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A Decision Theory View of Auditing

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The major objective of the field of applied statistics is to help solve decision problems in the face of uncertainty. This help has traditionally been provided by making inferences based on a probability model. These probability models are the statistician's models of the uncertainty faced by a real world problem-solver. The field of auditing has been the beneficiary over the past ten to fifteen years of increasing assistance from the field of applied statistics. This paper will review these contributions and then consider a new contribution that is a logical next step.

Dealing with Uncertainty

The auditor is continually making choices in the face of uncertainty. The first statistical recognition of this fact occurred with the use of classical statistics in evaluating the results of random sampling.[1] The significance of this approach was not that uncertainty was first recognized, but that the risks associated with one particular aspect of auditing were made explicit. That is, the classical statement of confidence interval and level (e.g., $\pm$ 50 at 95% confidence) specifies the risk of sampling error.[2] Thus one element of the uncertainty faced by an auditor with which he has always had to treat was now disclosed in statistical terms. Given this beginning contribution, expansion of the potential uses of applied statistics to auditing, comparable to other disciplines facing uncertainty, should follow.

In using classical sampling, the contribution of statistics is restricted to the evaluation of evidence obtained by random sampling. Incorporation of this evidence with other evidence is left to the auditor's judgment. More recently a method for combining sample evidence with other auditing evidence has been proposed.[3] Inferential methods in Bayesian statistics are based on a posterior probability distribution which is a combination of a prior probability distribution, representing evidence the auditor has evaluated up to the point of sampling, and a likelihood function, representing the information in the sample. By subjectively specifying the results of evidence evaluated up to a point of time as a probability function, the auditor has expanded the explicit recognition of the uncertainty he faces in carrying out an audit. Again, this uncertainty previously existed but was considered only through intuition and judgment. The advantages for the auditor that result from being more precise in considering risk have been argued by Roberts.[4]

While classical sampling methods have met with some acceptance by the auditing profession, Bayesian sampling methods have not. One major reason for this lack of acceptance is the need for a practical method of expressing the prior probability distribution. While some research has been carried out, a confidence-inspiring method still awaits development.[5] Another source of resistance to Bayesian methods is the "subjective" nature of the prior distribution. The use of classical sampling has been "sold" to some members of the auditing profession on the basis that it is more objective. Since the result of the audit process is an opinion or judgment decision, over-stating applied statistics as a source of objectivity can be misleading. Statistical methods discussed in this paper can make the parameters or bases of judgment more explicit.[6] But even if these approaches are carried to their full extent, judgment wil be required as a critical input to the model. The prior probability distribution is an example of an input based on judgment.

Both classical and Bayesian methods discussed above are methods of inference. The next logical step in the use of applied statistics is to move from inference to action. An audit action or decision can be addressed by use of statistical decision theory. This methodology requires as an input a payoff function in addition to the requirements for inference. This payoff function is a specification of the consequences of each possible outcome of the audit to the auditor. The use of this method allows the auditor to maximize in the sense that he will make the decision that has the highest expected payoff.

In addition to the problems discussed above in applying Bayesian methods, the use of decision theory also requires an auditor to specify his payoff function. For each possible outcome of the audit he must specify the "value" (possibly in monetary terms) to him.[7] In the auditor's complex environment this specification of outcome consequences will be quite difficult. For example, consider that an outcome consequence to an auditor will probably represent a combination or matching of the form of his opinion and the discovery or lack thereof of a material error with the reaction of the firm (fee bargaining, lawsuits, future business), the reaction of users (lawsuits), the reactions of the regulators (right to practice, criminal prosecution), and the reaction of the rest of the auditor's environment (professional regulation, loss of other clients).

The remainder of this paper will illustrate the application of decision theory to a relatively constrained audit decision followed by a discussion of the problems involved in relaxing the constraints and the related need for research. Some discussion of the reasons for the author's bias that such inquiry is needed is incorporated in these comments.

A Decision Theory Model

Audit decision making can be described as a series of choices beginning with the acceptance of the client, followed by a series of choices as to type and quantity of evidence, and may conclude with the choice of opinion. The evolution of these choices is likely to be complex. For purposes of this discussion a single artificially isolated audit decision will be modeled.

