



Robust cross-country analysis of inequality of opportunity

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HIGHLIGHTS

- Gap curves are useful for robust cross-country analysis of inequality of opportunity.
- We propose a formal test for equality of opportunity in each country.
- We propose a formal test for differences in inequality of opportunity between countries.
- We illustrate our approach using EU-SILC data.
- We can robustly rank the 16 countries in about half of cross-country comparisons.

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ABSTRACT

International rankings of countries based on inequality of opportunity indices may not be robust vis-à-vis the specific metric adopted to measure opportunities. Indices often aggregate relevant information and neglect to control for normatively irrelevant distributional factors. This paper shows that gap curves can be estimated from cross-sectional data and adopted to test hypotheses about robust cross-country comparisons of (in)equality of opportunity.

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1. Introduction

Equality of opportunity (EOP) theory distinguishes between illegitimate sources of inequality that deserve compensation (e.g. parental background) and legitimate ones (e.g. effort) – see [Roemer and Trannoy \(2016\)](#). Opportunities are unequally distributed if some individuals enjoy an illegitimate advantage with respect to others, in relation to circumstances beyond their control. Inequality of Opportunity (IOP) indices operationalize this notion ([Ramos and Van de gaer, 2016](#)).

Empirical studies focusing on earnings inequality have found that IOP is only a small fraction of total inequality. This might be due to observability constraints ([Niehues and Peichl, 2014](#)), as well as to heterogeneity in earnings opportunities. Firstly, IOP indices *aggregate* heterogeneity in the distribution of illegitimate advantage across circumstances groups, thus discarding

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potentially relevant information. Secondly, IOP indices *neglect* the role of covariates which are not illegitimate drivers of inequality (such as age and marital status), but that are correlated with circumstances (older cohorts have on average less educated parents) and explain earnings heterogeneity (older cohorts/married individuals display higher earnings, see [Balcázar, 2015](#)). Parametric models have been developed ([Ferreira and Gignoux, 2011](#)) to account for the role of covariates, at the cost of introducing specification bias.

International rankings based on IOP indices may hence not be robust vis-à-vis the selected IOP metric. We address this issue by introducing *gap curves* in the cross-country analysis of IOP. A gap curve depicts the gap between opportunity profiles attributed to different circumstances in a given country. When there is no gap in opportunity profiles, there is strong evidence of EOP. Otherwise, IOP prevails. Our first contribution is to show that gap curves (i) can be used to test hypothesis about EOP in each country and (ii) can be contrasted across countries to test for differences in IOP. The normative underpinnings of the ordering induced by non-intersecting gap curves have been detailed in

Andreoli et al. (2019). Our second contribution shows that unconditional gap curves for each country can be flexibly estimated using distribution regression methods, controlling for the effect of irrelevant covariates on opportunity profiles. We tests EOp and IOp for earnings in Europe using the European Union-Survey on Income and Living Conditions (EU-SILC).

2. Concepts

Let earnings $y_s(c, \varepsilon)$ in country s depend on circumstances $c \in \{c_1, \dots, c_N\}$, defining types, and on distributional factors ε , characterizing within-type earning heterogeneity. Some factors in ε are rewarded by the market, such as effort or talent, while others are irrelevant from a normative perspective, such as age and marital status. The object of interest is the *opportunity set*, depicting the distribution of potential earnings accruing to type- c individuals. The set coincides with the conditional cdf $F_s(y|c)$ for a group of homogeneous individuals. In applied analysis, $F_s(y|c)$ is often (non-parametrically) estimated from data about earnings and circumstances, neglecting the contribution of potential earnings heterogeneity driven by normatively irrelevant covariates.

