Micro & small caps liquidity and Stock Return: An analysis of the US Market

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Abstract

In this study we analyze the impact of different liquidity factors on the expected returns of the small caps listed in the USA stock market and find evidence that the mentioned effect exists. We show the existence of a premium driven by small caps that is not captured by the size factor but instead by their liquidity; this was done through performing linear regressions on models built as an extension of the Fama & French three-factor model. We learned that the liquidity factor exists and is bigger and statistically significant in the small caps than it is in the big caps, which corroborates what is stated in the literature. The importance of this investigation lays down in the potential application when deciding to build a portfolio that takes advantage of the liquidity effect of the small caps in addition to the standard 'size effect'.

Keywords: Liquidity Factor, Small Caps, Excess Return.

Preface

The basis for this research originally stemmed from my interest for understanding better the behavior of the stock market and the factors that influence it. As the market fluctuates according to diverse components and circumstances, the investors look restlessly for greater returns, and in order to do that, they try to find models that explain and predict their portfolio's returns. In this dissertation we analyze the liquidity as an important factor in the market behavior, especially Small Caps, which have been of interest since the 80's.

Chapter 1: Introduction

The arguments for including micro and small caps securities in an investment portfolio are diverse; from the most common like portfolio diversification to more complexes and meticulously designed strategies, which not only takes into consideration the performance of the stock but also the advantages derived from exploiting the lack of liquidity of this sort of security, since asset pricing is affected by the liquidity factor (Chan & Faff, 2005), (Acharya & Pedersen, 2005). These strategies must consider the investors' profile, and here we are not just referring to the widely known risk profile but to the investment horizon (in fact, these terms are slightly related, but not the same), since an investor with a longer investment time horizon is able to tolerate the illiquidity of some securities and therefore expect some additional return for this risk (Bailey, 2005).

When referring to Micro and Small Cap Stocks, we are talking about shares of companies that are generally in the low spectrum of the market when classified by their market value (market capitalization). Tags as Micro, Small, Mid, Big or Huge Cap are subjective and relative to the stock market in which they trade and tend to change over time along with the evolution of the entire financial ecosystem. For the purposes of this investigation, we would be using the current classification for the US Stock Market: Micro-Caps (Less than $300MM), Small-Caps ($300MM – $2B), Mid-Caps ($2B-$10B) and Big-Caps ($10B and greater) and would be infer this measures for the previous periods making an inference based on their proportional size against the total size of the market, thereby we avoid any size bias derived associated to the growth of the economy.

The Micro and Small caps stocks have some peculiarities which make them of interest for many investors, these peculiarities comprise: the fact that they have a higher probability of lacking institutional sponsorship and therefore an unexploited potential for the ones that get them in an early stage (Chordia, et al., 2011), the lack of coverage from analysts which can be associated with low excess valuation (Doukas, et al., 2008), the higher bid-ask spread due to a thinly market (Amihud & Mendelson, 1986) and even the higher probability of being bought by other companies (related to the mentioned low valuation) which triggers an improvement in price posterior at the moment of the buying announcement.

According to Reuters (McCrank & Mikolajczak, 2016), the risks of acquiring shares of small caps in the US Stock Market are getting higher while their liquidity is decreasing, and this phenomenon is happening since the 2008 crisis. Another affirmation that triggered the interest for the topic is one from Morningstar (Bryan, 2014), which states that even if the premium for small caps in the US Market still exists, it is not reliable and should not be counted on for assuring long-term performance. These asseverations constitute enough motivation to undertake this analysis in order to determine if such statements are valid, not only because of their apparent contradiction with what is intuitive for us (relation between liquidity and risk) but also due to the importance and urgency of the issue ($203.13B in Market Capitalization for Micro and Small Caps in the NASDAQ and NYSE in total and 31.4% of the listed companies in September 2017), and furthermore to establish its consequences for the current investors and give some guidance for the future ones.

This research aims to analyze the impact of the micro and small caps' liquidity in the return of stocks listed on the US Stock Exchange, taking into account not only the evolution of price but also the risk-return factor in order to have a fair measure of the return. The term liquidity will be used as a synonym for Market Liquidity throughout this document and is no other than the tolerance of the market to endure a buying or selling operation of an asset without inflicting a significant change in the asset's price or, to put it in other words, how fast and easy the asset can be shorted and yet get a reasonable price (Bodie, et al., 2013).

According to Acharya & Pedersen (2005), liquidity is inversely correlated to risk, hence to expected returns. In addition, their model indicates that high illiquidity forecasts high future returns and that liquidity and returns move at the same time; however, according to the investigation of Amihud et al. (2015), this co-movement has a lag on the illiquidity factor, which can imply Granger-causation. Subsequently, Acharya & Pendersen (2005) declares that liquidity is not constant and is strengthened by several factors like necessity to trade frequently (depending on the investor's strategy, asymmetric information, institutional effects, taxes) or a sudden necessity to trade (for fulfilling its obligations or a specific opportunity to seize), this constitutes what is called liquidity risk.

The findings of Amihud and Mendelson (1986) stated that there is something that they call clientele effect, which means that stocks with higher Bid-Ask spread are normally held by investors with a higher time horizon, and *"as a result of the clientele effect returns on higher-spread stocks are less spread-sensitive, giving rise to a concave return-spread relation"* (Amihud & Mendelson, 1986, p. 246); this is especially true for micro & small caps, which demonstrates to have bigger bid-ask spreads, since there is a negative relationship between firm size and spread (Amihud & Mendelson, 1986).

This work focus on the US Stock Exchange (NYSE, AMEX & NASDAQ) considering that it is the largest stock market in the world by far according to the World Federation of Exchanges (WFE, 2016). Our target is to analyze the performance of the stocks related to Micro & Small Caps compared to Big & Huge Caps based on the information gathered for a period of 30 years (1987-2016). It is reasonable to think that a 30-year period is enough to detect any tendency that will disturb the illiquidity premium over the stock returns that we analyze.

The Capital Asset Pricing Model (CAPM) is one of the most popular models in finance and represents the relationship between the market risk (undiversifiable risk) and the expected return of a specific portfolio (Sharpe, 1964) and despite the emergence of new approaches for pricing assets and portfolio selection, it is still widely used due to its simplicity; the trade-off between the complexity of a model and its usefulness has to be taken into account as part of any decision-making process, and this work is no exception to the rule. The three-factor model (Fama & French, 1993) is derived from the CAPM (Sharpe, 1964), and also takes into consideration size (SMB: Stands for High Minus Big, and reflects the difference between the simple average of the returns on small caps and big caps) and value (HML: Stands for High Minus Low, and represents the difference between the simple average of the returns on high and low Book to Market Ratio portfolios) factors for measuring how they also influence the excess return and if these variables are significant to the model.

To answer if this liquidity has an impact on the returns and its possible magnitude and tendency, we classify the assets in deciles according to their market capitalization and proceed to collect transactional data aggregated in weekly intervals and proceed to segment according to 5-year periods in order to analyze the evolution of the market.

We hypothesize about the existence of a liquidity premium that is independent to the Size factor or the Value factor of the Fama & French model; for this, we use the Turnover Ratio which was used in the investigation of Lam & Tam (2011) and that is an extension of model of Amihud & Mendelson (1986), we also take into account the Illiquidity factor, which is proposed by Amihud (2002) as an illiquidity proxy, and the Return to Turnover Ratio proposed by Florackis et al. (2011). The relation of these three factors is tested in different combinations and is the Return to Turnover Ratio the one that gives us significative results (it is mainly because the Illiquidity factor is size biased and the Turnover Ratio gives us the flow of shares negotiated compared to total shares, but is an indicator that does not provide the impact in the price/return after those transactions). It is with the convergence of the three-factor model and the liquidity factor (Return to Turnover Ratio) that we proceed to construct a model and run a regression for measuring how they influence the excess return and if their significance to the model. Afterwards, a comparison is made for each 5-year period is made between the micro/small caps and the mid/large caps.

The final result comprises the verification of the relation between the liquidity in Micro & Small Caps and their performance and a detailed analysis of its strength and direction measurement, proving the existence of a liquidity factor not captured by the firm size, which is statistically significative for the small caps and not for the large caps.

The contribution of this investigation gravitates around two points: The use of weekly indicators instead of the monthly data used in the work of Lam & Tam (2011) and Amihud (2002) , also another addition was the optimization of the model containing the liquidity factor, since the simulation was done with 3 models with different liquidity proxies (Illiquidity for Amihud, Turnover Ratio from Lam & Tam, and Illiquidity + Turnover Ratio), proving that Turnover Ratio was the best suited for the task because of the significance of its coefficients.

We consider that the institutional sponsorship should be measured in further research as part of the model since it is a factor that affects the analyst coverage (Chordia, et al., 2011) and therefore, the liquidity of the asset. It should be studied also the impact derived from the small caps being upgraded because a merger or a buyout since they were excluded from the data gathering.

In Chapter 2 we immerse the subject by defining liquidity, its drivers and the different proxies for quantifying its magnitude (which is useful when we want to create a benchmark between types of assets or the same asset in a different period). Then we take a look at the liquidity – return relationship and examine closely the liquidity in small caps (which is our main study subject), proceeding next to develop our hypothesis. In Chapter 3 we dedicate to explain how we gather, filter and process the data and which methodology we use to prove our hypothesis. The application of statistical tests and presentation of the results related to our hypothesis is made in Chapter 4. Finally, we develop a summary, make recommendations about the model and review some open questions for further research on Chapter 5.

Chapter 2: Literature Review

2.1. Definition of liquidity

The term liquidity of an asset has a widely accepted definition and was formally introduced by John Maynard Keynes (1930), being defined as how easily an asset can be traded in the market without affecting the market itself. This relation between the asset and the market has three components: the speed at which the transaction is performed, the costs derived from executing the transaction and the price impact in the rest of the similar assets remaining in the market.

Similarly, Beaupain and Joliet (2011) building on Kyle (1985, p. 85) state that for a market to be liquid, it "should be tight, deep and resilient". While tightness is related to the cost of the transaction, depth is the capacity of the market to transact a large order and resilience is the capability of the market to return to the long-term averages regarding its transaction costs and depth after an alteration.

2.2. The drivers of liquidity

According to Amihud et al. (2006, p. 270), the drivers of liquidity are *"exogenous transaction costs, demand pressure and inventory risk, and the handling of private information"*. The exogenous transaction costs consist of the expenses derived from buying or selling the securities in every transaction along the lifespan of the asset, which is also anticipated by the transaction actors and is included in advance in the pricing (essentially order processing costs, brokerage fees, and taxes).

The demand pressure refers to the constraints associated with the timing in which the participants in the market are present and the constraint that it represents to a seller who wants to close its position rapidly and has to pay a prime for it (by selling at a lower price). The inventory risk surges from this scenario, in which generally a market maker is the one who buys the stock and assumes the risk of possessing a large inventory of assets and is compensated by the lower price mentioned before.

Amihud et al. (2006) also mention that trading with private information; or more precisely, the assumption that the counterpart is trading based on private information is something that affects the bid and the ask prices of a security. In this scenario, for example, a buyer (seller) could be afraid that the seller (buyer) has some private information (regarding the fundamentals of the company or a close future action of the company that would modify its value) that gives him the opportunity to take advantage by selling (buying) an overpriced (underpriced) asset. It is due to this sort of risk that the bid/ask price is modified and with this, the spread. Brennan and Subrahmanyam (1995, p. 364) also comment on this source of liquidity, adducing that *“the market depth increases with the number of informed traders”* and that private information is reflected in the pricing of securities.

2.3. Quantifying Liquidity

Having defined liquidity as the easiness to trade an asset without affecting its price in the market, it is necessary to assess the magnitude in which an asset is more or less liquid than other. A liquid market is a sign of a healthy market.

2.3.1. Bid-ask Spread

Early works like the one of Demsetz (1968), Branch & Freed (1977) start talking about the relation between the relative spread (the difference between the bid and ask price compared to the price of the security) and the liquidity of the market.

Given the premise that the market is constituted by risk averter participants, the inventory risk is closely related to a higher spread since their positions would be fixed in relation to the assets (Branch. & Freed, 1977).

The investigation of Amihud and Mendelson (1986) focuses on the study of portfolios and uses the relative spread as a representation of the transaction cost in their model for estimating the return of a specific portfolio, implicating that the higher the spread, the higher the expected return. They also found something they called ‘the clientele effect’, which is the effect caused by the transaction costs on the holding period of the securities; this effect states that *"in equilibrium, assets with higher transaction costs are allocated to the agents with longer holding horizons"* (Amihud & Mendelson, 1986, p. 228) and as a consequence of this, the relation between spread and return is represented as a concave function, since *“the longer the holding period, the smaller the compensation required for a given increase in the spread”* (Amihud & Mendelson, 1986, p. 229).

