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A Multiple Dependent State Repetitive Sampling Plan Based on Performance Index for Lifetime Data with Type II Censoring

MUHAMMAD ASLAM[1], CHING-HO YEN[2], CHIA-HAO CHANG[3], ALI HUSSEIN AL-MARSHADI[1], AND CHI-HYUCK JUN[4]

[1]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia
[2]Department of Industrial Engineering and Management Information, Huafan University, Taipei 22301, Taiwan
[3]Department of Nursing, Chang Gung University of Science and Technology, Chiayi Campus, Chiayi 333, Taiwan
[4]Department of Industrial and Management Engineering, Pohang University of Science and Technology, Pohang 37673, South Korea

Corresponding author: Muhammad Aslam (aslam_ravian@hotmail.com)

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ABSTRACT In this paper, a multiple dependent state repetitive (MDSR) sampling plan based on the lifetime performance index C_L is proposed for lifetime data with type II censoring when the lifetime of a product follows the exponential distribution or Weibull distribution. The optimal parameters of the proposed plan are determined by minimizing the average sample number while satisfying the producer's risk and consumer's risk at corresponding quality levels. Besides, the performance of the proposed plan is compared with that of the existing lifetime sampling plan in terms of the average sample number and operating characteristic curve. Two illustrative examples are given for the demonstration of the proposed plan.

INDEX TERMS Multiple dependent state repetitive, lifetime performance index, type II censoring, average sample number, operating characteristic curve.

I. INTRODUCTION

The high quality of the product cannot be achieved without any effort. To achieve this goal, long term work of quality control planning is needed from the raw material to the finished product [1]. As mentioned by [2], manufacturers need quality (reliability) information before releasing products and potential customers require this information before purchasing products. Therefore, quality inspection/reliability testing is an essential part to ensure high quality/reliability products. In quality control, an acceptance sampling plan is extensively used on the inspection of raw material and finished products. For most electronic products, the lifetime can be said to be the most important quality characteristic. The longer lifetime means that the product has a better reliability, and this will attract consumers to purchase the product. In order to ensure that the lifetime can meet the requirement of a customer, the lifetime testing plan should be carried out before the goods are delivered.

The lot sentencing of non-electronic products can be done easily, while it may be difficult for electronic products. Since lifetime testing is usually destructive in nature, it is not possible to test/inspect the lifetimes of all items to reach the lot sentencing. Even though the inspection/testing is non-destructive, collecting full information from all units in one lot is time-consuming and costly. Therefore, it is essential to design an effective lifetime testing plan that can save much cost and time. For this purpose, two types of lifetime testing methods are commonly applied, called type-I censoring test and type-II censoring test. In type-I censoring test, the test is terminated when a preassigned time is reached. Acceptance or rejection of a lot is based on the observed number of failures during the testing time. In type-II censoring test, the test is terminated when the preassigned number of failures occurs in the chosen sample. Acceptance or rejection of a lot is based on the accumulated time of items when the testing is terminated. Both the two censoring schemes are extensively applied in the industry to reduce time and inspection cost. Reference [3] proposed a Bayesian variable plan for the type-I censoring. Reference [4] designed an accelerated sampling plan under the type-I censoring scheme. More details about sampling plans under censoring can be seen in, [3]–[9] and [10].

The lifetime performance index C_L is applied to measure the performance of the product lifetime with a non-negative lifetime characteristics T which set a lower specification limit L. Reference [1] used the exact and approximation

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approach to design the sampling plan for the exponential distribution. Reference [2] extended this work under the type-II censoring scheme. Some more works on the parameters estimation and testing of hypothesis for C_L can be seen in [11]–[13].

Recently, [14]–[16] introduced the more flexible sampling schemes, called the repetitive sampling and multiple dependent state (MDS) sampling in the area of acceptance sampling plans. In repetitive sampling, a random sample is repetitively selected until a decision is made for the submitted lot of the product. The MDS sampling scheme utilizes the information on the current lot as well as the previously accepted lots to make a decision for a lot having in-decision state at the first sample. These sampling schemes are more efficient than single sampling plans in terms of the average sample number. A variety of works on the repetitive and MDS sampling plans can be seen in [17]–[20] and [21], [22] introduced the generalization of the repetitive sampling plan and MDS plan, called the multiple dependent state repetitive (MDSR) plan. The MDSR is more efficient than the repetitive sampling plan and MDS sampling plan in terms of the average sample number. Reference [23] proposed the sampling plan using MDSR sampling for the normal distribution. Yen et al. [24] worked on the repetitive sampling plan based on one-sided specification limit. Aldosari et al. [25] introduced MDSR in the area of process control. More information of sampling plans can be seen in Aslam et al. [26], Sohn et al. [27] and Li et al. [28].

By exploring the literature and best of our knowledge, there is no work on the sampling plan based on life performance index under the MDSR sampling. In this paper, we will design the MDSR lifetime sampling plan using a life performance index for type II censored data under exponential and Weibull distributions. In addition, the performance of the proposed plan is compared with the existing sampling plans in terms of operating characteristic curve and average sample size.

The rest of paper is organized as follows. The life performance index is introduced in Section 2. In Section 3, the design of lifetime sampling plans is presented. Section 4 compares the proposed plan with the existing lifetime sampling plan. In Section 5, two examples are used to illustrate the proposed methodology. Conclusions are finally made in Section 6.

II. LIFETIME PERFORMANCE INDEX

In this section, we will introduce a lifetime performance index under the type II censoring for the exponential distribution and the Weibull distribution.

A. ESTIMATOR OF C_L FOR EXPONENTIAL DISTRIBUTION WITH TYPE II CENSORING

Suppose that T represents the lifetime of products having the lower specification limit L, which follows the exponential distribution having the probability density function (pdf) given by

$$f(t;\theta) = (1/\theta)e^{-t/\theta}, \quad t > 0 \tag{1}$$

where θ is the mean of the exponential distribution.

Then, the lifetime performance index can be defined as follows [24]:

$$C_L = \frac{\mu_T - L}{\sigma_T} \tag{2}$$

where μ_T and σ_T are the mean and standard deviation of lifetime, respectively.

From [2], we have

$$C_L = 1 - L/\theta, \quad -\infty < C_L < 1. \tag{3}$$

The lifetime non-conforming rate p is given by

$$p = P(T \le L) = 1 - e^{-L/\theta} = 1 - e^{C_L - 1}. \tag{4}$$

It is important to note that θ is unknown in practice. Suppose that under type II censoring, the s failures are observed from n items. [2] provided an asymptotically unbiased estimator $\hat{C}_L$ as follows

$$\hat{C}_L = 1 - L/\hat{\theta} = 1 - \frac{(s-1)L}{\sum_{i=1}^{s} t_{(i)} + (n-s)t_{(s)}} \tag{5}$$

where $t_{(i)}$ is the i-th failure time.

Let

$$w = \sum_{i=1}^{s} t_{(i)} + (n-s)t_{(s)}.$$

Then the statistic w is sufficient for θ which leads to

$$2w/\theta \sim \chi^2_{2s},$$

where χ^2_{2s} is a chi-squared distribution with $2s$ degree of freedom [2]. Later we will use the equation (5) to design the lifetime sampling plan for the exponential distribution.

B. ESTIMATOR OF C_L FOR WEIBULL DISTRIBUTION WITH TYPE II CENSORING

Suppose now that lifetime T follows the Weibull distribution which is a generalized form of the exponential distribution. The pdf of the Weibull distribution having mean $\mu_T = \theta\Gamma\left(1+\frac{1}{k}\right)$ and variance $\sigma_T^2 = \theta^2\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]$ is given by

$$f(t;\theta,k) = \frac{k}{\theta}\left(\frac{t}{\theta}\right)^{k-1}\exp\left[-\left(\frac{t}{\theta}\right)^k\right] \tag{6}$$

where k is shape parameter and θ is a scale parameter. The Weibull distribution reduces to an exponential distribution when k =1. From [2], the lifetime performance index under the Weibull distribution is given by

$$C_L = \frac{\mu_T - L}{\sigma_T} = \frac{1}{A}\left[\Gamma\left(1+\frac{1}{k}\right) - \frac{L}{\theta}\right], \quad -\infty < C_L < \Gamma\left(1+\frac{1}{k}\right) \tag{7}$$

where $A = \sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}$

The lifetime non-conforming rate p is given by

$$\mathrm{p} = 1 - P(T \geq L) = 1 - exp\left[-\left(\frac{L}{\theta}\right)^k\right] = 1 - \exp\left\{-\left[\Gamma\left(1+\frac{1}{k}\right) - \mathrm{A}C_L\right]^k\right\} \quad (8)$$

The asymptotically unbiased estimator $\hat{C}_L$ for the Weibull distribution is given by

$$\hat{C}_L = \frac{1}{\mathrm{A}}\left[\Gamma\left(1+\frac{1}{k}\right) - \frac{L * \Gamma(s)}{D^{\frac{1}{k}}\Gamma\left(s-\frac{1}{k}\right)}\right] \quad (9)$$

Let $D = \sum_{i=1}^{s}(n-i+1)\left(t_{(i)}^k - t_{(i-1)}^k\right)$. Then, according to [2]

$$2\theta^{-k}D \sim \chi_{2s}^2.$$

Here equation (9) will be used to design the lifetime sampling plan for the Weibull distribution.

