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A REMARK ON BISHOP'S MULTIVALUED PROJECTIONS

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Abstract. We present a proof of an extension theorem for holomorphic functions based on holomorphic multivalued projections.

1. Introduction. Let M be a complex submanifold of a Stein manifold X. It is known (cf. e.g. [**3**], Chapter VIII, Section A, Theorem 18) that $\mathcal{O}(X)|_M = \mathcal{O}(M)$, i.e. each function holomorphic on M extends holomorphically to X. In [**2**] E. Bishop proposed a proof of the above result based on the use of special analytic polyhedra (without sheaves methods). The central part of Bishop's proof is to show that for every relatively compact domain $U \subset X$ with $U \cap M \neq \varnothing$ we have $\mathcal{O}(U)|_{U\cap M} = \mathcal{O}(M)|_{U\cap M}$. At the end of his proof E. Bishop suggested that an alternative proof may be performed in the language of holomorphic multivalued projections. The aim of our note is to realize this idea.

The paper is organized as follows. First, in Sections 2 and 3 we recall some basic facts related to holomorphic functions on symmetric products. The definition of a system of holomorphic multivalued projections is given is Section 4. The main result of the paper is the following theorem.

If M is an analytic submanifold of a Stein manifold X and U is a relatively compact domain with $U \cap M \neq \varnothing$, then there exists a system of holomorphic multivalued projections $U \longrightarrow M$. Consequently, there exists a linear continuous extension operator $L : \mathcal{O}(M) \longrightarrow \mathcal{O}(U)$.

Section 5 collects some auxiliary results. The proof of the main theorem is in Section 6.

2. Symmetric products. The aim of this section is to present some properties of the symmetric products. For details see [**5**, Appendix V].

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Let X be a topological Hausdorff space. We define an equivalence relation on X^k by $(x_1,\dots,x_k) \sim (y_1,\dots,y_k) \overset{\text{def}}{\Longleftrightarrow} (y_1,\dots,y_k)$ is a reordering of $(x_1,\dots,x_k)$. We will denote the quotient set $X^k/\sim$ with $\overleftrightarrow{X}^k$ and call it the k-symmetric product of X. In the case $k=1$, we get $\overleftrightarrow{X}^1 = X$. Now, we define the projection $\pi : X^k \longrightarrow \overleftrightarrow{X}^k$, $\pi(x) := [x]$. We put $[x_1,\dots,x_k] := [(x_1,\dots,x_k)]$, $\{[x_1,\dots,x_k]\} := \{x_1,\dots,x_k\}$. Moreover, we put

$$[x_1 : \mu_1,\dots,x_\ell : \mu_\ell] := [\overbrace{x_1,\dots,x_1}^{\mu_1\text{-times}},\dots,\overbrace{x_\ell,\dots,x_\ell}^{\mu_\ell\text{-times}}],$$

provided that $x_j \neq x_t$ for $j \neq t$, $\mu_1,\dots,\mu_\ell \in \mathbb{N}$, $\mu_1 + \dots + \mu_\ell = k$. We define

$$[A_1,\dots,A_k] := \Big\{[x_1,\dots,x_k] : x_i \in A_i, \quad i = 1,\dots,k\Big\},$$

The topology on $\overleftrightarrow{X}^k$ is defined by the basis

$$[U_1,\dots,U_m], \quad U_i \text{ is open in } X, \quad i = 1,\dots,k.$$

Observe that π is continuous and open, and $\overleftrightarrow{X}^k$ is Hausdorff.

Definition 2.1. Let Y be Hausdorff topological space and let $F : X \longrightarrow \overleftrightarrow{Y}^n$ be continuous. Then we put

$$X_F^{(k)} := \{x \in X : \#\{F(x)\} = k\},$$
$$\chi_F := \max\{k : X_F^{(k)} \neq \varnothing\}, \quad X_F := X_F^{(\chi_F)}.$$

Note that X_F is open.

Proposition 2.2. *Let F be as above. Suppose that*

$$a \in X_F, \quad F(a) = [b_1 : \mu_1,\dots,b_k : \mu_k], \quad k := \chi_F.$$

Then there is a neighborhood $U \subset X_F$ of a and there are uniquely defined continuous functions $f_i : U \longrightarrow Y$, $i = 1,\dots,k$, such that

$$F(x) = [f_1(x) : \mu_1,\dots,f_k(x) : \mu_k], \quad x \in U.$$

In the above situation, we will write $F = \mu_1 f_1 \oplus \dots \oplus \mu_k f_k$ on U.

