

Cyclostationary noise analysis of superregenerative oscillators

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Abstract—A rigorous analysis of noise effects in superregenerative oscillators (SRO) operating in linear mode is presented. The analysis takes into account the cyclostationary nature of the SRO response to the noise sources, due to the effect of the quench signal. It is based on the determination of an envelope-domain linear-time-variant (LTV) transfer function with respect to each noise source, plus the application of a detailed stochastic analysis of the SRO output. Initially, the autocorrelation of the output signal is calculated, which varies at two different time scales, and is periodic with respect to the quench signal, so it can be expressed in terms of the frequency-dependent harmonic components of the LTV transfer functions. This enables the calculation of the output spectral density, depending on these harmonic components. Once the spectral density is known, the signal-to-noise ratio can be obtained in a straightforward manner. The analysis method has been validated with both independent circuit-level simulations and measurements.

Keywords—superregenerative oscillator, stability, noise.

I. INTRODUCTION

Superregenerative oscillators (SROs) use the exponential growth of an oscillation signal to obtain high gain amplification. Due to the high gain values attained, SROs have been used to replace amplifier chains in receivers [1]-[2] and, more recently, have also been implemented in active transponders [3]. The oscillation is controlled by a quench signal that periodically switches the oscillator on and off (Fig. 1), by shifting the critical pair of complex-conjugate poles from the left-hand side of the complex plane (LHS) to the right-hand side (RHS) and then back to the LHS. The SRO is sensitive to the input signal only during a fraction of the quench-signal period, located about the time value at which the critical pair of complex-conjugate poles crosses to the RHS [1]-[2]. The signal grows as long as this critical pair of poles is located on the RHS, so the maximum output amplitude is obtained when the poles cross again to the LHS [1]-[2]. Thus, the SRO responds with an output pulse, controlled by the quench signal (Fig. 1). The input signal provides an initial condition that places the system at a certain level of the initial transient and, in the so-called linear mode, the amplitude of the output pulse is proportional to the amplitude of this input signal [1]. However, the performance of the SRO may be degraded by the noise perturbations, which may even give rise to non-negligible output pulses in the absence of an intended input signal. Thus, a reliable prediction of the signal-to-noise ratio (S/N) at the design stage is essential.

Several previous works [4]-[5] have presented a calculation of the signal-to-noise ratio of SROs in linear mode. However, most of them consider a simplified second-order system, usually described in terms of a cubic nonlinearity and a resonator. On the other hand, they do not accurately account for the cyclostationary properties of the circuit response to the noise sources, resulting from the action of the quench signal.

These properties enable an intuitive analysis of the cyclostationary process, which facilitates the calculation of both the output noise power spectral density (PSD) and the output noise power. The analysis presented here will make use of the numerical black-box model proposed in [6], which enables the determination of the SRO response to any arbitrarily modulated input signal. It is based on the calculation of an envelope-domain time-variant transfer function [7], relating the output and input of the SRO, and extracted from circuit-level envelope-transient simulations [8]-[9]. Here the black-box modelling technique will be used to obtain a linear time variant (LTV) transfer function with respect to each noise source. Then, a stochastic analysis will be carried out, taking into account the cyclostationary nature of the autocorrelation function of the output signal. This will enable an accurate prediction of the noise power spectral density and a straightforward determination of the signal-to-noise ratio.

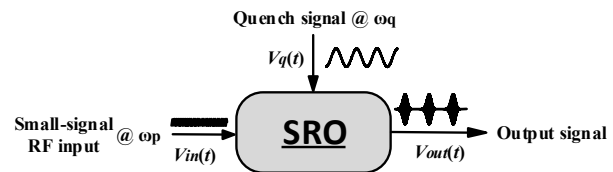


Fig. 1. Schematic representation of the SRO operation.