III. PROPOSED SAMPLING PLANS BASED ON LIFE PERFORMANCE INDEX

The proposed sampling plan called multiple dependent state repetitive (MDSR) sampling plan, is operated as follows:

Step 1: Choose the producer's risk and consumer's risk. Determine the quality level requirements p_{AQL} and p_{LTPD} at two risks (α, β) for the lifetime characteristic with a given lower lifetime limit L

Step 2: Choose the plan parameters (k_a and k_r) of the sampling plan. Take a random sample of size n $(n \geq s)$ to implement the lifetime testing until the s-th failure time occurs. Then, compute $\hat{C}_L$

Step 3: Make a decision on the lot as follows:

1) Accept the lot if $\hat{C}_L > k_a$
2) Reject the lot if $\hat{C}_L < k_r$
3) if $k_r \leqq \hat{C}_L \leqq k_a$, then accept the lot if$m$ preceding lots have been accepted under the condition of $\hat{C}_L > k_a$. Otherwise, repeat Step 2.

Before developing the MDSR lifetime sampling plan, we first define some notations for the easiness of readability.

NOMENCLATURE

- P_a: Lot acceptance probability in an MDS sampling plan based on a single sample
- P_r: Lot rejection probability in an MDS sampling plan based on a single sample
- P_{rep}: The probability of re-sampling after each sampling
- $L(p)$: Lot acceptance probability in the proposed MDSR sampling plan

A. MDSR SAMPLING PLAN FOR EXPONENTIAL DISTRIBUTION WITH TYPE II CENSORING

The lot acceptance probability and rejection probability in an MDS plan based on a single sample can be respectively expressed as

$$P_a = P\left(\hat{C}_L \geq k_a|C_L\right) + P\left\{k_r < \hat{C}_L \leq k_a|C_L\right\}\left[P\left(\hat{C}_L \geq k_a|C_L\right)\right]^m$$

and

$$P_r = P\left(\hat{C}_L < k_r|C_L\right).$$

When the lifetime follows the exponential distribution, P_a and P_r can be derived as

$$P_a = P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-\mathrm{p})}{(1-k_a)}\right) + \left\{P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-\mathrm{p})}{(1-k_r)}\right) - P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-\mathrm{p})}{(1-k_a)}\right)\right\} \times \left[P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-\mathrm{p})}{(1-k_a)}\right)\right]^m$$

and

$$P_r = 1 - P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-\mathrm{p})}{(1-k_r)}\right).$$

According to [24], the lot acceptance probability in the proposed MDSR sampling plan is written as when the quality level is p

$$L(p) = \frac{P_a}{P_a + P_r} \quad (10)$$

Therefore, the lot acceptance probability in the MDSR sampling plan for the exponential distribution can be expressed in (11), as shown at the bottom of this page.

$$L(p) = \frac{P_a}{P_a + P_r} = \frac{P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-p)}{(1-k_a)}\right) + \left\{P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-p)}{(1-k_r)}\right) - P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-p)}{(1-k_a)}\right)\right\}\left[P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-p)}{(1-k_a)}\right)\right]^{\mathrm{m}}}{1 - \left(\left\{P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-p)}{(1-k_r)}\right) - P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-p)}{(1-k_a)}\right)\right\}\left(1 - \left[P\left(\chi_{2s}^2 \geq \frac{-2(s-1)\ln(1-p)}{(1-k_a)}\right)\right]^m\right)\right)} \quad (11)$$

TABLE 1. MDSR for exponential distribution under $m = 1$.

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.0001	**0.0002**	14	21.707	0.999888	0.999806	8	13.093	0.999908	0.999814	7	11.615	0.999906	0.999808
	0.0003	7	9.738	0.999867	0.999741	4	5.697	0.999901	0.999771	3	5.212	0.999916	0.999736
	0.0004	5	6.668	0.999848	0.999686	2	4.001	0.999939	0.999676	2	3.591	0.999928	0.999692
	0.0005	4	5.270	0.999833	0.999634	2	3.034	0.999917	0.999699	2	2.827	0.999901	0.999708
0.001	**0.002**	14	21.679	0.99888	0.99806	8	13.083	0.99908	0.99814	8	11.767	0.99898	0.99818
	0.003	7	9.704	0.99866	0.99741	4	5.693	0.99901	0.99771	3	5.158	0.99915	0.99737
	0.004	5	6.668	0.99847	0.99685	2	3.997	0.99939	0.99676	2	3.588	0.99928	0.99692
	0.005	4	5.257	0.99832	0.99634	2	3.031	0.99917	0.99699	2	2.825	0.99901	0.99708
0.005	**0.010**	14	21.444	0.99435	0.99030	8	12.856	0.99534	0.99073	7	11.406	0.99523	0.99042
	0.015	7	9.657	0.99325	0.98702	4	5.647	0.99499	0.98855	3	5.114	0.99571	0.98686
	0.020	5	6.607	0.99226	0.98427	3	3.885	0.99444	0.98742	2	3.545	0.99634	0.98463
	0.025	4	5.216	0.99148	0.98168	2	2.982	0.99575	0.98508	2	2.809	0.99501	0.98540
0.010	**0.020**	14	21.288	0.98862	0.98056	8	12.75	0.99061	0.98144	7	11.316	0.99039	0.98082
	0.030	6	9.577	0.98778	0.97151	4	5.603	0.98987	0.97709	3	5.067	0.99133	0.97372
	0.040	5	6.558	0.98432	0.96846	3	3.860	0.98874	0.97480	2	3.507	0.99257	0.96928
	0.050	4	5.171	0.98268	0.96327	2	2.964	0.99137	0.96999	2	2.773	0.98973	0.97082
0.020	**0.040**	14	21.048	0.97701	0.96094	9	12.741	0.97966	0.96451	7	11.167	0.98052	0.96152
	0.060	6	9.410	0.97508	0.94280	4	5.527	0.97933	0.95406	3	4.984	0.98230	0.94736
	0.080	5	6.457	0.96780	0.93664	3	3.810	0.97688	0.94945	2	3.435	0.98471	0.93858
	0.100	4	5.081	0.96418	0.92621	2	2.904	0.98214	0.93999	2	2.725	0.97873	0.94152
0.030	**0.060**	14	20.732	0.96497	0.94116	8	12.403	0.97109	0.94399	7	11.024	0.97039	0.94209
	0.090	6	9.247	0.96186	0.91384	4	5.454	0.96837	0.93090	3	4.901	0.97288	0.92091
	0.120	5	6.359	0.95037	0.90451	3	3.759	0.96436	0.92395	2	3.365	0.97638	0.90788
	0.150	4	4.992	0.94436	0.88879	2	2.837	0.97221	0.91039	2	2.681	0.96701	0.91206
0.050	**0.100**	13	20.127	0.94211	0.89809	8	12.072	0.95052	0.90606	7	10.739	0.94927	0.90284
	0.150	6	8.929	0.93372	0.85517	4	5.310	0.94503	0.88414	3	4.737	0.95274	0.86774
	0.200	4	6.061	0.92841	0.81152	3	3.661	0.93709	0.87242	2	3.228	0.95813	0.84641
	0.250	4	4.819	0.90004	0.81284	2	2.729	0.95003	0.84999	2	2.585	0.94028	0.85273

Generally, the design of an acceptance sampling plan is determined based on the principle of two points on the OC curve. Because there are several combinations of the parameters for the proposed plan which satisfy the two inequalities, we choose the parameters which lead to the smallest average sample number (*ASN*). For the MDSR sampling plan under the exponential distribution, the *ASN* can be derived as

$$ASN = \frac{s}{1 - P_{rep}},$$

where

$$\begin{aligned}P_{rep} &= 1 - P_a - P_r = P\left\{k_r < \hat{C}_L \leq k_a | C_L\right\} \\ &\quad \times \left(1 - \left[P\left(\hat{C}_L \geq k_a | C_L\right)\right]^m\right) \\ &= \left\{P\left(\chi^2_{2s} \geq \frac{-2\,(s-1)\ln(1-\mathrm{p})}{(1-k_r)}\right)\right. \\ &\quad \left. - P\left(\chi^2_{2s} \geq \frac{-2\,(s-1)\ln(1-\mathrm{p})}{(1-k_a)}\right)\right\} \\ &\quad \times \left(1 - \left[P\left(\chi^2_{2s} \geq \frac{-2\,(s-1)\ln(1-\mathrm{p})}{(1-k_a)}\right)\right]^m\right)\end{aligned} \quad (12)$$

Through the principle of two points on the OC curve, the plan parameters can be obtained using the following non-linear optimization problem.

