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Conservation laws, classical symmetries and exact solutions of the generalized KdV-Burgers-Kuramoto equation

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Abstract: For a generalized KdV-Burgers-Kuramoto equation we have studied conservation laws by using the multiplier method, and investigated its first-level and second-level potential systems. Furthermore, the Lie point symmetries of the equation and the Lie point symmetries associated with the conserved vectors are determined. We obtain travelling wave reductions depending on the form of an arbitrary function. We present some explicit solutions: soliton solutions, kinks and antikinks.

Keywords: generalized KdV-Burgers-Kuramoto, Lie symmetries, multipliers, potential symmetries, travelling wave solutions.

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1 Introduction

This paper considers the generalized KdV-Burgers-Kuramoto equation

$$u_t + f(u)_x = \mu u_{xx} + \delta u_{xxx} - \lambda u_{xxxx} = 0, \qquad (1)$$

where $\mu > 0$ represents a dissipative effect, $\delta \in \mathbb{R}$ is a stroboscopic coefficient, $\lambda \geq 0$ represents an unstable function and f is a nonlinear function. It is a dissipative, stroboscopic and unstable system in physics [2], which generalizes the KdV-Burgers-Kuramoto equation

$$u_t + uu_x + \alpha u_{xx} + \beta u_{xxx} + \gamma u_{xxxx} = 0 \qquad (2)$$

where α, β and γ are constants.

Both equations have been in focus of many researchers due to their applications to many phenomena that are simultaneously involved in nonlinearity, dissipation, dispersion and instability.

For example, some papers have analyzed the rarefaction waves of this equation in order to find a relation between its stability and its strength and initial perturbation. In [1] is shown that rarefaction waves are nonlinearly stable, provided that the initial perturbation is large. However, in [2] is shown that nonlinearly stable rarefaction waves are associated with sufficiently small strength and initial perturbation.

On the other hand, in [3] exact solutions are obtained by using trigonometric function expansion method, exact travelling wave solutions have been obtained also in [4] by a generalized F-expansion method and, moreover, in [5] the first available numerical solutions are obtained by an homotopy analysis method.

In addition lots of works even have studied the fractional KdV-Burgers-Kuramoto equation. For instance, in [6] its solution is obtained by using the He's variational iteration method and Adomian's decomposition method.

Conservation laws have important uses in the study of PDEs in which certain physical properties do not change in the course of time, especially for determining conserved quantities and constants of motion. They are also useful for detecting integrability and linearizations, finding potentials and nonlocally-related systems, as well as checking the accuracy of numerical solution methods.

Moreover, as part of this analysis of (1), we have studied the conservation laws of the equation. Anco and Bluman proposed the multiplier method [7–9] that gave a general treatment of a direct conservation law for partial differential equations and we have applied it. Few examples of the multiplier method can be found in [10–13].

We have applied the classical Lie method to (1). Lie classical method [14] is a successful method with a wider applicability in physics due to its important applications in the context of differential equations [15]. There are many researchers that applied this method to partial differential

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equations to understand and study in depth several phenomena.

As example, Ndlovu and Moitsheki studied the Lie point symmetries admitted by the transient heat conduction problem [16], Gandarias and Khalique worked with symmetries in a generalization of the damped externally excited KdV equation [17], De la Rosa and Bruzón obtained classical and nonclassical symmetries of a generalized Gardner equation [18], Garrido and Bruzón made an analysis of the Generalized Drinfeld-Sokolov System [19], and so on.

As Alex J. Dragt said "One of the key discoveries of modern physics is that Lie groups are important for the description of Nature" [20]. In fact, Lie method may be used to reduce the number of independent variables of the partial differential equations (PDEs); in particular we might reduce the PDEs to ordinary differential equations (ODEs). The ODEs may also have symmetries that allow us to reduce the order of the equation, and we can integrate to find exact solutions. A great progress has being made in the development of methods and their applications for finding solitary traveling-wave solutions of nonlinear evolution equations. Many solutions of nonlinear partial differential equations have been found by one or other of these methods [21–24].

Our present work has the following aims. We show that the generalized KdV-Burgers-Kuramoto equation (1) admits only trivial local conservation law. We use the conservation law to obtain the associated potential systems. We investigate classical and potential symmetries. From reduced equation, we obtain exact solutions.