The gap curve $\Gamma_s(c, c', p)$ depicts the empirical distribution of the unfair gap between opportunity sets of types c and c' , and is defined:

$$\Gamma_s(c, c', p) := F_s^{-1}(p|c) - F_s^{-1}(p|c') \quad \forall c \neq c' \text{ and } p \in [0, 1],$$

where $F_s^{-1}(p|c)$ is the conditional quantile function corresponding to population share p (Fig. 1 displays an example). EOp imposes linear restrictions on the gap curve, leading to testable hypothesis:

$$H_0^{EOp}: \Gamma_s(c, c', p) = 0, \quad \forall c \neq c', \quad \forall p \in [0, 1].$$

EOp holds in country s (H_0^{EOp} not rejected) whenever opportunity sets coincide across all pairs $c \neq c'$. Otherwise, a form of IOp prevails. Indices can be used to rank countries by IOp. Many IOp indices are related to gap curves (see Andreoli and Fusco, 2017), including the Gini-opportunity index by Lefranc et al. (2008):

$$GO(s) := I(\mu_{c_1} \cdot (1 - G_{c_1}), \dots, \mu_{c_N} \cdot (1 - G_{c_N})) \\ = \frac{1}{2\mu_s} \sum_i \sum_j w_{c_i} \cdot w_{c_j} \cdot \left| \int_0^1 (1-p) \cdot \Gamma_s(c_i, c_j, p) dp \right|,$$

where w_c is circumstance c weight and μ_s the average earnings in the country. The GO incorporates efficiency (μ_c) and equity (Gini index G_c) concerns about the distribution of opportunities $F_s(y|c)$.

Rankings of countries based on IOp indices, including GO , are not robust to the selected IOp metric. Gap curves allow to compare countries on the basis of the whole distribution of opportunity gaps between any pair $c \neq c'$. Our baseline null hypothesis is that gap curves in countries s and s' coincide:

$$H_0^{IOp}: \Delta \Gamma(c, c', p) = \Gamma_s(c, c', p) - \Gamma_{s'}(c, c', p) = 0, \\ \forall c \neq c' \quad \forall p \in [0, 1].$$

Two countries are indistinguishable from a IOp perspective whenever H_0^{IOp} cannot be rejected. Rejection implies that fairness gaps between types differ across countries, albeit in an unrestricted way. Gap curves for types $c \neq c'$ may cross, in which case the distribution of unfair advantage depends on relevant distributional factors and countries cannot be robustly ordered. Alternatively, the gap curve for types $c \neq c'$ in country s dominates that of country s' , i.e., $\Gamma_s(c, c', \cdot)$ is never below and it is sometimes above $\Gamma_{s'}(c, c', \cdot)$. If there is strong dominance across all pairs $c \neq c'$ for which $\Delta \Gamma(c, c', \cdot) \neq 0$, then there is robust evidence that opportunities are more unequally distributed in country s compared to s' (Andreoli et al., 2019).

3. Estimation of unconditional gap curves

We estimate gap curves at a finite number of intercepts $p \in \{p_1, \dots, p_M\}$ using Recentered Influence Function (RIF) approximations of the quantile function (Firpo et al., 2009). The influence function of a quantile, $IF(F^{-1}(p)) = \frac{1-p}{f(F^{-1}(p))}$, measures the effect (by linearizing the inverse function) of a contamination in the data on that specific quantile. The RIF estimator yields unbiased estimates of the unconditional quantiles of the distribution. We provide RIF estimators for unconditional gap curves. For a given country and circumstance type, we first estimate linear probability regressions of an indicator $1(y_i \geq y)$, taking value 1 when observed income y_i is larger than a predetermined threshold y , on parental circumstances and covariates X_i (such as age and marital status):

$$\forall s, \forall i = 1, \dots, C: 1(y_i \geq y) = \alpha_i^s(y) \\ + \sum_{j \neq i} \beta_{ij}^s(y) \cdot 1(i \text{ is of type } c_j) + X_i \cdot \gamma_i^s(y) + u_i(y).$$

Income thresholds y in model i coincide with the observed quantiles of the conditional distributions $F_s(y|c_i)$ for each type separately.

