

BEST RANK- k APPROXIMATIONS FOR TENSORS: GENERALIZING ECKART-YOUNG

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Joint work with Jan Draisma and Giorgio Ottaviani. Given a tensor f in a Euclidean tensor space, we are interested in the critical points of the distance function from f to the set of tensors of rank at most k , which we call the critical rank-at-most- k tensors for f . When f is a matrix, the critical rank-one matrices for f correspond to the singular pairs of f . The critical rank-one tensors for f lie in a linear subspace H_f , the critical space of f . Our main result is that, for any k , the critical rank-at-most- k tensors for a sufficiently general f also lie in the critical space H_f . This is the part of Eckart-Young Theorem that generalizes from matrices to tensors. Moreover, we show that when the tensor format satisfies the triangle inequalities, the critical space H_f is spanned by the complex critical rank-one tensors. Since f itself belongs to H_f , we deduce that also f itself is a linear combination of its critical rank-one tensors. For simplicity, we will focus on binary forms during the talk.

REFERENCES

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