2 Multiplier method. Conservation laws

A local conservation law of equation (1) is a continuity equation

$$D_t C^1 + D_x C^2 = 0 \qquad (3)$$

that holds for the whole set of solutions $u(x, t)$, where the conserved density C^1 and the spatial flux C^2 are functions of x, t, u, and derivatives of u. Here D_t, D_x denote total derivatives with respect to t, x. The pair of expressions (C^1, C^2) is called a conserved current.

The multipliers method provides a way to find all local conservation laws admitted by any given evolution equation [14, 25]. Since equation (1) is a dispersive nonlinear evolution equation, its conservation laws of physical importance come from low-order multipliers [26, 27]. Consequently, the results in Ref.[8, 9, 14] show that all non-trivial conservation laws arise from multipliers.

Now, we expressed every non-trivial local conservation law (3) as its characteristic form

$$D_t \bar{C}^1 + D_x \bar{C}^2 = \left(-u_t - f(u)_x + \mu u_{xx} + \delta u_{xxx}\right) \Lambda \qquad (4)$$

where $\Lambda(x, t, u, u_t, u_x, u_{xx}, u_{xxx})$ is the called multiplier and $\left(\bar{C}^1, \bar{C}^2\right)$ is equivalent to $\left(C^1, C^2\right)$ because they just differ by a trivial conserved current.

Moreover, the function $\Lambda(x, t, u, u_t, u_x, u_{xx}, u_{xxx})$ is a multiplier if it verifies that $\left(-u_t - f(u)_x + \mu u_{xx} + \delta u_{xxx}\right) \Lambda$ is a divergence expression for all function $u(x, t)$, not only solutions of equation (1). Divergence condition can be characterized as follows

$$\frac{\delta}{\delta u}\left(\left(-u_t - f(u)_x + \mu u_{xx} + \delta u_{xxx}\right) \Lambda \right) = 0. \qquad (5)$$

So, splitting equation (5) with respect to the variables $u, u_t, u_x, u_{xx}, u_{xxx}$ we obtained a linear determining system for $\Lambda(x, t, u, u_t, u_x, u_{xx}, u_{xxx})$, which can be solved by the same algorithmic method used to solve the determining equation for infinitesimal symmetries.

In this case, for equation (1) the multiplier obtained is $\Lambda = 1$. And finally, given the multiplier Λ, we have integrated the characteristic (4) and we have obtained the corresponding conserved density and flux

$$\begin{aligned} C^1 &= u, \\ C^2 &= \lambda u_{xxx} - \delta u_{xx} - \mu u_x + f(u). \end{aligned} \qquad (6)$$

3 Lie symmetries

The Lie symmetry analysis is performed for the equation (1) by applying its classical method. It is considered a one-parameter Lie group of infinitesimal transformations in (x, t, u) given by

$$\begin{aligned} x^* &= x + \epsilon\xi(x, t, u) + O(\epsilon^2), \\ t^* &= t + \epsilon\tau(x, t, u) + O(\epsilon^2), \\ u^* &= u + \epsilon\eta(x, t, u) + O(\epsilon^2), \end{aligned} \qquad (7)$$

where ϵ is the group parameter. This transformation requires leaving invariant the set of solutions of the equation (1). Applying this condition determines an overdetermined, linear system of equations for the infinitesimals $\xi(x, t, u)$, $\tau(x, t, u)$ and $\eta(x, t, u)$. The associated Lie algebra of infinitesimal symmetries is the set of vector fields of the form

$$V = \xi(x, t, u)\partial_x + \tau(x, t, u)\partial_t + \eta(x, t, u)\partial_u. \qquad (8)$$

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Essentially, the functions $u = u(x, t)$, which are invariant under the infinitesimal transformations (8), are solutions of the following equation named as the invariant surface condition:

$$\eta(x,t,u) - \xi(x,t,u)u_x - \tau(x,t,u)u_t = 0.$$

The set of solutions of the equation (1) is invariant under the transformations (7) provided that

$$\mathrm{pr}^{(4)}V(\Delta) = 0 \quad \text{when} \quad \Delta = 0,$$

where $\mathrm{pr}^{(4)}V$ is the fourth prolongation of the vector field (8).