II. NOISE ANALYSIS

A. LTV transfer functions with respect to the noise sources

The envelope-domain analysis of the SRO is performed considering the oscillator free-running frequency ω_p as the carrier frequency. Then, the SRO output due to noise is expressed as:

$$n_{out}(t) = |N_{out}(t)| \cos(\omega_p t + \phi_{out}(t)) = \text{Re} \left[N_{out}(t) e^{j\omega_p t} \right] \quad (1)$$

In turn, the M independent noise sources are expressed as $N_m(t) e^{j\omega_p t}$, where $m = 1$ to M . To calculate the LTV transfer function [7] with respect to the noise source $N^m(t)$, this source is replaced with a deterministic small-signal source, having the general representation $G e^{j(\omega_p + \Omega)t}$, where the constant envelope G may correspond to either a voltage or current. The LTV transfer function associated to this source is given by [7]:

$$H^m(t, \Omega + \omega_p) = V_{out}(t, \Omega + \omega_p) / G \quad (2)$$

where V_{out} is the envelope of the output voltage. The above function is calculated sweeping Ω in $G e^{j(\omega_p + \Omega)t}$ and performing a circuit-level envelope transient analysis at each Ω step.

Because the quench signal is periodic, $H^m(t, \Omega + \omega_p)$ is periodic too, with the same period T_q . To illustrate the meaning of $H^m(t, \Omega + \omega_p)$, the case of a single arbitrary noise source $N^m(t)$ is initially considered. The output envelope $N_{out}(t)$ due this source is calculated discretising the time variable in $H^m(t, \omega_p + \Omega)$ and performing an IFT integral at each time t_n :

$$N_{out}(t_n) = \frac{1}{2\pi} \int_{\omega_{\min} - \omega_p}^{\omega_{\max} - \omega_p} H^m(t_n, \Omega + \omega_p) N^m(\Omega) e^{j\Omega t_n} d\Omega \quad (3)$$

When the SRO operates in linear mode, only the device white noise sources will have a relevant effect.

B. Stochastic analysis of the SRO output

Let a noise source with the envelope $N(t)$ be considered, where the superscript m has been dropped for notation clarity. For simplicity, the integration frequency interval about ω_p will be assumed to be symmetrical, so expression (3) is rewritten as:

$$N_{out}(t) = \frac{1}{2\pi} \int_{-B/2}^{B/2} H(t, \Omega + \omega_p) N(\Omega) e^{j\Omega t} d\Omega \quad (4)$$

where $N(\Omega) = N_r(\Omega) + jN_i(\Omega)$ and B is the input noise bandwidth. To obtain the power spectral density (PSD) of the process $N_{out}(t)$ one must first calculate the correlation function $R(t, \tau)$, with a double time dependence. In the case of a white-noise source $N(t)$, this is obtained as:

$$\begin{aligned} R(t, \tau) &= E[N_{out}(t + \tau) N_{out}(t)^*] = \\ &= \Gamma \int_{-B/2}^{B/2} \int_{-B/2}^{B/2} R_H(t, \tau; \Omega_1, \Omega_2) \delta(\Omega_1 - \Omega_2) e^{j((\Omega_1 - \Omega_2)t + \Omega_1 \tau)} d\Omega_1 d\Omega_2, \quad (5) \\ R_H(t, \tau; \Omega_1, \Omega_2) &\equiv H(t + \tau, \Omega_1 + \omega_p) H(t, \Omega_2 + \omega_p)^* / (2\pi)^2 \end{aligned}$$

where $E[N(\Omega_1) N(\Omega_2)^*] = \Gamma \delta(\Omega_1 - \Omega_2)$ and Γ is the constant PSD. Operating the delta function, the above equation simplifies as:

$$R(t, \tau) = \left(\frac{1}{2\pi} \right)^2 \Gamma \int_{-B/2}^{B/2} H(t + \tau, \Omega + \omega_p) H(t, \Omega + \omega_p)^* e^{j\Omega \tau} d\Omega \quad (6)$$