Minimize

$$ASN = \frac{1}{2}\left[\frac{s}{1 - P_{rep}\left(p_{AQL}\right)} + \frac{s}{1 - P_{rep}\left(p_{LTPD}\right)}\right] \quad (13a)$$

Subject to

$$L\left(p_{AQL}\right) \geq 1 - \alpha \quad (13b)$$

$$L\left(p_{LTPD}\right) \leq \beta \quad (13c)$$

Tables 1-3 display the sampling plan parameters values for the exponential distribution under various quality levels (p_{AQ}L, p_{LTPD}), with specified risks (α, β) for

TABLE 2. MDSR for exponential distribution under $m = 2$.

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.0001	**0.0002**	15	21.870	0.99988	0.999811	8	13.828	0.999904	0.999811	8	12.045	0.999893	0.999817
	0.0003	7	9.820	0.999861	0.99974	4	6.104	0.999901	0.999764	3	5.406	0.999909	0.999731
	0.0004	5	6.698	0.99984	0.999685	3	4.003	0.999885	0.999745	3	3.749	0.999864	0.99975
	0.0005	4	5.291	0.999824	0.999633	2	3.128	0.999911	0.999693	2	2.883	0.999893	0.999704
0.001	**0.002**	15	21.844	0.99880	0.99811	8	13.818	0.99904	0.99811	8	12.038	0.99893	0.99817
	0.003	7	9.785	0.99860	0.99740	4	6.099	0.99901	0.99764	4	5.367	0.99879	0.99774
	0.004	5	6.689	0.99840	0.99685	3	4.000	0.99885	0.99745	3	3.747	0.99864	0.99750
	0.005	4	5.277	0.99823	0.99633	2	3.108	0.99910	0.99694	2	2.871	0.99892	0.99704
0.005	**0.010**	14	21.735	0.99414	0.99027	9	13.495	0.99483	0.99107	8	11.847	0.99458	0.99087
	0.015	7	9.746	0.99301	0.98699	4	5.821	0.99472	0.98838	3	5.320	0.99537	0.98658
	0.020	5	6.632	0.99188	0.98424	3	3.962	0.99415	0.98727	2	3.706	0.99601	0.98416
	0.025	4	5.231	0.99103	0.98165	2	3.077	0.99544	0.98473	2	2.843	0.99452	0.98524
0.010	**0.020**	14	21.550	0.98820	0.98051	9	13.402	0.98959	0.98211	8	11.774	0.98909	0.98171
	0.030	7	9.634	0.98575	0.97395	4	5.774	0.98934	0.97675	3	5.261	0.99063	0.97318
	0.040	5	6.578	0.98356	0.96842	3	3.932	0.98815	0.97451	2	3.645	0.99185	0.96843
	0.050	4	5.183	0.98178	0.96323	2	3.042	0.99072	0.96948	2	2.817	0.98886	0.97047
0.020	**0.040**	14	21.241	0.97610	0.96086	9	13.284	0.97901	0.96408	7	11.564	0.97946	0.96108
	0.060	7	9.502	0.97101	0.94767	4	5.687	0.97827	0.95341	4	5.255	0.97501	0.95450
	0.080	5	6.475	0.96628	0.93657	3	3.875	0.97569	0.94892	2	3.561	0.98323	0.93698
	0.100	4	5.091	0.96238	0.92613	2	2.977	0.98082	0.93899	2	2.768	0.97701	0.94085
0.030	**0.060**	13	21.023	0.96483	0.93920	8	13.015	0.96988	0.94316	7	11.401	0.96879	0.94145
	0.090	7	9.365	0.95562	0.92115	4	5.601	0.96676	0.92998	4	5.181	0.96162	0.93156
	0.120	5	6.374	0.94810	0.90441	3	3.818	0.96257	0.92321	2	3.482	0.97412	0.90561
	0.150	4	5.001	0.94168	0.88869	2	2.910	0.97021	0.90863	2	2.713	0.96418	0.91119
0.050	**0.100**	13	20.390	0.93987	0.89780	8	12.620	0.94848	0.90476	7	11.075	0.94657	0.90185
	0.150	6	8.998	0.93023	0.85478	4	5.437	0.94234	0.88276	4	5.057	0.93337	0.88514
	0.200	4	6.101	0.92388	0.81095	3	3.709	0.93413	0.87137	2	3.325	0.95417	0.84309
	0.250	4	4.825	0.89568	0.81272	2	2.783	0.94656	0.84787	2	2.612	0.93563	0.85150

$m = 1, 2, 3m = 1, 2, 3$, respectively. Based on the given tables, the practitioner can know the required sample size and corresponding critical values for the decision of lot sentencing. For example, if the benchmarking quality level (p_{AQL}, p_{LTPD}) is set to (0.001, 0.002) with $\alpha = 0.01$ and $\beta = 0.05$ for $m = 2$, then the required sample size, critical acceptance value and critical rejection value can be obtained as $(s, k_a, k_r) = (14, 0.99888, 0.99806)$, and the corresponding *ASN* is 21.679. This implies that we would accept the lot when the lifetime testing is not terminated until the occurrence of the fourteenth failure with $\hat{C}_L > 0.99888$ or while $0.99806 < \hat{C}_L < 0.99888$ and the previous two lots were accepted. Otherwise, the lot will be rejected. It is noted that the critical acceptance values and critical rejection values seem to decrease simultaneously as the value of m increases since the successive lots of good quality give the reward.

B. MSDR SAMPLING PLAN FOR WEIBULL DISTRIBUTION WITH TYPE II CENSORING

Similarly to the derivation of the exponential distribution, when the lifetime obeys the Weibull distribution, the acceptance probability and rejection probability in the MDS sampling plan based on a single sample can be respectively expressed as

$$P_a = P\left(\chi^2_{2s} \geq \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - \mathrm{A}k_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right) + \left\{P\left(\chi^2_{2s} \geq \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - \mathrm{A}k_r\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right) - P\left(\chi^2_{2s} \geq \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - \mathrm{A}k_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right\} \times \left[P\left(\chi^2_{2s} \geq \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - \mathrm{A}k_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right]^m$$

TABLE 3. MDSR for exponential distribution under $m = 3$.

