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Risks and Performances of Leveraged Exchange Traded Funds.

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Introduction

Leveraged and inverse ETFs (LETFs) are products designed to offer a daily[1] multiple either positive or negative of the return of a reference index. They accomplish this by using leverage, normally obtained through different derivatives (futures, forwards and swaps).[2] A LETF normally follows the value of an index, another ETF or a basket of stocks with the additional feature that it uses this leverage. The following paper is intended to carry on a deep analysis on the structure, performance and risks evolving these products.

LETFs are powerful alternatives that let investors to amplify the returns on their investment. While higher returns always sound more attractive, simultaneously there is always higher risk implied, therefore LETFs are very specialized financial tools that should be treated with caution. The rapid growth of these types of ETFs has taken the attention of investors in the market and the media and generated some debate about how and when this new investment alternative best fits in the pool of assets available to investors.

Even if at first glance Non-leveraged ETFs (normal ETFs) and LETFs behave the same way, it is indeed this embedded leverage that make them fundamentally different. It is mandatory to understand this main characteristic, and how it is treated to maintain the promised return. In particular, the daily rebalancing mechanism that leads to return deviation from the benchmark index in holding periods longer than one day and how this deviation performs based on the multiple, the holding period, etc.

Part 1: Leveraged ETF

History

LETF's are relatively new players in the market, they were introduced for the first time in June 2006, when the U.S. ProShares issued a line of six leveraged and inverse products that allowed investors to take positions in popular benchmarks like Nasdaq and S&P 500[3].

[1] Although there are ETF that promise leverage return over more-than-one-day holding period, more than 82% delivers leveraged return over one day return. Source: Bloomberg data extracted on March 7, 2019.
[2] Charupat, N. and P. Miu (2011). "The pricing and performance of leveraged exchange-traded funds", Journal of Banking and Finance, 35, 966-977.
[3] Cummans, J. (2013, October 7). "The Most Significant Events in ETF History". Retrieved from https://etfdb.com/the-most-significant-events-in-etf-history/

In Europe, the first leveraged ETF was launched by Lyxor that replicates DAX index and delivers double positive return (Lyxor DAILY LevDAX UCITS ETF)[4]. Later, more LETFs were launched and became rapidly popular.

Since their presentation in June 2006 until the end of last year, LETFs attracted more than $53 billion of assets in the U.S.—about 10 percent of all U.S. ETF assets. As of mid-2017, there were 180 leveraged and inverse ETFs, covering a broad range of equity, sector, international, fixed-income, commodity and currency markets.

Mechanism of Leverage Return (Portfolio Composition)

Managers of LETFs have two ways to deliver the stated multiples: physical replication and synthetic replication. By doing physical replication, the managers of the fund buy the underlying benchmark in a quantity that reflects the promised multiple. For example, if the fund offers 2x the S&P 500 index, the managers should buy two times the amount of the funds in S&P index. The additional money needed for this operation is obtained by borrowing. In contrast, in order to pursue a synthetic replication, managers buy derivatives as futures contracts or total return swaps. In this case, the money of the fund is used to pay premiums to the counterparties of the derivatives contracts, who compromises to deliver the agreed multiple return over the underlying.[5]

The most common method is the synthetic replication[6], which implies that the creation/redemption process differs from not leveraged ETFs. All ETFs have authorized participants (AP) that allow the creation and the redemption of shares with the ETF issuer in order to launch or withdraw shares to the market. Usually, an AP only can create or redeem a minimum number of shares (the amount varies according to ETF). Normal ETFs have an “in kind” creation/redemption process that consist in exchanging shares of ETF by the underlying: this way, ETF issuer can replicate in more or less quantity the underlying followed by the ETF. By contrast, LETF that uses synthetic replication have an “in cash” creation/redemption process: this is that the AP exchange LETF shares by cash. This is important for the LETF issuer because it uses cash to replicate the leveraged return promised to investors.

[4] LYXOR, (2017, April). “Leveraged ETFs”. Retrieved from https://www.lyxoretf.co.uk/pdfDocuments/DTP108575%20Lyxor%20ID%20card%20-%20leveraged%20ETFs%20240417.pdf

[5] Charupat, Narat, Miu, Peter, 2xxx. Leveraged Exchange-Traded Funds: A Comprehensive Guide to Structure, Pricing and Performance.

[6] 91% of leverage ETF uses synthetic replication. Bloomberg data extracted on March 7, 2019.

Rebalance process and compounded return

After the leveraged return for one period is replicated and because of the change in the value of the underlying, the exposure of the LETF must be rebalanced in order to assure the leveraged ratio for the next period[7]. Let saythat we managed funds in a LETF of 100 million dollars and we promise double exposure over S&P 500. The first day we have an exposure of 200 million dollars and the index increases its value in 10 %. Now we have an increase of 20 million over the initial amount of 100 million: this is a 20 %return that matches the double exposure over the underlying return (in this case 10 %) promised by the fund. In the beginning of the second day, the exposure must be 240 million as this is the double of 120 million, which implies a rebalance of 20 million added to the fund. The rebalance process is needed either the underlying increases or decreases its value and is valid for leveraged and inverse-leveraged ETFs

The following equations (8) will explain how the rebalancing process is computed.

The return of the underlying index of an LETF is given by the following equation

$$R_{t_i,t_{i+1}} = \frac{S_{t_{i+1}}}{S_{t_i}} - 1$$

Equation 1

If β is defined as the leveraged multiple of a LETF, then according with Charupat and Miu (2010) the compounded return form holding this ETF over N days is:

$$R_{t_0,t_N} = \prod_{i=0}^{N-1}(1 + \beta R_{t_i,t_{i+1}})$$

Equation 2

The Net Asset Value (NAV) of the LETF at time t_i can be represented as L_{t_i}, so the NAV at the close of t_{i+1} day will be:

$$L_{t_{i+1}} = L_{t_i}(1 + \beta R_{t_i,t_{i+1}})$$

Equation 3

Which can be generalized using Equation 2 as

$$L_{t_N} = L_{t_0}\prod_{i=0}^{N-1}(1 + \beta R_{t_i,t_{i+1}})$$

[7] Usually, the period of leveraged return that LEFTs promise is one day. This mean rebalance must be done in a daily basis.

Equation 4

Then, *"the notional amount of the total return swaps exposure that is required before the market opens on the next day to replicate the intended leveraged return of the index for the fund from calendar time* t_n *to time* t_{n+1} *"*[8] will be given by the following equation

$$L_{t_n} = xA_{t_n}$$

Equation 5

Which means that the notional amount of the total return swaps necessary before the open of the markets, to maintain the constant exposure is equal to the NAV times the leveraged or inverse multiple.

$$L_{t_{n+1}} = xA_{t_{n+1}} = xA_{t_n}(1 + xR_{t_n,t_{n+1}})$$

Equation 6

Now, given the return of the underlying index for the day $n + 1$, the exposure of the total return swaps, denoted by $E_{t_{n+1}}$ is equal to

$$E_{t_{n+1}} = L_{t_n}(1 + R_{t_n,t_{n+1}}) = xA_{t_n}(1 + R_{t_n,t_{n+1}})$$

Equation 7

The difference between equation 6 and equation 7 gives the amount by which the exposure of the total return swaps to be rebalanced at time $n + 1$

$$\Delta_{t_{n+1}} = L_{t_{n+1}} - E_{t_{n+1}}$$

$$\Delta_{t_{n+1}} = xA_{t_n}(1 + xR_{t_n,t_{n+1}}) - xA_{t_n}(1 + R_{t_n,t_{n+1}})$$

$$\Delta_{t_{n+1}} = A_{t_n}(x^2 - x)R_{t_n,t_{n+1}}$$

Equation 8

Possible Sources of Return Deviations over different holding period[9]

The tracking error is commonly defined as the difference between the return on net asset values of ETF and the return on the underlying. As seeing before, the primally tracking error source come due to the compounded effect, which is not under the control of the management. The chances of tracking error due to compounding effect are grater when

[8] Cheng, M. and A. Madhavan (2009), "The dynamics of leveraged and inverse Exchange-traded funds", Journal of Investment Management, 7, 43-62.

[9] Charupat, N. and P. Miu (2011). "The pricing and performance of leveraged exchange-traded funds", Journal of Banking and Finance, 35, 966-977.

the holding period is great; or, given a holding period, when the multiple (in absolute value) or the volatility of the underlying are high.