Due to the time periodicity of $H(t, \Omega + \omega_p)$ for each Ω , this function can be expanded in a Fourier series:

$$H(t, \Omega + \omega_p) = \sum_k H_k(\Omega) e^{jk\omega_q t}, \quad \omega_q = 2\pi/T_q \quad (7)$$

Replacing (7) into (6) one obtains:

$$\begin{aligned} R(t, \tau) &= E[N_{out}(t + \tau) N_{out}(t)^*] = \\ &= \frac{\Gamma}{(2\pi)^2} \sum_{k,l} \int_{-B/2}^{B/2} H_k(\Omega) H_l(\Omega)^* e^{j(k-l)\omega_q t} e^{j(k\omega_q + \Omega)\tau} d\Omega = \quad (8) \\ &= \sum_{k,l} U_{k,l}(\tau) e^{j(k-l)\omega_q t} = \sum_s R_s(\tau) e^{js\omega_q t}, \quad R_s(\tau) \equiv \sum_{k-l=s} U_{k,l}(\tau) \end{aligned}$$

where the components $U_{k,l}(\tau)$ are given by:

$$U_{k,l}(\tau) = \frac{\Gamma}{(2\pi)^2} \int_{-B/2}^{B/2} H_k(\Omega) H_l(\Omega)^* e^{j(k\omega_q + \Omega)\tau} d\Omega \quad (9)$$

From (8), the autocorrelation function $R(t, \tau)$ satisfies:

$$R(t, \tau) = R(t + T_q, \tau) \quad (10)$$

On the other hand, and taking into account the periodicity of $H(t, \Omega + \omega_p)$, it is easily seen from (4) that the time average $E[N_{out}(t)]$ satisfies:

$$E[N_{out}(t)] = E[N_{out}(t + T_q)] \quad (11)$$

The fulfilment of (10) and (11) indicates that $N_{out}(t)$ is a cyclostationary stochastic process. Then, the PSD of this process can be calculated using the Fourier series expansion (8) of the correlation function as [10]-[11]:

$$S(\Omega) = \left\langle |N_{out}(\Omega)|^2 \right\rangle = \int_{-\infty}^{\infty} R_0(\tau) e^{-j\Omega \tau} d\tau \quad (12)$$

This is because, as demonstrated in [10]-[11], only the term with $s=0$ of summation (8) contributes to the cyclostationary process PSD. Now, comparing (8) and (9), the term $R_0(\tau)$, which provides the output PSD, is given by:

$$\begin{aligned} R_0(\tau) &= \sum_{k=-N}^N U_{k,-k}(\tau) = \frac{\Gamma}{(2\pi)^2} \sum_{k=-P}^P e^{jk\omega_q \tau} g_k(\tau), \quad (13) \\ g_k(\tau) &\equiv \int_{-\infty}^{\infty} |H_k(\Omega)|^2 e^{j\Omega \tau} d\Omega \end{aligned}$$

where it has been assumed that $H_k(\Omega) \approx 0$ for $|\Omega| > B/2$. Finally, introducing (13) in (12), the PSD of the output noise is:

$$\begin{aligned} S_n(\Omega) &= \frac{\Gamma}{(2\pi)^2} \sum_{k=-P}^P \int_{-\infty}^{\infty} g_k(\tau) e^{-j(\Omega - k\omega_q)\tau} d\tau = \\ &= \frac{\Gamma}{(2\pi)^2} \sum_{k=-P}^P |H_k(\Omega - k\omega_q)|^2 \quad (14) \end{aligned}$$

Expression (14) provides an intuitive approach to the output noise PSD $S_n(\Omega)$. Indeed, this equation shows that $S_n(\Omega)$ is shaped by the Fourier components $H_k(\Omega)$ of the LTV transfer function $H(t, \Omega + \omega_p)$. Note that the contribution of each component $H_k(\Omega)$ is shifted to the k -th harmonic of the quench frequency ω_q .