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.0001	**0.0002**	16	22.295	0.999877	0.999815	10	14.265	0.999892	0.999827	8	12.492	0.999893	0.999815
	0.0003	7	9.898	0.99986	0.999739	4	6.374	0.999901	0.999759	4	5.520	0.99988	0.999771
	0.0004	5	6.742	0.99984	0.999684	3	4.101	0.999885	0.999741	3	3.798	0.999864	0.999748
	0.0005	4	5.299	0.999823	0.999633	3	3.427	0.999849	0.999751	2	3.064	0.999901	0.999694
0.001	**0.002**	16	22.271	0.99877	0.99815	10	14.257	0.99892	0.99827	8	12.485	0.99893	0.99815
	0.003	7	9.886	0.99860	0.99739	4	6.110	0.99896	0.99763	4	5.494	0.99879	0.99771
	0.004	5	6.722	0.99839	0.99684	3	4.098	0.99885	0.99741	3	3.796	0.99864	0.99748
	0.005	4	5.292	0.99823	0.99633	3	3.420	0.99848	0.99751	2	3.061	0.99901	0.99694
0.005	**0.010**	15	22.087	0.99397	0.99050	10	14.174	0.99457	0.99134	8	12.274	0.99458	0.99077
	0.015	7	9.829	0.99301	0.98696	4	6.034	0.99473	0.98818	4	5.451	0.99390	0.98857
	0.020	5	6.664	0.99187	0.98421	3	4.049	0.99415	0.98710	3	3.764	0.99310	0.98742
	0.025	4	5.250	0.99102	0.98162	2	3.187	0.99545	0.98432	2	2.905	0.99451	0.98499
0.010	**0.020**	14	21.981	0.98820	0.98043	10	14.063	0.98907	0.98267	8	12.191	0.98909	0.98151
	0.030	7	9.713	0.98574	0.97388	4	5.982	0.98936	0.97635	4	5.410	0.98767	0.97712
	0.040	5	6.609	0.98354	0.96836	3	4.015	0.98815	0.97419	3	3.737	0.98601	0.97482
	0.050	4	5.201	0.98175	0.96317	2	3.148	0.99075	0.96869	2	2.876	0.98884	0.96999
0.020	**0.040**	15	21.616	0.97542	0.96178	10	13.886	0.97787	0.96523	8	12.10	0.97801	0.96287
	0.060	7	9.567	0.97094	0.94755	4	5.880	0.97831	0.95265	4	5.337	0.97486	0.95413
	0.080	5	6.502	0.96624	0.93646	3	3.951	0.97570	0.94831	3	3.688	0.97129	0.94949
	0.100	4	5.106	0.96233	0.92603	2	3.072	0.98087	0.93755	2	2.817	0.97690	0.93999
0.030	**0.060**	13	21.520	0.96484	0.93890	10	13.715	0.96639	0.94767	8	11.865	0.96642	0.94420
	0.090	7	9.427	0.95557	0.92098	4	5.781	0.96682	0.92889	4	5.267	0.96154	0.93099
	0.120	5	6.397	0.94803	0.90427	3	3.887	0.96258	0.92236	3	3.639	0.95578	0.92401
	0.150	4	5.013	0.94159	0.88856	2	2.996	0.97028	0.92470	2	2.760	0.96410	0.90999
0.050	**0.100**	13	20.822	0.93988	0.89734	9	13.213	0.94535	0.90840	8	11.556	0.94255	0.90640
	0.150	6	9.108	0.9302	0.85416	4	5.590	0.94241	0.88115	4	5.129	0.93322	0.88433
	0.200	4	6.162	0.92382	0.8101	3	3.768	0.93414	0.86999	2	3.455	0.95428	0.83870
	0.250	4	4.833	0.89552	0.81256	2	2.851	0.94664	0.84499	2	2.647	0.93544	0.84994

and

$$P_r = 1 - P\left(\chi^2_{2s} \geq \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - Ak_r\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right).$$

Therefore, the lot acceptance probability in the proposed MDSR sampling plan for Weibull distribution can be expressed in (14), as shown at the bottom of the next page, where A= $\sqrt{\Gamma(1+2/k) - \Gamma^2(1+1/k)}$

Under Weibull distribution, the *ASN* can be derived as

$$ASN = \frac{s}{1-P_{rep}},$$

where

$$P_{rep} = 1 - P_a - P_r = P\left\{k_r < \hat{C}_L \leq k_a | C_L\right\} \times \left(1 - \left[P\left(\hat{C}_L \geq k_a | C_L\right)\right]^m\right)$$

$$= \left\{P\left(\chi^2_{2s} \geq \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - Ak_r\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right) - P\left(\chi^2_{2s} \geq \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - Ak_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right\}$$
$$\times \left(1 - \left[P\left(\chi^2_{2s} \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right) - Ak_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right]^m\right) \quad (15)$$

As mentioned above, based on the principle of two points on the OC curve, the plan parameters can be obtained using the following non-linear optimization solution.

Minimize

$$ASN = \frac{1}{2}\left[\frac{s}{1-P_{rep}(p_{AQL})} + \frac{s}{1-P_{rep}(p_{LTPD})}\right] \quad (16a)$$

Subject to

$$L(p_{AQL}) \geq 1-\alpha \quad (16b)$$
$$L(p_{LTPD}) \leq \beta \quad (16c)$$

Tables 4-6 show the sampling plan parameters values of Weibull distribution having shape parameter $k = 2$

TABLE 4. MDSR for Weibull distribution under $m = 1$ ($k = 2$).

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.005	**0.010**	15	21.542	1.7467	1.7013	7	13.026	1.7688	1.6939	7	11.406	1.7608	1.6973
	0.015	7	9.650	1.7319	1.6620	4	5.641	1.7537	1.6724	3	5.116	1.7627	1.6498
	0.020	5	6.608	1.7171	1.6337	3	3.888	1.7419	1.6554	2	3.543	1.7656	1.6111
	0.025	4	5.216	1.7054	1.6085	2	2.982	1.7543	1.6156	2	2.797	1.7401	1.6190
0.010	**0.020**	14	21.276	1.6805	1.6092	8	12.746	1.7001	1.6137	7	11.318	1.6970	1.6078
	0.030	6	9.576	1.6684	1.5395	4	5.603	1.6866	1.5725	3	5.098	1.7001	1.5404
	0.040	5	6.557	1.6342	1.5176	3	3.861	1.6694	1.5485	2	3.507	1.7031	1.4861
	0.050	4	5.171	1.6169	1.4818	2	2.952	1.6866	1.4923	2	2.773	1.6663	1.4970
0.020	**0.040**	14	20.997	1.5821	1.4824	8	12.588	1.6101	1.4890	7	11.171	1.6055	1.4807
	0.060	6	9.412	1.5637	1.3837	4	5.537	1.5901	1.4306	3	4.986	1.6076	1.3861
	0.080	5	6.457	1.5135	1.3525	3	3.810	1.5639	1.3967	2	3.436	1.6119	1.3094
	0.100	4	5.081	1.4872	1.3018	2	2.894	1.5873	1.3179	2	2.725	1.5578	1.3240

TABLE 5. MDSR for Weibull distribution under $m = 2$ ($k = 2$).

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.005	**0.010**	15	21.745	1.7439	1.7011	8	13.543	1.7598	1.6998	7	11.842	1.7567	1.6960
	0.015	7	9.703	1.7279	1.6618	4	5.815	1.7495	1.6706	3	5.314	1.7567	1.6471
	0.020	5	6.632	1.7124	1.6335	3	3.959	1.7373	1.6540	2	3.685	1.7586	1.6069
	0.025	4	5.231	1.7001	1.6083	2	3.076	1.7485	1.6121	2	2.843	1.7327	1.6171
0.010	**0.020**	14	21.546	1.6763	1.6088	8	13.441	1.6957	1.6113	8	11.768	1.6835	1.6159
	0.030	7	9.639	1.6501	1.5573	4	5.772	1.6807	1.5700	3	5.264	1.6908	1.5369
	0.040	5	6.578	1.6275	1.5173	3	3.932	1.6630	1.5464	2	3.649	1.6932	1.4800
	0.050	4	5.183	1.6093	1.4816	2	3.041	1.6784	1.4876	2	2.816	1.6559	1.4945
0.020	**0.040**	14	21.253	1.5761	1.4818	9	13.222	1.5947	1.4977	7	11.568	1.5972	1.4782
	0.060	6	9.506	1.5543	1.3828	4	5.687	1.5814	1.4274	4	5.245	1.5567	1.4332
	0.080	5	6.477	1.5042	1.3521	3	3.875	1.5550	1.3940	2	3.563	1.5977	1.3015
	0.100	4	5.092	1.4767	1.3015	2	2.976	1.5757	1.3116	2	2.764	1.5432	1.3208

under various quality levels (p_{AQL}, p_{LTPD}), with specified risks (α,β) for $m = 1, 2, 3m = 1, 2, 3$, respectively. Also, Tables 7-9 list the proposed sampling plan parameters values of Weibull distribution having shape parameter $k = 3$ under various quality levels (p_{AQL}, p_{LTPD}), with specified risks (α, β) for $m = 1, 2, m = 1, 2, 3$, respectively. Using these tables, the practitioner can decide the required sample size and corresponding critical values for the disposition of lots. For example, if the benchmarking quality level (p_{AQL}, p_{LTPD}) is set to (0.01, 0.02) with $\alpha = 0.05$ and $\beta = 0.05$ for $m = 3$ ($k = 2$), then the required sample size, critical acceptance value and critical rejection value can be obtained as $(s, k_a, k_r) = (9, 1.6901, 1.6177)$, and the *ASN* is 14.106. This means that the lot will be accepted if the lifetime testing is to be terminated upon the occurrence of the ninth failure with $\hat{C}_L > 1.6901$ or while $1.6177 < \hat{C}_L \mathrm{k_r} \le \hat{L}_e < \mathrm{k_a} < 1.6901$ and the previous three lots were accepted. Otherwise, the lot will be rejected. It is also noted that the critical acceptance values and critical rejection values seem to decrease simultaneously as the value of m increases since the successive lots of good quality give the reward.