Proposition 2.3. *Let $F : X^k \longrightarrow Y$ be continuous. Then there exists a continuous function $\overleftrightarrow{F} : \overleftrightarrow{X}^k \longrightarrow Y$ such that $F = \overleftrightarrow{F} \circ \pi$ if and only if F is symmetric.*

3. Holomorphic multivalued functions.

Definition 3.1. Let M, N be complex manifolds and let M be connected. We say a continuous mapping $F : M \longrightarrow \overleftrightarrow{N^n}$ is *holomorphic on* M ($F \in \mathcal{O}(M, \overleftrightarrow{N^n})$) if:

- $M \setminus M_F$ is thin, i.e. every point $x_0 \in M \setminus M_F$ has an open connected neighborhood $V \subset M$ and exists a function $\varphi \in \mathcal{O}(V)$, $\varphi \not\equiv 0$, such that $(M \setminus M_F) \cap V \subset \varphi^{-1}(0)$,
- for every $a \in M_F$, if $F = \mu_1 f_1 \oplus \cdots \oplus \mu_k f_k$ on V as in Proposition 2.2, then $f_1, \ldots, f_k \in \mathcal{O}(V)$.

If M is disconnected, then we say that F is *holomorphic on* M, if $F|_C \in \mathcal{O}(C, \overleftrightarrow{N^n})$ for any connected component $C \subset M$.

Proposition 3.2. *Let M, N, K be complex manifolds and let $f \in \mathcal{O}(M, N)$, $g \in \mathcal{O}(N, \overleftrightarrow{K^n})$. Assume that $f(M) \cap N_g \neq \varnothing$ and M is connected. Then $g \circ f \in \mathcal{O}(M_{g\circ f}, \overleftrightarrow{K^n})$.*

Proof. Observe that $\chi_{g\circ f} = \chi_g$. We may assume that $M_{g\circ f}$ is connected. Fix an $x_0 \in M_{g\circ f}$. Then $f(x_0) \in N_g$. Thus $g = \mu_1 g_1 \oplus \cdots \oplus \mu_k g_k$ on U, where U is an open neighborhood of $f(x_0)$. Therefore, $g \circ f = g_1 \circ f \oplus \cdots \oplus g_k \circ f$ on V, where $V := f^{-1}(U)$. □

Proposition 3.3. *Let $f \in \mathcal{O}(M, \overleftrightarrow{N^n})$ and $g \in \mathcal{O}(N^n, K)$ be symmetric. Then $\overleftrightarrow{g} \circ f \in \mathcal{O}(M, K)$.*

Proof. We may assume that M is connected. By Definition 3.1 for $a \in M_f$, we get $f = \mu_1 f_1 \oplus \cdots \oplus \mu_k f_k$ on U, where U is a neighborhood of a. We see that $\overleftrightarrow{g} \circ f = g(\overbrace{f_1, \ldots, f_1}^{\mu_1\text{-times}}, \ldots, \overbrace{f_k, \ldots, f_k}^{\mu_k\text{-times}})$ on U. Hence $\overleftrightarrow{g} \circ f \in \mathcal{O}(M_f, K) \cap \mathcal{C}(M, K)$. Since $M \setminus M_f$ is thin, we obtain $\overleftrightarrow{g} \circ f \in \mathcal{O}(M, K)$. □

4. Bishop's multivalued projections.

Definition 4.1. Let M be an analytic submanifold of a manifold X. Let $U \subset X$ be a domain such that $U \cap M \neq \varnothing$. We say a holomorphic function

$$\Delta : U \longrightarrow \overleftrightarrow{(M \times \mathbb{C})^n}$$

is a *holomorphic multivalued projection* $U \longrightarrow M$ if for any $x \in U \cap M$ such that $\Delta(x) = [\![(x_1, z_1), \ldots, (x_n, z_n)]\!]$ we have $x_{j_0} = x$ for some $j_0 \in \{1, \ldots, n\}$ and $z_j = 0$ for each $j \in \{1, 2, \ldots, n\} \setminus \{j_0\}$.

Let $\mathcal{P}$ denote the set of all holomorphic multivalued projections $U \longrightarrow M$. Then we define the map

$$\Xi : (U \cap M) \times \mathcal{P} \longrightarrow \mathbb{C}, \quad \Xi(x, \Delta) := z_{j_0}.$$

Observe that Ξ is well defined.

