofer diffraction from a single slit. In that case, the total intensity radiated from the slit must be multiplied by the factor $(\sin\eta/\eta)^2$ if viewed from a direction such that the phase difference between wavelets emanating from the two sides of the slit is 2η.[35] This remark accounts for the factor $(\sin\frac{1}{2}\omega_0\Theta/\frac{1}{2}\omega_0\Theta)^2$ in Eq. (77). As before, the effect of the image sources can be accounted for by the dipole factor $k\delta\cos\theta$, where δ is the depth of the active region defined above. According to the basic laws of monopole radiation

$$p' \sim \frac{1}{c^2 x}\frac{\partial q}{\partial t}, \quad (78)$$

per unit source volume. Using the relation (49) for q, we then have

$$p' \sim \frac{\rho}{c^2 x}\frac{\partial^2}{\partial t^2}\int \beta\, d^3y.$$

In the approximation of Eq. (77), the time derivative is equivalent to multiplication by ω_0 and

$$\int \beta'\, d^3y \sim (N_0\Theta)(Ll\delta)(R^3X_0),$$

where the parentheses have been introduced to identify entities having a clear physical meaning, namely the total number of bubbles per unit volume generated in the active "box," $N_0\Theta$, the volume of the "box," $Ll\delta$, and the volume change of each bubble, R^3X_0. Collecting the separate parts we find

$$p' \sim (\rho/c^2x)\omega_0^2(N_0\Theta)(Ll\delta)(R^3X_0)k\delta\cos\theta \times(\sin\tfrac{1}{2}\omega_0\Theta/\tfrac{1}{2}\omega_0\Theta). \quad (79)$$

An oscillation with amplitude p' at ω_0 damped at the rate b has, in the neighborhood of ω_0, a spectral density of magnitude

$$\hat{p} \sim p'/b, \quad (80)$$

while $\hat{p} \sim 0$ away from ω_0. Upon substitution of (79) and (80) into the expression for the intensity,

$$\hat{I} = (c^3/T\rho)|\hat{p}|^2, \quad (81)$$

the previous result (77) is recovered up to some numerical factors.

III. COLLECTIVE BUBBLE OSCILLATION

Ambient noise data exhibit a broad maximum at a frequency around 500 Hz.[1,3] Bubbles having a natural frequency in this range have diameters of the order of 1 cm, and it seems unlikely that such large bubbles could be generated in great numbers in practically any condition as is implied by the data. However, we believe that acoustic emissions from the collective oscillations of bubble clouds could help explain the observations.

As with any system of coupled oscillators, a cloud of bubbles is capable of pulsating according to collective modes.[36,37] The frequency of the lowest ones of these modes is much smaller than that of the individual bubbles, since most bubbles pulsate in phase and therefore the inertia of the liquid has a large effect. Exact results are available for the case of two equal bubbles that, when touching, have a frequency $(\log 2)^{1/2} \simeq 0.83$ smaller than that of either component of the pair.[36,38]

In the case of N bubbles, a simple estimate of the effect can be obtained as follows. Consider a bubble cloud having linear dimensions of order L. The lowest natural frequency of oscillation of the system, $f_{\min}$, must be of the order c_m/L, where c_m is the speed of sound in the bubbly mixture given in (12). We can therefore write, neglecting a term of order β^2,

$$f_{\min} \simeq (1/L)(p_0/\rho\beta)^{1/2}. \quad (82)$$

If the cloud, idealized as a sphere, contains N bubbles of equal radius R, we have

$$\beta = (R/L)^3 N, \quad (83)$$

since β is the fraction of volume occupied by the gas. It may be expected that a similar relation would remain valid for other geometrical shapes of the cloud up to a numerical constant of order one. For slow oscillations, which are essential-