The coefficients α, β and γ can be estimated using cross-sectional data. For country s and population share p , we estimate two effects: $\beta_{ij}^s(y)$ and $\beta_{ji}^s(y)$. The first effect provides an estimate of the probability gap $F_s(y|c_j) - F_s(y|c_i)$ at earnings $y = F_s^{-1}(p|c_i)$. Similarly, the second effect measures $F_s(y|c_i) - F_s(y|c_j)$ at earnings $y = F_s^{-1}(p|c_j)$. We apply the IF formula above to obtain estimates of the gap curve coordinates, expressed in the space of earnings. Since the quantiles $F_s^{-1}(p|c_i)$ and $F_s^{-1}(p|c_j)$ do not generally coincide, we use the average effect as a reliable estimate of the unconditional gap curve. This gives:

$$\hat{\Gamma}_s(c_i, c_j, p) = \frac{1}{2} \left[\frac{\beta_{ji}^s(F_s^{-1}(p|c_j))}{f_s(F_s^{-1}(p|c_j)|c_j)} - \frac{\beta_{ij}^s(F_s^{-1}(p|c_i))}{f_s(F_s^{-1}(p|c_i)|c_i)} \right] \\ \forall c \neq c' \text{ and } p \in [0, 1],$$

where $f_s(y|c_i)$ is the density of the conditional distribution (non-parametrically identified) of type c_i earnings opportunities.

Gap curves are estimated at earnings deciles ($M = 10$), their variance-covariance matrices are bootstrapped. Assuming normality, H_0^{EOp} and H_0^{IOp} can be tested against an unrestricted alternative using χ_{M-1}^2 -distributed joint equality tests for vectors of quantiles estimates. We use t-tests of quantile-specific differences in gap curves to test $\Gamma_s(c, c', p) - \Gamma_{s'}(c, c', p) \geq 0, \forall p \in [0, 1]$ (dominance) for those pairs $c \neq c'$ for which H_0^{IOp} is rejected among countries s and s' . $GO(s)$ is estimated from empirical gap curves via numerical integration methods, thus controlling for normatively irrelevant factors.

4. Empirical illustration

We use the 2011 EU-SILC module on “intergenerational transmission of disadvantage” to test EOp and IOp for earnings acquisition across 16 European countries. Parental education (high, medium, low) defines three types. Our sample includes male full-time employed aged 30–50 (see Andreoli and Fusco, 2017 for details). Estimates are always conditional on age and marital status.

Countries in Table 1 are arranged by increasing GO . Countries that display similar GO levels are statistically indistinguishable, while the ranking of the other countries stems from marginal differences in IOp (below diagonal, “=” indicates insignificant differences at 5% level). We use gap curves to qualify these results.