2.3.2. Quantity traded (TOR)

The trading activity is another important component directly related to the liquidity and has been studied for determining its relationship to the spread; in this context, Stoll (1978) affirms that asymmetry of information would be linked to the size of the transaction.

In addition, Amihud & Mendelson (1986) propose Turnover Ratio as a proxy for liquidity; Lam & Tam (2011), Pereira & Zhang (Pereira & Zhang, 2010) and Chordia et al. (2011) building on Chordia et al. (2001) use also this approach and propose that turnover ratio is directly proportional to the stock liquidity. The Turnover Ratio is represented by:

$$TOR_{it} = \frac{VolSHR_{it}}{TotalSHR_{it}}$$

Where TOR_{it} is the Turnover Ratio for stock *i* on period *t*, $VolSHR_{it}$ is the quantity of shares traded for stock *i* on period *t*, and $TotalSHR_{it}$ is the total quantity of shares (on average) for stock *i* on period *t*.

2.3.3. Amihud measure (Illiquidity)

A key part of defining liquidity was the impact that it has on the price of the security in the market, and since John Maynard Keynes (1930) is stated that the trading of a liquid asset should not affect the price in the market significantly, and this affectation on the price in the market is decreasing with the increase of the liquidity.

Amihud (2002) proposes a measure that acts as a proxy for the price impact for studying the daily liquidity of the NYSE for more than 30 years. This measure is called Illiquidity and is represented as the average ratio of the absolute return to the trading volume (in dollars) of the day.

$$ILLIQ_{iyt} = \frac{|R_{iyt}|}{VOLD_{iyt}}$$

Where $ILLIQ_{iyt}$ is the illiquidity measure for stock *i* on day *t* of year *y*, R_{iyt} represents the return on stock *i* on day *t* of year *y*, and $VOLD_{iyt}$ is the volume traded in dollars on stock *i* on day *t* of year *y*.

2.3.4. RtoTR

An alternative price impact measure is proposed by Florackis et al. (2011) and constructed based on the findings of Amihud (2002), called RtoTR (return to turnover ratio) and expressed as follows:

$$RtoTR_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|R_{itd}|}{TR_{itd}}$$

Where $RtoTR_{itd}$ is the Return to Turnover Ratio of stock *i* on day *d* for month *t*, TR_{itd} is the Turnover Ratio of stock *i* on day *d* for month *t*, R_{itd} is the return of stock *i* on day *d* for month *t*, and D_{it} is the number of valid observation days in month *t* for stock *i*.

This measure is considered by Florackis et al. (2011) as an improvement for the Amihud measure because of its advantage of eliminating the size bias (given that TR is a relative magnitude).

2.3.5. Zeros

Another measure of liquidity, directly related to the easiness to trade is proposed in the work of Lesmond et al. (1999) that is expressed by the proportion of days in which the stock experienced zero returns. The logic behind is that days with zero returns are most likely to be non-trade days, which would be caused by high trading costs or the unattractiveness of the asset, and a stock with more ‘zeros’ would, therefore, be more illiquid. It is a straightforward concept; however, it could take as part of the indicator a trading-day with zero return since it does not control this factor.

2.4. Liquidity – Return Relation

Copeland & Galai (1983) study the effects of the inventory risk at which the market makers are exposed and its relationship with the spread, which in fact is a two-way relationship since having a more liquid market gives the ability to market makers to have a lighter stock and therefore reduce the effective spread, but also a tighter spread will have the effect of making a market more liquid (Beaupain & Joliet, 2011).

The work of Amihud & Mendelson (1986) explores the relationship between liquidity, which is represented there by the bid-ask spread and the expected returns on a stock market composed by rational and risk-averter participants who possess different investment horizons (holding periods). Furthermore, the higher the cost of trading (in this case, the effective spread), the higher the holding period of the investor; which in consequence has the effect of decreasing the liquidity due to a lesser trading activity of the stock (Amihud & Mendelson, 1986).

One of the most popular models that represent the relationship between stock returns and liquidity is the one from Amihud (2002), which introduces the concept of Illiquidity in a multi-factor model (based on Fama & French), examining the effect of illiquidity on stocks in the NYSE for the period 1963-1997 using daily data from CRSP. This study demonstrates that *"expected market illiquidity has a positive effect on ex-ante stock excess return"* (Amihud, 2002, p. 32), but also analyzes the effects of an 'unexpected illiquidity', which has the opposite effect over current stock returns. The explanation for this effect is that when there is a shock of unexpected illiquidity for a stock, it triggers an increase of short-term illiquidity, which also affects the expected returns positively, and finally it causes a decreasing in stock prices, which leads to an immediate decrease in stock returns.

The connection between liquidity and returns are also evaluated in the research of Brennan & Subrahmanyam (1996), in which are analyzed 25 portfolios from the NYSE for the period 1984-1991. They show the existence of a return premium related to the cost of trading.

2.5. Liquidity in Small Caps

One of the first mentions to a 'small-firm anomaly' was stated by Banz (1981, p. 16), who suggested *"a negative relation between risk-adjusted mean returns on stocks and their market value"*; using data from the stocks quoted on the NYSE for at least five years in the period 1926 to 1975, he extended the CAPM model by adding the effect of the market value of a security in its expected return. The suggested model is represented as the following:

$$E(R_i) = \gamma_o + \gamma_1\beta_1 + \gamma_2(\frac{(\phi_i - \phi_m)}{\phi_m})$$

Where $E(R_i)$ is the expected return on security i, γ_o is the risk-free rate, γ_1 is the expected market risk premium, ϕ_i represents the market value of security *i*, and ϕ_m represents the average market value. The firm effect is captured by γ_2, which measures the contribution of the market value in the return of the security, demonstrating in his work that the small firms have bigger risk-adjusted returns.

A distinct approach regarding the effect that information about a company has on the liquidity of its stocks is suggested in the work of Klein & Bawa (1977), and added to the asseveration of Hong et al. (2000, p. 267) that *"firm size is a useful measure of the information diffusion"* and therefore analyst coverage (a highly valuated company has a bigger coverage and availability of information), it is safe to state that firm size has an effect on its stock liquidity.

Finally, the tick size plays a role in the liquidity of the market, especially for those who work with small limit orders (Goldstein & Kavajecz, 2000). This effect of the tick size is also relative to the stock price, since according to Bourguelle & Declerck (2004), the bigger the relative tick size (smaller price), the bigger 'the priority cost' and this affects the liquidity provision by providing fewer limit orders and increasing size in the limit orders offered.

2.6. Hypothesis Development

This section aims to examine whether there is a liquidity premium in the expected excess return on stocks and it is not captured by the spread in returns between Small and Big Capitalization firms (SMB from Fama & French (1993)) or the Value Factor (HML). Thus, here we provide a theoretical explanation for the relationship and a hypothesis is provided.

The three-factor model of Fama and French (1993) extends the CAPM model (Sharpe, 1964) by adding the difference in returns due to their nature in size (SMB – Small minus Big) and the difference in returns because of their book-to-market ratio (HML – High minus Low) which represents its position in the business cycle (value or growth firm). The model is as follows:

$$R_t - RF_t = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \varepsilon_t$$

Where (R_t - RF_t) is the expected premium return, α refers to the excess return not captured by the model, β is the sensitivity of the expected excess asset returns to the expected excess market returns, (RM_t - RF_t) represents the market premium, and s and h are the sensitivity of the expected excess asset returns to the spreads SMB and HML.

A modification of the Fama & French Model is proposed by Amihud (Amihud, 2002) in which are introduced factors that capture the liquidity (specifically in this case, illiquidity) of the asset; he proposes the decomposition of the illiquidity factor in two: one that represents the expected illiquidity and depends on the past illiquidity, and the other one is an unexpected illiquidity (or a liquidity shock). He studies a period of 408 months (1963 - 1997) using data from stocks traded on the NYSE. The model used is as follows:

$$(RM - Rf)_y = g_o + g_1 * lnAILLIQ_{y-1} + g_2 * lnAILLIQ_y^U + \omega_y$$

Where $(RM - Rf)_y$ is the excess premium return in year y, $lnAILLIQ_{y-1}$ is the logarithm of the average daily illiquidity ratio (defined in 2.3.3) of the security in year y-1, and $lnAILLIQ_y^U$ stands for the logarithm of the unexpected average illiquidity in year y.

Posteriorly Amihud et al. (2015) used a modification of this model to study the effects of illiquidity for 43 countries around the globe on the period 1999-2010 in which they consider regional specific variables and no longer use the unexpected illiquidity variable. In this study, the population is divided in deciles according to its size and is country specific in order to control for difference in regional factors and market microstructure (Amihud, et al., 2015).

Finally, according to a most recent research from Fong et al. (2017) demonstrates that the best proxy for liquidity per dollar using low-frequency data is represented by the Amihud factor; thus, in order to standardize the factor and drop the size bias, we divide it by the size and obtain the RtoTR (Florackis, et al., 2011).

This study tries to demonstrate that the liquidity factor is priced and has a bigger return premium in the small caps.

Ho: *There is no premium in the expected return of the small caps related to their liquidity.*

Ha: *There is a premium in the expected stock returns directly related to the liquidity for small caps, which is not captured by the size factor (SMB – Small Minus Big) nor the value factor (HML – High Minus Low).*

Chapter 3: Data and Methodology

The effect of stock liquidity on its returns is analyzed for stocks traded in the US Market (NYSE, AMEX & NASDAQ) from January 1987 to December 2016, using weekly observations from Thomson Reuters databases. This kind of low-frequency data is considered a good proxy for intraday data according to Goyenko et al. (2009); this and the fact that low frequency data is hardly available are the reasons why we chose to work with weekly records. The sample discriminates firms according to yearly criteria and groups them into deciles by their market capitalization (from 1 to 10, where '1' corresponds to the group with the 10% of companies with biggest market capitalization in a specific year), and includes all the data of a specific year *y* for a firm *j* when:

(i) The stock of the firm *j* traded at least 4 weeks in the year *y*.

(ii) The price of the stock *j* was less than $10000 per share (in order to avoid a bias on stocks that are designed to be illiquid due to the lack of stock split).

(iii) The stock *j* is not listed initially nor delisted in the year *y*.

(iv) The absolute value of the difference in the decile assigned for a stock *j* in year *y* and year *y-1* is less or equal than 2. This restriction is put in order to control for big changes in value that could distort the analysis.

(v) The market capitalization of *j* in year *y* is bigger than $0.5 million.

The grouping of the deciles is made by a weighted average of the indicators Turnover Ratio, Illiquidity Factor, Return; also indicators like the Market Value and Volume Traded were grouped by summing them. Variables like Size Factor, Value Factor, Risk Free Rate and Market Premium do not need to be aggregated since they are global values (constant for every portfolio in a same instance of time).

Next, we use the deciles to create portfolios of securities that we segment as small caps (deciles 6, 7, 8 & 9 since it contains the range between $43 million and $800 million for the year 2016) and big caps (decile 1, which contains firms with market capitalization superior a $10.9 billion for the year 2016). Even though the data is measured weekly, in order to capture the changes of the market in the general model, we divide the 30-year period into 6 periods of 5 years each. The descriptive statistics of the sample are shown in **Appendix A**.

The data corresponding to the SMB and HML factors were obtained from the section 'U.S. Research Returns Data' in the Kenneth French's Website (French, 2017) and we use the return given for the one-month Treasury bill as the Risk Free Rate.

In order to capture the premium in the return given the liquidity of the asset, we test the correlation between the liquidity factors (Turnover Ratio -TOR-, Amihud factor -ILLIQ- and Return to Turnover Ratio -RtoTR-) with other significant candidate variables for our model.

Table 3.1

Correlation coefficients corresponding to the main variables involved in the models analyzed in this investigation. (R-Rf) is the excess return and (Rm-Rf) represents the market premium.