DEFINITION 4.2. We say $\Pi = (\Delta_j)_{s=1}^k$ is a *system of holomorphic multivalued projections* $U \longrightarrow M$, if $\Delta_s : U \longrightarrow \overleftrightarrow{(M \times \mathbb{C})^{k_s}}$, $s = 1, \ldots, k$, are holomorphic multivalued projections and $\sum_{s=1}^k \Xi(x, \Delta_s) = 1$ for any $x \in U \cap M$.

Now, we are going to construct a linear continuous extension operator $L_\Pi : \mathcal{O}(M) \longrightarrow \mathcal{O}(U)$, where $\mathcal{O}(M)$ and $\mathcal{O}(U)$ are endowed with the locally uniform convergence topologies.

THEOREM 4.3. *Assume that there exists a system* Π *of holomorphic multivalued projections on* U*. Then there exists a linear continuous operator*

$$L_\Pi : \mathcal{O}(M) \longrightarrow \mathcal{O}(U)$$

such that $L_\Pi(u)(x) = u(x)$ *for* $x \in U \cap M$*.*

PROOF. Let $\Pi := (\Delta_s)_{s=1}^k$ and let $u \in \mathcal{O}(M)$. Put

$$\overleftrightarrow{u_s}([(x_1, \lambda_1), \ldots, (x_{k_s}, \lambda_{k_s})]) := \sum_{j=1}^{k_s} u(x_j)\lambda_j,$$

$$L_\Pi(u) := \sum_{s=1}^{k} \overleftrightarrow{u_s} \circ \Delta_s.$$

Obviously, L_Π is linear, $u_s \in \mathcal{O}((M \times \mathbb{C})^{k_s})$ and for $x \in U \cap M$ we have

$$L_\Pi(u)(x) = \sum_{j=1}^{k} u(x)\Xi(x, \Delta_j) = u(x) \sum_{j=1}^{k} \Xi(x, \Delta_j) = u(x).$$

By Proposition 3.3 $L_\Pi(u) \in \mathcal{O}(U)$. Moreover, L is a continuous operator. Indeed, let $K \subset U$ be compact. We see that the functions

$$\Lambda_s : \overleftrightarrow{(M \times \mathbb{C})^{k_s}} \ni [(x_1, \lambda_1), \ldots, (x_{k_s}, \lambda_{k_s})] \longmapsto [x_1, \ldots, x_k] \in \overleftrightarrow{M^{k_s}},$$

$$\widehat{\Lambda}_s : \overleftrightarrow{(M \times \mathbb{C})^{k_s}} \ni [(x_1, \lambda_1), \ldots, (x_{k_s}, \lambda_{k_s})] \longmapsto [\lambda_1, \ldots, \lambda_k] \in \overleftrightarrow{\mathbb{C}^{k_s}},$$

$s = 1, \ldots, k$, are continuous. Obviously, $\Lambda_s \circ \Delta_s(K) \subset [K_1^s, \ldots, K_{k_s}^s]$, where K_j^s is compact for $j = 1, \ldots, k_s$, and $s = 1, \ldots, k$. Similarly, $\widehat{\Lambda}_s \circ \Delta_s(K) \subset [\widehat{K}_1^s, \ldots, \widehat{K}_{k_s}^s]$, where $\widehat{K}_j^s$ is compact for $j = 1, \ldots, k_s$, and $s = 1, \ldots, k$. Put $L := \bigcup_{s=1}^k \bigcup_{j=1}^{k_s} K_j^s$ and $\widehat{L} := \bigcup_{s=1}^k \bigcup_{j=1}^{k_s} \widehat{K}_j^s$. Note that the above sets are compact. Next, we set $C := k \max\{|\lambda| : \lambda \in \widehat{L}\}$. We obtain

$$\|L_\Pi(u)\|_K \leq C\|u\|_L, \quad u \in \mathcal{O}(M).$$

$\square$

In view of Theorem 4.4 it is natural to try to find a class of triples (X, M, U) for which a system of holomorphic multivalued projections $U \longrightarrow M$ exists. Using Bishop's method we will get the following result.

THEOREM 4.4. *Let M be an analytic submanifold of a Stein manifold X. Let U be a relatively compact domain of X such that $U \cap M \neq \varnothing$. Then there exists a system of multivalued holomorphic projections $U \longrightarrow M$.*

The proof of Theorem 4.4 will be presented in Section 6. Theorems 4.5 and 4.4 immediately imply the following result.