In the case of M white-noise sources, with the spectral densities Γ_m , where $m = 1 \dots M$, the output noise process in (4) would be given by the sum of all integral contributions corresponding to each input noise source. The analysis should be carried out in the same way as (4)-(14), producing a total output-noise spectral density of the form $S_n(\Omega) = \sum_{m,n} S_n^{m,n}(\Omega)$,

with $S_n^{m,n}(\Omega)$ being the terms arising from the correlation between the m -th and n -th noise sources.

C. Signal to noise ratio

To evaluate the S/N ratio one should first obtain the noise power about the carrier frequency ω_p . The case of a single white-noise source with the spectral density Γ will be initially considered. An expression for the noise power can be derived using the result (14) for the output noise PSD as:

$$\begin{aligned} P_n &= \frac{\langle n_{out}(t)^2 \rangle}{R} = \frac{\langle \left(\text{Re} \left(N_{out}(t) e^{j\omega_p t} \right) \right)^2 \rangle}{R} = \frac{\langle |N_{out}(t)|^2 \rangle}{2R} \\ &= \frac{1}{2R} \int_{-\infty}^{\infty} S_n(\Omega) d\Omega = \frac{1}{2R} \frac{\Gamma}{(2\pi)^2} \sum_k \int_{-\infty}^{\infty} |H_k(\Omega - k\omega_p)|^2 d\Omega = (15) \\ &= \frac{1}{2R} \frac{\Gamma}{(2\pi)^2} \int_{-\infty}^{\infty} \sigma_{H^i}^2(\Omega) d\Omega \end{aligned}$$

where expression (1) has been applied and $\sigma_{H^i}^2(\Omega) = \langle |H^i(t, \Omega + \omega_p)|^2 \rangle$ is the time average of the square value of the LTV transfer function. Note that the mean square value of the envelope carrier contributes with the factor $\langle \sin^2 \omega_p t \rangle = \langle \cos^2 \omega_p t \rangle = 1/2$ in (15). On the other hand, the signal power is obtained considering a single RF input tone, which will fulfill $V_{in}(\Omega) = A\delta(\Omega - \Omega_0)$, where Ω_0 represents a frequency offset from the carrier at ω_p . Using the black-box model in [6], the signal power is:

$$\begin{aligned} P &= \frac{\langle |V_{out}(t)|^2 \rangle}{2R} = \frac{1}{2R} \left\langle \left| \int_{-B/2}^{B/2} \frac{H^i(t, \Omega + \omega_p)}{2\pi} V_{in}(\Omega) e^{j\Omega t} d\Omega \right|^2 \right\rangle \\ &= \frac{1}{2R} \left(\frac{A}{2\pi} \right)^2 \langle |H^i(t, \Omega_0 + \omega_p)|^2 \rangle = \frac{1}{2R} \left(\frac{A}{2\pi} \right)^2 \sigma_{H^i}^2(\Omega_0) \end{aligned} \quad (16)$$

where $H^i(t, \Omega + \omega_p)$ is the LTV transfer function with respect to the input signal. Finally, the signal-to-noise (S/N) ratio is calculated using (15) and (16) as:

$$S/N = P/P_n = \frac{A^2}{\Gamma} \frac{\sigma_{H^i}^2(\Omega_0)}{\int_{-\infty}^{\infty} \sigma_{H^i}^2(\Omega) d\Omega} \quad (17)$$