	SMB	HML	ILLIQ	RtoTR	TOR	R-Rf	Rm-Rf
SMB	1						
HML	-0.2029	1					
ILLIQ	-0.0223	-0.0069	1				
RtoTR	-0.0547	0.0083	0.1867	1			
TOR	0.0244	-0.0091	-0.3224	-0.2433	1		
R-Rf	0.2709	-0.0229	-0.0002	0.0604	-0.0126	1	
Rm-Rf	0.0787	-0.0684	-0.0008	0.0357	-0.0240	0.8878	1

Given that the TOR, ILLIQ and the RtoTR are important factors regarding our model, we will simulate the three of them as the liquidity proxies and propose three models in which we test the different combinations of them; the objective when doing this is to find the best model taking into account the trade-off between complexity and usefulness. In order to do this we compare how much each factor contribute in explaining the model and take into account important factors as autocorrelation of error terms (using Durbin-Watson test) and the significance of the coefficients. We compare the R-squares of the models and analyze them separately since according to Table 3.1 there is a correlation between TOR, ILLIQ and RtoTR.

$$R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \varphi_p * ILLIQ_{pt} + \varepsilon_t \quad \textbf{(Model 1)}$$

$$R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \omega_p * RtoTR_{pt} + \varepsilon_t \quad \textbf{(Model 2)}$$

$$R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \gamma_p * TOR_{pt} + \varepsilon_t \quad \textbf{(Model 3)}$$

Where TOR_{pt} is the average weekly turnover ratio (volume traded divided by total shares) of all stocks in portfolio *p* in week *t*.
$ILLIQ_{pt}$ represents the weekly illiquidity factor (Amihud, 2002), computed as the absolute value of the weekly return, and divided by the volume in dollars traded of all stocks in portfolio *p* in week *t*.

$RtoTR_{pt}$ is the average weekly return to turnover ratio of all stocks in portfolio *p* in week *t*. This factor is computed as the absolute value of the weekly return, divided by the weekly turnover ratio.

We also assemble three equations to analyze the difference in return between Big and Small caps and model the relation between the return spread between the two portfolios and the spread in the liquidity factors.

$$R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \psi * (ILLIQ_{bt} - ILLIQ_{st}) + \varepsilon_t \quad \textbf{\textit{(Model 4)}}$$

$$R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \delta * (RtoTR_{bt} - RtoTR_{st}) + \varepsilon_t \quad \textbf{\textit{(Model 5)}}$$

$$R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \phi * (TOR_{bt} - TOR_{st}) + \varepsilon_t \quad \textbf{\textit{(Model 6)}}$$

Where $R_{bt} - R_{st}$ represents the spread in returns between Big and Small Caps portfolios (in that order) in period *t*. Also, $(ILLIQ_{bt} - ILLIQ_{st}), (RtoTR_{bt} - RtoTR_{st})\ and\ (TOR_{bt} - TOR_{st})$ are the spreads for the liquidity factor given between portfolios (Big and Small Caps) for the period *t*.

Finally, in order to study the possibility of direct explanation of the excess return given the liquidity of the entire market, we also generate one model for each market liquidity factor, which the dependent variable is represented as the excess return of the Big and Small Caps portfolios.

$$R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + l * Mkt\_ILLIQ_t + \varepsilon_t \quad \textbf{\textit{(Model 7)}}$$

$$R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + r * Mkt\_RtoTR_t + \varepsilon_t \quad \textbf{\textit{(Model 8)}}$$

$$R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + o * Mkt\_TOR_t + \varepsilon_t \quad \textbf{\textit{(Model 9)}}$$

Where Mkt_ILLIQ$_t$, Mkt_RtoTR$_t$ and Mkt_TOR$_t$ are the market liquidity factors in the period *t.*

It is important to control for perturbations in the model, such as the effects of financial crisis (burst of the Dot Com bubble in 2000 and the subprime crisis in 2008 are the two specific cases in our 30-year period analyzed). Ben-Rephael (2017) states that in periods of financial crisis, the phenomenon called flight-to-liquidity takes place. Given that the investors are uncertain about the market, they prefer to have their investments allocated on very liquid assets, which produces a massive selling of illiquid assets and their correspondent decrease in price. This decrease in price represents a short-term reduction in return for the illiquid assets, rewarding at the same time the liquid ones. This behavior is opposite at what the literature shows as the standard of the market and is taken into account when modeling the data.

In order to analyze the model we execute a regression with the data and compare the coefficients for each period (portfolio big cap versus portfolio small cap) in addition to the comparison of the coefficients for *s*. We do this for each model.

In line with our hypothesis, the liquidity factor coefficients for the Small Caps should be statistically significant at least at the 10% level. Then:

$$\gamma_{sc} <> \gamma_{bc}$$
$$\varphi_{sc} <> \varphi_{bc}$$
$$\omega_{sc} <> \omega_{bc}$$

Where γ_{sc} refers to the coefficient of the TOR for small caps (the sensitivity of the excess return given the turnover ratio), γ_{bc} refers to the coefficient of the TOR for big caps, φ_{sc} represents the coefficient of the Amihud's Illiquidity factor (ILLIQ) for small caps, φ_{bc} is the coefficient of ILLIQ for the big caps, and ω_{sc} is the coefficient of the Return to Turnover Ratio (RtoTR) for small capitalization firms while ω_{bc} is the coefficient of the RtoTR for big caps.

Chapter 4: Empirical Analysis and Results

This thesis analyzes theoretically and empirically the relationship between stock liquidity and return, emphasizing the investigation in small caps listed in the USA market (NYSE, NASDAQ & AMEX) and how they have a premium on their expected returns linked to a liquidity factor, which is stronger and more significant than the one found in the large caps.

4.1. Univariate Analysis

4.1.1. Descriptive statistics

The **Table A.2** shows the descriptive weekly statistics of the factors that are taking into account to be part of the model (including Amihud's Illqiduity factor, Turnover Ratio and Return to Turnover Ratio). Where *R-Rf* is the excess return of the portfolio (Big or Small Cap) for the specific period described at the left, *Rm-Rf* is the market premium for the mentioned period. We observe from the data that in the majority of cases (4 out of the 6 periods), the small caps outperform the small caps, having in total an excess return of 10.91% for the 30-year period and 16.09% for the last 15-year period. The skewness for both, the excess returns for big caps and the excess return for small caps is negative in all the 5-year periods analyzed, which implies that the distribution is asymmetric and skewed to the left

Additionally, we observe in **Table A.2** that ILLIQ (Amihud's Illiquidity factor) is significantly bigger in small caps than it is in big caps. This is mainly due to the form this indicator is built, since the numerator (represented by the absolute value of the return) is smaller and the denominator (represented by the volume traded in dollars) is bigger for big caps; evidently, this changes when we divide by size (resulting in the RtoTR factor), here we observe that this factor changes over time, and being 33.27% bigger for small caps than for big caps in the last period. Regarding to TOR (Turnover Ratio), we observe that it is more volatile for the small caps in every period expect for 2007-2011 (fifth period, having a total average of 21.48% more volatility.

It is important to also point out that in the period comprised between 2007 and 2011 we notice the highest weekly volatility in the market premium (3.36%), excess return on Big Caps (3.2%), excess return on Small Caps (4.01%) and the second highest volatility for the weekly SMB factor (0.0131). This abnormal behavior for the fifth period is mainly driven by the called Great Recession in 2008 (subprime mortgage crisis after the collapse of the housing bubble) and the Flash Crash in 2010 (due to market manipulation through spoofing algorithms).

4.1.2. Unit root Test

One of the causes of the phenomenon known as spurious relationship (or spurious regression) is the existence of variables with unit-root, which means that these variables have a stochastic trend that can produce results that may wrongly indicate the existence of a relationship in our estimation by ordinary least squares. We then run the test in Eviews:

Table 4.1.2.1

Data obtained from applying unit root tests to the different variables used in the models generated. It shows that only TOR has a unit root and it this phenomenon is present in the 'crisis years', without this effect, none of the variables has a unit root.
The test was executed including the trend and intercept in the equation and a Maxlag equal to 23.

		Period 1998 - 2016			**Period 1998 - 2016 without 2000 & 2008**		
		Dickey-Fuller statistic	**Critical value**	**p-value**	**Dickey-Fuller statistic**	**Critical value**	**p-value**
Big Caps	**R-Rf**	-43.0196	-3.9639	0	-40.3434	-3.9643	0
	ILLIQ	-5.8569	-3.9640	0	-5.6716	-3.9644	0
	RtoTR	-19.1644	-3.9639	0	-18.2344	-3.9643	0
	TOR	-2.7706	-3.9640	0.2085	-3.7421	-3.9644	0.0199
Small Caps	**R-Rf**	-24.4302	-3.9639	0	-23.3901	-3.9643	0
	ILLIQ	-5.1389	-3.9639	0.0001	-4.9495	-3.9644	0.0002
	RtoTR	-9.0723	-3.9639	0	-10.1068	-3.9644	0
	TOR	-7.7011	-3.9639	0	-6.5619	-3.9644	0
Market	**Rm-Rf**	-42.1422	-3.9639	0	-39.5126	-3.9643	0
	ILLIQ	-4.5988	-3.9639	0.001	-4.4840	-3.9644	0.0016
	RtoTR	-10.9289	-3.9639	0	-17.6241	-3.9643	0
	TOR	-2.8188	-3.9640	0.1906	-3.8257	-3.9644	0.0155
	SMB	-38.5510	-3.9639	0	-39.0595	-3.9643	0
	HML	-24.6371	-3.9639	0	-23.3989	-3.9643	0

According to the results obtained, we proceed to reject the null hypothesis of the existence of unit root for all the variables included in the model (we even analyze both portfolios separately).

4.2. Bivariate Analysis

In **Table A.2** we showed, among other things, the difference in weekly excess returns between big and small caps. Since the observed differences shown by the means gives us a clue about the general performance of the two portfolios, it is important to also analyze the statistical significance of the data. Given that we are managing two different samples, we have to run the analysis under the 'unpaired sample' configuration, and we operate under the null hypothesis that the difference between the means of both groups is equal to zero.

We interpret the p-values obtained and conclude that the null hypothesis is not rejected according to the results. Thus, there is not statistically significant proof that the means are different in any of the periods.

Table 4.2.1

Validation of equality using the T-test (two-tail unpaired sample)

(BC & SC excess return)	**p-value**	
Period	**No exclusion**	**Excluding 'crisis years'**
1987-1991 (Period 1)	0.6694	0.6694
1992-1996 (Period 2)	0.5643	0.5643
2997-2001 (Period 3)	0.7586	0.6755
2002-2006 (Period 4)	0.1452	0.1452
2007-2011 (Period 5)	0.9747	0.9148
2012-2016 (Period 6)	0.8843	0.8843
1987-2016	0.6991	0.6511

We proceed to evaluate the Turnover Ratio using the same approach, and we realize that the null hypothesis is rejected for all time periods except the fourth; it is interpreted as the existence of a statistically significant difference between the means of the weekly Turnover Ratio of the big and small caps (except in the fourth period, as we said).

Table 4.2.2

Validation of equality using the T-test (two-tail unpaired sample)

(BC & SC Turnover Ratio)

	p-value	
Period	**No exclusion**	**Excluding 'crisis years'**
1987-1991 (Period 1)	0.0002	0.0002
1992-1996 (Period 2)	0	0
2997-2001 (Period 3)	0.0003	0
2002-2006 (Period 4)	0.9919	0.9919
2007-2011 (Period 5)	0	0
2012-2016 (Period 6)	0.0003	0.0003
1987-2016 (Whole Period)	0.0021	0.0781

Replicating the analysis for Amihud's Illiquidity factor, we obtain concrete results that reject the null hypothesis, demonstrating a statistically significance in the difference between the Illiquidity factors concerning small and big caps.

Table 4.2.3

Validation of equality using the T-test (two-tail unpaired sample)

(BC & SC Illiquidity Factor)

	p-value	
Period	**No exclusion**	**Excluding 'crisis years'**
1987-1991 (Period 1)	0	0
1992-1996 (Period 2)	0	0
2997-2001 (Period 3)	0	0
2002-2006 (Period 4)	0	0
2007-2011 (Period 5)	0	0
2012-2016 (Period 6)	0	0
1987-2016 (Whole Period)	0	0

Analyzing the Return to Turnover Ratio (RtoTR), we obtain results that show statistically significant difference between small and big caps for all periods except 1997-2001, which is affecting the result for the total range of time.

Table 4.2.4

Validation of equality using the T-test (two-tail unpaired sample)

(BC & SC RtoTR)		
	p-value	
Period	**No exclusion**	**Excluding 'crisis years'**
1987-1991 (Period 1)	0	0
1992-1996 (Period 2)	0	0
2997-2001 (Period 3)	0.5781	0.1991
2002-2006 (Period 4)	0.0002	0.0002
2007-2011 (Period 5)	0	0
2012-2016 (Period 6)	0.0001	0.0001
1987-2016 (Whole Period)	0.7001	0.5139

4.3. Application of the Model

In chapter 3 we introduced the seven models to test for testing our hypothesis. In this section the models are presented and tested.

One important assumption for a linear regression is that the standard deviations of the error terms are constant, which is known as homoscedasticity. The most suited test, since we are working with time-varying volatility series is ARCH model (Engle, 1982).