THEOREM 4.5. *Let M be an analytic submanifold of a Stein manifold X. Let U be a relatively compact domain of X such that $U \cap M \neq \varnothing$. Then exists a linear continuous extension operator $L : \mathcal{O}(M) \longrightarrow \mathcal{O}(U)$.*

5. Auxiliary Results. Let M be a d-dimensional submanifold of a Stein manifold X.

DEFINITION 5.1. Let $f \in \mathcal{O}(X, \mathbb{C}^k)$. We say that a set $P \subset P_0 := M \cap f^{-1}(\mathbb{D}^k)$ is an *analytic polyhedron* in M ($P \in \mathcal{P}(M, k, f)$) iff $P \subset\subset M$ and P is the union of a family of connected components of P_0.

We say that an analytic polyhedron $P \in \mathcal{P}(M, k, f)$ is *special* if $d = k$.

THEOREM 5.2 (cf. [**2**]). *Assume that $P \in \mathcal{P}(M, k, f)$, $S \subset P$, $T \subset f^{-1}(\mathbb{D}^k)$ are compact. Then there exists a special analytic polyhedron $Q \in \mathcal{P}(M, d, g)$ such that $S \subset Q \subset P$ and $g(T) \subset \mathbb{D}^d$.*

THEOREM 5.3 (cf. [**2**]). *Assume that X is Stein, $T \subset X$ is compact, and U is an open neighborhood of T such that $(U \setminus T) \cap \hat{T}_{\mathcal{O}(X)} = \varnothing$. Let $\mathcal{A} := \mathrm{cl}_{\mathcal{C}(T)}(\mathcal{O}(U)|_T)$. Then $\mathrm{Spec}(\mathcal{A}) = T$, i.e. every non-zero character (homomorphism) $\xi : \mathcal{A} \longrightarrow \mathbb{C}$ is an evaluation (i.e. there exists an $x_0 \in T$ such that $\xi(f) = f(x_0)$ for every $f \in \mathcal{A}$).*

Consequently (cf. [**1**]*, Chapter I, Section II, Corollary 10), if $w_1, \dots, w_m \in \mathcal{A}$ have no common zeros on T, then there exist $c_1, \dots, c_m \in \mathcal{A}$ such that $c_1 w_1 + \dots + c_m w_m = 1$.*

THEOREM 5.4 (cf. [**2**]; see also [**4**], Chapter 7). *Assume that $P \in \mathcal{P}(M, d, f)$ is special. Then there exist a $k \in \mathbb{N}$ and a holomorphic mapping $\omega : \mathbb{D}^d \longrightarrow \overset{\leftrightarrow}{P^k}$ such that:*

- *$f^{-1}(z) \cap P = \{\omega(z)\}$, $z \in \mathbb{D}^d$,*
- *$\#\{\omega(z)\} = k$ for $z \in \mathbb{D}^d \setminus \Sigma'$, where Σ' is a proper analytic set.*

PROPOSITION 5.5. *Let ω, f, X,P be as above. Additionally assume that $f(U) \subset \mathbb{D}^d$, where $U \subset X$ is a domain and $U \cap P \neq \varnothing$. Then $\omega \circ f|_U \in \mathcal{O}(U, \overset{\leftrightarrow}{P^k})$.*

Proof. By Proposition 3.2, it is sufficient to show that $U \setminus U_{\omega\circ f_{|U}}$ is thin. Recall (Theorem 5.4) that $(\mathbb{D}^d)_\omega = \mathbb{D}^d \setminus \Sigma'$, where $\Sigma' \subset \mathbb{D}^d$ is a proper analytic set. Since U is connected and $U \cap P \neq \varnothing$, we conclude that $f^{-1}(\Sigma') \cap U$ is a proper analytic set. It remains to observe that $U \setminus U_{\omega\circ f} \subset f^{-1}(\Sigma') \cap U$. $\square$

Proposition 5.6. *Let ω, f, X,P be as above. Then* $\omega \circ f|_P \in \mathcal{O}(P, \overleftrightarrow{P}^k)$.