III. APPLICATION TO A FET-BASED SRO

The schematic and photograph of the FET-based SRO are shown in Fig. 2. This SRO operates at the oscillation frequency $f_p = 2.7$ GHz, with the quench signal frequency $f_q = 8$ MHz. The main white-noise contributions are due to the input 50 Ohm resistor, modeled with a voltage source N_1 , and the transistor noise, modelled with an equivalent drain-to-source current source N_2 . The PSD of the input noise source and the transistor drain source are $\Gamma_1 = 8 \cdot 10^{-19}$ V²/Hz and $\Gamma_2 = 4 \cdot 10^{-20}$ A²/Hz, respectively. Two envelope-domain LTV transfer functions $H^m(t, \Omega + \omega_p)$, where $m = 1, 2$, have been calculated with respect to these two noise sources. For illustration, Fig. 3(a)

shows the transfer function with respect to the drain noise source (which is the dominant contribution), evaluated at $\Omega/(2\pi) = -42.6$ MHz. As can be seen, it is periodic at the quench-signal period and has a pulsed shape. The Fourier components $H_k(\Omega)$ of this signal for $k=0, 1, 2$ are shown in Fig. 3(b). The PSD of the output noise due to N_2 has been calculated using expression (14) with $P=32$ components. with the result of Fig. 4.

An input signal of power $P_{in} = -60$ dBm and frequency offset $\Omega_0 \approx 0$ will be considered. To obtain the S/N ratio, the mean square values $\sigma_{H^{1,2}}^2(\Omega)$, corresponding to the noise sources, and $\sigma_{H^1}^2(0) = \langle |H^1(t, \omega_p)|^2 \rangle$, corresponding to the RF input signal are calculated. The resulting values of P_n , P and S/N using (15)-(17) are shown in Table I. In the following, these results will be compared with both circuit-level envelope-transient simulations and measurements of the noise response.

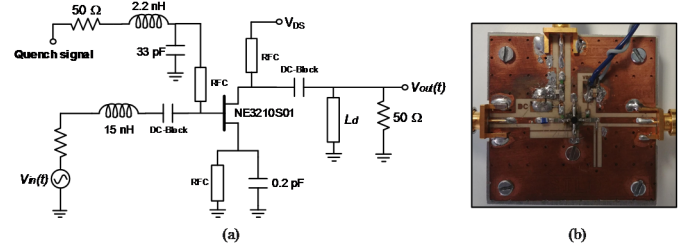


Fig. 2. Schematic and photograph of the FET-based SRO. The oscillation and quench frequency values are $f_p = 2.7$ GHz and $f_q = 8$ MHz, respectively.

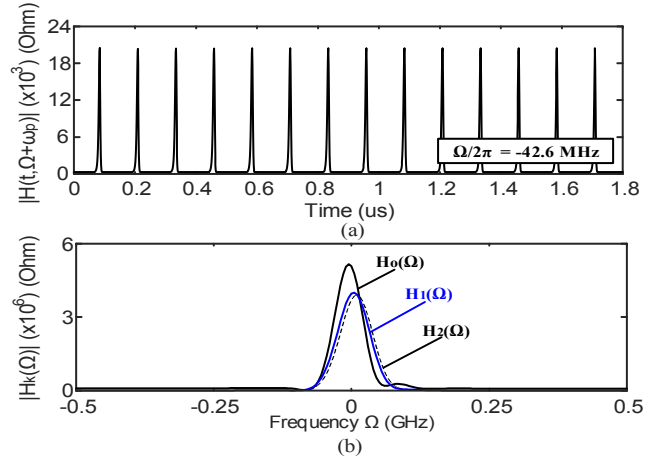


Fig. 3. Calculation of the output noise PSD. (a) Magnitude of the LTV transfer function $H^2(t, \Omega + \omega_p)$ evaluated at $\Omega/(2\pi) = -42.6$ MHz. This function is calculated with respect to the current noise source N_2 . (b) Fourier coefficients $H_k(\Omega)$ of $H^2(t, \Omega + \omega_p)$, for $k=0, 1, 2$.

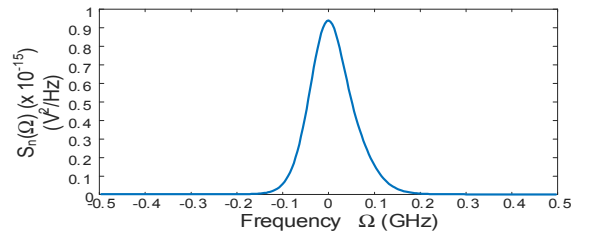


Fig. 4. PSD of the output noise due to a white-noise current source N_2 . The PSD of the source is $\Gamma_2 = 4 \cdot 10^{-20}$ A²/Hz.