This yields to the following overdetermined linear system:

$$\begin{aligned} \tau_u &= 0,\\ \tau_x &= 0,\\ \xi_u &= 0,\\ \eta_{uu} &= 0,\\ \tau_t - 4\xi_x &= 0,\\ 4c\eta_{ux} - 6c\xi_{xx} - b\xi_x &= 0,\\ 6c\eta_{uxx} - 3b\eta_{ux} - 4c\xi_{xxx} & \\ +3b\xi_{xx} - 2a\xi_x &= 0,\\ c\eta_{xxxx} - b\eta_{xxx} - a\eta_{xx} + f\eta_x + \eta_t &= 0,\\ 4c\eta_{uxxx} - 3b\eta_{uxx} - 2a\eta_{ux} + f_u\eta + 3\xi_x f & \\ -c\xi_{xxxx} + b\xi_{xxx} + a\xi_{xx} - \xi_t. & \end{aligned} \tag{9}$$

The solutions of the system (9) depend on the constants a, b, c and on the function $f = f(u)$. By solving the system (9), four different cases have been obtained. Hence, the classification of the Lie symmetries is the following:

– *Case 1.* For $f(u)$ an arbitrary function and a, b, c arbitrary constants, with $c \neq 0$, the infinitesimal generators are

$$V_1 = \partial_x$$
$$V_2 = \partial_t$$

– *Case 2.* If $f = k\,(\mathrm{Ln}\,(u) - 1)\,u + m\,u + n$ and a, b, c are arbitrary constants, with $c \neq 0$, the infinitesimal generators are $V_1,\ V_2$ and

$$V_3^2 = t\partial x + \left(\frac{1}{k}\right) u\,\partial u$$

4 Classical potential symmetries

In [28, 29] Bluman *et al.* introduced a method to find a new class of symmetries for a PDE. They are called potential symmetries and can be obtained for any differential equation which can be written as a conservation law.

It means that given scalar PDE of second order

$$F(x,t,u,u_x,u_t,u_{xx},u_{xt},u_{tt}) = 0, \tag{10}$$

where the subscripts denote the partial derivatives of u, it can be written as a conservation law

$$\frac{D}{Dt}F_1(x,t,u,u_x,u_t) - \frac{D}{Dx}F_2(x,t,u,u_x,u_t) = 0, \tag{11}$$

for some functions F_1 and F_2 of the indicated arguments and where $\frac{D}{Dx}$ and $\frac{D}{Dt}$ are the total derivative operators defined by

$$\begin{aligned} \frac{D}{Dx} &= \frac{\partial}{\partial x} + u_x\frac{\partial}{\partial u} + u_{xx}\frac{\partial}{\partial u_x} + u_{xt}\frac{\partial}{\partial u_t} + \ldots,\\ \frac{D}{Dt} &= \frac{\partial}{\partial t} + u_t\frac{\partial}{\partial u} + u_{xt}\frac{\partial}{\partial u_x} + u_{tt}\frac{\partial}{\partial u_t} + \ldots. \end{aligned}$$

Then if we introduce a potential variable $v = v(x, t)$ for the PDE written in the conserved form (6), we obtain a potential system (system approach) that we call S

$$\left\{\begin{aligned} v_x &= F_1(x,t,u,u_x,u_t),\\ v_t &= F_2(x,t,u,u_x,u_t). \end{aligned}\right. \tag{12}$$

Furthermore, for many physical equations one can eliminate u from the potential system (12) and form an auxiliary integrated or potential equation (integrated equation approach)

$$G(x,t,v,v_x,v_t,v_{xx},v_{xt},v_{tt}) = 0, \tag{13}$$

for some function G of the indicated arguments.