$$L(p) = \frac{P_a}{P_a + P_r}$$

$$= \frac{P\left(\chi^2_{2s} \ge \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right)-Ak_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right) + \left\{P\left(\chi^2_{2s} \ge \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right)-Ak_r\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right) - P\left(\chi^2_{2s} \ge \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right)-Ak_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right\}\left[P\left(\chi^2_{2s} \ge \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right)-Ak_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right]^m}{1 - \left(\left\{P\left(\chi^2_{2s} \ge \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right)-Ak_r\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right) - P\left(\chi^2_{2s} \ge \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right)-Ak_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right\}\left(1 - \left[P\left(\chi^2_{2s} \ge \frac{-2\Gamma^k(s)\ln(1-p)}{\left[\Gamma\left(1+\frac{1}{k}\right)-Ak_a\right]^k \Gamma^k\left(s-\frac{1}{k}\right)}\right)\right]^m\right)\right)} \quad (14)$$

TABLE 6. MDSR for Weibull distribution under $m = 3$ ($k = 2$).

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.005	**0.010**	15	22.072	1.7439	1.7008	8	14.423	1.7601	1.6978	7	12.481	1.7569	1.6942
	0.015	7	9.789	1.7278	1.6614	4	6.034	1.7497	1.6684	4	5.450	1.7373	1.6725
	0.020	5	6.666	1.7123	1.6332	3	4.049	1.7374	1.6522	3	3.762	1.7222	1.6555
	0.025	4	5.253	1.7001	1.6080	2	3.186	1.7487	1.6081	2	2.903	1.7326	1.6148
0.010	**0.020**	14	21.970	1.6763	1.6082	9	14.106	1.6901	1.6177	8	12.183	1.6835	1.6143
	0.030	7	9.717	1.6499	1.5568	4	5.984	1.6810	1.5670	4	5.412	1.6633	1.5727
	0.040	5	6.610	1.6273	1.5169	3	4.016	1.6631	1.5441	3	3.738	1.6415	1.5486
	0.050	4	5.202	1.6091	1.4812	2	3.149	1.6788	1.4820	2	2.874	1.6557	1.4913
0.020	**0.040**	15	21.628	1.5716	1.4872	9	13.808	1.5950	1.4951	8	12.025	1.5864	1.4899
	0.060	6	9.654	1.5542	1.3814	4	5.882	1.5817	1.4234	4	5.338	1.5563	1.4311
	0.080	5	6.502	1.5039	1.3517	3	3.952	1.5551	1.3909	2	3.735	1.5984	1.2913
	0.100	4	5.106	1.4763	1.3011	2	3.071	1.5761	1.3044	2	2.816	1.5429	1.3166

TABLE 7. MDSR for Weibull distribution under $m = 1$ ($k = 3$).

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.005	**0.010**	14	21.409	2.1977	2.0888	8	12.836	2.2284	2.0940	7	11.392	2.2230	2.0850
	0.015	7	9.651	2.1582	2.0140	4	5.640	2.2045	2.0316	3	5.110	2.2232	1.9845
	0.020	5	6.607	2.1250	1.9580	3	3.886	2.1756	1.9953	2	3.541	2.2251	1.9030
	0.025	4	5.222	2.1001	1.9092	2	2.981	2.1985	1.9113	2	2.797	2.1660	1.9177
0.010	**0.020**	14	21.276	2.0523	1.9159	8	12.745	2.0911	1.9228	7	11.315	2.0842	1.9114
	0.030	7	9.583	2.0017	1.8217	4	5.602	2.0601	1.8441	3	5.068	2.0836	1.7849
	0.040	5	6.557	1.9588	1.7510	3	3.860	2.0228	1.7984	2	3.506	2.0853	1.6825
	0.050	4	5.170	1.9248	1.6895	2	2.952	2.0510	1.6929	2	2.775	2.0101	1.7006
0.020	**0.040**	14	20.997	1.8669	1.6972	8	12.576	1.9159	1.7063	7	11.171	1.9071	1.6919
	0.060	6	9.410	1.8315	1.5379	4	5.527	1.8747	1.6072	3	4.983	1.9042	1.5331
	0.080	5	6.457	1.7440	1.4890	3	3.809	1.8253	1.5495	2	3.435	1.9042	1.4048
	0.100	4	5.086	1.7001	1.4114	2	2.894	1.8589	1.4175	2	2.725	1.8057	1.4266

TABLE 8. MDSR for Weibull distribution under $m = 2$ ($k = 3$).

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.005	**0.010**	14	21.680	2.1909	2.0882	9	13.473	2.2110	2.1034	7	11.818	2.2135	2.0824
	0.015	7	9.714	2.1501	2.0136	4	5.815	2.1950	2.0280	4	5.340	2.1678	2.0342
	0.020	5	6.630	2.1149	1.9576	3	3.960	2.1655	1.9924	2	3.684	2.2086	1.8951
	0.025	4	5.229	2.0876	1.9089	2	3.074	2.1851	1.9048	2	2.842	2.1496	1.9143
0.010	**0.020**	14	21.535	2.0438	1.9152	9	13.390	2.0692	1.9345	7	11.737	2.0723	1.9081
	0.030	7	9.632	1.9906	1.8212	4	5.773	2.0482	1.8396	3	5.263	2.0662	1.7783
	0.040	5	6.588	1.9463	1.7500	3	3.931	2.0102	1.7948	2	3.643	2.0645	1.6728
	0.050	4	5.183	1.9107	1.6891	2	3.041	2.0341	1.6849	2	2.816	1.9890	1.6965
0.020	**0.040**	14	21.247	1.8563	1.6963	9	13.217	1.8884	1.7211	7	11.569	1.8921	1.6879
	0.060	7	9.499	1.7869	1.5776	4	5.689	1.8601	1.6018	4	5.244	1.8161	1.6111
	0.080	5	6.475	1.7284	1.4885	3	3.874	1.8097	1.5453	2	3.561	1.8778	1.3932
	0.100	4	5.092	1.6810	1.4109	2	2.975	1.8377	1.4081	2	2.765	1.7801	1.4217

TABLE 9. MDSR for Weibull distribution under $m = 3$ ($k = 3$).

		α=0.01, β=0.05				α=0.05, β=0.05				α=0.05, β=0.10			
p_{AQL}	p_{LTPD}	s	ASN	ka	kr	s	ASN	ka	kr	s	ASN	ka	kr
0.005	**0.010**	14	22.119	2.1909	2.0873	9	14.112	2.2114	2.1005	8	12.269	2.2016	2.0949
	0.015	7	9.786	2.1492	2.0129	4	6.032	2.1954	2.0237	4	5.446	2.1675	2.0319
	0.020	5	6.662	2.1146	1.9571	3	4.048	2.1656	1.9891	3	3.761	2.1323	1.9955
	0.025	4	5.248	2.0872	1.9084	2	3.187	2.1858	1.8972	2	2.903	2.1493	1.9098
0.010	**0.020**	15	21.905	2.0376	1.9223	9	14.007	2.0697	1.9310	8	12.188	2.0572	1.9238
	0.030	7	9.708	1.9903	1.8204	4	5.981	2.0487	1.8344	4	5.409	2.0134	1.8445
	0.040	5	6.607	1.9458	1.7500	3	4.015	2.0103	1.7907	3	3.736	1.9681	1.7986
	0.050	4	5.201	1.9103	1.6885	2	3.148	2.0349	1.6757	2	2.874	1.9886	1.6911
0.020	**0.040**	14	21.639	1.8563	1.6950	9	13.800	1.8889	1.7168	8	12.028	1.8731	1.7076
	0.060	7	9.567	1.7866	1.5767	4	5.880	1.8604	1.5956	4	5.337	1.8155	1.6078
	0.080	5	6.502	1.7280	1.4878	3	3.951	1.8098	1.5405	3	3.687	1.7560	1.5498
	0.100	4	5.106	1.6804	1.4103	2	3.071	1.8385	1.3973	2	2.816	1.7794	1.4155

TABLE 10. Comparison of several methods for exponential distribution under $\alpha = 0.01$ and $\beta = 0.05$.