There are other sources of tracking error that are under the control of the management like management fees, custodian fees, licensing fees and other. All these fees are condensed in the expense ratio that is stated as an annual ratio. The higher the expense ratio, the higher the tracking error.

Also, the fund incurs in transaction costs related to the creation/redemption process. Unlike common ETFs, LETFs usually incur in higher transactions cost because the need of derivatives contracts. In general, transaction costs are higher when: the underlying is more volatile or less liquid, the leverage ratio is higher, and the frequency of creation/redemption is higher.

Finally, there may be a source of tracking error in the replication strategy used by the fund. The underlying value of the derivatives used for replication may not be perfectly correlated to the value of the underlying followed by the LETF. This effect is more common in illiquid underlying that relies more on OTC derivatives.

Compounding effect in a simulated framework

In order to illustrate the compounding effect, a Monte Carlo simulation method is applied through the open source programming language Python. The code creates 5,000 simulated paths for the underlying benchmark from which is calculated the compounded return and Net Asset Value of the LETF. It is assumed that the fund has a daily rebalancing frequency and three different investor’s holding period are evaluated: weekly, monthly and yearly.

The steps for the simulations are the following:

1. **Simulation of Index Prices:** the underlying benchmark is assumed to follow a jump-diffusion process[10]

$$S_{(t+\Delta t)} = S_{(t)} \exp\left[\left(\mu - \lambda k - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon + N\left(\mu_J - \frac{\delta^2}{2}\right) + \delta \sum_{i=1}^{N} \varepsilon_i\right]$$

Equation 9

[10] A jump-diffusion process is obtained by the combination of a Wiener process and a jump process (Shonkwiler, 2013)

Where,

μ = index expected return.

σ^2 = index volatility.

λ =rate at which the jumps occur (Poisson parameter).

$k = e^{\mu_j} - 1$

μ_j =mean of the jump.

σ_j^2 = standard deviation of the jump.

dt = small period of time.

Z = standard normal variable.

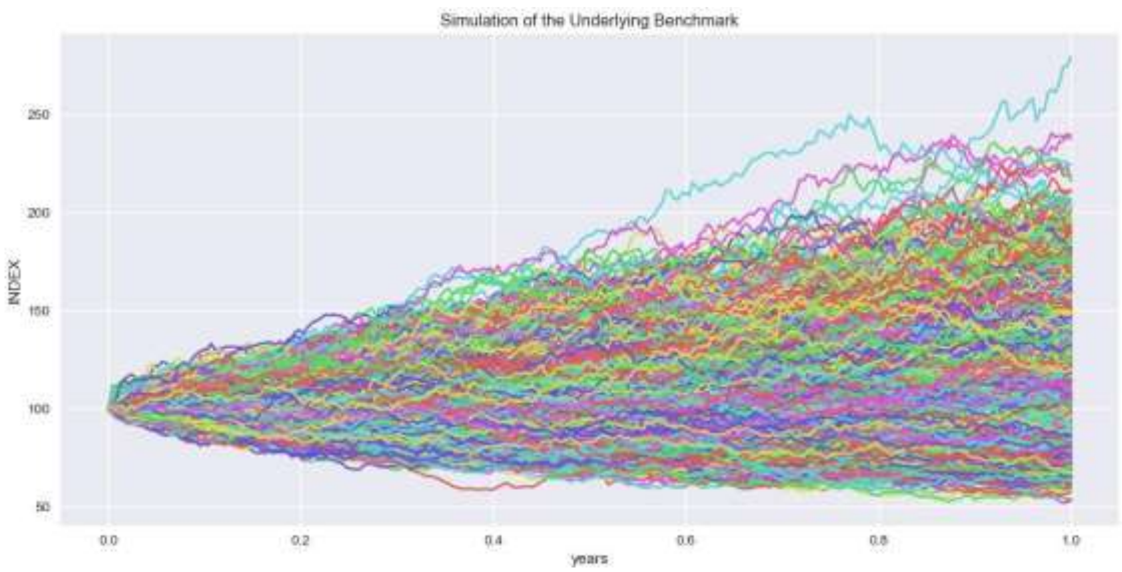
N =Poisson distributed random variable.

For the drift part of equation, the index expected return used is the 14.39% which is the average return of the Nasdaq 100 index and the volatility is calculated using a GARCH (1,1), (see appendix 2). Also, $dt = 1$ which means that the change in time is annually.

For the Poisson diffusion part of the equation, the average size of the jump (μ_j) used is 5.7% and the standard deviation of 3.1%. The lambda parameter is assumed to be 0.58, which we can interpreter as the expected number of jumps times in a year (see appendix 3).

For the simulation the initial value for this index is supposed to be 100 and we used 5,000 iterations. Also, this model assumes that the daily returns of the index are normally distributed and are independently and identically distributed; this means that the return of a particular day is not correlated with the return of the previous day. The Python's function written to this matter is called **GBM()**.

Figure 1. Simulation of the Underlying Index



2. **<u>Calculation of the Daily Return:</u>** Once the underlying benchmark is calculated the Equation 1 is used to obtain the daily return. In the Python's code the function written is called **Index_Return().**
3. **<u>Calculation of the Compounded Return:</u>** According with Charupat and Miu (2010), the Equation 2 is used to obtain the compounded return of the fund, with different leveraged multiples. In this step the Python's function written is named **Compounded_Return().**
4. **<u>Calculation of the Net Asset Value (NAV):</u>** Finally, the NAV is calculated using the Python function **Leverage_ETF_NAV()**, that takes as a parameter the initial value and the compounded return. This function is written following the Equation 4.

Results

The results are presented for a LETF with different multiples (positive and negative) that follow the underlying benchmark that was calculated in the first step of the simulation.

Payoff of an investment in a LETF with a multiple of 2 and -2

In the Figure 2 is illustrated the distribution of the possible payoffs of a $100 investment in a LETF with a multiple of 2 and -2 and it is possible to observe the different results for the holding period of one week, one month and one year.

Figure 2. Payoff of a $100 investment for different holding periods in a Leverage ETF with a multiple of 2 and -2

In Table 1, the descriptive statistics of the simulation are presented. The mean payoff for the multiple of 2 is 134.04 and for the inverse multiple is 75.74. In this table is also possible to observe that the standard deviation is increasing along with the holding period of investment.

In Figure 2. Payoff of a $100 investment for different holding periods in a Leverage ETF with a multiple of 2 and -2, it is notorious the skewness of the distributions, which can be confirmed looking to the skewness indicators in Table 1. According with the statistics for the 1-year investment period, the distributions are positively skewed for both multiples; but for the rest of distributions they are fairly symmetrical.

Table 1. Descriptive Statistics for a $100 investment in a Leverage ETF for different holding periods in a Leverage ETF with a multiple of 2 and -2

Stat.	**Multiple of 2**			**Inverse Multiple of -2**		
	1 Year	1 Month	1 Week	1 Year	1 Month	1 Week
mean	134.04	102.28	100.39	75.74	97.87	99.62
std	60.94	10.88	4.29	35.22	10.40	4.27
min	26.96	68.26	87.00	11.15	60.50	72.58
25%	91.90	94.82	97.43	51.10	90.83	96.81
50%	122.30	101.42	100.28	68.80	97.68	99.59
75%	163.09	109.00	103.17	91.66	104.53	102.49
max	745.10	150.18	130.33	309.77	144.27	114.25
Skewness	1.51	0.46	0.42	1.48	0.13	-0.23
Kurtosis	4.81	0.42	1.71	3.50	0.20	1.47

Payoff of an investment in a LETF with a multiple of 3 and -3

The results of the simulation for a multiple of 3 and -3 for holdings periods of one week, one month and one year are presented in the Figure 3. Payoff of a $100 investment for different holding periods in a Leverage ETF with a multiple of 3 and -3

Figure 3. Payoff of a $100 investment for different holding periods in a Leverage ETF with a multiple of 3 and -3

Also, in Table 2. Descriptive Statistics for a $100 investment in a Leverage ETF for different holding periods in a Leverage ETF with a multiple of 3 and -3, the descriptive statistics for this simulation are presented. In this case, the mean payoff for a one-year investment is 155.92 for the positive multiple and 66.39 for the negative multiple. In this table is also possible to observe that the standard deviation is increasing along with the holding period of investment (as it occurs with the multiple of 2).