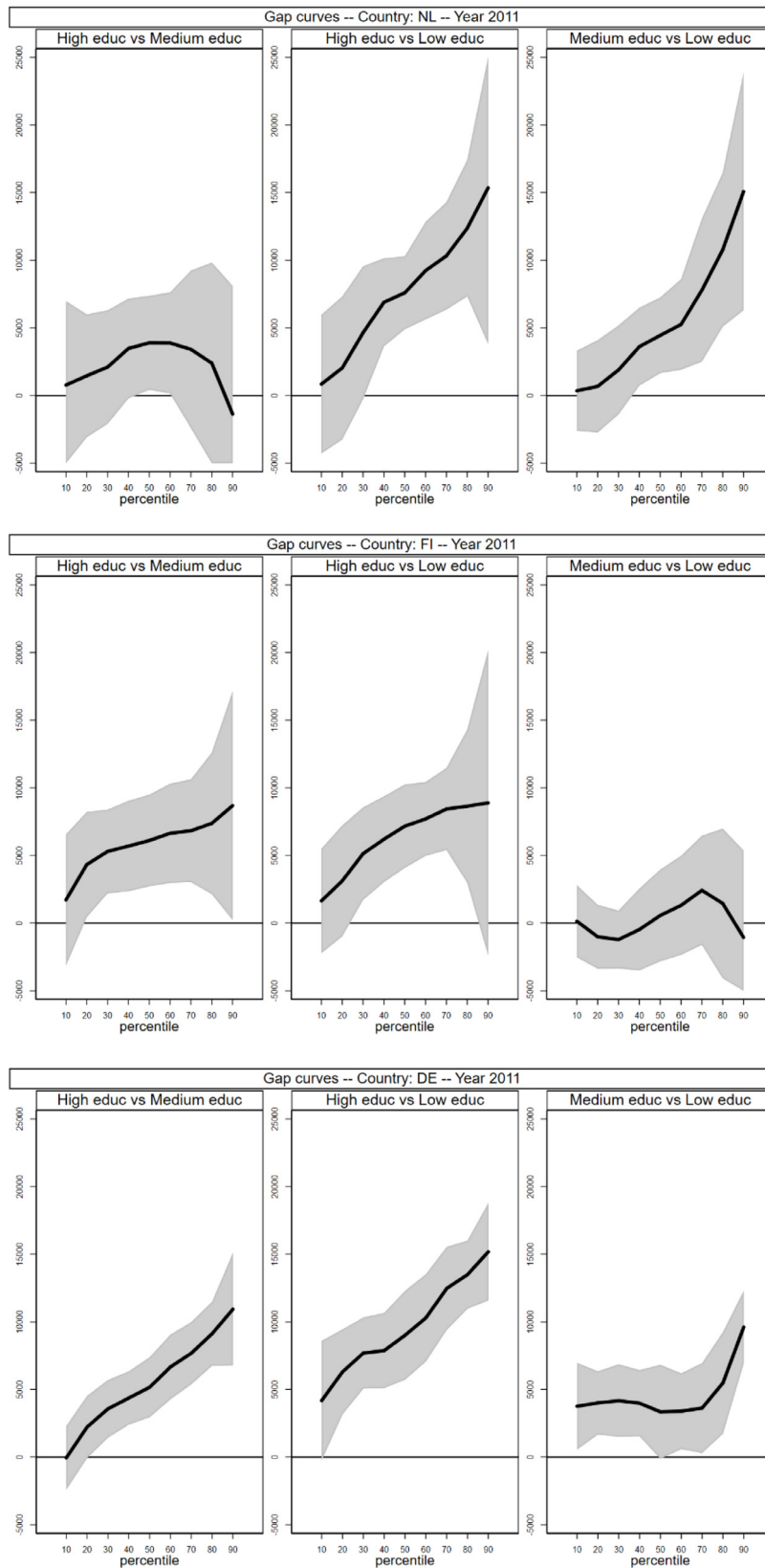


Fig. 1. Gap curves in selected countries (with 95% CI).

Our first result (diagonal in Table 1) is that in about half of the countries, EOp is rejected across all three comparisons: high vs medium, high vs low and medium vs low parental education. In the remaining countries (including the Nordic), H_0^{EOp} is not

rejected for only one pair of types. This is strong evidence against EOp in Europe. We contrast gap curves across countries to test for robust IOp orderings.

Table 1
Tests for EOp and IOp, 16 EU-SILC countries, 2011.