		MDSR			repetitive	Single (Wu et al., 2018)
		ASN			ASN	ASN
p_{AQL}	p_{LTPD}	m=1	m=2	m=3		
0.005	**0.010**	21.444	21.735	22.087	23.625	35
	0.015	9.657	9.746	9.829	10.379	15
	0.020	6.607	6.632	6.664	6.985	10
	0.025	5.216	5.231	5.25	5.472	8
0.010	**0.020**	21.288	21.55	21.981	23.459	35
	0.030	9.577	9.634	9.713	10.283	15
	0.040	6.558	6.578	6.609	6.916	10
	0.050	5.171	5.183	5.201	5.412	7

IV. ADVANTAGES OF THE PROPOSED SAMPLING PLAN

A well-designed sampling plan should have the property that there is a high probability of accepting a lot with good quality level and a low probability of accepting a lot with bad quality level. Besides, a sampling plan having smaller sample size or average sample number is also considered as a better one in comparison to others with the same level of protection to both the producer and the consumer. In this section, we compare the proposed sampling plan with two different existing sampling plans based on C_L by means of OC curve and average sample number (*ASN*) when the lifetime of products follow the exponential distribution. Figures 1-2 depict the OC curves of three sampling plans with some quality levels, for specified risks $(\alpha, \beta) = (0.01, 0.05)$ and $(0.05, 0.10)$, respectively. Overall, these graphs have no significant difference, which implies that their discriminatory power of lots is very close to each other.

Furthermore, Tables 10 and 11 present the *ASN* values for the three sampling plans under the same circumstance as mentioned above. According to outputs in the tables, we can observe the proposed plan provides less *ASN* than two other sampling plans for all combinations of quality levels with given risks. For example, when $(p_{AQL}, p_{LTPD}) = (0.005, 0.01)$ and $(\alpha, \beta) = (0.05, 0.10)$, then the required *ASN* values of the proposed plans for $m = 1, 2, 3$m $= 1, 2, 3$ are only 11.406, 11.847 and 12.274, respectively. Relatively, the required *ASN* values for repetitive sampling and single sampling (Wu et al., 2018) are 13.475 and 19, respectively. Notably, the proposed plan can significantly reduce the *ASN* as compared with single sampling for cases where the difference between p_{AQL} and p_{LTPD} is very slight. Based on the analysis of the OC curve and *ASN*, we know the MDSR sampling plan should be recommended for lifetime testing of products since it can reduce the testing cost effectively.

V. TWO ILLUSTRATIVE EXAMPLES

To show the application of the proposed sampling plan, two examples are given for illustration.

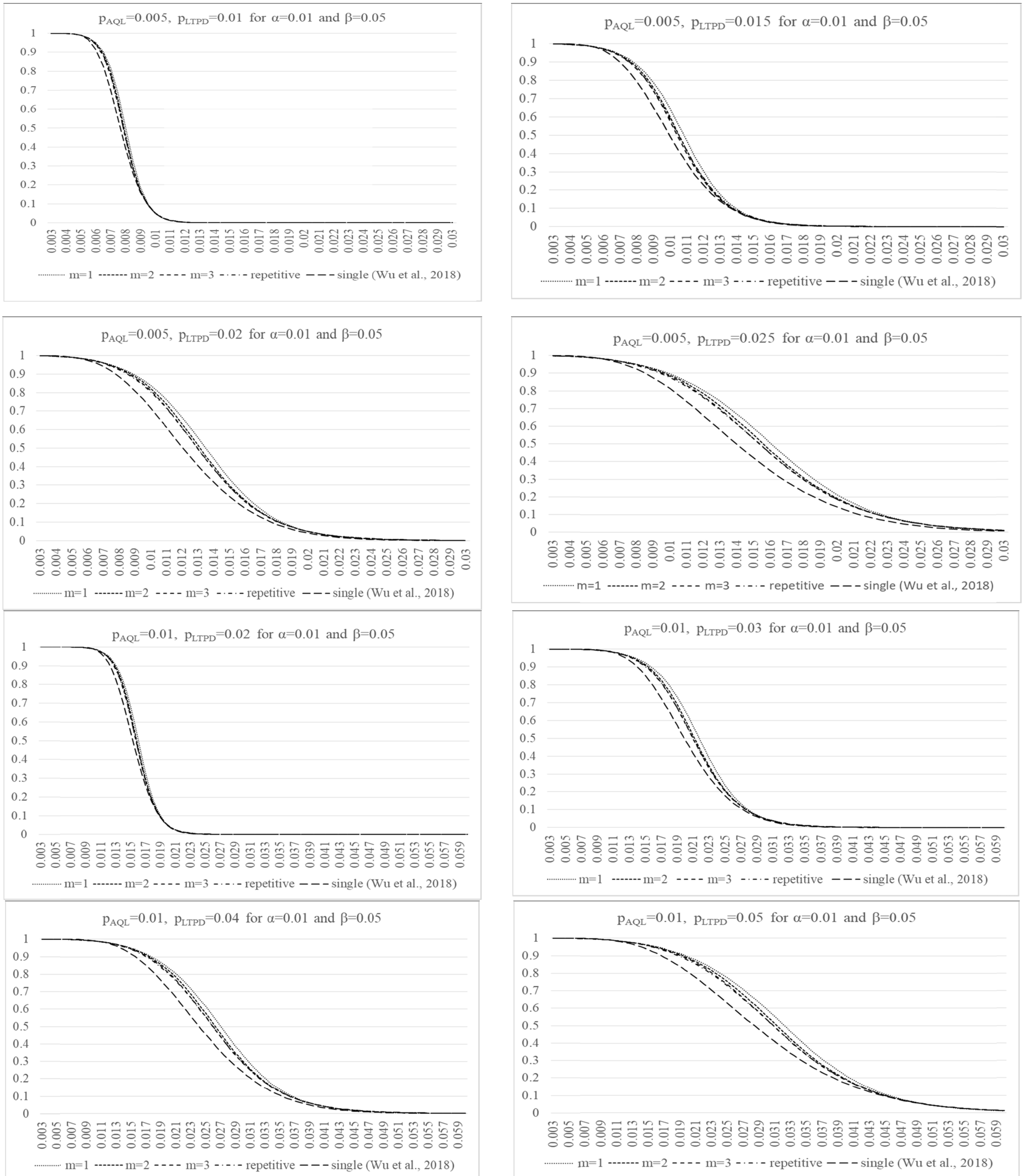

FIGURE 1. **The operating characteristic curves for specified quality levels with $\alpha = 0.01$ and $\beta = 0.05$.**

A. TRANSISTOR TEST WITH EXPONENTIAL DISTRIBUTION

A transistor is a semiconductor device used to amplify or switch electronic signals and electrical power. Since the lifetime of a transistor is an important quality characteristic, a life testing has to be conducted before shipping to customers. The required quality levels ($p_{AQ}L$, p_{LTPD}) and allowable risks