Another important assumption for a linear regression is the absence of autocorrelation of error terms (a relationship between values separated from each other by a given time lag), for providing these results, we use the Breusch-Godfrey Serial Correlation LM Test.

First, in each subsection of the section 4.3 we present the results of the regression using a specific model, then we test for autocorrelation and heteroscedasticity, and if one of those conditions is present, we use the Newey-West HAC method from Eviews. It is important to emphasize that in this adjustment we also drop the data from years 2000 and 2008, since we validate that the data from this specific years are not significant for our model, given the existence of a phenomenon that makes the market behave differently as usual (Ben-Rephael, 2017).

4.3.1. Model 1 – Regression

In Table 4.3.1 we observe the results of the regression applied using Model 1 on the data, where the liquidity factor analyzed is ILLIQ and the dependent variable is the excess return. Each analyzed portfolio is modelled.

Table 4.3.1

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \varphi_p * ILLIQ_{pt} + \varepsilon_t$

	1987-1991 (Period 1)	1992-1996 (Period 2)	1997-2001 (Period 3)	2002-2006 (Period 4)	2007-2011 (Period 5)	2012-2016 (Period 6)	1987-2016 (Total)
Big Caps							
C	0.0006	0.0002	-0.0001	0.0000	0.0003	-0.0017	-0.0001
(p-value)	*(0.0025)*	*(0.2472)*	*(0.7475)*	*(0.7812)*	*(0.1203)*	*(0.3357)*	*(0.4273)*
Rm-Rf	0.9850	0.9866	0.9919	0.9894	0.9777	0.6715	0.9513
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2125	-0.2063	-0.1209	-0.1436	-0.1489	-0.0229	-0.1514
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.6856)*	*(0.0000)*
HML	-0.0420	-0.0463	-0.0506	-0.0460	-0.0386	0.0431	-0.0514
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.4519)*	*(0.0000)*
ILLIQ	-1.5360	-0.7982	0.0194	-1.5633	-14.8738	122.1830	0.1734
(p-value)	*(0.0002)*	*(0.1169)*	*(0.9443)*	*(0.1336)*	*(0.0040)*	*(0.1911)*	*(0.6601)*
Adj. R-squared	0.9977	0.9971	0.9925	0.9980	0.9976	0.6811	0.9579
Durbin-Watson	2.0090	2.0249	2.1883	2.1558	2.1341	2.0568	2.0139
Small Caps							
C	0.0000	-0.0002	-0.0017	0.0005	-0.0008	0.0000	-0.0002
(p-value)	*(0.9978)*	*(0.7553)*	*(0.2213)*	*(0.4099)*	*(0.1816)*	*(0.9933)*	*(0.3914)*
Rm-Rf	0.9210	0.8717	0.7391	0.8071	0.9612	0.6458	0.8439
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	1.0723	0.9653	0.8027	0.9011	0.8709	1.2354	0.9319
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	0.2163	0.2461	0.2941	0.3073	0.2639	0.2733	0.3464
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
ILLIQ	0.0006	0.0019	0.0055	0.0038	0.0035	0.0121	0.0016
(p-value)	*(0.3951)*	*(0.1790)*	*(0.2156)*	*(0.1504)*	*(0.2110)*	*(0.4549)*	*(0.0505)*
Adj. R-squared	0.9568	0.9030	0.8467	0.9113	0.9677	0.8097	0.9013
Durbin-Watson	1.9371	1.8296	1.5523	1.7272	2.1785	1.9193	1.8905

4.3.1.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.1.1

Heteroscedasticity Test (Arch) on Model 1. Dependent variable: Resid^2

Big Caps	F-statistic	0.0010	Prob. F(1,1563)	0.9745
	Obs*R-squared	0.0010	Prob. Chi-Square(1)	0.9745
Small Caps	F-statistic	0.0530	Prob. F(1,1563)	0.8180
	Obs*R-squared	0.0531	Prob. Chi-Square(1)	0.8178

Taking into account the p-values shown in Table 4.3.1.1, the null hypothesis of absence of heteroscedasticity is not rejected; therefore, we assume homoscedasticity of the error terms for our model and ensure that the least-squares estimators are each a best linear unbiased estimator of the respective population parameter.

4.3.1.2. Error Term Analysis – Autocorrelation Test

Table 4.3.1.2

Breusch-Godfrey Serial Correlation LM Test on Model 1

	Big Caps		**Small Caps**	
	F-statistic	**p-value**	**F-statistic**	**p-value**
1987-1991 (Period 1)	0.337905	0.7136	0.879742	0.4162
1992-1996 (Period 2)	0.277907	0.7576	1.85768	0.1581
1997-2001 (Period 3)	1.440204	0.2388	9.373956	0.0001
2002-2006 (Period 4)	1.19612	0.3041	1.646658	0.1947
2007-2011 (Period 5)	1.048356	0.352	1.097107	0.3354
2012-2016 (Period 6)	0.110909	0.8951	0.201754	0.8174
1987-2016 (Total)	0.059043	0.9427	4.162641	0.0157

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis for the Small Caps and have to adjust the model for the periods where it is detected (period 3 and the 30-year period).

4.3.1.3. Adjusted Model

Table 4.3.1.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \varphi_p * ILLIQ_{pt} + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	0.0006	0.0002	-0.0001	0.0000	-0.0001	-0.0017	-0.0001
(p-value)	*(0.0025)*	*(0.2472)*	*(0.8105)*	*(0.7812)*	*(0.5155)*	*(0.3357)*	*(0.5166)*
Rm-Rf	0.9850	0.9866	0.9892	0.9894	0.9859	0.6715	0.9456
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2125	-0.2063	-0.1475	-0.1436	-0.1428	-0.0229	-0.1656
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.6856)*	*(0.0000)*
HML	-0.0420	-0.0463	-0.0437	-0.0460	-0.0276	0.0431	-0.0394
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0042)*	*(0.0000)*	*(0.0000)*	*(0.4519)*	*(0.0003)*
ILLIQ	-1.5360	-0.7982	0.0395	-1.5633	-3.2094	122.1830	0.1625
(p-value)	*(0.0002)*	*(0.1169)*	*(0.8919)*	*(0.1336)*	*(0.5626)*	*(0.1911)*	*(0.6894)*
Adj. R-squared	0.9977	0.9971	0.9912	0.9980	0.9978	0.6811	0.9484
Durbin-Watson	2.0090	2.0249	2.0283	2.1558	2.0769	2.0568	2.0063
Small Caps							
C	0.0000	-0.0002	**-0.0030**	0.0005	-0.0006	0.0000	**0.0000**
(p-value)	*(0.9978)*	*(0.7553)*	***(0.0536)***	*(0.4099)*	*(0.3083)*	*(0.9933)*	***(0.8820)***
Rm-Rf	0.9210	0.8717	**0.7806**	0.8071	0.9048	0.6458	**0.8300**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
SMB	1.0723	0.9653	**0.8771**	0.9011	0.9540	1.2354	**0.9831**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
HML	0.2163	0.2461	**0.3236**	0.3073	0.2785	0.2733	**0.2940**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
ILLIQ	0.0006	0.0019	**0.0109**	0.0038	0.0070	0.0121	**0.0017**
(p-value)	*(0.3951)*	*(0.1790)*	***(0.0768)***	*(0.1504)*	*(0.0202)*	*(0.4549)*	***(0.0295)***
Adj. R-squared	0.9568	0.9030	**0.8491**	0.9113	0.9687	0.8097	**0.8994**
Durbin-Watson	1.9371	1.8296	**1.3245**	1.7272	2.0503	1.9193	**1.7918**

The model shows a strong explanation of the dependent variable (94.8% for Big Caps and 89.9% for Small Caps), and while it shows statistical significance for almost all variables, it is not the case for the liquidity factor (ILLIQ).

4.3.2. Model 2 – Regression

In Table 4.3.2 we observe the results of the regression applied using Model 2 on the data, where the liquidity factor analyzed is RtoTR and the dependent variable is the excess return. Each analyzed portfolio is modelled.

Table 4.3.2

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \omega_p * RtoTR_{pt} + \varepsilon_t$

	1987-1991 (Period 1)	1992-1996 (Period 2)	1997-2001 (Period 3)	2002-2006 (Period 4)	2007-2011 (Period 5)	2012-2016 (Period 6)	1987-2016 (Total)
Big Caps							
C	0.0002	-0.0001	-0.0001	0.0001	-0.0001	-0.0007	-0.0003
(p-value)	*(0.1346)*	*(0.3913)*	*(0.7197)*	*(0.3038)*	*(0.5906)*	*(0.4322)*	*(0.1339)*
Rm-Rf	0.9860	0.9868	0.9917	0.9901	0.9774	0.6704	0.9504
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2138	-0.2060	-0.1204	-0.1435	-0.1478	-0.0228	-0.1496
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.6845)*	*(0.0000)*
HML	-0.0427	-0.0472	-0.0503	-0.0464	-0.0395	0.0447	-0.0508
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0001)*	*(0.0000)*	*(0.0000)*	*(0.4333)*	*(0.0000)*
RtoTR	-0.0002	0.0000	0.0000	-0.0003	-0.0002	0.0030	0.0002
(p-value)	*(0.0031)*	*(0.9419)*	*(0.8451)*	*(0.0144)*	*(0.4321)*	*(0.0949)*	*(0.1501)*
Adj. R-squared	0.9977	0.9971	0.9925	0.9980	0.9975	0.6824	0.9579
Durbin-Watson	1.9976	2.0417	2.1844	2.1894	2.0930	2.0482	2.0121
Small Caps							
C	-0.0001	-0.0006	-0.0010	0.0004	-0.0003	-0.0007	-0.0003
(p-value)	*(0.7133)*	*(0.1476)*	*(0.2949)*	*(0.5034)*	*(0.6730)*	*(0.4876)*	*(0.2867)*
Rm-Rf	0.9249	0.8761	0.7434	0.7996	0.9617	0.6441	0.8443
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	1.0718	0.9694	0.8050	0.8963	0.8681	1.2143	0.9299
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	0.2092	0.2529	0.2979	0.3116	0.2654	0.2555	0.3441
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0001)*	*(0.0000)*
RtoTR	0.0005	0.0018	0.0009	0.0011	0.0000	0.0026	0.0006
(p-value)	*(0.0792)*	*(0.0006)*	*(0.2384)*	*(0.0997)*	*(0.9490)*	*(0.0824)*	*(0.0221)*
Adj. R-squared	0.9572	0.9067	0.8466	0.9115	0.9675	0.8115	0.9014
Durbin-Watson	1.9378	1.9029	1.5833	1.7553	2.1711	1.9212	1.8916

4.3.2.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.2.1

Heteroscedasticity Test (Arch) on Model 2. Dependent variable: Resid^2				
Big Caps	F-statistic	0.0010	Prob. F(1,1563)	0.9744
	Obs*R-squared	0.0010	Prob. Chi-Square(1)	0.9743
Small Caps	F-statistic	0.0515	Prob. F(1,1563)	0.8205
	Obs*R-squared	0.0516	Prob. Chi-Square(1)	0.8204

Taking into account the p-values shown in Table 4.3.2.1, the null hypothesis of absence of heteroscedasticity is not rejected; therefore, we assume homoscedasticity of the error terms for our model and ensure that the least-squares estimators are each a best linear unbiased estimator of the respective population parameter.

4.3.2.2. Error Term Analysis – Autocorrelation Test

Table 4.3.2.2

Breusch-Godfrey Serial Correlation LM Test on Model 2

	Big Caps		**Small Caps**	
	F-statistic	**p-value**	**F-statistic**	**p-value**
1987-1991 (Period 1)	0.496529	0.6092	1.14105	0.3211
1992-1996 (Period 2)	0.325159	0.7227	1.701849	0.1844
1997-2001 (Period 3)	1.400999	0.2482	7.962557	0.0004
2002-2006 (Period 4)	1.407972	0.2465	1.368483	0.2564
2007-2011 (Period 5)	1.188237	0.3064	1.158075	0.3157
2012-2016 (Period 6)	0.081895	0.9214	0.188493	0.8283
1987-2016 (Total)	0.051132	0.9502	4.064035	0.0174

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis for the Small Caps and have to adjust the model for the periods where it is detected (period 3 and the 30-year period).