Just like the simulation with a multiple of 2, in the case of the multiple of 3 with a holding period of 1 year the distributions are positively skewed for positive and negative multiples, however the rest of distributions are fairly symmetrical.

Table 2. Descriptive Statistics for a $100 investment in a Leverage ETF for different holding periods in a Leverage ETF with a multiple of 3 and -3

Stat.	Multiple of 3			Inverse Multiple of -3		
	1 Year	1 Month	1 Week	1 Year	1 Month	1 Week
mean	155.92	103.48	100.59	66.39	96.85	99.44
std	113.21	16.57	6.45	49.54	15.48	6.40
min	13.03	56.05	80.95	3.47	42.82	58.93
25%	82.74	91.98	96.12	33.97	86.26	95.21
50%	126.70	101.79	100.38	53.52	96.22	99.34
75%	195.44	113.38	104.73	82.20	106.58	103.71
max	1,901.36	181.71	146.63	510.62	172.35	121.86
Skewness	2.80	0.61	0.47	2.40	0.28	-0.19
Kurtosis	18.72	0.67	1.81	9.26	0.34	1.44

Distribution of the return deviation.

To define the return deviation, first we have to state the naïve return. The naïve return is the return of the benchmark (for any holding period) times the multiple of leverage. So, the return deviation is the difference between the LETF return and the naïve return. Taking the Equation 1 and Equation 2 we have

$$\text{Return deviation} = (L_{t_0,t_n} - x R_{t_0,t_n}) = \prod_{i=0}^{n-1}(1 + x r_{t_i,t_{i+1}}) - 1 - x\left(\frac{P_{t_n}}{P_{t_0}} - 1\right)$$

Equation 10

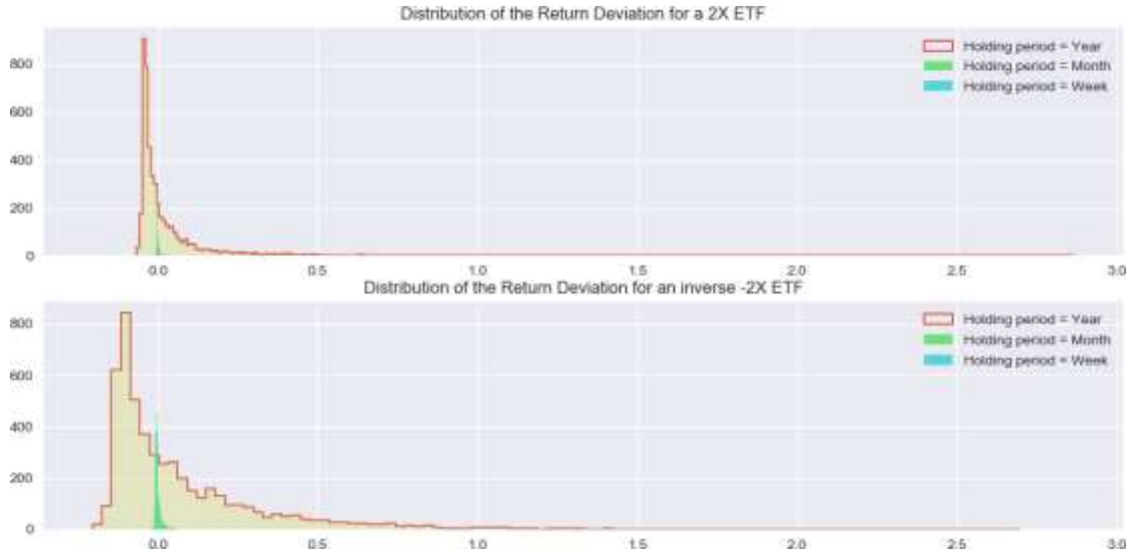
Where:

$L_{t_0,t_n} = \prod_{i=0}^{n-1}(1 + x r_{t_i,t_{i+1}})$ = the leverage compounded return from day t_0 to day t_n

$R_{t_i,t_j} = \left(\frac{P_{t_j}}{P_{t_i}} - 1\right) =$ the naïve return from day t_i to day t_j

The results of the return deviation for different holding periods and different leverage multiples are presented in the following Figures and Tables.

Figure 4. Distribution of the return deviation for a Leverage ETF with a multiple of 2 and -2



An interest result is that all the distributions obtained are positively skewed and have a high kurtosis which is an indicator that the data set have many positive outliers. Another aspect is that the standard deviation is increasing along with the holding period of investment for each leverage multiple.

Table 3. Descriptive Statistics of the return deviation for a Leverage ETF with a multiple of 2 and -2

Stat.	**Multiple of 2**			**Inverse Multiple of -2**		
	1 Year	1 Month	1 Week	1 Year	1 Month	1 Week
mean	2.87%	0.04%	0.00%	6.91%	0.11%	0.01%
std	13.43%	0.41%	0.07%	24.81%	1.19%	0.21%
min	-6.45%	-1.93%	-0.55%	-20.36%	-6.64%	-1.77%
25%	-3.71%	-0.19%	-0.03%	-9.98%	-0.56%	-0.10%
50%	-1.73%	-0.09%	-0.01%	-2.30%	-0.27%	-0.03%
75%	3.76%	0.13%	0.03%	15.37%	0.39%	0.08%
max	286.64%	4.47%	1.12%	269.61%	10.82%	3.20%
Skewness	5.50	2.86	2.62	2.32	2.49	2.46
Kurtosis	61.22	14.15	21.80	8.25	10.65	19.74

Figure 5. Distribution of the return deviation for a Leverage ETF with a multiple of 3 and -3

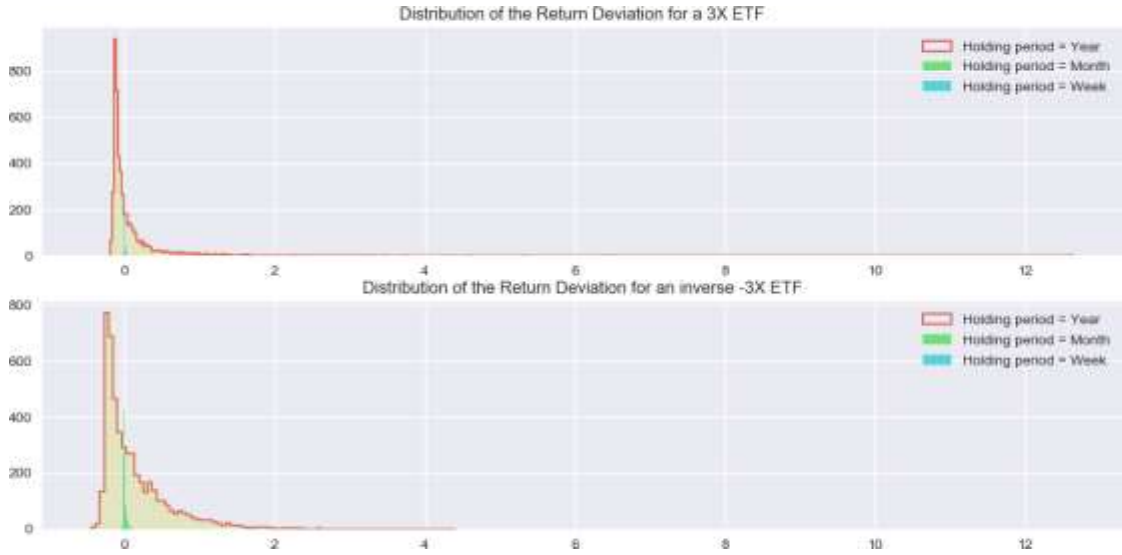


Table 4. Descriptive Statistics of the return deviation for a Leverage ETF with a multiple of 3 and -3.