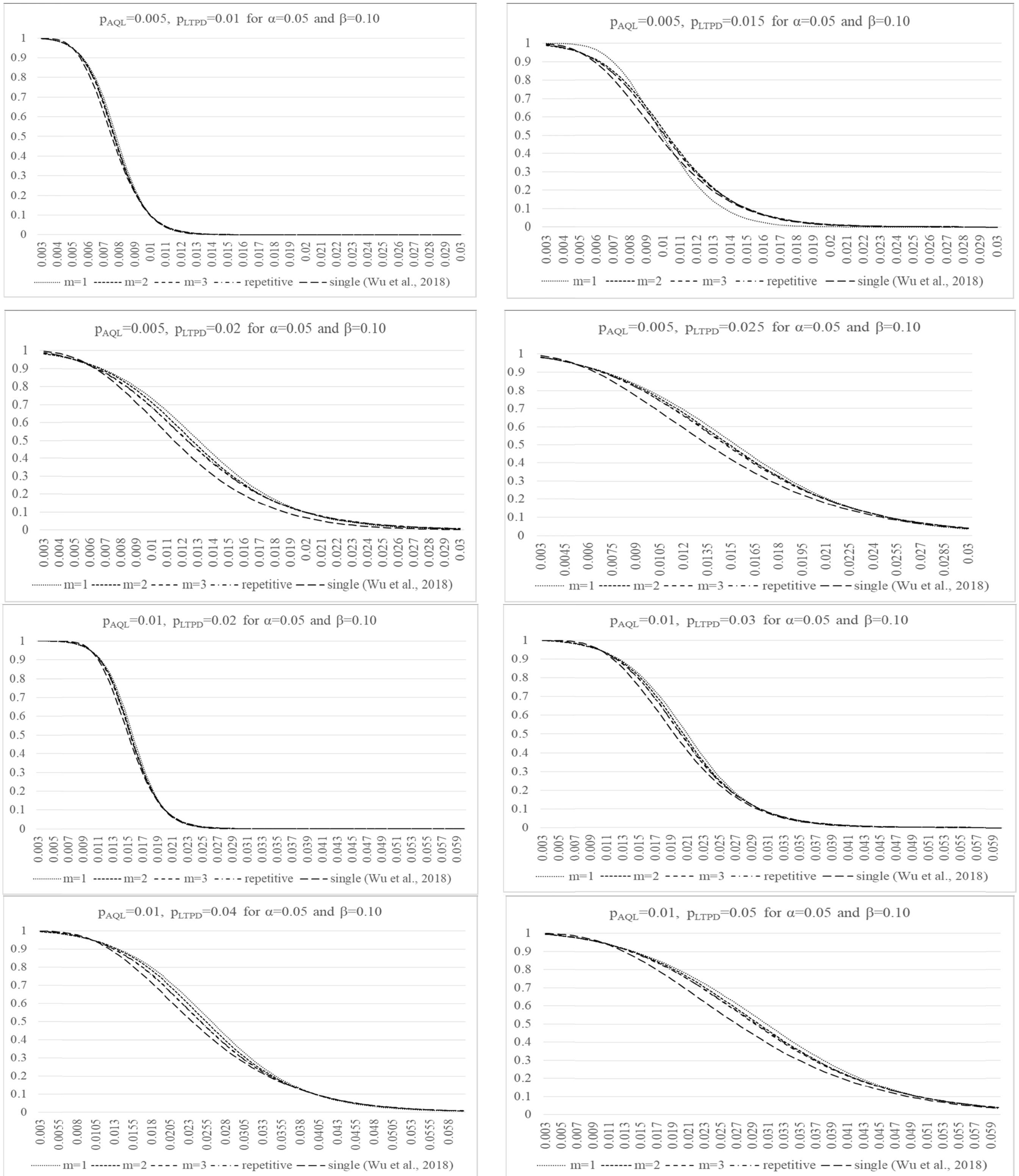

FIGURE 2. The operating characteristic curves of for specified quality levels with $\alpha = 0.05$ and $\beta = 0.10$.

(α, β) are designated as (0.005, 0.01) and (0.01, 0.05) respectively for a lifetime of a particular model of a transistor with a specific lower limit $L = 200$. Now, a sample of 30 transistors is chosen for life testing. Suppose that the proposed sampling plan with m = 2 is applied. Then, we could find the required number of failures, the critical acceptance value, and the

TABLE 11. Comparison of several methods for exponential distribution under $\alpha = 0.05$ and $\beta = 0.10$.

		MDSR			repetitive	Single (Wu et al., 2018)
		ASN			ASN	ASN
p_{AQL}	p_{LTPD}	m=1	m=2	m=3		
0.005	0.010	11.406	11.847	12.274	13.475	19
	0.015	5.114	5.32	5.451	5.890	8
	0.020	3.545	3.706	3.764	4.002	6
	0.025	2.809	2.843	2.905	3.167	4
0.010	0.020	11.316	11.774	12.191	13.385	19
	0.030	5.067	5.261	5.41	5.844	8
	0.040	3.507	3.645	3.737	3.969	5
	0.050	2.773	2.817	2.876	3.124	4

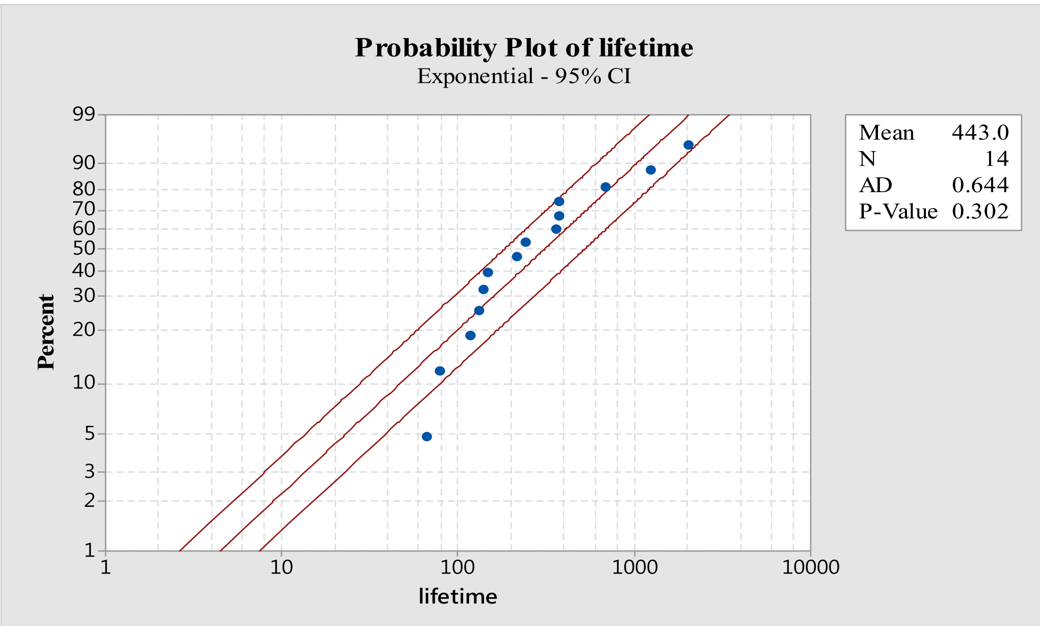

FIGURE 3. The probability plot for the failure times of transistors.

critical rejection value of the sampling plan as $(s,k_a,\ k_r) = (14, 0.99414, 0.99027)$ from Table 2. The test is terminated after fourteen failures with no replacement. Suppose now that the 14 failure times are 66.78, 79.15, 117.97, 131.61, 139.18, 147.06, 217.2, 241.98, 359.55, 371.79, 377.6, 691.7, 1228.12 and 2032.95. The probability plot of failure time shown in Figure 3 shows that an exponential distribution fits the data.

From the observations, the value of $\hat{C}_L$ can be calculated as

$$\hat{C}_L = 1 - \frac{(s-1)\,L}{\sum_{i=1}^{s} t_{(i)} + (n-s)\,t_{(s)}} = 1 - \frac{13 \times 200}{6202.64 + 16 \times 2032.95} = 0.932868.$$

Because the value of 0.932868 is smaller than the critical rejection value k_r =0.99027 significantly, this lot would be rejected.

B. CAPACITOR TEST WITH WEIBULL DISTRIBUTION

A capacitor is a two-terminal electrical component that stores potential energy in an electric field, which is widely used as parts of electrical circuits in many common electrical devices. Based on past experiences, the lifetime of a capacitor can be modelled by the Weibull distribution with the shape parameter $k = 2$. In the contract from the vendor and the buyer, the required quality levels (p_{AQL}, p_{LTPD}) and allowable risks $(\alpha,\ \beta)$ are set as (0.01, 0.02) and (0.01, 0.05) respectively for the specified type of capacitor with a lower limit $L = 300$. Now, life testing is implemented for a sample

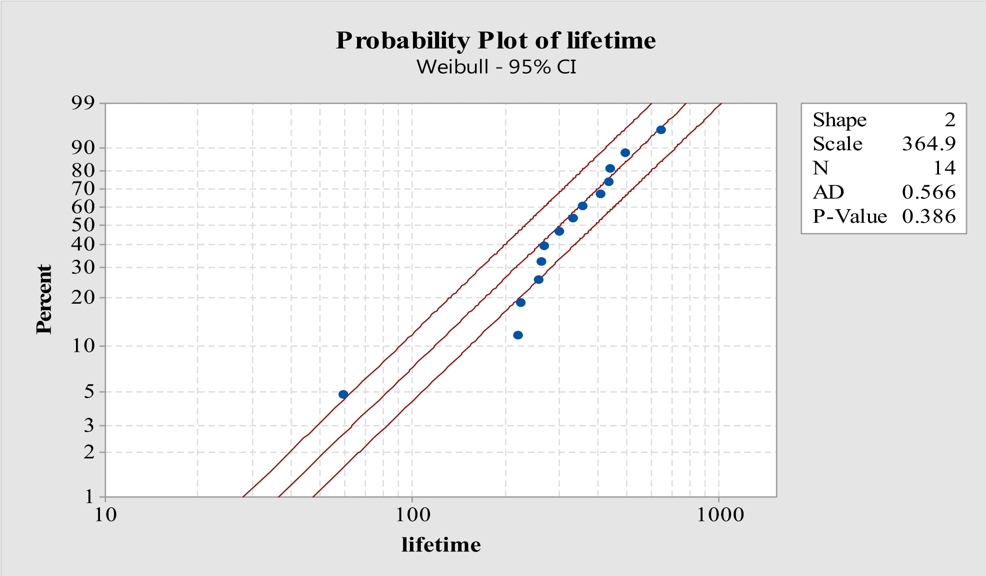

FIGURE 4. The probability plot for the failure times of capacitors.