On the other hand, any Lie group of transformations for (12)

$$\begin{aligned} X_S = \; & \xi(x,t,u,v)\partial_x + \tau(x,t,u,v)\partial_t + \psi(x,t,u,v)\partial_u\\ & +\varphi(x,t,u,v)\partial_v. \end{aligned}$$

induces a nonlocal symmetry, *potential symmetry*, for the given PDE (10) when at least one of the infinitesimals which correspond to the variables x and u depends explicitly on the potential v. So, we obtain potential symmetries if and only if the following condition is satisfied

$$\xi_v^2 + \tau_v^2 + \phi_v^2 \neq 0. \tag{14}$$

In order to find potential symmetries of (1) and from the conservation law (11), we consider the equation in conserved form and the associated potential system is given by

$$\left\{\begin{aligned} & v_x = u,\\ & v_t = \mu u_x + \delta u_{xx} - \lambda u_{xxx} - f(u). \end{aligned}\right. \tag{15}$$

In the present work, we present the point symmetries of (15) and we study which symmetries induce potential

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symmetries of equation (1). These symmetries are such that the condition (14) is satisfied. If the above relation does not hold, then the point symmetries of (15) project into point symmetries of (1).

A Lie point symmetry admitted by $S(x,t,u,v)$ is a symmetry characterized by an infinitesimal transformation of the form

$$\begin{aligned} x^{\star} &= x+\epsilon\xi(x,t,u,v)+\mathcal{O}(\epsilon^2)\\ t^{\star} &= t+\epsilon\tau(x,t,u,v)+\mathcal{O}(\epsilon^2)\\ u^{\star} &= u+\epsilon\eta(x,t,u,v)+\mathcal{O}(\epsilon^2)\\ v^{\star} &= v+\epsilon\varphi(x,t,u,v)+\mathcal{O}(\epsilon^2) \end{aligned} \quad (16)$$

admitted by system (15). System (15) admits Lie symmetries if and only if

$$\mathrm{pr}^{(1)}X(v_x-u)=0,$$

$$\mathrm{pr}^{(3)}X(v_t-\mu u_x-\delta u_{xx}+\lambda u_{xxx}+f(u))=0,$$

where $\mathrm{pr}^{(3)}V$ is the third extended generator of

$$\begin{aligned} X_S = {} & \xi(x,t,u,v)\partial_x+\tau(x,t,u,v)\partial_t+\psi(x,t,u,v)\partial_u\\ & +\varphi(x,t,u,v)\partial_v. \end{aligned}$$

In other words, we require that the infinitesimal generator leaves invariant the set of solutions of (15). This yields to an overdetermined system of thirteen equations for the infinitesimals $\xi(x,t,u,v)$, $\tau(x,t,u,v)$, $\psi(x,t,u,v)$ and $\varphi(x,t,u,v)$.
From this system we have obtained that

$$\xi=\xi(x,t),\qquad \tau=\tau(t),\quad \psi=\alpha(x,t,v)u+\beta(x,t,v),$$
$$\varphi=\varphi(x,t,v)$$

where ξ,τ,α,β and φ must satisfy the following equations

$$\begin{aligned} a\alpha_v &= 0,\\ \varphi_v-\alpha-\tau_t+3\xi_x &= 0,\\ 4a\alpha_v u+a\beta_v+3a\alpha_x-3a\xi_{xx}-b\xi_x &= 0,\\ \varphi_v u-\alpha u-\xi_x u+\varphi_x-\beta &= 0,\\ 6a\alpha_{vv}u^2+3a\beta_{vv}u+9a\alpha_{vx}u-3b\alpha_v u &\\ +3a\beta_{vx}-b\beta_v+3a\alpha_{xx}-2b\alpha_x &\\ -a\xi_{xxx}+b\xi_{xx}-2c\xi_x &= 0,\\ a\alpha_{vvv}u^4+a\beta_{vvv}u^3+3a\alpha_{vvx}u^3 &\\ -b\alpha_{vv}u^3+3a\beta_{vvx}u^2-b\beta_{vv}u^2+3a\alpha_{vxx}u^2 &\\ -2b\alpha_{vx}u^2-c\alpha_v u^2+3a\beta_{vxx}u-2b\beta_{vx}u &\\ -c\beta_v u+a\alpha_{xxx}u-b\alpha_{xx}u-c\alpha_x u &\\ +f_u\alpha u-\xi_t u+\varphi_t+a\beta_{xxx}-b\beta_{xx} &\\ -c\beta_x+f_u\beta-f\alpha+3\xi_x f &= 0 \end{aligned} \quad (17)$$