Country	GO	Comparison country																
		NL	FI	DE	SK	NO	SE	IS	AT	BE	PL	UK	EE	LT	HU	IE	LU	
NL	0.023	2	0	0	n.o.	0	0	0	0	0	0	1	0	0	0	1	2	
FI	0.026	=	2	1	0	0	0	0	1	1	0	0	0	1	1	2		
DE	0.028	=	=	3	n.o.	0	0	0	0	0	n.o.	1	0	n.o.	1	0	2	
SK	0.028	=	=	=	2	0	0	1	2	2	1	2	0	1	1	2	2	
NO	0.028	=	=	=	=	2	0	0	0	0	0	0	0	n.o.	0	1	2	
SE	0.032	=	=	=	=	=	3	0	0	0	0	0	0	n.o.	0	1	2	
IS	0.036	=	=	=	=	=	=	2	0	0	0	0	0	0	0	0	2	
AT	0.042	>	=	=	=	=	=	=	3	0	n.o.	0	0	1	n.o.	1	2	
BE	0.043	>	=	>	>	=	=	=	=	3	0	0	0	1	0	1	2	
PL	0.045	>	>	>	>	>	=	=	=	=	3	1	0	n.o.	0	2	2	
UK	0.046	>	>	>	>	>	=	=	=	=	=	3	n.o.	n.o.	n.o.	1	2	
EE	0.053	>	>	>	>	>	=	=	=	=	=	=	2	0	0	2	2	
LT	0.055	=	=	=	=	=	=	=	=	=	=	=	=	3	1	2	3	
HU	0.062	>	>	>	>	>	>	>	>	>	>	>	=	=	=	3	1	2
IE	0.070	>	>	>	>	>	>	>	>	>	>	>	=	=	=	=	2	0
LU	0.101	>	>	>	>	>	>	>	>	>	>	>	>	>	>	>	>	3

Note: Earnings opportunities of three types: low, medium and high parental education.

Our second result (above diagonal in Table 1) concerns the IOp ranking: H_0^{IOP} is not rejected in 63 over 120 pairwise comparisons of countries (in this case, we report “0”). These countries display similar levels of IOp as their gap curves coincide for all pairs $c \neq c'$. The result contrasts the ordering produced by GO (for instance, $GO(UK) > GO(FI)$ although H_0^{IOP} is not rejected between these two countries), thus unveiling the consequences of aggregating heterogeneity. In each of the remaining cases (57), there exists at least a pair of types for which gap curves do not coincide (H_0^{IOP} rejected). If the gap curves cross, the two countries are not robustly ordered (“n.o.”) in terms of IOp. Otherwise, gap curves are clearly ordered, with column-countries in Table 1 robustly displaying more IOp than row-countries (the table reports the cases in which dominance in gap curves holds). In a large majority of comparisons for which H_0^{IOP} is rejected (45/57), countries can be robustly ordered. In particular, Luxembourg and Ireland are the most opportunity-unequal countries in Europe, while the UK and Belgium are robustly ranked as more opportunity unequal than all the low-IOp countries.

The graphs of the gap curves depict the full heterogeneity in opportunity gaps within and across countries. Fig. 1 displays gap curves for the least opportunity-unequal countries (all equal in terms of GO). In the Netherlands, IOp originates from the earnings penalty attributable to low-educated parents. Conversely, unfair advantage in Finland is clustered on children raised by high-educated parents. In both countries, unfair gaps increase with earnings opportunities, suggesting complementarity between parental background and distribution factors. Patterns of disadvantage in Germany resemble that in Finland, with an important difference: children with low-educated parents suffer a significant earnings penalty with respect to children with middle-educated parents, albeit disadvantage is unrelated to distributional factors. Gap curves dominance allows to conclude that Germany displays robustly more IOp than Finland, an evidence not captured by IOp indices. Many other cross-country comparisons display similar patterns.

5. Conclusions

Gap curves are useful to identify and test for robust IOp rankings of countries. Using distribution regression methods, we are able to (i) estimate the full distribution of the fairness gaps implied by a gap curve while (ii) controlling for normatively irrelevant covariates, two aspects neglected by IOp indices. Our empirical illustration shows that (i) EOp in Europe is strongly rejected, (ii) in about half of cross-country comparisons, we are

able to robustly rank countries by IOp, and (iii) even in least opportunity-unequal countries, gap curves reveal substantial differences in the way high or low educated parental background induces advantages or penalties in earnings, and in the way (dis)advantage correlates with effort/talents.

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