of 30 capacitors. For the proposed sampling plan with m=1, the required number of failures, the critical acceptance value, and the critical rejection value of the proposed plan can be determined as $(s, k_a, k_r) = (14, 1.6805, 1.6092)$ from Table 4. The test is terminated after fourteen failures with no replacement. The failure times are 59.63, 220.78, 225.61, 257.13, 264.98, 268.97, 302.42, 332.62, 358.22, 408.87, 438.82, 443.03, 496.36 and 647.33. Figure 4 depicts the probability plot of failure times of capacitors, which shows that the Weibull distribution with shape parameter $k = 2$ is suitable for the data.

From the observations, the value of $\hat{C}_L$ can be calculated as

$$\hat{C}_L = \frac{1}{A}\left[\Gamma\left(1+\frac{1}{k}\right) - \frac{L\Gamma(s)}{D^{\frac{1}{k}}\Gamma\left(s-\frac{1}{k}\right)}\right]$$
$$= \frac{1}{0.463251}\left[\Gamma\left(1+\frac{1}{2}\right) - \frac{300 * \Gamma(14)}{8568291^{\frac{1}{2}} * \Gamma\left(14-\frac{1}{2}\right)}\right]$$
$$= 1.35327.$$

Because the value of 1.35327 is smaller than the critical rejection value $k_r = 1.6092$ significantly, this lot would be rejected.

VI. CONCLUSIONS

Lifetime is a very critical concern to customers. For many electronic products, life testing is required to be executed to assure the reliability of products before products are sent to customers. The life performance index C_L can assess the lifetime performance efficiently, which provides a one-to-one mathematical relationship for the conforming rate of products.

In this paper, we propose the MDSR lifetime testing plan based on C_L for exponential distribution data and Weibull distribution data with type II censoring. The plan parameters of the proposed method are tabulated for specified risk combinations with corresponding quality levels. In addition, we investigate the performance of the proposed sampling plan over that of the existing single lifetime testing plan based on C_L, which shows that the MDSR lifetime testing plan can reduce the sample size significantly as compared with the existing lifetime testing plan for all cases. For lifetime testing of experiments, the proposed plan can be recommended since it can save the cost of lifetime testing significantly. It is noted that the failed items are not permitted to be replaced while applying the proposed methodology. In the direction of future research, we can consider type I censoring test or other lifetime distribution functions to develop new lifetime testing plans for lot sentencing.

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MUHAMMAD ASLAM received the M.Sc. degree in statistics from GC University Lahore, in 2004, with the Chief Minister of the Punjab Merit Scholarship, the M.Phil. degree in statistics from GC University Lahore, in 2006, with the Governor of the Punjab Merit Scholarship, and the Ph.D. degree in statistics from the National College of Business Administration and Economics Lahore, in 2010, under the kind supervision of Prof. Dr. M. Ahmad.

He was a Lecturer of statistics with the Edge College System International, from 2003 to 2006. He was a Research Assistant with the Department of Statistics, GC University Lahore, from 2006 to 2008. Then, he joined the Forman Christian College University as a Lecturer, in 2009, where he was an Assistant Professor (2010–2012), and also an Associate Professor (2012–2014). He was an Associate Professor of statistics with the Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia, from 2014 to 2017. He taught summer course as a Visiting Faculty of statistics with Beijing Jiaotong University, China, in 2016. He is currently a Full Professor of statistics with the Department of Statistics, King Abdul-Aziz University. He has published more than 315 research papers in national and international well-reputed journals, including, for example, the IEEE Access, the *Journal of Applied Statistics*, the *European Journal of Operation Research*, the *Information Sciences*, the *Journal of Process Control*, the *Journal of the Operational Research Society*, the *Applied Mathematical Modeling*, the *International Journal of Fuzzy Systems*, *Symmetry*, the *International Journal of Advanced Manufacturer Technology, Communications in Statistics*, the *Journal of Testing and Evaluation*, and the *Pakistan Journal of Statistics*. His papers have been cited more than 2600 times with a h-index 28 and i-10 index 78 (Google Scholar). His papers have been cited more than 1200 times with h-index 21 (Web of Science Citations). He is the author of a book published in Germany. He has been the HEC approved Ph.D. supervisor, since 2011. He has supervised five Ph.D. theses, more than 25 M.Phil. theses, and three M.Sc. theses. He is currently supervising a Ph.D. thesis and more than five M.Phil. theses in statistics. He is a Reviewer of more than 50 well reputed international journals. He has reviewed more than 140 research papers for various well reputed international journals. His areas of interests include reliability, decision trees, industrial statistics, acceptance sampling, rank set sampling, neutrosophic statistics, and applied statistics. He introduced the area of Neutrosophic Statistical Quality Control (NSQC) for the first time. His is the Founder of neutrosophic inferential statistics (NIS) and NSQC. His contribution is the development of neutrosophic statistics theory for the inspection and process control. He originally developed theory in these areas under the Neutrosophic Statistics. He extended the Classical Statistics theory to Neutrosophic Statistics originally, in 2018.

Dr. Aslam is a member of the editorial board of the *Electronic Journal of Applied Statistical Analysis*, the *Asian Journal of Applied Science and Technology*, and the *Pakistan Journal of Commence and Social sciences*. He is also a member of the Islamic Countries Society of Statistical Sciences. He has appointed as an External Examiner for the 2016/2017–2018/2019 triennium at The University of Dodoma, Tanzania. He has received Meritorious Services Award in research from the National College of Business Administration and Economics Lahore, in 2011. He received the Research Productivity Award for the year 2012 by the Pakistan Council for Science and Technology. His name is listed at the 2nd Position among Statistician in the Directory of Productivity Scientists of Pakistan 2013, and is listed at the 1st Position among Statistician in the Directory of Productivity Scientists of Pakistan 2014. He got 371 positions in the list of top 2210 profiles of Scientist of Saudi Institutions 2016. He is selected for the Innovative Academic Research and Dedicated Faculty Award 2017 by SPE, Malaysia. He has received the King Abdulaziz University Excellence Awards in Scientific Research for the paper entitled ''Aslam, M., Azam, M., Khan, N., and Jun, C.-H. (2015). *A New Mixed Control Chart to Monitor the Process*; and the *International Journal of Production Research*, 53 (15), 4684-4693''. He received the King Abdulaziz University Citation Award for the paper entitled ''Azam, M., Aslam, M., and Jun, C.-H. (2015). *Designing of a Hybrid Exponentially Weighted Moving Average Control Chart Using Repetitive Sampling*; and the *International Journal of Advanced Manufacturing Technology*, 77:1927-1933, in 2018''.

CHING-HO YEN is currently a Professor with the Department of Industrial Engineering and Management Information, Huafan University, Taipei, Taiwan.

CHIA-HAO CHANG is currently a Professor with the Department of Nursing, Chang Gung University of Science and Technology, Chiayi Campus, Chiayi, Taiwan.

ALI HUSSEIN AL-MARSHADI is currently a Full Professor with the Department of Statistics, King Abdulaziz University. His fields of interests are simulation and applied statistics.

CHI-HYUCK JUN received the B.S. degree in mineral and petroleum engineering from Seoul National University, the M.S. degree in industrial engineering from the Korea Advanced Institute of Science and Technology, and the Ph.D. degree in operations research from the University of California at Berkeley. Since 1987, he has been with the Department of Industrial and Management Engineering, Pohang University of Science and Technology (POSTECH), where he is currently a Professor with the Department of Industrial and Management Engineering. He is interested in data mining and reliability/quality.