4.3.2.3. Adjusted Model

Table 4.3.2.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \omega_p * RtoTR_{pt} + \varepsilon_t$

	1987-1991 (Period 1)	1992-1996 (Period 2)	1997-2001 (Period 3)	2002-2006 (Period 4)	2007-2011 (Period 5)	2012-2016 (Period 6)	1987-2016 (Total)
Big Caps							
C	0.0002	-0.0001	0.0000	0.0001	-0.0004	-0.0007	-0.0003
(p-value)	*(0.1346)*	*(0.3913)*	*(0.9493)*	*(0.3038)*	*(0.0079)*	*(0.4322)*	*(0.1489)*
Rm-Rf	0.9860	0.9868	0.9892	0.9901	0.9837	0.6704	0.9447
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2138	-0.2060	-0.1477	-0.1435	-0.1397	-0.0228	-0.1634
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.6845)*	*(0.0000)*
HML	-0.0427	-0.0472	-0.0439	-0.0464	-0.0269	0.0447	-0.0387
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0043)*	*(0.0000)*	*(0.0001)*	*(0.4333)*	*(0.0004)*
RtoTR	-0.0002	0.0000	0.0000	-0.0003	0.0004	0.0030	0.0003
(p-value)	*(0.0031)*	*(0.9419)*	*(0.9520)*	*(0.0144)*	*(0.1253)*	*(0.0949)*	*(0.1321)*
Adj. R-squared	0.9977	0.9971	0.9912	0.9980	0.9978	0.6824	0.9484
Durbin-Watson	1.9976	2.0417	2.0300	2.1894	2.0058	2.0482	2.0039
Small Caps							
C	-0.0001	-0.0006	**-0.0023**	0.0004	-0.0002	-0.0007	**-0.0004**
(p-value)	*(0.7133)*	*(0.1476)*	***(0.0132)***	*(0.5034)*	*(0.7682)*	*(0.4876)*	***(0.2563)***
Rm-Rf	0.9249	0.8761	**0.7948**	0.7996	0.9022	0.6441	**0.8305**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
SMB	1.0718	0.9694	**0.8857**	0.8963	0.9534	1.2143	**0.9796**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
HML	0.2092	0.2529	**0.3308**	0.3116	0.2799	0.2555	**0.2906**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0001)*	***(0.0000)***
RtoTR	0.0005	0.0018	**0.0024**	0.0011	0.0006	0.0026	**0.0010**
(p-value)	*(0.0792)*	*(0.0006)*	***(0.0223)***	*(0.0997)*	*(0.2203)*	*(0.0824)*	***(0.0356)***
Adj. R-squared	0.9572	0.9067	**0.8523**	0.9115	0.9681	0.8115	**0.9002**
Durbin-Watson	1.9378	1.9029	**1.3881**	1.7553	2.0019	1.9212	**1.8087**

When applying the last model to our data we came across with a strong explanation of the dependent variable (94.84% for big caps and 90.02% for small caps).

The estimated coefficients for the market premium (the portfolio beta) are statistically significant at the 1% level for Small and Big Caps portfolios in all the periods analyzed, observing also that the coefficients for the Big Caps are close to 1 for almost all periods; this is intuitive according to the CAPM (Sharpe, 1964) given that the Big Caps represent more than 70% (78.43% in average for the period comprised between 1987 and 2016) of the market regarding total market value (taking into account the segmentation we execute in this dissertation, which is equivalent to the first decile of companies ranked by market value) and therefore the movement of the market is closely related to them.

The coefficients for the size factor (SMB) are statistically significant at the 1% level for Small Caps in all periods and for Big Caps in 5 out of 6 sub periods. The value of the coefficients shows a clear pattern that are consistent with the model and suggests the existence of a return premium inversely correlated to the firm size (Fama & French, 1993), given that these coefficients are big and positive for the Small Caps (average of 0.98) and negative for Big Caps (average of -0.17 for the periods with demonstrated statistical significance).

The value factor (HML) plays also an important role in the model, proving to be statistically significant at the 1% level for Small Caps in all six five-year periods and in the case of the Big Caps, the coefficients are statistically (also at the 1% level) significant in 5 out of 6 periods. The value of the coefficients in the Small Caps portfolio suggests that the value factor is positively related to the excess return (average equal to 0.29) and negatively related to Big Caps (average of -0.05 for the periods 1 to 5, for which we have significant coefficients).

We observe a statistically significant coefficient for the liquidity factor RtoTR for almost all sub periods and for the entire 28-year period (without crisis years) when analyzing small caps. This result shows a positive coefficient for the existing liquidity factor on the small caps, while for the big caps it is not statistically significant.

From this result, we find that the evidence from the data is consistent with the work done by Florackis et al. (2011), which proposed RtoTR as an alternative measure for Amihud's Illiquidity (given that it is considered size biased by the author) and confirms alternative hypothesis.

4.3.3. Model 3 – Regression

In Table 4.3.3 we observe the results of the regression applied using Model 3 on the data, where the liquidity factor analyzed is TOR and the dependent variable is the excess return. Each analyzed portfolio is modelled.

Table 4.3.3

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \gamma_p * TOR_{pt} + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	-0.0002	-0.0005	0.0008	-0.0004	0.0002	0.0003	0.0001
(p-value)	*(0.4134)*	*(0.0086)*	*(0.1383)*	*(0.1694)*	*(0.6469)*	*(0.9314)*	*(0.5937)*
Rm-Rf	0.9860	0.9866	0.9920	0.9896	0.9764	0.6715	0.9510
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2103	-0.2046	-0.1210	-0.1430	-0.1465	-0.0125	-0.1515
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.8239)*	*(0.0000)*
HML	-0.0448	-0.0461	-0.0490	-0.0432	-0.0391	0.0492	-0.0516
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0001)*	*(0.0000)*	*(0.0000)*	*(0.3928)*	*(0.0000)*
TOR	0.0102	0.0337	-0.0349	0.0118	-0.0071	0.0069	-0.0080
(p-value)	*(0.6498)*	*(0.0169)*	*(0.0983)*	*(0.2477)*	*(0.3354)*	*(0.9390)*	*(0.3356)*
Adj. R-squared	0.9976	0.9972	0.9926	0.9980	0.9975	0.6789	0.9579
Durbin-Watson	1.9750	2.0410	2.2147	2.1642	2.0958	2.0478	2.0142
Small Caps							
C	-0.0019	-0.0027	-0.0063	0.0023	0.0030	0.0054	0.0003
(p-value)	*(0.0995)*	*(0.0051)*	*(0.0001)*	*(0.0709)*	*(0.1187)*	*(0.0649)*	*(0.5605)*
Rm-Rf	0.9215	0.8586	0.7247	0.8047	0.9552	0.6433	0.8439
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	1.0716	0.9420	0.7820	0.8998	0.8785	1.2321	0.9309
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	0.2217	0.2399	0.2762	0.3057	0.2715	0.2841	0.3458
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
TOR	0.1667	0.1721	0.2839	-0.0432	-0.0959	-0.1367	-0.0058
(p-value)	*(0.0437)*	*(0.0007)*	*(0.0000)*	*(0.3464)*	*(0.0790)*	*(0.0969)*	*(0.7591)*
Adj. R-squared	0.9574	0.9066	0.8559	0.9109	0.9679	0.8114	0.9010
Durbin-Watson	1.9485	1.8326	1.6103	1.7183	2.1898	1.9390	1.8882

4.3.3.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.3.1

Heteroscedasticity Test (Arch) on Model 3. Dependent variable: Resid^2

Big Caps	F-statistic	0.0010	Prob. F(1,1563)	0.9746
	Obs*R-squared	0.0010	Prob. Chi-Square(1)	0.9745
Small Caps	F-statistic	0.0578	Prob. F(1,1563)	0.8101
	Obs*R-squared	0.0578	Prob. Chi-Square(1)	0.8100

Taking into account the p-values shown in Table 4.3.3.1, the null hypothesis of absence of heteroscedasticity is not rejected; therefore, we assume homoscedasticity of the error terms for our model and ensure that the least-squares estimators are each a best linear unbiased estimator of the respective population parameter.

4.3.3.2. Error Term Analysis – Autocorrelation Test

Table 4.3.3.2

Breusch-Godfrey Serial Correlation LM Test on Model 3

	Big Caps		**Small Caps**	
	F-statistic	**p-value**	**F-statistic**	**p-value**
1987-1991 (Period 1)	0.424871	0.6543	0.80415	0.4486
1992-1996 (Period 2)	0.378016	0.6856	1.977379	0.1406
1997-2001 (Period 3)	1.784976	0.1699	6.585322	0.0016
2002-2006 (Period 4)	1.345754	0.2622	1.72957	0.1794
2007-2011 (Period 5)	1.207801	0.3006	1.218707	0.2973
2012-2016 (Period 6)	0.093557	0.9107	0.119901	0.8871
1987-2016 (Total)	0.056382	0.9452	4.41779	0.0122

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis for the Small Caps and have to adjust the model for the periods where it is detected (period 3 and the 30-year period).

4.3.3.3. Adjusted Model

Table 4.3.3.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \gamma_p * TOR_{pt} + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	-0.0002	-0.0005	0.0011	-0.0004	0.0006	0.0003	0.0001
(p-value)	*(0.4134)*	*(0.0086)*	*(0.0965)*	*(0.1694)*	*(0.0872)*	*(0.9314)*	*(0.5847)*
Rm-Rf	0.9860	0.9866	0.9878	0.9896	0.9845	0.6715	0.9455
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2103	-0.2046	-0.1474	-0.1430	-0.1414	-0.0125	-0.1658
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.8239)*	*(0.0000)*
HML	-0.0448	-0.0461	-0.0463	-0.0432	-0.0265	0.0492	-0.0397
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0024)*	*(0.0000)*	*(0.0001)*	*(0.3928)*	*(0.0003)*
TOR	0.0102	0.0337	-0.0503	0.0118	-0.0174	0.0069	-0.0085
(p-value)	*(0.6498)*	*(0.0169)*	*(0.0754)*	*(0.2477)*	*(0.0166)*	*(0.9390)*	*(0.3748)*
Adj. R-squared	0.9976	0.9972	0.9913	0.9980	0.9979	0.6789	0.9484
Durbin-Watson	1.9750	2.0410	2.0565	2.1642	2.0646	2.0478	2.0062
Small Caps							
C	-0.0019	-0.0027	**-0.0062**	0.0023	0.0021	0.0054	**0.0006**
(p-value)	*(0.0995)*	*(0.0051)*	***(0.0198)***	*(0.0709)*	*(0.2716)*	*(0.0649)*	***(0.3196)***
Rm-Rf	0.9215	0.8586	**0.7684**	0.8047	0.9019	0.6433	**0.8298**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
SMB	1.0716	0.9420	**0.8565**	0.8998	0.9556	1.2321	**0.9820**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
HML	0.2217	0.2399	**0.2971**	0.3057	0.2822	0.2841	**0.2928**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0001)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
TOR	0.1667	0.1721	**0.3049**	-0.0432	-0.0514	-0.1367	**-0.0071**
(p-value)	*(0.0437)*	*(0.0007)*	***(0.0390)***	*(0.3464)*	*(0.3500)*	*(0.0969)*	***(0.7404)***
Adj. R-squared	0.9574	0.9066	**0.8492**	0.9109	0.9680	0.8114	**0.8991**
Durbin-Watson	1.9485	1.8326	**1.3479**	1.7183	1.9737	1.9390	**1.7864**

The model shows a strong explanation of the dependent variable (94.8% for Big Caps and 89.9% for Small Caps), and while it shows statistical significance for almost all variables, it is not the case for the spread on the liquidity factor (ILLIQ) in the majority of sub-periods for Big Caps and for none of the 28-year periods, which is a sign of lack of robustness of the model.

4.3.4. Model 4 – Regression

In Table 4.3.4 we observe the results of the regression applied using Model 4 on the data, where the dependent variable is the spread between the Big and Small Caps portfolio return and the liquidity factor analyzed is the difference between the Big and Small Caps portfolios ILLIQ.