From system (17) we have considered the following cases:

- The parameters a, b, c are arbitrary constants, with $c\neq 0$, and f is an arbitrary function. From system (17) we have obtained the infinitesimals:

  $$\xi=k_1,\qquad \tau=k_2,\qquad \varphi=k_3,\qquad \psi=0.$$

  However, it is not a potential symmetry of the equation (1) because the condition (14) is not satisfied.
- If $f=k\,(\mathrm{Ln}\,(u)-1)\,u+m\,u+n$, from system (17) we have obtained the infinitesimals:

  $$\xi=k_1 t+k_2,\qquad \tau=k_3,\qquad \psi=-k_1\,u,$$

  $$\varphi=k_1\,v+t\,((1-k)\,k_1\,u+k_1\,n)+r.$$

  And again, it is not a potential symmetry of the equation (1) because the condition (14) is not satisfied.

Consequently, we have concluded that the equation (1) does not admit potential symmetries.

5 Similarity Reductions

Having determined the infinitesimals, the symmetry variables are found by solving the characteristic equation which is equivalent to solving the invariant surface condition

$$\eta(x,t,u)-\xi(x,t,u)u_x-\tau(x,t,u)u_t=0. \quad (18)$$

Case 1. For a, b, c and $f(u)$ arbitrary, the only symmetries admitted by (1) are the group of space and time translations, which are defined by the infinitesimal generators

$$\mathbf{v}_1=\partial_x,\qquad \mathbf{v}_2=\partial_t.$$

Substituting the infinitesimals in the invariant surface condition (18) we obtain the similarity variable and the similarity solution

$$\begin{aligned} z &= \mu x-\omega t,\\ u(x,t) &= h(z). \end{aligned} \quad (19)$$

Substituting (22) into (1) we obtain

$$\lambda\,\mu^4\,h'''' -\delta\,\mu^3\,h''' -\mu^3\,h'' +\mu\,f_h\,h' -\omega\,h' =0. \quad (20)$$

Integrating once with respect to z we get

$$\lambda\,\mu^4\,h''' -\delta\,\mu^3\,h'' -\mu^3\,h' +\mu\,f-\omega\,h+A=0, \quad (21)$$

where A is an integrating constant.
Case 2. If $f=k\,(\mathrm{Ln}\,(u)-1)\,u+m\,u+n$ and a, b, c are arbitrary constants, with $c\neq 0$. We provide next the generators of the nontrivial one-dimensional optimal system which

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are $V_1,\ V_2+V_3$. Substituting the infinitesimals of the subalgebra V_2+V_3 into the invariant surface condition (18) we obtain the similarity variable and the similarity solution

$$\begin{aligned} z &= x-\tfrac{t^2}{2}, \\ u(x,t) &= h(z)\exp(\tfrac{t}{k}). \end{aligned} \tag{22}$$

Substituting (22) into (1) we obtain

$$\lambda h'''' - \delta h''' - \mu h'' + m h' + k \log(h)\ h' + \frac{h}{k} = 0. \tag{23}$$

6 Travelling wave solutions

Let us assume that equation (21) has solution of the form

$$h = aH^b(z), \tag{24}$$

where a, b are parameters and $H(z)$ can be: a solution of Jacobi equation

$$(H')^2 = r + pH^2 + qH^4, \tag{25}$$

with r, p and q constants; an exponential function or a polynomial function.

If H is solution of equation (25) we can distinguish three subcases: *(i)* H is the Jacobi elliptic sine function, $\mathrm{sn}(z,m)$, *(ii)* H is the Jacobi elliptic cosine function, $\mathrm{cn}(z,m)$, *(iii)* H is the Jacobi elliptic function of the third kind $\mathrm{dn}(z,m)$. We substitute the solution H into ODE y we determine $f(h)$. In the following we give four examples of equations, which are solutions with physical interest:

- For

$$f(h) = \frac{}{\sqrt{1-h}}\Big[60\lambda h^2 - 12\delta\sqrt{1-h}h - 60\lambda h + 3h + 4\delta\sqrt{1-h} + \sqrt{1-h} + 8\lambda - 2\Big]$$

where $h(z)=\mathrm{cn}^2(z,1)$ is a solution of (21) and taking into account that $\mathrm{cn}(z,1)=\mathrm{sech}(z)$, we obtain that for $\omega=\mu=1$

$$u(x,t) = \mathrm{sech}^2(x-t) \tag{26}$$

is a solution of equation (1) with $f(u) = \frac{60\lambda u^2-12\delta\sqrt{1-u}u-60\lambda u+3u+4\delta\sqrt{1-u}+\sqrt{1-u}+8\lambda-2}{\sqrt{1-u}}$. In Figure 1, we plot a solution (26), which describes a soliton.

- For

$$f(h) = \frac{1}{288\sqrt{5}\sqrt{1-\sqrt{h}}h^{\frac{3}{4}}}\Big[800\sqrt{3}\lambda h + 245^{\frac{3}{2}}h - 965^{\frac{3}{2}}\delta\sqrt{1-\sqrt{h}}h^{\frac{3}{4}} + 288\sqrt{5}\sqrt{1-\sqrt{h}}h^{\frac{3}{4}} - 2503^{\frac{3}{2}}\lambda\sqrt{h} - 185^{\frac{3}{2}}\sqrt{h} + 365^{\frac{3}{2}}\delta\sqrt{1-\sqrt{h}}h^{\frac{1}{4}} + 253^{\frac{3}{2}}\lambda\Big],$$

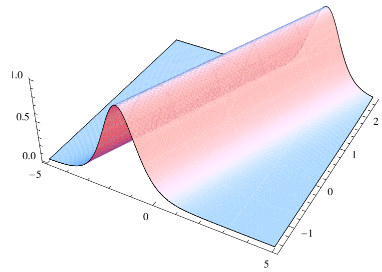

Figure 1: Solution (26)

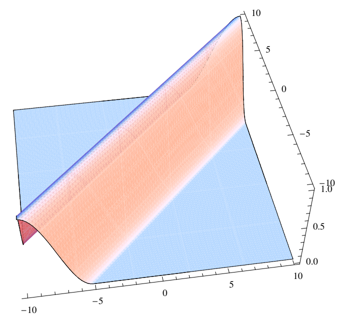

Figure 2: Solution (27)

where $h(z)=\mathrm{cn}^4(z,0)$ is a solution of (21), and taking into account that $\mathrm{cn}(z,0)=\cos(z)$, we obtain that for $\mu=\omega=\frac{k}{2}$ and $k=\sqrt{\frac{5}{12}}$

$$u(x,t) = \begin{cases} \cos^4\left[\frac{k}{2}(x-t)\right] & |x-t| \le \frac{\pi}{k}, \\ 0 & |x-t| > \frac{\pi}{k} \end{cases} \tag{27}$$

is a solution of Eq. (1) with

$$f(u) = \frac{1}{288\sqrt{5}\sqrt{1-\sqrt{u}}u^{\frac{3}{4}}}\Big[800\sqrt{3}\lambda u + 245^{\frac{3}{2}}u - 965^{\frac{3}{2}}\delta\sqrt{1-\sqrt{u}}u^{\frac{3}{4}} + 288\sqrt{5}\sqrt{1-\sqrt{u}}u^{\frac{3}{4}} - 2503^{\frac{3}{2}}\lambda\sqrt{u} - 185^{\frac{3}{2}}\sqrt{u} + 365^{\frac{3}{2}}\delta\sqrt{1-\sqrt{u}}u^{\frac{1}{4}} + 253^{\frac{3}{2}}\lambda\Big].$$

In Figure 2, we plot a solution (27), which is a compacton solution with a single peak.