Stat.	**Multiple of 3**			**Inverse Multiple of -3**		
	1 Year	1 Month	1 Week	1 Year	1 Month	1 Week
mean	9.17%	0.12%	0.01%	13.14%	0.21%	0.02%
std	47.52%	1.26%	0.21%	45.25%	2.36%	0.42%
min	-18.73%	-5.56%	-1.63%	-44.32%	-13.69%	-3.60%
25%	-11.37%	-0.57%	-0.10%	-19.07%	-1.12%	-0.20%
50%	-5.89%	-0.28%	-0.03%	-2.74%	-0.55%	-0.06%
75%	9.81%	0.38%	0.08%	30.83%	0.79%	0.16%
max	1263.69%	14.19%	3.42%	441.14%	20.71%	6.30%
Skewness	7.72	2.99	2.66	2.04	2.43	2.43
Kurtosis	126.04	15.58	22.38	6.10	10.31	19.30

In Appendix 5 the scatter plot of the return and the leverage compounded return for each holding period and leverage multiple are presented. Here, it is possible to observe that for a larger holding period the relation between these two returns in not linear and, on the contrary, it takes a convex form.

Return Deviation Regression Analysis in a Simulated Framework.

Finally, we proceeded to evaluate the performance of the returns of the LETF versus the returns of the benchmark, in the simulated framework.

We evaluated the returns of the LETFs over different length of holding periods – one day, one week (5 days), one month (21 days), and one year (252 days). We computed the returns based on changes in the funds' NAV; with the formula

$$R^{i}_{t-1,t} = \frac{NAV^{i}_{t}}{NAV^{i}_{t-1}} - 1$$

Equation 11

Where $R^{i}_{t-1,t}$ is the return on the fund i during the period from $t-1$ to t

Next, these holding-period returns were regressed against the underlying benchmark returns measured over the same periods. Then, we tested whether the intercepts are significantly different from zero, and whether the slope coefficients are significantly different from 2, 3 or -2, -3. If LETFs deliver their promised ratio returns, then the intercepts should be zero. If, as mentioned earlier, tracking errors increase with the length of the holding period, then we expect the slope coefficients to be different from stated multiples for long holding periods. The longer the holding period, the further the slope coefficient from the stated multiple is expected to be.

The results for all holding periods are presented in Table 5.

Table 5. Regression results. -Simulated Data-.
This table summarizes the regressions of the returns of the Leveraged ETFs over various holding periods on the returns of the underlying benchmarks from the simulated data, both the coefficients and P-values are displayed. The P-values for the intercepts are based on whether they are equal to zero. The P-values for the slope coefficients are based on whether they are equal to 2.

Regression Results Benchmark ETF and Leveraged ETF -Simulated Data-.						
Multiple			**One day**	**One Week**	**One month**	**One Year**
x2	**Intersection**	Coefficient	0	0	0.0001	-0.0285
		P-Value	0.745	0.0133	0.0825	0.00
	Slope	Coefficient	2	2.005	2.026	2.367
		P-Value	1	0	0	0
-x2	**Intersection**	Coefficient	0	0.0001	0.0003	0.0489
		P-Value	0.745	0.0136	0.0837	0.00
	Slope	Coefficient	-2	-1.985	-1.926	-1.242
		P-Value	1	0	0	0
x3	**Intersection**	Coefficient	0	0.0001	0.0003	-0.0989
		P-Value	0	0.0132	0.0828	0.00
	Slope	Coefficient	3	3.015	3.079	4.22
		P-Value	1	0	0	0
-x3	**Intersection**	Coefficient	0	0.0001	0.0006	0.0858
		P-Value	0.1847	0.0137	0.0845	0.00
	Slope	Coefficient	-3	-2.97	-2.854	-1.605
		P-Value	1	0	0	0

For a 1-day period return, all leveraged ETFs produce returns that are exactly to what they promise. As for the intercepts, all of them are not different from zero at the 5% level.

As a result, we conclude that the LETFs in our simulation are successful in delivering the promised daily performance. This is not surprising because of the formula used to compute them.

The same can be said about weekly returns. The differences are very small, but the coefficients are still close to 2,3 and -2, -3. Therefore, it appears that over a holding period of one week, these LETFs still deliver returns that are very close to their leverage ratios.

The differences become larger when we look at monthly returns. The slope coefficients, especially those for the inverse ones, move away from the leverage ratios. And here we can see that the multiple also has some influence, the x3 and -x3 tend to have a higher tracking error.

Finally, the tracking errors for one-year returns are even larger. All have slope coefficients that significantly differ from the leverage ratios. For example, the X3 has a slope of (4.22) and the inverse (-3) -1.66.

It is clear at this point that investors who keep the LETFs for periods around one year will not receive a return that corresponds to the leverage ratio.

Part 2: Performance of LETFs in the "almost" real world.

After analyzing the simulated data, in this part of the paper we wanted to analyze how a selected LETF has performance over the time in the real world. Since investors care about longer returns more than one-day returns, we need to study the suitability of the product over different investment horizons.

In order to do so, we selected one benchmark index and its corresponding double ETF. The benchmark index is the *Nasdaq 100 index*. We use the ETF that track the corresponding index as our proxy of benchmark, in our case is the Invesco QQQ Trust (QQQ)

Using this ETFs as the benchmark allows us to make a better comparison with LETF because from the view of an investor both the ETF and the LETFs are potentially tradeable.

The LETF in this study is the double (2X) ETF ProShares Ultra QQQ (ticker: QLD). See Appendix 4 to have a full description of the LEFT.

We selected this index because it is widely used as benchmark in the industry and its corresponding double ETF has a significant trading volume. For instance, on March 29, 2019, the last day of our sample period, the trading volume for the ETF QQQ was 35.2 million.[11]

The period selected for the study starts on June 22, 2006, which was the date of introduction to the market of the chosen ETF. We end our sample period on March 29, 2019. The data was obtained from Bloomberg Terminal. Table 6 reports some summary statistics for the daily returns.

The last column shows the average holding period; the LETF is held for a shorter period (8 days) than the regular ETF (13 days), and this tells us that investors are prone to have a very short-term strategy handling LETFs.

Table 6. Summary statistics of daily returns
This table reports summary statistics of daily NAV returns for the benchmark ETF (the first row) and the 2x Leveraged ProShares Ultra (the second row) for the sample period. The last column is the average holding period12.

Benchmark ETF and Leveraged ETF (Daily Returns) (June 22/2006 to March 29/2019)								
Summary of Statistics								**Avg Holding Period (days)**
	Ticker	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**	**Max**	**Min**	
Benchmark	QQQ	0.000572	0.013421	-0.01202	8.15088	0.1259	-0.105195	12.91
LETF	QLD	0.001085	0.026837	-0.01499	8.11717	0.251574	-0.208411	7.95

Performance of the LETF

As we did for the simulated LETF we proceeded to evaluate the performance of the LETF [QLD] versus the naïve return for different holding periods.

We evaluated the returns of the LETF over different length of holding periods – one day, two-day, one week (5 days), one month (21 days), one quarter (63 days), and one year

[11] Bloomberg Terminal 2019
[12] The average holding period is calculated over the sample period as follow. First, the average number of outstanding shares every month is divided by the number of fund units traded in that month, and then multiplied by 30 to obtain the holding period (in terms of days) in that month. These holding periods are then averaged to arrive at the figures in the table, we used the same calculation method as the paper we reproduce from.

(252 days). We computed the returns based on changes in the funds' NAVs; with the same formula

$$R^{i}_{t-1,t} = \frac{NAV^{i}_{t}}{NAV^{i}_{t-1}} - 1$$

Equation 12

Where $R^{i}_{t-1,t}$ is the return on the fund i during the period from $t - 1$ to t

To show an example of different holding period returns, Appendix 6 shows scatter plots of the LEFT [QLD] against the naive returns of the benchmark [QQQ] over different holding periods, as we did with the simulated one, but this time we include more holding periods for the LETF.

Distribution of Return Deviation for the ETF QLD

Once we have the naïve return and the return of the ETF QLD, we can proceed to calculate the return deviation, just as we did in the first part. In the Figure 6, the distribution of this deviation is presented. In this case, skewness is low (below 0.5) so it could be considered as fairly symmetrical (see

Table 7). Also, kurtosis increase when the holding period decrease, but it is not too large. Lastly, just as the results in the first part, the standard deviation is increasing along with the holding period of investment.