Proof. We apply Proposition 5.5 to each connected component of P. $\square$

6. Proof of Theorem 4.4. We point out that the main idea of the proof is due to E. Bishop ([**2**]).

Proof. Since X is Stein and U is relative compact, we can find an analytic polyhedron $Q \in \mathcal{P}(X, n, \widetilde{f})$ such that $\overline{U} \subset Q$. Thus $Q \cap M \in \mathcal{P}(M, n, \widetilde{f})$ and by Theorem 5.2 there exists an $f \in \mathcal{O}(X, \mathbb{C}^d)$ such that $f(\overline{U}) \subset \mathbb{D}^d$ and $U \cap M \subset P$ for some special polyhedron $P \in \mathcal{P}(M, d, f)$. Let $r_0 \in (0, 1)$ be such that $f(\overline{U}) \subset \mathbb{D}^d(r_0)$. Fix $r_0 < r < r_1 < 1$ and define $T := P \cap f^{-1}(\overline{\mathbb{D}}^d(r))$. Note that T is compact. One can easily prove that $(P \setminus T) \cap \widehat{T}_{\mathcal{O}(M)} = \varnothing$.

Take a $p \in T$. Since X is Stein, there exists an $h \in \mathcal{O}(X, \mathbb{C}^d)$ giving local coordinates on M at p. One can prove that there exists an $\eta \in \mathbb{C}$, $0 < |\eta| << 1$ such that:

- the mapping $g := f + \eta h$ gives local coordinates on M at p,
- $g(\overline{U}) \subset \mathbb{D}^d(r_0)$,
- the set $R_1 := P \cap g^{-1}(\mathbb{D}^d(r_1))$ is a special analytic polyhedron in M, and
- $U \cap M \subset T \subset R_1 \subset P$.

In particular, by Theorem 5.4, there exist a $k \in \mathbb{N}$ and holomorphic mapping $\omega : \mathbb{D}^d(r_1) \longrightarrow \overleftrightarrow{R_1^k}$ such that:

- $g^{-1}(z) \cap R_1 = \{\omega(z)\}, z \in \mathbb{D}^d(r_1)$,
- $\#\{\omega(z)\} = k$ for $z \in \mathbb{D}^d(r_1) \setminus H$, where H is an analytic set.

For $x \in V_1 := g^{-1}(\mathbb{D}^d(r_1))$, let $\omega(g(x)) = [x_1 : \mu_1, \dots, x_s : \mu_s]$, where $\mu_1 + \cdots + \mu_s = k$. To simplify notations we assume that $x_1 = x$ for $x \in R_1$. Observe that $p = p_1$ must be of multiplicity 1 ($\mu_1 = 1$). Since X is Stein, there exists a $w \in \mathcal{O}(X)$ such that $w(p_1) \neq w(p_j)$, $j = 2, \dots, k$. We define

$$\widetilde{w}(x) := \sum_{j=1}^{k} \prod_{\substack{\mu \in \{1,\dots,k\} \\ \mu \neq j}} (w(x) - w(x_\mu)), \quad x \in R_1.$$

We have

$$\widetilde{w}(x) = \sum_{\nu=0}^{k-1} \overleftrightarrow{S}_\nu(\omega(g(x)))w^\nu(x), \quad x \in R_1,$$

where $S_\nu : R_1^k \longrightarrow \mathbb{C}$ is given by following formulas

$$S_{k-1}(t) := k,$$
$$S_\nu(t) := (-1)^{k-1-\nu} \sum_{j=1}^{k} \sigma_{k-1-\nu}(w(t_1), \dots, w(t_{j-1}), w(t_{j+1}), \dots, w(t_k)),$$
$$\nu = 0, \dots, k-2, \quad t = (t_1, \dots, t_k) \in R_1^k,$$

and $\sigma_1, \dots, \sigma_{k-1} : \mathbb{C}^{k-1} \longrightarrow \mathbb{C}$ are standard symmetric polynomials. Observe that each function $S_\nu : R_1^k \longrightarrow \mathbb{C}$ is holomorphic and symmetric. Thus, by Proposition 3.3 and Proposition 5.6, $\widetilde{w} \in \mathcal{O}(R_1)$.

Obviously, $\widetilde{w}(p) \neq 0$. Notice that in fact all the above objects depend on p. Consequently, we will write $R_1^{(p)}, k^{(p)}, x_j^{(p)}, w^{(p)}, \widetilde{w}^{(p)}, x_j^{(p)}, g^{(p)}, \omega^{(p)}$. Since T is compact and $\widetilde{w}^{(p)}(p) \neq 0$, there exists a finite number of points $p^1, \dots, p^m \in T$ such that the functions $\widetilde{w}^{(p^1)}, \dots, \widetilde{w}^{(p^m)}$ have no common zeros on T. Now, we apply Theorem 5.3 with $(X, T, U) := (M, T, W)$, where $W := \bigcap_{s=1}^m R_1^{(p^s)}$. Consequently, we get $c_1, \dots, c_m \in \mathcal{A} \subset \mathcal{O}(\operatorname{int}_M T)$ such that $c_1 \widetilde{w}^{(p^1)} + \dots + c_m \widetilde{w}^{(p^m)} = 1$ on T.