A. Numerical simulations

From (15)-(16), the S/N ratio can be expressed in terms of the mean square values of the envelope signals $V_{out}(t)$ and $N_{out}(t)$ as $S/N = \langle |V_{out}(t)|^2 \rangle / \langle |N_{out}(t)|^2 \rangle$. The mean square value $\langle |N_{out}(t)|^2 \rangle$ is obtained through a costly circuit-level envelope-transient simulation, performed during a time interval of 12 μ s. The output noise signal $N_{out}(t)$ is shown in Fig. 5(a). It presents oscillation pulses of small amplitude, due to the presence of noise perturbations in the SRO sensitivity interval. The resulting S/N value is compared in Table I with the one provided by the stochastic analysis (15)-(16). There is a very good agreement between both analyses, with less than 1 dB of difference between the two calculations.

B. Measurement results

The experimental characterization of the SRO is initially performed without the RF deterministic input signal. The resulting output signal, due to noise perturbations only, is processed to extract the envelope of the output pulses, with the result of Fig. 5(b). The amplitude levels of the envelope pulses in $N_{out}(t)$ are very similar to those obtained in the simulation of Fig. 5(a). In the measurement, the S/N value is directly calculated from the SRO output with and without input signal. The S/N values obtained in measurement and simulation are compared in Table I.

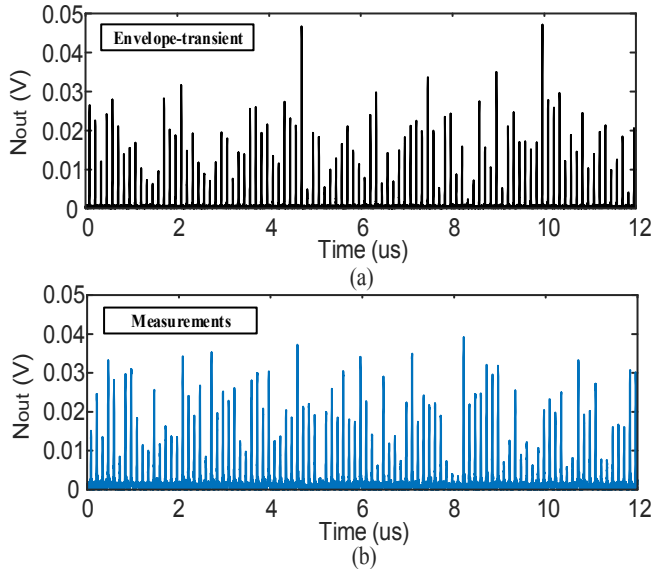


Fig. 5. Noise power calculation. Envelope of the output signal in the presence of noise perturbations only. (a) Envelope-transient simulation. (b) Measurements.

Table I. Signal to noise ratio calculation.

	P(μ W)	P_n (μ W)	$P+P_n$ (μ W)	S/N (dB)
Eq.(15)– Eq. (16)	5.789	0.1071	5.896	17.32
Simulation	5.786	0.1173	5.903	16.93
Measurements	-	0.3241	13.23	16.01

IV. CONCLUSION

A stochastic analysis of the output signal of SROs in the presence of noise sources has been presented. It is applicable to SROs of arbitrary topology, since it is based on a black-box model of the SRO, in terms of a linear time variant (LTV) transfer function with respect to each noise source. The stochastic analysis takes into account the cyclostationary nature of the autocorrelation function of the output signal, enabling a straightforward determination of the output noise power spectral density and the signal-to-noise ratio. The results of the stochastic analysis have been successfully compared with circuit-level envelope-transient simulations, in the presence of noise sources, and with measurements.

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