Figure 6. Distribution of the Return Deviation in the ETF QLD

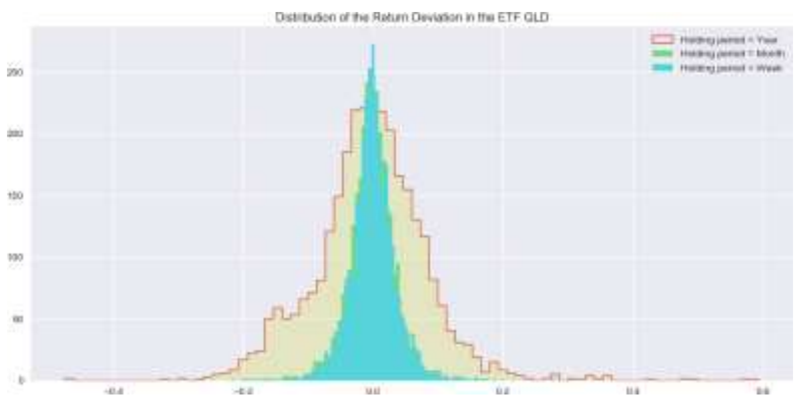


Table 7. Descriptive Statistics of the return deviation for different holding periods in the ETF QLD

Stat.	1 Year	1 Month	1 Week
count	2962	3193	3209
mean	-0.44%	-0.20%	-0.04%
std	8.77%	3.77%	3.79%
min	-47.44%	-26.70%	-29.02%
25%	-5.11%	-2.13%	-1.95%
50%	-0.35%	-0.16%	-0.09%
75%	4.54%	1.79%	1.85%
max	59.40%	30.60%	27.94%
Skewness	0.39	0.05	0.22
Kurtosis	3.63	5.28	6.53

We also create the distribution of the possible payoffs of a $100 investment in the QLD ETF for the holding period of one week, one month and one year. Just as before, the standard deviation is increasing along with the holding period of investment. Results are presented in Figure7 and Table 8.

Figure 7. Payoff of an investment of 100 in the ETF QLD

Table 8. Descriptive Statistics for a $100 investment in a Leverage ETF for different holding periods in the ETF QLD

Stat.	1 Year	1 Month	1 Week
count	2962	3193	3209
mean	128.46	102.14	100.51
std	41.02	10.51	5.44
min	20.97	45.79	68.95
25%	105.67	96.73	97.83
50%	133.85	103.22	101.00
75%	153.88	108.64	103.54
max	314.59	149.56	129.81
Skewness	-0.18	-0.78	-0.52
Kurtosis	1.13	2.62	3.20

Return Deviation Regression Analysis.

Finally, we simulated the analysis done by Charupat, N. and P. Miu (2011), for Canadian market. Different holding period returns are regressed against the returns of the benchmark measured over the same periods. We then tested if the intercepts are significantly different from zero, and whether the slope coefficients are significantly different from the stated multiple in, this case 2 for our selected ETF.

If the ETF delivers its promised ratio returns, then the intercepts should be zero, and the slope coefficients should be 2. As mentioned earlier, tracking errors increase with the length of the holding period; We expect for the longer the periods a slope significantly different from 2.

Given that we have a relatively short time series and our objective is to analyze the long-term performance of LETF, we overlapped data to compute the quarterly and yearly returns. For instance, when studying the behavior of 63-day returns, we first calculate the 63-days return of the benchmark and double ETF starting from the beginning of the time period. Then we move down one day to the second value and calculate 63-days return again. These new 63-days return become the second observation of the long-term performance analysis. Note that these two sets of one quarter returns are calculated from the same 62 days plus a different (first day or) sixty-third day. Hence, we lose the least amount of data when we study the one quarter and one year holding periods.

The results for all holding periods are presented in Table 8.

Table 8. Regression results. -Historical Data-.
This table summarizes the regressions of the NAV returns of the QLD LETF over various holding periods on the returns of the underlying benchmarks, both the coefficients and P-values are displayed. The P-values for the intercepts are based on whether they are equal to zero. The P-values for the slope coefficients are based on whether they are equal to 2. The Quarterly and Annual returns are all calculated using overlapped data

Regression Results Benchmark ETF [QQQ] and Leveraged ETF [QLD]							
(June 22/2006 to March 29/2019)							
		One day	**Two Day**	**One Week**	**One month**	**One Quarter**	**One Year**
Intersection	Coefficient	-0.0001	-0.0002	-0.0004	-0.0021	-0.0055	-0.0255
	P-Value	0.0394	0.0053	0.0058	0.00	0.00	0.00
Slope	Coefficient	1.996	1.994	1.998	2.01	2.009	2.16
	P-Value	0.0665	0.06857	0.7525	0.00029	0.00604	0

For daily periods, the LETF produces returns that are extremely close to what they promise. We see a p-value of 0.06 on the slope coefficients, meaning it is not different from 2. However, intercepts are statistically different from zero at the 5% level, but the magnitude is very small and not likely to be economically significant., We conclude that the LETF in our sample is successful in delivering the promised daily performance.

For the most part, the same can be said about two-day and weekly returns. The fund has slope coefficients that are not significantly different (at the 5% level) from 2. Therefore, it appears that over a holding period of one week, this LETF, [QLD], still delivers returns that are very close to its leverage ratio.

The differences appear when we move to monthly returns. The slope coefficient slightly drifts away from 2 the promised ratio. The p-value (0.00029) shows that the difference is significant. The same happens for quarterly returns, with acoefficient close to 2 but with p-value (0.0069) lower than 5%.

Finally, the coefficients for yearly returns are even farther away, has a slope coefficient that significantly differ from the leverage ratios 2.16 (p-value=0). It is evident that investors who hold this leveraged ETF over a year-long horizon will not receive a return that corresponds to the stated leverage ratio.

The conclusions from the regression are consistent with the scatter plots showed in Appendix 6, where returns between benchmark and LETF are proportional up to the weekly holding period, afterwards, the plots start to show unproportioned returns. This is also seen in the average holding period for LETF QLD (8days), which is close to a one-week period and significantly lower than regular ETF holding period (more than 1.6 times).

The results in Table 8 apply to the sample data from the period mentioned earlier. The fact that we use overlapping on the quarterly and annual returns to investigate the relation between the long-term performance of leveraged ETFs and benchmarks may lead to some bias. To avoid this bias, another method to resample the distribution of the actual data could be useful (bootstrapping for example), this is subject for future analysis.

Part 3: The Pricing Efficiency.

As we saw in Part I, due to the creation /redemption mechanism on ETFs (leverage and non-leveraged) the prices remain close to the NAVs, and this is one of the reasons why ETFs are attractive for some investors. However, there are small differences in prices and these could leave room for arbitrage advantageous opportunities. If, for example, the ETF price is below its NAV, traders can buy fund units, redeem them for the NAV and capture the difference (minus transaction costs and imposed fees). This process for LETF, unlike regular ETFs, is simpler and implies lower transaction costs, because as mentioned earlier the redemption/creation process is done in-cash, so no stocks exchange is necessary.

We must note that this arbitrage strategy is not straightforward and may have a degree of risk. This is because a time lag between the order placing time and time the price is set. Creation/redemption orders have to be submitted prior to the end of a trading day (for LEFTs is 9:30am) in order for the orders to be executed at that day's NAV (which is determined at the end of a trading day).

In such a case, arbitragers would have to hedge the price risk, most of the times by using futures contracts on the underlying index, which of course makes the arbitrage transactions more costly and less precise.

Previous studies have examined the pricing efficiency of LETFs in the U.S. market and in the Canadian market (Charupat and Miu,2011). They found that, on average, deviations are small; but large premiums and large discounts are not uncommon. However, this studies where carried out almost one decade ago.

That is why in this part of the paper we wanted to see the behavior of these price's mismatches on the selected LETF up to current date and analyze if they are big enough to be exploited.

Price deviations are defined as the differences between the fund's closing prices and their net asset values (NAVs). We are going to handle them as percentages of NAVs, computed as follow:

$$\square_{\square}^{\square\square\square\square} = \frac{\square_{\square}^{\square\square\square\square} - \square\square\square_{\square}^{\square\square\square\square}}{\square\square\square_{\square}^{\square\square\square\square}} \times 100 \quad \textit{Equation 10}$$

Where $□_{□}^{□□□□}$ is the closing price of the LETF [QLD] on day t, and $□□□_{□}^{□□□□}$ is the net asset value (NAV) of the fund on the same day. Accordingly, $□_{□}^{□□□□}$ is the price deviation (in percentage term) observed on QLD on day t. If $□_{□}^{□□□□}$ is positive it is a premium, if negative, it is a discount.