Put $k_s := k^{(p^s)}, \omega_s := \omega^{(p^s)}, g_s := g^{(p^s)}, x_{s,j} := x_j^{(p^s)}, w_s := w^{(p^s)}, s = 1, \dots, m$. Let $x_0 \in U_{\omega_s \circ g_s|_U}$. Therefore, by Proposition 5.5, $\omega_s \circ g_s|_U = \widetilde{F}_{s,1} \oplus \dots \oplus \widetilde{F}_{s,k_s}$ on A_s, where A_s is a neighborhood of x_0, $s = 1, \dots, m$. Set

$$F_{s,j}(x) := x_{s,j},$$
$$G_{s,j}(x) := c_s(x_{s,j}) \prod_{\substack{\mu \in \{1,\dots,k_s\} \\ \mu \neq j}} (w_s(x) - w_s(x_{s,\mu})),$$
$$\widetilde{G}_{s,j}(x) := c_s(\widetilde{F}_{s,j}(x)) \prod_{\substack{\mu \in \{1,\dots,k_s\} \\ \mu \neq j}} (w_s(x) - w_s(\widetilde{F}_{s,\mu}(x))), \quad x \in A_s,$$
$$\widehat{G}_{s,j}(x, (y_1, \dots, y_{k_s})) := c_s(y_j) \prod_{\substack{\mu \in \{1,\dots,k_s\} \\ \mu \neq j}} (w_s(x) - w_s(y_\mu)).$$

We define the projections

$$\Delta_s := (F_{s,1}, G_{s,1}) \oplus \dots \oplus (F_{s,k_s}, G_{s,k_s}) \text{ on } U, \quad s = 1, \dots, m.$$

We see that

$$(F_{s,1}, G_{s,1}) \oplus \dots \oplus (F_{s,k_s}, G_{s,k_s}) = (\widetilde{F}_{s,1}, \widetilde{G}_{s,1}) \oplus \dots \oplus (\widetilde{F}_{s,k_s}, \widetilde{G}_{s,k_s}) \quad \text{on } A_s,$$

$s = 1, \dots, m$. Therefore, we get $\Delta_s \in \mathcal{O}(U_{\omega_s \circ g_s|_U}, \overleftrightarrow{(M \times \mathbb{C})^{k_s}})$.

It remains to check that Δ_s, $s = 1, \dots, m$, are continuous on U. For $x_0 \in U \setminus U_{\omega_s \circ g_s|_U}$, let $(x_n)_{n=1}^\infty \subset U$ be an arbitrary sequence such that $\lim_{n\to\infty} x_n = x_0$.

We obtain $\omega_s \circ g_s|_U(x_n) = [y_n^1, \dots y_n^{k_s}]$. Put $y_n := (y_n^1, \dots, y_n^{k_s})$. Obviously, we may assume that $\lim_{n\to\infty} y_n^j = y_0^j$ for $j = 1, \dots, k_s$, where $\omega_s \circ g_s|_U(x_0) = [y_0^1, \dots, y_0^{k_s}]$. We have $\Delta_s(x_n) = [(y_n^1, \widehat{G}_{s,1}(x_n, y_n)), \dots, (y_n^{k_s}, \widehat{G}_{s,k_s}(x_n, y_n)] \longrightarrow [(y_0^1, \widehat{G}_{s_1}(x_0, y_0)), \dots, (y_0^{k_s}, \widehat{G}_{s,k_s}(x_0, y_0))] = \Delta_s(x_0)$. Thus Δ_s is continuous on U for any $s = 1, \dots, m$.

Note that, $U_{\omega_s \circ g_s|_U} \subset U_{\Delta_s}$ and thus $\Delta_s \in \mathcal{O}(U, \overleftrightarrow{(M \times \mathbb{C})^{k_s}})$, $s = 1, \dots, k$. Hence $\Pi := (\Delta_s)_{s=1}^m$ is a system of holomorphic projections. Indeed, we observe that $\Xi(x, \Delta_s) = c_s(x)\widetilde{w}^{(p^s)}(x)$ for $x \in U \cap M$, $s = 1, \dots, m$, hence $\sum_{s=1}^m \Xi(x, \Delta_s) = 1$. $\square$

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