Suppose an audit of a single balance, B, such that $\sum_{i=1}^{100} b_i = B$. The auditor's choice in this examination is to give either a clean opinion (a_1), or require an adjustment (a_2), to B. The existence (s_1), or absence (s_2), of a "material error" in B is the criterion which the auditor wishes to employ. This specification of possible actions, a_i, and states of the balance, s_i, provides the basis for the construction of a payoff table as follows:

	Error S_1	No error S_2
Clean opinion a_1		
Adjustment a_2		

In this payoff table the auditor will put the consequences or payoffs to him of each action-state combination. The objective at this point is to choose payoffs that, while arbitrary, have some intuitive appeal. The values in the following table represent dollars (in thousands).[8]

	S_1	S_2
Clean opinion a_1	-20	7
Adjustment a_2	3	-1

The $7,000 amount in the no error-clean opinion combination represents the fee net of ordinary expenses and is usually the most desired outcome. The no error adjustment combination is –$1,000 because it is assumed that the adjustment involves extra audit work for which the client will not pay. The $3,000 amount in the error adjustment combination represents extra work that in part is billed and collected from the client. The –$20,000 for the error clean opinion combination represents the impact of a settlement with the client (or a third party) to not pursue a suit for negligence.

The auditor plans to sample for evidence regarding the balance but before doing so, assesses his prior belief regarding the balance he is examining. Based on his knowledge of the client and of the system generating the balance, he states that S_1, a material error, has a .10 chance of existing and S_2 an absence of a material error, has a .90 chance of existing.

At this point the auditor could decide to not sample and simply make a choice based on his prior probability distribution and his payoff function as stated in the payoff table. (Such a decision might be correct in decision theory, but the auditor must also respond to professional conventions which will require at least some testing.) The criterion for choice is to select the action with the highest expected value. Using the auditor's prior probability distribution, these expected values are as follows:

$$E\,(a_1) = E\,(\text{clean opinion}) = .1\,(-20) + .9\,(7) = 4.3$$

$$E\,(a_2) = E\,(\text{adjustment}) = .1\,(3) + .9\,(-1) = -.6$$

The decision indicated at this point is a clean opinion.

The auditor's next step is to collect additional evidence and modify his prior distribution. In obtaining and using sample evidence in a decision theory framework some basic tools have been developed. The first is called the expected value of perfect information (EVPI). It indicates the upper limit on the value and thereby the amount that should be spent for additional information. The EVPI is computed by summing the values of the best action for each state, S_i, weighted by its probability of occurring and then deducting the value or payoff of the best decision (a clean opinion) under the prior distribution. The expected value of the best decision is $3(.1) + 7(.9) = 6.6$. Subtracting 4.3 results in an EVPI of 2.3. This indicates that no more than $2,300 should be spent on sampling.

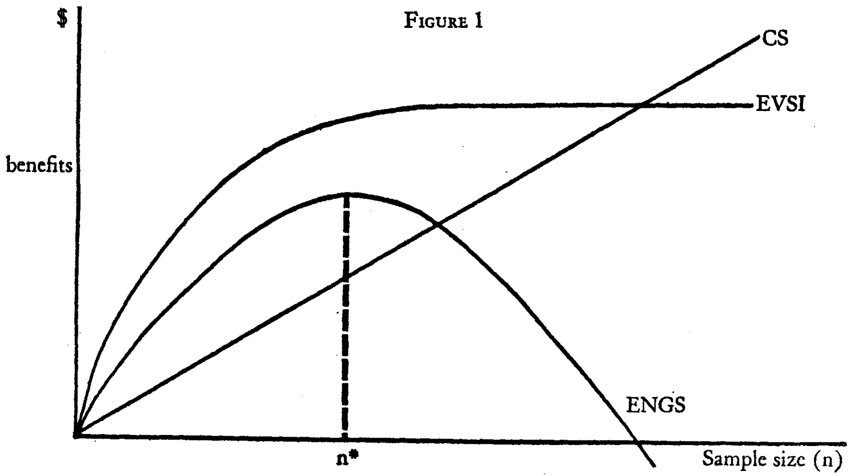
Perfect information is seldom available and, for this reason, the expected value of sample information (EVSI) is a useful number to the decision maker. To compute this value, the decision maker must have or assume some knowledge about the population from which he plans to sample. In this case we will continue making assumptions that keep the presentation and computations simple.

The computation of EVSI requires the use of Bayes' Model to combine the audit decision-maker's prior probability with each possible sample outcome, compute the expected values of each possible outcome, and then identify those sample outcomes that would indicate a change from the decision indicated by the auditor's prior distribution. The EVSI for a particular sample size is the sum of the expected values of all actions for all sample outcomes indicating a change in decision weighted by the probability of that sample outcome occurring. The Appendix summarizes the computations of EVSI for a sample of five from the b_i making up our balance, B. In this sample we have assumed that only two situations could exist in the balance B. Either a material error exists, defined as exactly 20 b_i's in error by their total amount, or there is no material error which is defined as exactly 5 b_i's in error. Sampling is defined to be with replacement to permit use of binomial tables. The computed EVSI is .443. If the cost of taking each sample item is twenty dollars (.02 in terms of the payoff matrix), the expected net gain of sampling (ENGS) to the auditor is EVSI less the cost of the sample or $.443 - 5(.02) = .343$. This value should be positive for a particular sample to be worthwhile. In this case the sample is worth $343 to the auditor in terms of his payoff table.