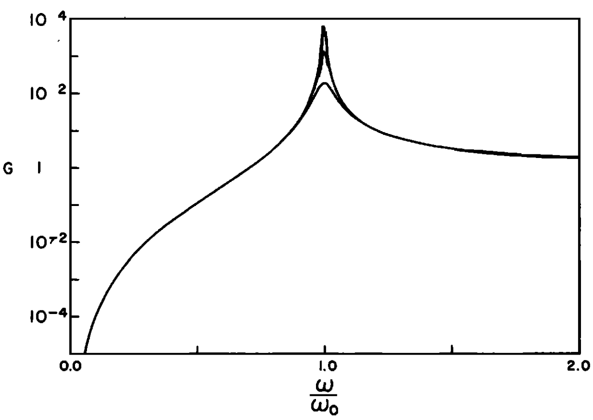

FIG. 4. The function $G(\omega)$ defined in Eq. (76) for air bubbles in water at 1 atm with an equilibrium radius R of 5 mm (top line), 1 mm (middle line), and 0.1 mm (bottom line).

ly isothermal, we can use (54) with $\kappa = 1$ to estimate the natural frequency f_0 of a single bubble. Combining these results and eliminating R/L by use of (83) we find

$$f_{\min}/f_0 \sim 1/\beta^{1/6} N^{1/3} . \qquad (84)$$

The void fraction appears raised to such a small power that this factor can be taken to be 1 (e.g., $\beta^{1/6} \simeq 0.7$ for $\beta = 0.1$). This relation therefore essentially demonstrates a decrease in frequency proportional to $N^{-1/3}$ that, although slow, can be sufficient to give frequencies in the range of several hundreds of Hz in realistic conditions. For example, a 2.5-mm-radius bubble has a natural frequency $f_0 = 1.302$ kHz approximately, and $N = 14$ is enough to result in $f_{\min} \simeq 500$ Hz. Incidentally, it may be noted that, for $N = 2$, $N^{-1/3} \simeq 0.79$, which is very close to the exact result quoted above.

For these low frequencies to appear, the bubbles in the cloud must be excited coherently. (Alternatively, one may say that, for the present purposes, the cloud is defined by the number of bubbles that are coherently excited.) That such a relatively long-range coherent excitation mechanism may exist is not unlikely in the presence of a breaking wave, where bubbles are generated by large (10–20 cm, say) masses of water toppling down the face of the wave.

The actual occurrence of this process in nature remains of course to be proven, but it certainly does not appear impossible and would furnish an attractive candidate to explain this intermediate-frequency part of the spectrum. Some indications that support this view have recently been reported.[39] We may therefore pursue this concept a little further trying to estimate the resulting noise intensities.

An approximate theoretical description can be based on the relations of the previous section many of which remain valid if they are referred to the normal modes of oscillation rather than to the individual bubbles. Indeed, essentially by definition, the amplitude of each mode satisfies a damped oscillator equation. In this approach X denotes the amplitude of a normal mode, ω_0 its frequency, and b its damping constant. Thus we may use Eq. (74) which, at $\omega = \omega_0$, becomes

$$\widehat{S}(\omega_0) = \frac{9}{16\pi^3} \frac{\rho\beta_0^2 X_0^2 V\delta^3}{Tc^3\Theta^2} \frac{\omega_0^6}{b^2(b^2 + 4\omega_0^2)} . \qquad (85)$$

It is easy to show, and it is intuitively obvious, that the damping constant is approximately equal to that of each constituent bubble *at the frequency of the mode*. The often-neglected fact that the damping constant exhibits a marked frequency dependence[32,33] is important here. For example, for bubbles having a radius $R = 2.5$ mm, the value of b at resonance is $b = 70.5$ Hz while, at 500 Hz, $b = 282$ Hz. Therefore, since now b is not small compared with ω_0, an extra term must be retained in the denominator of (85). Using $\omega_0/2\pi = 500$ Hz, $b = 300$ Hz, and the same values as in the previous section for the other quantities, we obtain from (85) the estimate $\widehat{S} = 1.16 \times 10^{-12}$ W s/m², i.e.,

$$\widehat{S} = 62.6 \text{ dB} .$$

The values quoted by Wilson for the source level at this frequency are 56.2 dB for 7.5-m/s winds, and 66.5 dB for 15 m/s. It may be concluded that the effect considered here can be significant, even after applying corrections for surface coverage. Note that the further correction arising from the averaging over radii mentioned in the previous section is not expected to have a large effect in this case since $b \sim \omega_0$.