The price deviations are calculated from the closing prices over the same time period as the part II: from June 22, 2006 to March 29, 2019. (Obtained from Bloomberg Terminal).

The results are presented in Table 9. The average price deviation is small (not exceeding 0.014% of NAVs), the average difference is not significant different from zero at a 95% confidence (T-test P-value= 0.99). We also can see that there is a higher % of discounts (57%) most of the times the prices are below the NAV levels.

This average (price deviation) is lower than the average bid/ask spread expressed as a percentage of NAV (0.02228).

While the average deviations are small, the standard deviation (0.23) shows that larger premiums or discounts are likely to happen, confirmed by the gap between the 5th and 95th percentiles, -0.31 and 0.24. We compared bid/ask spread expressed as % of NAV every day available and price deviations: these last ones are within or very close to the bid/ask spreads.

Moreover, if we add transactions costs and hedging costs to cover the price risk due to the time lag (order placement vs NAV computation), the arbitrage opportunities will be very complicated to exploit.

Table 9. Price Deviations Leveraged ETF [QLD] -Historical Data-.
This table summarizes price deviations (NAV versus Price) QLD.

Price Deviations Leveraged ETF [QLD] (June 22/2006 to March 29/2019)							
N	**Average (%)**	**5th Percentile (%)**	**95th Percentile (%)**	**Std. Dev**	**% Premium**	**% Discounts**	**Bid/Ask Spread as % of NAV**
3214	-0.014 (0.999)	-0.308	0.240	0.228	42.875	57.130	0.022

Conclusions

In this paper, we examined the financial product known as Leveraged ETF, its composition, performance and the pricing efficiency both in a simulated framework and with real world data. We selected a widely common and high volume traded index and its double ETF, we wanted to focus our analysis in the US market, we were aware that some research had been done for US based products in the past, however these studies are one decade old.

With respect to the performance, the returns obtained, both frameworks give similar results: the multiple is statistically respected in a holding period no longer than one week, whereas in longer periods the multiple differs statistically from the stated one. These results are coherent with the historical holding period of the LETF, which is 8 days.

In the simulated study we tested different multiples and also inverse multiples were included, we evidence that the magnitude and the direction of the multiple has impact, because the return deviations tended to be larger on triple multiples and even larger with inverse ones,

Regarding to the analysis of the price efficiency: this is, how close is the price of the LETF from its NAV. The observed results show that on average the deviation from the NAV in an amount is small o very close to the bid/ask spread, which is an indicator of price efficiency.

Finally, we conclude that Leveraged ETF’s, given the characteristics, behavior, and past performance are specialized products for a specific target of investors, with a high-risk profile, and a short time investment horizon, who are aware the exposure and potential wins and losses.

Appendix.

Appendix 1. Python Code.

The code was developed on Python, and will be attached separately.

Appendix 2. Volatility Modeling

In order to recreate the performance of the index trough Monte Carlo simulations, as a parameter we needed the volatility of the underlying index that should be prevailing in the future. Since volatility modeling could be a research topic itself, we decided to use a GARCH as a proxy of the volatility.

The index that we selected is the *Nasdaq 100 index.*

First, we extracted the data from Bloomberg. The data period selected for the sample study starts from June 22, 2006 and ends on March 29, 2019.

Next, we calculated the logarithmical daily returns based on the closing prices. Table 10 shows a summary of statistics. Then, we performed a Shapiro-Wilk test to evaluate normality in our data, which confirmed that our sample does not look Gaussian. With these results, we proceeded with the GARCH model.

Table 10. Summary statistics of daily log-returns of the Index
This table reports summary statistics of daily log-returns of the index. NASDAQ 100 Index

NASDAQ 100 Index - Log-returns						
(June 22/2006 to March 29/2019)						
Ticker	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**	**Max**	**Min**
NDX	1.000061	0.001754	-0.14	9.94	1.01658	0.985

The volatility of daily log-returns has been modelled using different GARCH (p, q) models; GARCH (1,1), GARCH (1,2), GARCH (2,1), GARCH (2,2). The different Information criteria (AIC -BIC) values are shown in Table 11.

Table 11. Summary Information criteria GARCH of daily log-returns of the Index
This table reports the AIC & BIC of the different GARCH models tested to get the volatility of the log-returns, the model that fits the best is the one with the lowest values.

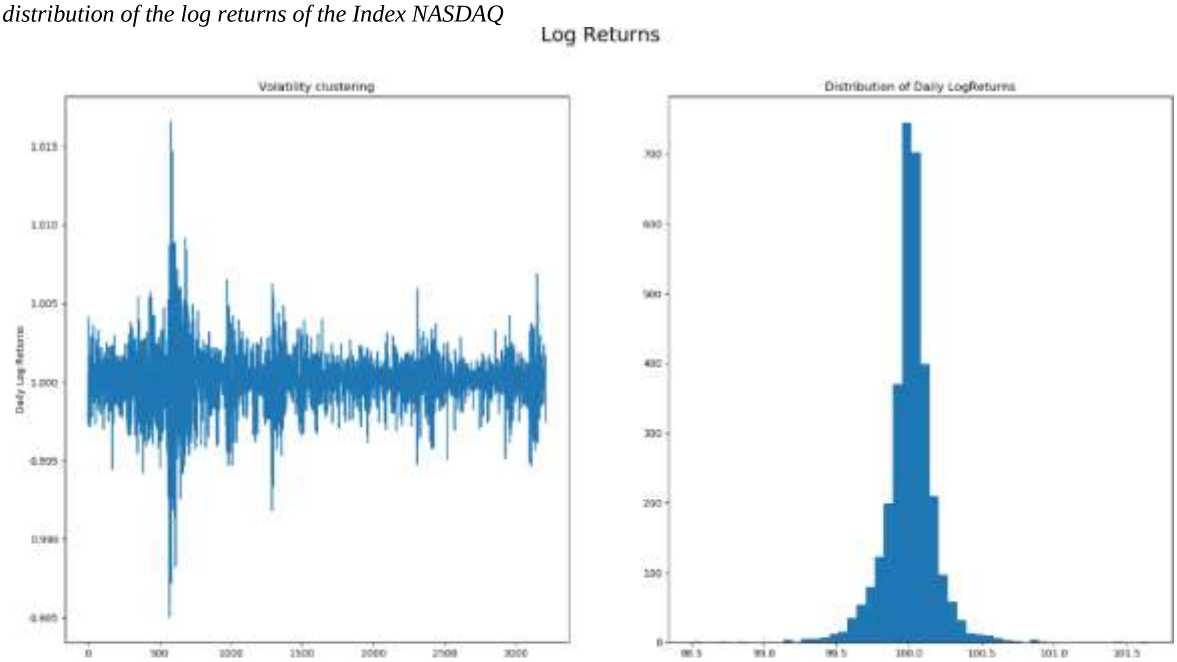
Information Criteria GARH (p, q) Log -Returns				
(June 22/2006 to March 29/2019)				
	GARCH **(1, 1)**	GARCH **(1, 2)**	GARCH **(2, 1)**	GARCH **(2, 2)**
AIC	-3560.95	-3558.95	-3560.67	-3560.07
BIC	-3536.65	-3528.58	-3530.39	-3524.62

From the empirical results obtained, we can conclude the following:

Firstly, it was found that the return series of the selected index are not normally distributed.

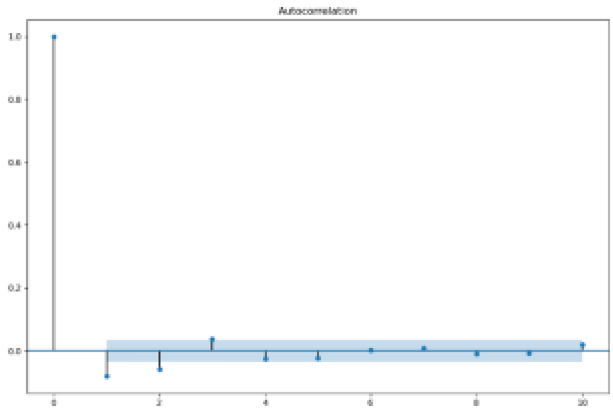
Secondly, the return series also exhibit volatility clustering and leptokurtosis seen from the high excess kurtosis values.