- For

$$F(h) = \frac{1}{2\sqrt{1-h}\sqrt{h+1}\sqrt{1-h^2}}\Big[12\delta\sqrt{1-h}h^2\sqrt{h+1}\sqrt{1-h^2} - 4\delta\sqrt{1-h}\sqrt{h+1}\sqrt{1-h^2} + \sqrt{1-h}\sqrt{h+1}\sqrt{1-h^2} - 48\lambda h^5 + 80\lambda h^3 + 4h^3 - 32\lambda h - 4h\Big],$$

where $h(z)=\frac{1}{4}\mathrm{sn}(z,1)$ is a solution of (21), and taking into account that $\mathrm{sn}(z,1)=\tanh(z)$, we obtain that for $\mu=1$ and $\omega=\frac{1}{2}$

$$u(x,t) = \frac{1}{4}\tanh\left(x-\frac{t}{2}\right) \tag{28}$$

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is a solution of Eq. (1) with

$$f(u) = \frac{1}{2\sqrt{1-h}\sqrt{h+1}\sqrt{1-h^2}}\Big[12\delta\sqrt{1-h}h^2\sqrt{h+1}\sqrt{1-h^2} -4\delta\sqrt{1-h}\sqrt{h+1}\sqrt{1-h^2} +\sqrt{1-h}\sqrt{h+1}\sqrt{1-h^2} - 48\lambda h^5 + 80\lambda h^3 + 4h^3 -32\lambda h - 4h\Big]$$

In Figure 3, we plot a solution (28), which describes a kink solution.

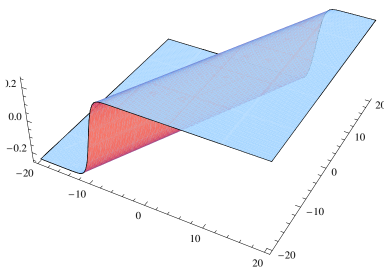

Figure 3: Solution (28)

- For $F(h) = \frac{Q_1}{54\sqrt{1-h}h^3\sqrt{h+1}\sqrt{1-h^2}}$ with

$$Q_1 = \Big[168\delta\sqrt{1-h}h^5\sqrt{h+1}\sqrt{1-h^2} -12\delta\sqrt{1-h}h^3\sqrt{h+1}\sqrt{1-h^2} +27\sqrt{1-h}h^3\sqrt{h+1}\sqrt{1-h^2} +60\delta\sqrt{1-h}h\sqrt{h+1}\sqrt{1-h^2} - 560\lambda h^8 +800\lambda h^6 + 72h^6 - 192\lambda h^4 - 36h^4 -208\lambda h^2 - 36h^2 + 160\lambda\Big],$$

where $h(z) = \mathrm{sn}^3(z,1)$ is a solution of (21), and taking into account that $\mathrm{sn}(z,1) = \tanh(z)$, we obtain that for $\mu = 1$ and $\lambda = \frac{1}{2}$

$$u(x,t) = \tanh^3\left(x - \frac{t}{2}\right) \tag{29}$$

is a solution of equation (1) for $f(u) = \frac{Q_2}{54\sqrt{1-u}u^3\sqrt{u+1}\sqrt{1-u^2}}$ and

$$Q_2 = 168\delta\sqrt{1-u}u^5\sqrt{u+1}\sqrt{1-u^2} -12\delta\sqrt{1-u}u^3\sqrt{u+1}\sqrt{1-u^2} +27\sqrt{1-u}u^3\sqrt{u+1}\sqrt{1-u^2} +60\delta\sqrt{1-u}u\sqrt{u+1}\sqrt{1-u^2} - 560\lambda u^8 + 800\lambda u^6 +72u^6 - 192\lambda u^4 - 36u^4 - 208\lambda u^2 - 36u^2 + 160\lambda.$$

In Figure 4, we plot a solution (29), which describes an anti-kink solution.

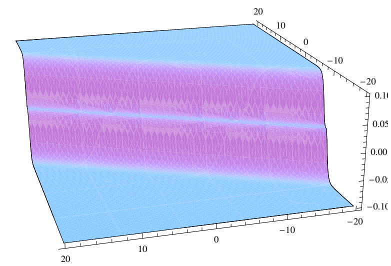

Figure 4: Solution (29)

7 Conclusions

By using the multipliers conservation laws method we exhibited that equation (1) only admits trivial conservation laws. The method of Lie group analysis is applied to the investigation of symmetry properties, as well as corresponding reduced ordinary differential equations. We have proved that the potential symmetries project into point symmetries of Eq. (1). We derive for some functions many exact solutions which are solitons, kinks, anti-kinks and compactons.

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