If the radius decreases, f_0 increases and the damping constant at a fixed frequency (e.g., $\omega_0/2\pi = 500$) increases also towards the asymptotic value[33]

$$b = [(\gamma - 1)/10\gamma](p_0/\rho D_G) , \qquad (86)$$

where γ is the ratio of specific heats of the gas, D_G is its thermal diffusivity, and surface tension, viscous, and acoustic effects have again been neglected for simplicity. For air at 1 atm, this formula gives $b = 137.8$ kHz. With this value of b, Eq. (85) gives, at 500 Hz, a totally negligible contribution, smaller than the reference intensity. Hence, we conclude that if the mechanism envisaged here is of any importance, the acoustically active clouds must contain bubbles greater than ~ 1 mm. (For $R = 1$ mm, b at 500 Hz is 3.76 kHz.)

IV. OTHER MECHANISMS AND CONCLUSIONS

In this article, we have tried to estimate the effect of bubbles on oceanic ambient noise for frequencies up to a few kHz. We have identified three frequency regions where bubbles can have a significant effect. The first one is up to 100–200 Hz where they can amplify the pressure oscillations induced by the water turbulence. The second one is in the kHz range, where single-bubble oscillations appear capable of accounting for most of the observed noise levels. The third one is in the intermediate range, where collective oscillations of bubble clouds may contribute significantly to the noise.

Bubble effects at still higher frequencies are not only possible but quite probable in certain conditions. Rain causes noise in the range 5–20 kHz[40–42] and, according to the results of Crum and Pumphrey,[31] the bubbles produced by the impact of water drops on the ocean surface appear to be responsible for most of the acoustic emission. A similar effect can be caused by sprays and, possibly at lower frequencies, by splashes. As a matter of fact, at least one model has been proposed in which the impact of sprays is held responsible for ambient noise not only at high frequency, but also in the range 1–1000 Hz.[43] This model extrapolates the results of Franz[30] very far away from the parameter region where they were obtained and does not appear to be supported by the data.[40–42] As for splashes, their occurrence must be much less frequent than that of wave breaking, at least for moderate wind speeds. Although we have not tried a quantitative estimate, this feature seems sufficient to dismiss them as a major noise source.

Snow causes noise around 40–60 kHz,[42] possibly by a process associated with the formation of bubbles. Here, the mechanism could be the entrapment of air as the flake melts. The tiny bubble thus produced would be full of air at 1 atm, far lower than the pressure $p_0 + 2\sigma/R$ it would have at equilibrium (here σ is the surface tension). The bubble would therefore start oscillating at its [very high, cf. Eq. (54)] natural frequency radiating part of the excess initial energy.

Still other noise-producing mechanisms associated with bubbles have been proposed. One has to do with "bubble cavitation"[2,3,5] and would cause noise in the same way as a collapsing cavitation bubble does. As discussed elsewhere,[44,45] this process is utterly impossible because it would require pressure fluctuations due to oceanic current and surface motion in excess of 1 bar. In part, the misunderstanding seems to have originated due to the confusion between the threshold for rectified diffusion and that for violent collapse.[46,47] The bursting or "popping" of bubbles at the ocean surface has also been suggested as a source of noise.[5] On the basis of some simple estimates,[45] it appears that the number of bubbles necessary to account for the observed levels would be of the order of hundreds of millions per square centimeter per second, which appears impossible. Finally, the turbulent wake of bubbles ascending by buoyancy has been proposed as a noise source.[3] This acoustic emission would have a quadrupole nature, as for ordinary turbulence, and would therefore be quite negligible. However, volume pulsations of the bubbles would be induced by the vortex shedding that takes place in these conditions, but their frequency could hardly be expected to exceed 10 Hz. The total contribution to the overall sound energy that such a process can produce also appears to be quite low.