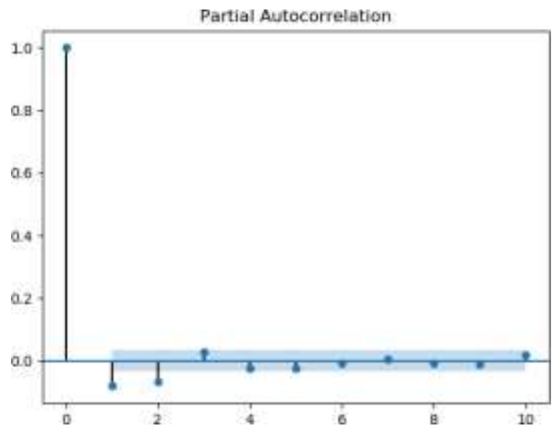
Figure 8. NASDAQ Log Returns Distribution.
This figure shows the distribution of the log returns to confirm, volatility clustering and the histogram of the distribution of the log returns of the Index NASDAQ



Over all, GARCH (1,1) performed best in modeling volatility of QLD US stock based LEFT returns. This can be confirmed with the autocorrelation and partial autocorrelation plots, where we see that after the first lag there is no much information contained.

Figure 9. Autocorrelation and Partial Autocorrelation





The results of the GARCH (1,1) are the following.

Omega is 0.00048, Alpha is 0.1058 and Beta is 0.8753. The p-values of all coefficients confirm that they are statistically different from zero.

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad \text{Equation 12}$$

Where the σ_t^2 is the variance, ω, α, β are the coefficients calculated above, and ε_{t-1}^2 and σ_{t-1}^2 are the return and the volatility from the previous period.

These results were plugged into the Monte Carlo simulation parameters in Part 1.

Appendix 3. Jump process Parameters

The Poisson Process is a counting process, which means that a random variable represents the total number of events that occur by time t (Ross, 2010). In this case, the event is a jump, and we define it as a return that exceed 4 standards deviations. To calculate the parameter for the Poisson distribution we use the method of moments. So first we count how many times in a year an event occurs, i.e. a return exceed 4 standards deviations, as is show in the following table.

Table 5. Number of jumps per year

Year	Events
1	0
2	1
3	0
4	0
5	1
6	0
7	1
8	0
9	1
10	1
11	1
12	1
Mean	0.58
Stand Dev	0.22

Then, as "*the method of moments estimates of* λ *is simply the arithmetic mean of the counts listed above*"[13], this means that the parameter estimate is $\hat{\lambda} = 0.58$. We use this estimation to calculate the standard deviation using the following equation

$$\sigma_{\hat{\lambda}} = \sqrt{\frac{\hat{\lambda}}{n}} = \sqrt{\frac{0.58}{12}} = 0.22$$

Once that we have this parameter, we take all the observations that are categorized as a jump and calculate the mean and the standard deviation of them, so we have an average of the jump and its dispersion. We obtain a value of 5.67% and 0.030% respectively.

[13] Rice, John A (2010). Chapter 8. Pag 261

Appendix 4. The leveraged ETF 2x ProShares Ultra QQQ – Ticker: [QLD US]

About the ETF

ProShares Ultra QQQ (the "Fund") is an Equity based Leveraged ETF that seeks daily investment results that correspond to twice the daily performance of the NASDAQ-100 Index (the "Index").

The Index: the NASDAQ-100 is a stock market index made up of a basket of the 100 largest, non-financial most actively traded U.S companies listed on the Nasdaq stock exchange.

The "single day": consist the period from the time the Fund computes its net asset value (NAV) until the time of the Fund's next NAV calculation.

Objective

The Fund seeks daily investment results, before fees and expenses, that correspond to two times (2x) the daily performance of the Index. The Fund does not seek to achieve its stated investment objective over a period of time greater than a single day.

Returns for periods longer than a single day are likely to the differ from the Fund's stated multiple (2x) due to the impact of volatility and leverage on the compounding investor's returns.

Fees and Expenses.

Annual Fund Operating Expenses. (expenses to be paid each year as a percentage of the value of the investment).

Investment Advisory Fees	0.75%
Other Expenses	0.24%
Fee Waiver/Reimbursement*	-0.04%
Total Annual Fund Operating Expenses	**0.95%**

*ProShare Advisors LLC ("ProShare Advisors") has agreed to waive and to reimburse Other Expenses to the extent Total Annual Fund Operating expenses, exceed 0.95%.

Funds Investment strategy

The Fund invests in financial instruments that ProShares Advisors believes, in combination, should produce daily returns consistent with the Fund's investment objective.

Equity Securities — The Fund invests in common stock issued by public companies.

Derivatives — The Fund uses derivatives as a substitute for investing directly in stocks in order to seek returns for a single day that are leveraged (2x) to the returns of the Index for that day. These derivatives principally include Swap Agreements and Future Contracts.

Money Market Instruments — The Fund invests in short-term cash instruments that have a remaining maturity of 397 days or less and exhibit high quality credit profiles for example: U.S. Treasury Bills and Repurchase Agreements.

ProShares Advisors fixes the type, quantity and mix of investment positions that it believes, in combination, the Fund should hold to produce daily returns consistent with the Fund's investment objective, through mathematical approach.

The Fund invests in or have exposure to a representative sample of the securities within the Index or even to securities not contained in the Index or in financial instruments, with the intent of obtaining exposure with aggregate characteristics similar to those of a multiple of the single day returns of the Index.

The Fund seeks to engage in daily rebalancing to position its portfolio so that its exposure to the Index is consistent with the Fund's daily investment objective. The Index's movements during the day will affect whether the Fund's portfolio needs to be rebalanced.

Management

The Fund is advised by ProShares Advisors. Michael Neches, Senior Portfolio Manager, and Devin Sullivan, Portfolio Manager, have jointly and primarily managed the Fund since October 2013 and April 2018, respectively.

Purchase and Sale of Fund Shares

The Fund will issue and redeem shares only to Authorized Participants in exchange for the deposit or delivery of a basket of assets (securities and/or cash) in large blocks, known as Creation Units, each of which is comprised of 50,000 shares. Retail investors may only purchase and sell shares on a national securities exchange through a broker-dealer. Because the Fund's shares trade at market prices rather than at NAV, shares may trade at a price greater than NAV (a premium) or less than NAV (a discount).[14]

[14] **ProShares Ultra QQQ** Summary Prospectus October 1, 2018.

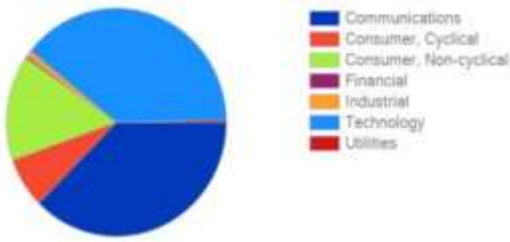
ProShares Ultra QQQ
As of (03/19/2019)

Fund Information			**Trading Information**		
Last Close	$	89.47	Bid Ask Spread		0.03
Total Assets (mill)	$	1,864.81	90-Day Average Volume (mill)		1.90
Inception Date		6/21/2006	Shares Outstanding (mill)		20.90
Benchmark		Nasdaq 100 Stock Index	**Fund Management information**		
Currency		USD	Creation Unit Size		50,000.00
Ticker		QLD US	Creation fee	$	250.00
ISIN		US74347R2067	Create Redeem Process		In-Kind/ Cash
Bloomberg Classification			Settlement Cycle		T+2
Fund Type		ETF	Open for new creations		Yes
Fund Asset Class Focus		Equity	Swap Based		Yes
Market Cap		Large-cap	Derivatives Based		Yes
Geographic Focus		U.S.	Currency hedged		No
Leverage		2x	Replication Strategy		Derivatives
Actively Managed		No	Rebalancing Frequency		Daily

*Source: Bloomberg Terminal.