Given acceptable means of assessing prior beliefs and payoff functions, statistical decision theory presents auditors with an interesting and potentially desirable alternative. Using the expected net gain from sampling as a criterion, the auditor could compute the value of alternative sample sizes and choose a sample size that is optimal in terms of his payoff function. The cost of sampling can be expected to increase in an approximately linear fashion while the EVSI will tend to increase rapidly and then level out. Figure 1 approximates the effect of increasing sample size on ENGS. In this figure n^* would be the optimal sample size.

After the auditor chooses his sample size, he will take the sample and evaluate it. His final decision is based on a terminal posterior probability distribution based on *actual* rather than expected sample results. If in the above example a sample of five were taken with two errors (as described above)

FIGURE 1



located, a specific terminal posterior distribution can be obtained. When this posterior is combined with the loss function (table), either a_1 or a_2 will have a higher payoff indicating the appropriate action for the auditor to take. In the example the posterior distribution on the states would be:

$P(S_1) = .515$

$P(S_2) = .485$

as indicated by the Appendix. The expected payoff of the two actions would be:

a_1: $-20\,(.515) + 7\,(.485) = -6.905$

a_2: $3\,(.515) + -1\,(.485) = 1.060.$

The indicated action is to require an adjustment to B.

Extensions and Research

The simplifications made in the above illustration can be relaxed to develop a model more closely fitting actual audit decisions. Without actually constructing an example, a modification particularly appropriate for audit decisions will be proposed in the following paragraphs.

In comparing the above example to the auditor's decision environment, the first point that might occur to the experienced auditor is "if only the real world were so simple!" Instead of a single decision in isolation, the auditor in examining a set of financial statements must make a series of complex, interrelated decisions as to the type and quantity of evidence to collect and evaluate. To deal with this complexity, the profession has relied on good, "intuition based" judgment developed through training and experience. A less charitable observer might add that auditors may tend to over-rely on conventional practices to deal with this complexity. For example, it has been observed that some practitioners do too much cash work. This event might be a result of relying on convention rather than good judgment.

To deal with the auditor's decision environment, the decision theorist needs a structure or sequential model of the auditor's decision or judgment processes.

Such a model has not been clearly exposed in the literature, but SAP No. 54 (now Section 320 of Statement on Auditing Standards No. 1) does implicitly seem to include the framework of such a model. One possible view of that framework is as follows:

1. The auditor engages in a process of learning the client's operations, operating environment, accounting systems, and personnel. In a decision theory context he is collecting general evidence so that he can claim to be an expert with regard to the client and begin his examination with non-diffuse or concentrated prior probability distributions on each material element in the financial statements.
2. For each significant class of transactions the firm is likely to have a separate information subsystem providing the basis for one or more balances or parts of balances in the financial statements. For each such subsystem, for the system handling miscellaneous transactions, and for the system combining the results into financial statements, the auditor evaluates the internal control. In a decision theory context the auditor is assessing his belief to this point regarding the probability distribution on each accounting subsystem generating and not correcting a material error.
3. Using the prior distribution developed in (2) above, the auditor will plan, both as to type and scope, systems (compliance) tests and output (substantive) tests. In a decision theory context he is engaging in assessing the expected net gain from sampling, ENGS, for both (1) different types of tests and (2) different sample sizes (up to and including a census). This assessment requires the use of a payoff function and is based on the expected results of sampling as the above example indicates.
4. The execution of the plan established in (3) above will in essence be a series of Bayesian revisions of the auditor's subjunctive beliefs regarding the financial statements based on the actual results of sampling. At each major step in execution the auditor should revise his remaining plans based on the results of the preceding evidence. Each posterior distribution becomes a prior probability distribution for the next evidence collection activity. Note that at the conclusion of systems testing for all accounting subsystems, the auditor must combine the results of one *or more* systems to complete his prior assessment of balances. For example, the accounts receivable balance may be the result of an accounting subsystem for credit sales being combined with a cash collection subsystem. The posterior distributions for both systems should be combined for use as a prior distribution in testing the accounts receivable balance. In addition, the interrelationship of financial statement balances would have to be considered. The results of tests of sales and cash balances could influence the posterior distribution on accounts receivable.
5. Finally, the auditor reports his opinion on the financial statements choosing from among those opinions proscribed by his profession. In a decision theory context, this would be a final decision based on the payoff function and his confidence in the balances as expressed in his terminal posterior distribution on the balances.