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APPENDIX

We give here a proof of (38) assuming the functional form (37) for the turbulence correlation function f and making the approximation $|\mathbf{K}|l \ll 1$. Exact results for the forms (34) and (35) will also be given

For the present purposes, the integration over τ in the definition (30) of $\widetilde{Q}$ is unimportant and we consider only the space integrations,

$$\overline{Q}(\mathbf{K},\tau) = \frac{1}{(2\pi)^2}\int_{-\infty}^{0} d\zeta \iint d\xi\, d\eta \exp(-i\mathbf{K}\cdot\boldsymbol{\xi})Q(\boldsymbol{\xi},\tau) . \quad \text{(A1)}$$

Passing to polar coordinates with the η axis directed along $\mathbf{K}$, this expression becomes

$$\overline{Q} = \frac{1}{(2\pi)^2}\int_0^\infty r^2\, dr \int_{\pi/2}^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi \times \exp(-i|\mathbf{K}|r\sin\theta\sin\phi)Q(r,\tau) ,$$

since Q only depends on $|\boldsymbol{\xi}|$ in isotropic conditions. The angular integrations are readily carried out using the results

$$\int_0^{2\pi} \exp(-i\zeta\sin\phi)d\phi = 2\pi J_0(\zeta) ,$$

$$\int_0^{z} J_0(\sqrt{z^2 - t^2})dt = \sin z .$$

After insertion of (32), (A1) then becomes

$$\overline{Q} = \frac{(\rho\,\overline{u'^2})^2}{\pi K}\int_0^\infty dr\sin Kr \int_r^\infty \left(\mu - \frac{r^2}{\mu}\right)\left(\frac{df}{d\mu}\right)^2 d\mu ,$$

where $K = |\mathbf{K}|$. Upon interchange of the order of integration, the integration over r can be carried out with the result

$$\overline{Q} = \frac{2(\rho\,\overline{u'^2})^2}{\pi K^4}\int_0^\infty \left[\left(\frac{3}{\mu K} - \mu K\right)\sin\mu K - 3\cos\mu K\right] \times \left(\frac{df}{d\mu}\right)^2 d\mu . \quad \text{(A2)}$$

Now, let $u = \mu/l$ and assume the form (37) for f, so that $df/d\mu = l^{-1}df/du$. Equation (A2) then becomes

$$\overline{Q} = \frac{2(\rho\,\overline{u'^2})^2}{\pi l^2 K^5}\int_0^\infty \left\{\left[\frac{3}{u} - (Kl)^2 u\right] \times \sin(Klu) - 3(Klu)\cos(Klu)\right\}\left(\frac{df}{du}\right)^2 du . \quad \text{(A3)}$$

Since df/du decays rapidly past $u \sim 1$, and since $Kl \ll 1$, the quantity in curly brackets in this expression can be approximated for small Kl to find

$$\{\cdots\} \simeq \tfrac{1}{15}(Kl)^5 u^4 ,$$

from which we find the result (36). For the specific form (34) of f, valid in the low Reynolds number case, the integral in (A3) can be evaluated exactly to find

$$\overline{Q} = [(\rho\,\overline{u'^2})^2/8\pi^{1/2}]l^3\exp(-l^2K^2/4) ,$$

while with the high Reynolds number expression (35) one finds

$$\overline{Q} = \frac{(\rho\,\overline{u'^2})^2}{2\pi K^3}\left(\frac{2}{Kl}\right)^2\left\{3\arctan\left(\frac{Kl}{2}\right) - \frac{(Kl/2)[3 + 5(Kl/2)^2]}{[1 + (Kl/2)^2]^2}\right\} .$$

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