Table 4.3.4

Model Equation: $R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \psi * (ILLIQ_{bt} - ILLIQ_{st}) + \varepsilon_t$

	1987-1991 (Period 1)	1992-1996 (Period 2)	1997-2001 (Period 3)	2002-2006 (Period 4)	2007-2011 (Period 5)	2012-2016 (Period 6)	1987-2016 (Total)
C	0.0000	0.0002	0.0019	-0.0006	0.0010	0.0001	0.0001
(p-value)	*(0.9244)*	*(0.7303)*	*(0.1791)*	*(0.3748)*	*(0.1200)*	*(0.8289)*	*(0.5948)*
Rm-Rf	0.0648	0.1153	0.2527	0.1825	0.0162	0.0259	0.1073
(p-value)	*(0.0001)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.3631)*	*(0.0804)*	*(0.0000)*
SMB	-1.2837	-1.1713	-0.9238	-1.0445	-1.0200	-1.2477	-1.0832
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	-0.2617	-0.2929	-0.3452	-0.3521	-0.3028	-0.2263	-0.3979
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
Diff ILLIQ	0.0009	0.0022	0.0065	0.0038	0.0060	0.0053	0.0014
(p-value)	*(0.2378)*	*(0.1291)*	*(0.1546)*	*(0.1684)*	*(0.0483)*	*(0.4715)*	*(0.0460)*
Adj. R-squared	0.9300	0.8956	0.7847	0.7948	0.7721	0.8913	0.8025
Durbin-Watson	1.9599	1.8870	1.5694	1.7450	2.1945	2.0064	1.9239

4.3.4.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.4.1

Heteroscedasticity Test (Arch) on Model 4. Dependent variable: Resid^2			
F-statistic	78.1001	Prob. F(1,1563)	0.0000
Obs*R-squared	74.4785	Prob. Chi-Square(1)	0.0000

Taking into account the p-values shown in Table 4.3.4.1, the null hypothesis of absence of heteroscedasticity is rejected; thus, we assume heteroscedasticity of the error terms for our model and cannot guarantee that the least-squares estimators the best linear unbiased estimator of the respective population parameter.

4.3.4.2. Error Term Analysis – Autocorrelation Test

Table 4.3.4.2

Breusch-Godfrey Serial Correlation LM Test on Model 4

	Big Caps	
	F-statistic	**p-value**
1987-1991 (Period 1)	1.258548	0.2858
1992-1996 (Period 2)	1.456963	0.2349
1997-2001 (Period 3)	9.366597	0.0001
2002-2006 (Period 4)	1.4435	0.238
2007-2011 (Period 5)	1.578381	0.2083
2012-2016 (Period 6)	0.014858	0.9853
1987-2016 (Total)	4.617743	0.01

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis and have to adjust the model for all the periods, since in the previous point we also found heteroscedasticity of the error terms.

4.3.4.3. Adjusted Model

Table 4.3.4.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \psi * (ILLIQ_{bt} - ILLIQ_{st}) + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
C	**0.0000**	**0.0002**	**0.0033**	**-0.0006**	**0.0006**	**0.0001**	**-0.0001**
(p-value)	***(0.9006)***	***(0.7524)***	***(0.0391)***	***(0.4174)***	***(0.2902)***	***(0.7976)***	***(0.7539)***
Rm-Rf	**0.0648**	**0.1153**	**0.2082**	**0.1825**	**0.0807**	**0.0259**	**0.1156**
(p-value)	***(0.0185)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0003)***	***(0.0682)***	***(0.0000)***
SMB	**-1.2837**	**-1.1713**	**-1.0247**	**-1.0445**	**-1.0966**	**-1.2477**	**-1.1486**
(p-value)	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***
HML	**-0.2617**	**-0.2929**	**-0.3695**	**-0.3521**	**-0.3058**	**-0.2263**	**-0.3333**
(p-value)	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***
Diff ILLIQ	**0.0009**	**0.0022**	**0.0122**	**0.0038**	**0.0084**	**0.0053**	**0.0015**
(p-value)	***(0.1562)***	***(0.2181)***	***(0.0526)***	***(0.3556)***	***(0.0134)***	***(0.3476)***	***(0.0192)***
Adj. R-squared	**0.9300**	**0.8956**	**0.8183**	**0.7948**	**0.8078**	**0.8913**	**0.8390**
Durbin-Watson	**1.9599**	**1.8870**	**1.3356**	**1.7450**	**2.0520**	**2.0064**	**1.7691**

The model shows a strong explanation of the dependent variable (83.9%), and while it shows statistical significance for almost all variables, it is not the case for the spread on the liquidity factor (ILLIQ) in the majority of sub-periods, which is a sign of lack of robustness of the model.

4.3.5. Model 5 – Regression

In Table 4.3.5 we observe the results of the regression applied using Model 5 on the data, where the dependent variable is the spread between the Big and Small Caps portfolio return and the liquidity factor analyzed is the difference between the Big and Small Caps portfolios RtoTR.

Table 4.3.5

Model Equation: $R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \delta * (RtoTR_{bt} - RtoTR_{st}) + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
C	-0.0007	-0.0008	0.0001	-0.0009	0.0005	-0.0001	-0.0002
(p-value)	*(0.0234)*	*(0.0042)*	*(0.8485)*	*(0.0375)*	*(0.3354)*	*(0.7244)*	*(0.2401)*
Rm-Rf	0.0597	0.1042	0.2396	0.1897	0.0160	0.0262	0.1033
(p-value)	*(0.0004)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.3709)*	*(0.0770)*	*(0.0000)*
SMB	-1.2755	-1.1606	-0.9110	-1.0362	-1.0160	-1.2447	-1.0727
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	-0.2588	-0.2887	-0.3397	-0.3538	-0.3038	-0.2242	-0.3912
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
Diff RtoTR	0.0006	0.0012	0.0015	0.0021	0.0009	0.0006	0.0011
(p-value)	*(0.0418)*	*(0.0034)*	*(0.0461)*	*(0.0135)*	*(0.1577)*	*(0.4853)*	*(0.0000)*
Adj. R-squared	0.9308	0.8982	0.7863	0.7981	0.7704	0.8913	0.8045
Durbin-Watson	1.9467	1.9199	1.6213	1.7658	2.1667	1.9964	1.9239

4.3.5.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.5.1

Heteroscedasticity Test (Arch) on Model 5. Dependent variable: Resid^2			
F-statistic	70.1905	Prob. F(1,1563)	0.0000
Obs*R-squared	67.2598	Prob. Chi-Square(1)	0.0000

Taking into account the p-values shown in Table 4.3.5.1, the null hypothesis of absence of heteroscedasticity is rejected; thus, we assume heteroscedasticity of the error terms for our model and cannot guarantee that the least-squares estimators the best linear unbiased estimator of the respective population parameter.

4.3.5.2. Error Term Analysis – Autocorrelation Test

Table 4.3.5.2

Breusch-Godfrey Serial Correlation LM Test on Model 5

	Big Caps	
	F-statistic	**p-value**
1987-1991 (Period 1)	1.657677	0.1926
1992-1996 (Period 2)	1.126055	0.3259
1997-2001 (Period 3)	7.400287	0.0008
2002-2006 (Period 4)	1.421285	0.2433
2007-2011 (Period 5)	1.476555	0.2304
2012-2016 (Period 6)	0.002508	0.9975
1987-2016 (Total)	4.217299	0.0149

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis and have to adjust the model for all the periods, since in the previous point we also found heteroscedasticity of the error terms.

4.3.5.3. Adjusted Model

Table 4.3.5.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \delta * (RtoTR_{bt} - RtoTR_{st}) + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
C	**-0.0007**	**-0.0008**	**0.0000**	**-0.0009**	**0.0000**	**-0.0001**	**-0.0005**
(p-value)	***(0.0505)***	***(0.0060)***	***(0.9645)***	***(0.0357)***	***(0.9933)***	***(0.7062)***	***(0.0153)***
Rm-Rf	**0.0597**	**0.1042**	**0.1796**	**0.1897**	**0.0810**	**0.0262**	**0.1108**
(p-value)	***(0.0355)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0001)***	***(0.0654)***	***(0.0000)***
SMB	**-1.2755**	**-1.1606**	**-0.9952**	**-1.0362**	**-1.0896**	**-1.2447**	**-1.1338**
(p-value)	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***
HML	**-0.2588**	**-0.2887**	**-0.3593**	**-0.3538**	**-0.3058**	**-0.2242**	**-0.3260**
(p-value)	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***
Diff RtoTR	**0.0006**	**0.0012**	**0.0030**	**0.0021**	**0.0014**	**0.0006**	**0.0013**
(p-value)	***(0.1608)***	***(0.0110)***	***(0.0097)***	***(0.0725)***	***(0.2036)***	***(0.4914)***	***(0.0002)***
Adj. R-squared	**0.9308**	**0.8982**	**0.8269**	**0.7981**	**0.8043**	**0.8913**	**0.8423**
Durbin-Watson	**1.9467**	**1.9199**	**1.4081**	**1.7658**	**1.9971**	**1.9964**	**1.7883**

When applying the last model to our data we came across with a strong explanation of the dependent variable (84.23%).

The estimated coefficients for the market premium (the portfolio beta) are statistically significant at the 1% level in almost all the periods analyzed and are statistically significant at the 10% level for all periods.

The coefficients for the size factor (SMB) and value factor (HML) are statistically significant at the 1% level in all sub periods.

We observe a statistically significant coefficient for the liquidity factor (represented as the difference between the RtoTR for Big and Small Caps) for almost half of the sub periods and for the entire 28-year period (without crisis years).

This result shows evidence of a relation between the spread in the return of portfolios and the difference in their liquidity factors.

4.3.6. Model 6 – Regression

In Table 4.3.6 we observe the results of the regression applied using Model 6 on the data, where the dependent variable is the spread between the Big and Small Caps portfolio return and the liquidity factor analyzed is the difference between the Big and Small Caps portfolios TOR.

Table 4.3.6

Model Equation: $R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \phi * (TOR_{bt} - TOR_{st}) + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
C	-0.0003	0.0008	-0.0006	-0.0013	0.0003	-0.0002	-0.0003
(p-value)	*(0.3451)*	*(0.0469)*	*(0.3759)*	*(0.0023)*	*(0.6502)*	*(0.4976)*	*(0.1070)*
Rm-Rf	0.0674	0.1361	0.2650	0.1875	0.0150	0.0263	0.1084
(p-value)	*(0.0001)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.4018)*	*(0.0771)*	*(0.0000)*
SMB	-1.2708	-1.1176	-0.9043	-1.0422	-1.0156	-1.2476	-1.0796
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	-0.2683	-0.2702	-0.3403	-0.3336	-0.3047	-0.2281	-0.3964
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
Diff TOR	0.1629	0.2949	0.2539	0.0431	-0.0139	-0.0087	0.0454
(p-value)	*(0.1359)*	*(0.0001)*	*(0.0001)*	*(0.4250)*	*(0.6870)*	*(0.8118)*	*(0.0099)*
Adj. R-squared	0.9302	0.9011	0.7951	0.7937	0.7687	0.8911	0.8028
Durbin-Watson	1.9363	1.8917	1.5871	1.7103	2.1882	2.0085	1.9148

4.3.6.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.6.1

Heteroscedasticity Test (Arch) on Model 6. Dependent variable: Resid^2			
F-statistic	82.1074	Prob. F(1,1563)	0.0000
Obs*R-squared	78.1092	Prob. Chi-Square(1)	0.0000

Taking into account the p-values shown in Table 4.3.6.1, the null hypothesis of absence of heteroscedasticity is rejected; thus, we assume heteroscedasticity of the error terms for our model and cannot guarantee that the least-squares estimators the best linear unbiased estimator of the respective population parameter.

4.3.6.2. Error Term Analysis – Autocorrelation Test

Table 4.3.6.2

Breusch-Godfrey Serial Correlation LM Test on Model 6

	Big Caps	
	F-statistic	**p-value**
1987-1991 (Period 1)	0.686251	0.5044
1992-1996 (Period 2)	1.213978	0.2987
1997-2001 (Period 3)	8.004917	0.0004
2002-2006 (Period 4)	2.045635	0.1314
2007-2011 (Period 5)	1.933772	0.1467
2012-2016 (Period 6)	0.01782	0.9823
1987-2016 (Total)	5.388736	0.0047

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis and have to adjust the model for all the periods, since in the previous point we also found heteroscedasticity of the error terms.

4.3.6.3. Adjusted Model

Table 4.3.6.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{bt} - R_{st} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \phi * (TOR_{bt} - TOR_{st}) + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	**-0.0003**	**0.0008**	**-0.0001**	**-0.0013**	**0.0005**	**-0.0002**	**-0.0005**
(p-value)	***(0.3342)***	***(0.0430)***	***(0.9075)***	***(0.0038)***	***(0.3632)***	***(0.4552)***	***(0.0170)***
Rm-Rf	**0.0674**	**0.1361**	**0.2159**	**0.1875**	**0.0811**	**0.0263**	**0.1157**
(p-value)	***(0.0161)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0001)***	***(0.0656)***	***(0.0000)***
SMB	**-1.2708**	**-1.1176**	**-1.0188**	**-1.0422**	**-1.0970**	**-1.2476**	**-1.1476**
(p-value)	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***
HML	**-0.2683**	**-0.2702**	**-0.3441**	**-0.3336**	**-0.3065**	**-0.2281**	**-0.3323**
(p-value)	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***	***(0.0000)***
Diff TOR	**0.1629**	**0.2949**	**0.0762**	**0.0431**	**-0.0783**	**-0.0087**	**-0.0043**
(p-value)	***(0.2242)***	***(0.0011)***	***(0.3517)***	***(0.5298)***	***(0.0373)***	***(0.7982)***	***(0.8855)***
Adj. R-squared	**0.9302**	**0.9011**	**0.8121**	**0.7937**	**0.8062**	**0.8911**	**0.8384**
Durbin-Watson	**1.9363**	**1.8917**	**1.3076**	**1.7103**	**2.0345**	**2.0085**	**1.7627**

The model shows a strong explanation of the dependent variable (83.8%), and while it shows statistical significance for almost all variables, it is not the case for the spread on the liquidity factor (TOR) in the majority of sub-periods and in the 28-year period, which is a sign of lack of robustness of the model.