Asset Allocation	As of (03/19/2019)
Equity	36.66%
Money Market	63.34%

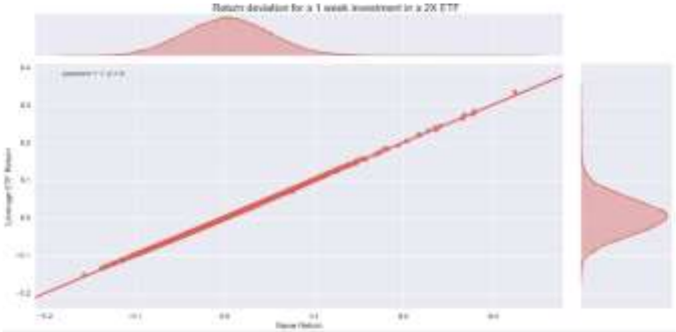
Sector Allocation	As of (03/19/2019)
Communications	13.76%
Consumer, Cyclical	2.62%
Consumer, Non -Cyclical	5.49%
Financial	0.10%
Industrial	0.34%
Technology	14.25%
Utilities	0.14%

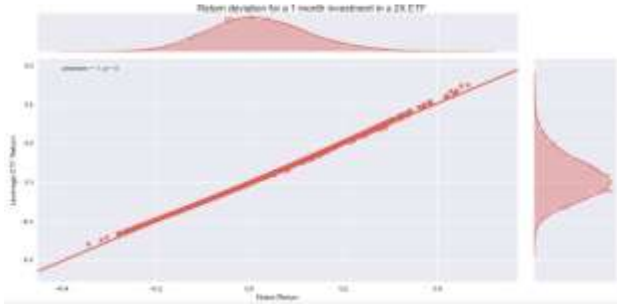


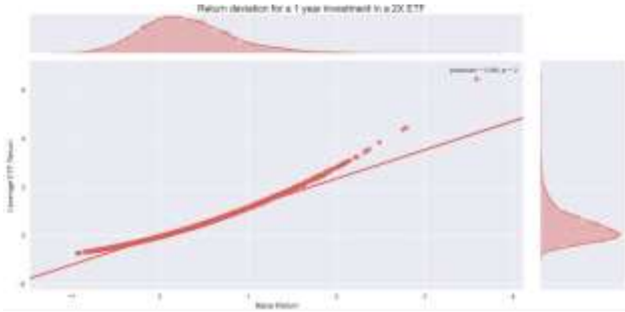
Appendix 5. Analysis of Distribution and Volatility -Simulated-

Week | Month | Year

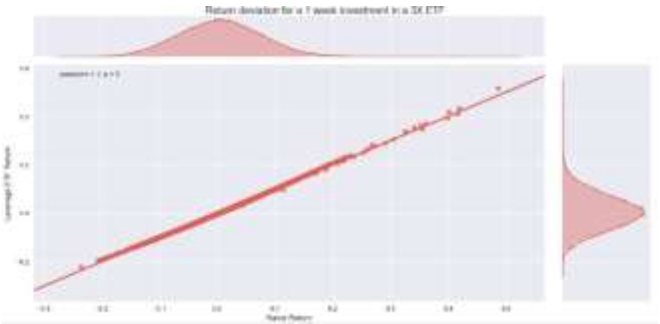
2X

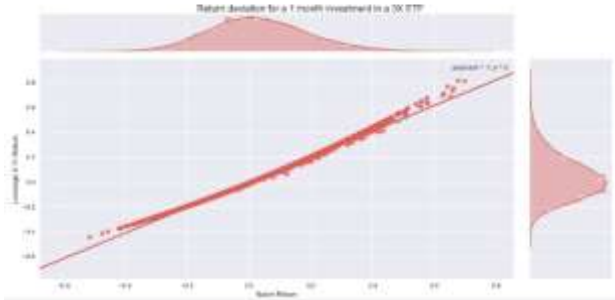


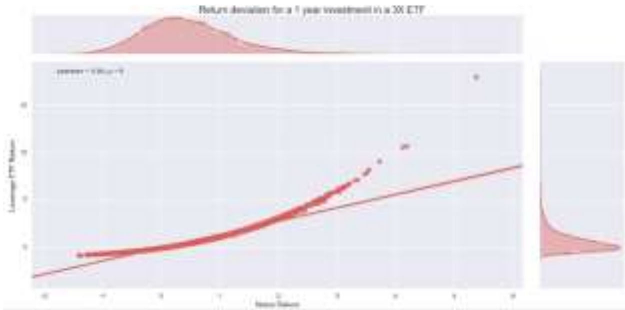




3X

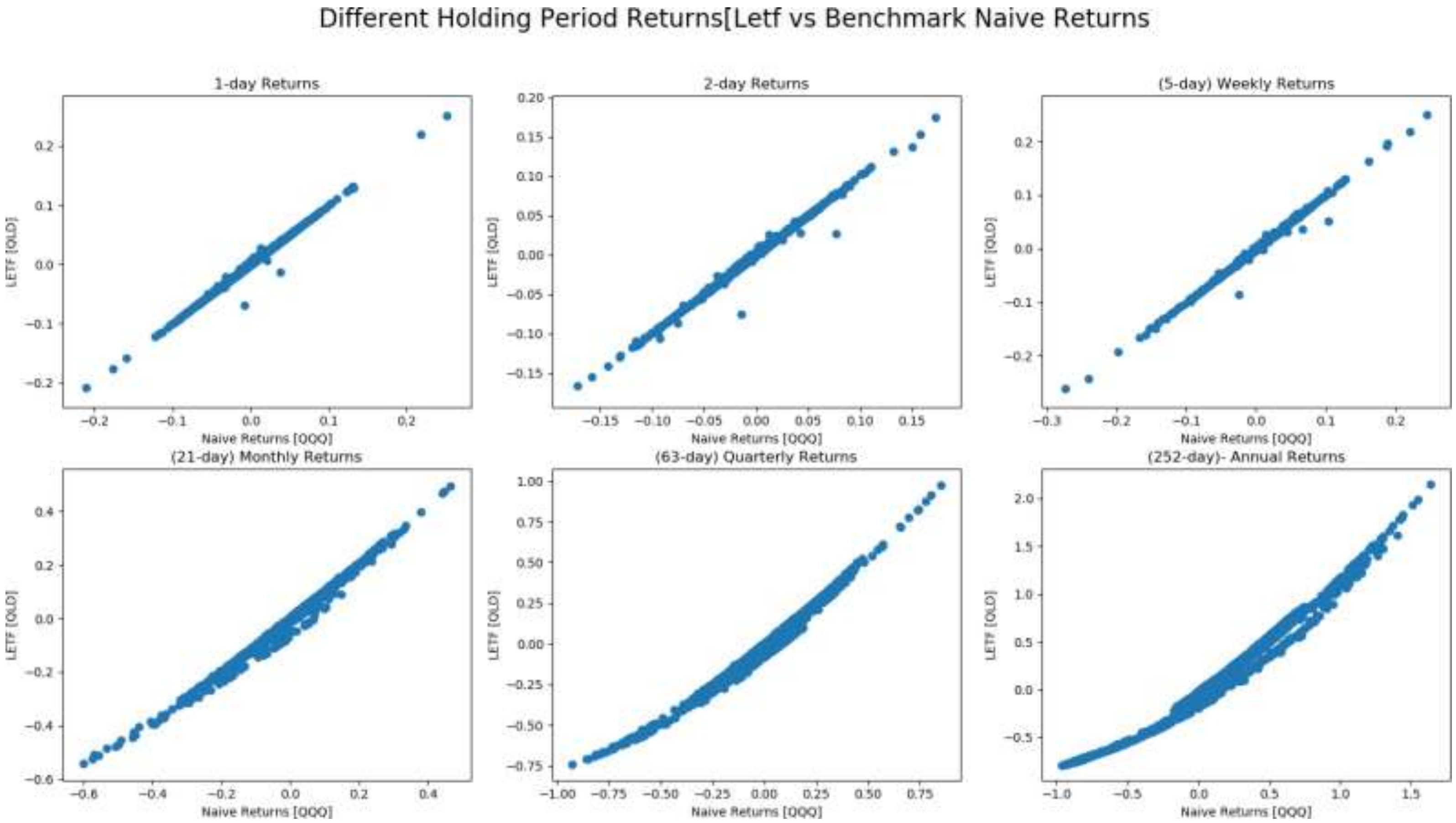






Appendix 6. Returns different holding periods [QLD] vs [QQQ]-Historical Data-

This figure shows the NAV returns of 2x Leveraged ProShares Ultra [QLD] versus returns of Benchmark [QQQ] over different holding periods. The sample period is from June 22, 2006 to March 29, 2019. The Quarterly and Annual returns are all calculated using overlapped data.



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