4.3.7. Model 7 – Regression

In Table 4.3.7 we observe the results of the regression applied using Model 7 on the data, where the liquidity factor analyzed is the market ILLIQ (represented as Mkt ILLIQ) and the dependent variable is the excess return. Each analyzed portfolio is modelled.

Table 4.3.7

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + \ l * Mkt\_ILLIQ_t + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	0.0000	0.0000	0.0003	-0.0001	0.0002	0.0001	-0.0001
(p-value)	*(0.9131)*	*(0.8095)*	*(0.2723)*	*(0.4616)*	*(0.1294)*	*(0.9330)*	*(0.6153)*
Rm-Rf	0.9856	0.9870	0.9912	0.9896	0.9772	0.6717	0.9512
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2112	-0.2060	-0.1209	-0.1435	-0.1484	-0.0127	-0.1514
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.8213)*	*(0.0000)*
HML	-0.0454	-0.0471	-0.0523	-0.0450	-0.0404	0.0492	-0.0514
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.3903)*	*(0.0000)*
Mkt ILLIQ	-0.0037	-0.0039	-0.0192	-0.0007	-0.0357	0.0917	0.0006
(p-value)	*(0.2635)*	*(0.3152)*	*(0.1293)*	*(0.8171)*	*(0.0007)*	*(0.6288)*	*(0.9397)*
Adj. R-squared	0.9976	0.9971	0.9926	0.9980	0.9976	0.6792	0.9579
Durbin-Watson	1.9983	2.0444	2.2087	2.1754	2.1558	2.0486	2.0133
Small Caps							
C	0.0001	0.0000	-0.0023	0.0007	-0.0008	0.0003	-0.0004
(p-value)	*(0.9186)*	*(0.9999)*	*(0.0532)*	*(0.1941)*	*(0.2327)*	*(0.7903)*	*(0.2073)*
Rm-Rf	0.9214	0.8720	0.7426	0.8054	0.9614	0.6455	0.8443
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	1.0715	0.9658	0.8019	0.9013	0.8697	1.2347	0.9314
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	0.2162	0.2484	0.3008	0.3111	0.2661	0.2775	0.3476
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
Mkt ILLIQ	0.0092	0.0212	0.1083	0.0269	0.0489	0.0863	0.0298
(p-value)	*(0.4838)*	*(0.3279)*	*(0.0347)*	*(0.2303)*	*(0.3101)*	*(0.6786)*	*(0.0161)*
Adj. R-squared	0.9568	0.9026	0.8485	0.9111	0.9676	0.8094	0.9014
Durbin-Watson	1.9320	1.8304	1.5601	1.7188	2.1849	1.9144	1.8938

4.3.7.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.7.1

Heteroscedasticity Test (Arch) on Model 7. Dependent variable: Resid^2

Big Caps	F-statistic	0.0010	Prob. F(1,1563)	0.9746
	Obs*R-squared	0.0010	Prob. Chi-Square(1)	0.9745
Small Caps	F-statistic	0.0487	Prob. F(1,1563)	0.8253
	Obs*R-squared	0.0488	Prob. Chi-Square(1)	0.8252

Taking into account the p-values shown in Table 4.3.7.1, the null hypothesis of absence of heteroscedasticity is not rejected; therefore, we assume homoscedasticity of the error terms for our model and ensure that the least-squares estimators are each a best linear unbiased estimator of the respective population parameter.

4.3.7.2. Error Term Analysis – Autocorrelation Test

Table 4.3.7.2

Breusch-Godfrey Serial Correlation LM Test on Model 7

	Big Caps		**Small Caps**	
	F-statistic	**p-value**	**F-statistic**	**p-value**
1987-1991 (Period 1)	0.184652	0.8315	0.857029	0.4256
1992-1996 (Period 2)	0.405733	0.6669	1.782626	0.1703
1997-2001 (Period 3)	1.663585	0.1915	9.227192	0.0001
2002-2006 (Period 4)	1.493384	0.2266	1.72544	0.1802
2007-2011 (Period 5)	1.060543	0.3478	1.165194	0.3135
2012-2016 (Period 6)	0.095714	0.9088	0.227232	0.7969
1987-2016 (Total)	0.055815	0.9457	3.896613	0.0205

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis for the Small Caps and have to adjust the model for the periods where it is detected (period 3 and the 30-year period).

4.3.7.3. Adjusted Model

Table 4.3.7.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + l * Mkt\_ILLIQ_t + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	0.0000	0.0000	0.0003	-0.0001	0.0000	0.0001	-0.0001
(p-value)	*(0.9131)*	*(0.8095)*	*(0.2848)*	*(0.4616)*	*(0.8447)*	*(0.9330)*	*(0.5959)*
Rm-Rf	0.9856	0.9870	0.9885	0.9896	0.9855	0.6717	0.9456
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2112	-0.2060	-0.1469	-0.1435	-0.1423	-0.0127	-0.1657
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.8213)*	*(0.0000)*
HML	-0.0454	-0.0471	-0.0466	-0.0450	-0.0280	0.0492	-0.0393
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0023)*	*(0.0000)*	*(0.0000)*	*(0.3903)*	*(0.0003)*
Mkt ILLIQ	-0.0037	-0.0039	-0.0191	-0.0007	-0.0225	0.0917	0.0019
(p-value)	*(0.2635)*	*(0.3152)*	*(0.1550)*	*(0.8171)*	*(0.0326)*	*(0.6288)*	*(0.8040)*
Adj. R-squared	0.9976	0.9971	0.9913	0.9980	0.9979	0.6792	0.9484
Durbin-Watson	1.9983	2.0444	2.0484	2.1754	2.1411	2.0486	2.0057
Small Caps							
C	0.0001	0.0000	**-0.0029**	0.0007	-0.0010	0.0003	**-0.0001**
(p-value)	*(0.9186)*	*(0.9999)*	***(0.0459)***	*(0.1941)*	*(0.1494)*	*(0.7903)*	***(0.5711)***
Rm-Rf	0.9214	0.8720	**0.7825**	0.8054	0.9046	0.6455	**0.8304**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
SMB	1.0715	0.9658	**0.8723**	0.9013	0.9527	1.2347	**0.9823**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
HML	0.2162	0.2484	**0.3271**	0.3111	0.2826	0.2775	**0.2953**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
Mkt ILLIQ	0.0092	0.0212	**0.1394**	0.0269	0.1281	0.0863	**0.0300**
(p-value)	*(0.4838)*	*(0.3279)*	***(0.0578)***	*(0.2303)*	*(0.0125)*	*(0.6786)*	***(0.0146)***
Adj. R-squared	0.9568	0.9026	**0.8507**	0.9111	0.9688	0.8094	**0.8996**
Durbin-Watson	1.9320	1.8304	**1.3393**	1.7188	2.0860	1.9144	**1.7960**

The model shows a strong explanation of the dependent variable (94.8% for Big Caps and 90.0% for Small Caps), and while it shows statistical significance for almost all variables, it is not the case for the liquidity factor (Market ILLIQ).

4.3.8. Model 8 – Regression

In Table 4.3.8 we observe the results of the regression applied using Model 8 on the data, where the liquidity factor analyzed is the market RtoTR (represented as Mkt RtoTR) and the dependent variable is the excess return. Each analyzed portfolio is modelled.

Table 4.3.8

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + r * Mkt\_RtoTR_t + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	0.0002	0.0000	0.0001	0.0001	0.0000	-0.0008	-0.0002
(p-value)	*(0.0543)*	*(0.9394)*	*(0.7949)*	*(0.2073)*	*(0.8107)*	*(0.4156)*	*(0.2291)*
Rm-Rf	0.9856	0.9873	0.9922	0.9903	0.9775	0.6705	0.9508
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2142	-0.2068	-0.1222	-0.1433	-0.1480	-0.0246	-0.1503
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.6628)*	*(0.0000)*
HML	-0.0424	-0.0478	-0.0518	-0.0466	-0.0393	0.0426	-0.0511
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.4559)*	*(0.0000)*
Mkt RtoTR	-0.0003	-0.0001	-0.0001	-0.0003	-0.0003	0.0033	0.0002
(p-value)	*(0.0007)*	*(0.4020)*	*(0.5807)*	*(0.0088)*	*(0.2458)*	*(0.0991)*	*(0.3063)*
Adj. R-squared	0.9977	0.9971	0.9925	0.9980	0.9976	0.6823	0.9579
Durbin-Watson	2.0247	2.0227	2.1995	2.1933	2.0965	2.0504	2.0124
Small Caps							
C	0.0006	0.0004	0.0000	0.0013	0.0006	-0.0007	0.0003
(p-value)	*(0.1958)*	*(0.3464)*	*(0.9751)*	*(0.0363)*	*(0.4186)*	*(0.5351)*	*(0.3860)*
Rm-Rf	0.9208	0.8722	0.7388	0.8040	0.9643	0.6443	0.8443
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	1.0689	0.9670	0.8003	0.9002	0.8639	1.2222	0.9297
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	0.2163	0.2496	0.2906	0.2985	0.2679	0.2706	0.3455
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
Mkt RtoTR	-0.0002	0.0000	-0.0002	-0.0003	-0.0018	0.0034	-0.0002
(p-value)	*(0.4966)*	*(0.9245)*	*(0.8770)*	*(0.7667)*	*(0.1140)*	*(0.1162)*	*(0.6297)*
Adj. R-squared	0.9568	0.9023	0.8458	0.9106	0.9678	0.8111	0.9011
Durbin-Watson	1.9143	1.8335	1.5669	1.7002	2.1662	1.9152	1.8870

4.3.8.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.8.1

Heteroscedasticity Test (Arch) on Model 8. Dependent variable: Resid^2

Big Caps	F-statistic	0.0010	Prob. F(1,1563)	0.9744
	Obs*R-squared	0.0010	Prob. Chi-Square(1)	0.9744
Small Caps	F-statistic	0.0578	Prob. F(1,1563)	0.8100
	Obs*R-squared	0.0579	Prob. Chi-Square(1)	0.8099

Taking into account the p-values shown in Table 4.3.8.1, the null hypothesis of absence of heteroscedasticity is not rejected; therefore, we assume homoscedasticity of the error terms for our model and ensure that the least-squares estimators are each a best linear unbiased estimator of the respective population parameter.

4.3.8.2. Error Term Analysis – Autocorrelation Test

Table 4.3.8.2

Breusch-Godfrey Serial Correlation LM Test on Model 8

	Big Caps		**Small Caps**	
	F-statistic	**p-value**	**F-statistic**	**p-value**
1987-1991 (Period 1)	0.382555	0.6825	0.749147	0.4738
1992-1996 (Period 2)	0.26997	0.7636	1.769578	0.1725
1997-2001 (Period 3)	1.528544	0.2188	8.799701	0.0002
2002-2006 (Period 4)	1.414131	0.245	2.071259	0.1282
2007-2011 (Period 5)	1.134823	0.3231	1.220778	0.2967
2012-2016 (Period 6)	0.088638	0.9152	0.213482	0.8079
1987-2016 (Total)	0.052921	0.9485	4.500571	0.0112

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis for the Small Caps and have to adjust the model for the periods where it is detected (period 3 and the 30-year period).

4.3.8.3. Adjusted Model

Table 4.3.8.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + r * Mkt\_RtoTR_t + \varepsilon_t$

	1987-1991 (Period 1)	**1992-1996 (Period 2)**	**1997-2001 (Period 3)**	**2002-2006 (Period 4)**	**2007-2011 (Period 5)**	**2012-2016 (Period 6)**	**1987-2016 (Total)**
Big Caps							
C	0.0002	0.0000	0.0002	0.0001	-0.0004	-0.0008	-0.0002
(p-value)	*(0.0543)*	*(0.9394)*	*(0.5882)*	*(0.2073)*	*(0.0105)*	*(0.4156)*	*(0.2316)*
Rm-Rf	0.9856	0.9873	0.9895	0.9903	0.9840	0.6705	0.9450
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2142	-0.2068	-0.1500	-0.1433	-0.1403	-0.0246	-0.1642
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.6628)*	*(0.0000)*
HML	-0.0424	-0.0478	-0.0455	-0.0466	-0.0271	0.0426	-0.0390
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0031)*	*(0.0000)*	*(0.0000)*	*(0.4559)*	*(0.0003)*
Mkt RtoTR	-0.0003	-0.0001	-0.0002	-0.0003	0.0004	0.0033	0.0002
(p-value)	*(0.0007)*	*(0.4020)*	*(0.4368)*	*(0.0088)*	*(0.1555)*	*(0.0991)*	*(0.2502)*
Adj. R-squared	0.9977	0.9971	0.9912	0.9980	0.9978	0.6823	0.9484
Durbin-Watson	2.0247	2.0227	2.0523	2.1933	2.0059	2.0504	2.0043
Small Caps							
C	0.0006	0.0004	**-0.0002**	0.0013	0.0004	-0.0007	**0.0004**
(p-value)	*(0.1958)*	*(0.3464)*	***(0.8562)***	*(0.0363)*	*(0.5510)*	*(0.5351)*	***(0.2442)***
Rm-Rf	0.9208	0.8722	**0.7774**	0.8040	0.9058	0.6443	**0.8297**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
SMB	1.0689	0.9670	**0.8775**	0.9002	0.9508	1.2222	**0.9818**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
HML	0.2163	0.2496	**0.3072**	0.2985	0.2794	0.2706	**0.2927**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
Mkt RtoTR	-0.0002	0.0000	**0.0001**	-0.0003	-0.0001	0.0034	**0.0000**
(p-value)	*(0.4966)*	*(0.9245)*	***(0.9081)***	*(0.7667)*	*(0.9097)*	*(0.1162)*	***(0.9718)***
Adj. R-squared	0.9568	0.9023	**0.8443**	0.9106	0.9678	0.8111	**0.8991**
Durbin-Watson	1.9143	1.8335	**1.3211**	1.7002	1.9533	1.9152	**1.7859**

The model shows a strong explanation of the dependent variable (94.8% for Big Caps and 90.0% for Small Caps), and while it shows statistical significance for almost all variables, it is not the case for the liquidity factor (Market RtoTR).

4.3.9. Model 9 – Regression

In Table 4.3.9 we observe the results of the regression applied using Model 9 on the data, where the liquidity factor analyzed is the market TOR (represented as Mkt TOR) and the dependent variable is the excess return. Each analyzed portfolio is modelled.

Table 4.3.9

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + o * Mkt\_TOR_t + \varepsilon_t$

	1987-1991 (Period 1)	1992-1996 (Period 2)	1997-2001 (Period 3)	2002-2006 (Period 4)	2007-2011 (Period 5)	2012-2016 (Period 6)	1987-2016 (Total)
Big Caps							
C	-0.0003	-0.0006	0.0007	-0.0005	0.0002	0.0006	0.0001
(p-value)	*(0.3442)*	*(0.0037)*	*(0.2158)*	*(0.1400)*	*(0.6075)*	*(0.8508)*	*(0.5785)*
Rm-Rf	0.9860	0.9863	0.9925	0.9896	0.9764	0.6714	0.9510
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2102	-0.2053	-0.1201	-0.1430	-0.1464	-0.0130	-0.1513
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.8178)*	*(0.0000)*
HML	-0.0447	-0.0461	-0.0487	-0.0436	-0.0391	0.0499	-0.0516
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0001)*	*(0.0000)*	*(0.0000)*	*(0.3866)*	*(0.0000)*
Mkt TOR	0.0143	0.0344	-0.0269	0.0122	-0.0073	-0.0034	-0.0076
(p-value)	*(0.5379)*	*(0.0075)*	*(0.1629)*	*(0.2028)*	*(0.3270)*	*(0.9682)*	*(0.3340)*
Adj. R-squared	0.9976	0.9972	0.9926	0.9980	0.9975	0.6789	0.9579
Durbin-Watson	1.9765	2.0476	2.2060	2.1658	2.0940	2.0479	2.0142
Small Caps							
C	-0.0003	-0.0005	-0.0019	0.0069	0.0013	0.0028	0.0008
(p-value)	*(0.8292)*	*(0.6986)*	*(0.3830)*	*(0.0019)*	*(0.4831)*	*(0.4230)*	*(0.0654)*
Rm-Rf	0.9216	0.8714	0.7371	0.8040	0.9587	0.6449	0.8434
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	1.0742	0.9679	0.8000	0.8933	0.8717	1.2320	0.9309
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
HML	0.2143	0.2511	0.2871	0.2833	0.2680	0.2826	0.3456
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
Mkt TOR	0.0478	0.0582	0.0646	-0.1818	-0.0301	-0.0588	-0.0223
(p-value)	*(0.6061)*	*(0.4252)*	*(0.4080)*	*(0.0087)*	*(0.3754)*	*(0.5328)*	*(0.0927)*
Adj. R-squared	0.9568	0.9025	0.8462	0.9129	0.9676	0.8096	0.9012
Durbin-Watson	1.9289	1.8342	1.5721	1.7239	2.1493	1.9181	1.8870

4.3.9.1. Error Term Analysis - Heteroscedasticity Test

Table 4.3.9.1

Heteroscedasticity Test (Arch) on Model 9. Dependent variable: Resid^2

Big Caps	F-statistic	0.0010	Prob. F(1,1563)	0.9745
	Obs*R-squared	0.0010	Prob. Chi-Square(1)	0.9745
Small Caps	F-statistic	0.0574	Prob. F(1,1563)	0.8106
	Obs*R-squared	0.0575	Prob. Chi-Square(1)	0.8105

Taking into account the p-values shown in Table 4.3.9.1, the null hypothesis of absence of heteroscedasticity is not rejected; therefore, we assume homoscedasticity of the error terms for our model and ensure that the least-squares estimators are each a best linear unbiased estimator of the respective population parameter.

4.3.9.2. Error Term Analysis – Autocorrelation Test

Table 4.3.9.2

Breusch-Godfrey Serial Correlation LM Test on Model 9

	Big Caps		**Small Caps**	
	F-statistic	**p-value**	**F-statistic**	**p-value**
1987-1991 (Period 1)	0.41761	0.6591	0.781909	0.4586
1992-1996 (Period 2)	0.446445	0.6404	1.81837	0.1644
1997-2001 (Period 3)	1.682917	0.1879	8.522098	0.0003
2002-2006 (Period 4)	1.402634	0.2478	1.666171	0.191
2007-2011 (Period 5)	1.231524	0.2936	0.964247	0.3827
2012-2016 (Period 6)	0.092717	0.9115	0.200003	0.8189
1987-2016 (Total)	0.05662	0.945	4.467922	0.0116

The null hypothesis of the test is that there is no autocorrelation detected. We reject the hypothesis for the Small Caps and have to adjust the model for the periods where it is detected (period 3 and the 30-year period).

4.3.9.3. Adjusted Model

Table 4.3.9.3

Adjusted model, excluding crisis years and using the Newey-West HAC method

Model Equation: $R_{pt} - RF_{pt} = \alpha + \beta(RM_t - RF_t) + s * SMB_t + h * HML_t + o * Mkt\_TOR_t + \varepsilon_t$

	1987-1991 (Period 1)	1992-1996 (Period 2)	1997-2001 (Period 3)	2002-2006 (Period 4)	2007-2011 (Period 5)	2012-2016 (Period 6)	1987-2016 (Total)
Big Caps							
C	-0.0003	-0.0006	0.0009	-0.0005	0.0007	0.0006	0.0002
(p-value)	*(0.3442)*	*(0.0037)*	*(0.1830)*	*(0.1400)*	*(0.0697)*	*(0.8508)*	*(0.5533)*
Rm-Rf	0.9860	0.9863	0.9888	0.9896	0.9844	0.6714	0.9456
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*
SMB	-0.2102	-0.2053	-0.1462	-0.1430	-0.1414	-0.0130	-0.1657
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.8178)*	*(0.0000)*
HML	-0.0447	-0.0461	-0.0445	-0.0436	-0.0265	0.0499	-0.0397
(p-value)	*(0.0000)*	*(0.0000)*	*(0.0034)*	*(0.0000)*	*(0.0001)*	*(0.3866)*	*(0.0003)*
Mkt TOR	0.0143	0.0344	-0.0385	0.0122	-0.0179	-0.0034	-0.0082
(p-value)	*(0.5379)*	*(0.0075)*	*(0.1533)*	*(0.2028)*	*(0.0145)*	*(0.9682)*	*(0.3560)*
Adj. R-squared	0.9976	0.9972	0.9913	0.9980	0.9979	0.6789	0.9484
Durbin-Watson	1.9765	2.0476	2.0465	2.1658	2.0585	2.0479	2.0063
Small Caps							
C	-0.0003	-0.0005	**-0.0030**	0.0069	-0.0027	0.0028	**0.0003**
(p-value)	*(0.8292)*	*(0.6986)*	***(0.4897)***	*(0.0019)*	*(0.1387)*	*(0.4230)*	***(0.4791)***
Rm-Rf	0.9216	0.8714	**0.7786**	0.8040	0.9087	0.6449	**0.8298**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
SMB	1.0742	0.9679	**0.8726**	0.8933	0.9491	1.2320	**0.9816**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
HML	0.2143	0.2511	**0.3084**	0.2833	0.2763	0.2826	**0.2927**
(p-value)	*(0.0000)*	*(0.0000)*	***(0.0000)***	*(0.0000)*	*(0.0000)*	*(0.0000)*	***(0.0000)***
Mkt TOR	0.0478	0.0582	**0.1149**	-0.1818	0.0619	-0.0588	**0.0015**
(p-value)	*(0.6061)*	*(0.4252)*	***(0.5526)***	*(0.0087)*	*(0.0843)*	*(0.5328)*	***(0.9314)***
Adj. R-squared	0.9568	0.9025	**0.8454**	0.9129	0.9683	0.8096	**0.8991**
Durbin-Watson	1.9289	1.8342	**1.3406**	1.7239	2.0205	1.9181	**1.7860**

The model shows a strong explanation of the dependent variable (94.8% for Big Caps and 89.9% for Small Caps), and while it shows statistical significance for almost all variables, it is not the case for the liquidity factor (Market TOR).

Chapter 5: Conclusion

This dissertation is addressed to analyze the impact of the liquidity of small caps on the excess return of a portfolio, which is not captured by the firm size factor (SMB) or the value factor (HML) for stocks listed in the USA Stock Market (NYSE, NASDAQ & AMEX) for the period from January 1987 to December 2016. We extracted data from Thompson Reuters using weekly observations, clean the data using the criteria explained in Chapter 3 and divided into deciles according to their market value, then constructed a Small Caps and a Big Caps portfolio by assigning those specific deciles, basing the designation in the extrapolation of the current segmentation (given that the small and big cap concept is relative to the size of the entire market).

We used three different models and analyzed how they explained the dependent variable and how significant were the coefficients in order to fin the optimal, which is the Model 2 (Chapter 3). The data obtained from the chosen model is consistent with the literature and exhibits the presence of a liquidity factor related to the excess return of a given portfolio; this liquidity factor, which was defined as the Turnover Ratio, is found to be statistically significant only for the small caps portfolio and is independent of the firm size factor and the value factor.

The model is tested successfully and demonstrates its robustness according to the analysis on Chapter 4. The robustness is demonstrated by showing evidence which is consistent with the literature for the sub-periods as well as the entire period.

We believe that this liquidity factor is a useful tool for practitioners when deciding on the construction of a portfolio; however, it must be taken into consideration, as we can see in the research, the behavior of the market, specifically in the scenario of a financial crisis. A crisis scenario would inverse the relationship between liquidity and excess return, given that the investors start selling illiquid assets in an effort to cash out if the scene worsens, which drives prices downward for this assets and upward for the liquid ones; in the short term, illiquid assets get a negative return and liquid assets improve their performance.

We leave for further research the study of each market (NYSE, NASDAQ & AMEX) individually, since it is possible that the differences in the market characteristics can account as a factor that influences liquidity; characteristics like the quantity of market-makers (NYSE has seven specialist firms while NASDAQ has nearly 300 market-makers), the modality of trading (auction in NYSE vs electronic trading, though this is transparent to the final investor due to the extended use of electronic platforms) and the type of companies listed in each of the markets (since NASDAQ is a younger market with more growth-oriented technological firms). The impact of institutional sponsorship and the impact of the upgrade of small caps derived of a merger or a buyout should be also subject of supplementary investigation and quantify their relationship with the liquidity.

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