While representing an untested suggestion, the above process clearly indicates that a modeling of this complex series of decisions is a challenging task.

In practice the computations and analysis suggested by this process would require computer algorithms.

In addition to the usual advantages of modeling judgment processes to gain insights for improvements and further productive research, decision theory seems to promise another possibility.[9] The auditor's current environment is litigation prone and many cases suggest that trouble for the auditor may have been the result of slow response to a changing environment. An auditor may be undesirably slow to change because of the "weight" of professional conventions. A decision theory approach to an audit may encourage and help justify change in the face of this pressure from conventional practices because it provides a means of comparing alternative sources of evidence in terms of criteria that should be convincing.

Additional benefits that a decision theory approach to auditing may provide are in the area of communication. In the application of the current intuition/judgment-based approach to scope and evidence source decisions, it is often difficult to articulate clearly the criteria used in making decisions. If decision theory could make these criteria more explicit, it is likely that the on-the-job training and supervision of inexperienced assistants could be facilitated. In addition, communication between experienced auditors is less likely to be garbled if it is based on explicit agreement on risk and payoffs. Another aspect of communication relates to the evaluation of our services by society. While certainly not a panacea, a decision theory approach may facilitate the documentation of decisions and criteria that will be more convincing and less "mystic" to outsiders (such as attorneys and regulators).

Concluding Observations

In concluding an exploratory discussion of an untested source of new techniques, it is appropriate to reinforce the problem areas that must be carefully researched before an evaluation of their usefulness can be made. There are at least three significant problems. The first is identification of the structure of the process discussed above. Second, as noted above, some research on assessing prior probability distributions has been published. But before such techniques can be considered practical for auditors, considerable additional effort in developing appropriate distributions and means of training professionals in their use is needed. Third, the payoff function (table) used above needs considerable expansion and testing on auditors before any use of decision theory can be seriously considered. Basic texts in decision theory do develop the continuous payoff and probability function relationship that could be appropriate for auditors. But they need testing and evaluation in the auditor's environment. Further, the use of monetary values in an auditor's payoff function does not seem reasonable.[10] Because of the extremely large amounts that a decision-state combination resulting in a lawsuit might involve and the nonmonetary, or at least indirect, effects on reputation, a utility-based payoff function seems more reasonable.

In summary, decision theory offers considerable promise. Its basic promise that decisions under uncertainty are best made based on a probabilistic collection and evaluation of sample evidence structured in terms of economic criteria (the expected payoffs) is appealing as a model for the audit process. Whether or not the application of decision theory to auditing will result in better audit decision-making, better communication between auditors and their public, and better communication between auditors can be answered only through research. The outlook is promising.

Footnotes

1. See, for example, Richard M. Cyert and H. Justin Davidson, *Statistical Sampling for Accounting Information,* Prentice-Hall, Inc., 1962, and *An Auditor's Approach to Statistical Sampling,* American Institute of Certified Public Accountants. (Five-volume Individual Study Program.)

2. Note that the risk discussed here is only for estimation or confidence interval purposes. Sampling risk in a hypothesis testing environment is specified by making explicit (or controlling) both the risk of rejecting an acceptable population and the risk of accepting an unacceptable population.

3. See, for example, William H. Kraft, Jr., "Statistical Sampling for Auditors: A New Look," *The Journal of Accountancy,* August 1968, and James E. Sorensen, "Bayesian Analysis in Auditing," *The Accounting Review,* January 1969.

4. Donald M. Roberts, "A Statistical Interpretation of SAP No. 54," *The Journal of Accountancy,* March 1974, pp. 47-53.

5. See John C. Corless, "Assessing Prior Distributions for Applying Bayesian Statistics in Auditing," *The Accounting Review,* July 1972, and Robert L. Winkler, "The Assessment of Prior Distributions in Bayesian Analysis," *The Journal of the American Statistical Association,* pp. 775-800.

6. See Kenneth A. Smith, "The Relationships of Internal Control Evaluation and Audit Sample Size," *The Accounting Review,* April 1972, pp. 260-269 for a discussion of this issue.

7. See, for a non-technical discussion of value in this model, Howard Raiffa, *Decision Analysis,* Addison-Wesley, 1968.

8. As is discussed briefly later in this paper, the use of monetary values in a payoff function represents an assumption. The payoffs in a payoff function should be values from a personal utility function. The use of utility gives recognition that dollars can have different worth to different people. Monetary values will be used in this example, but this aspect of payoff functions needs to be analyzed in the auditor's environment.

9. For examples of research in the use of other judgment-oriented disciplines see C. E. Gorry and G. O. Barnett, "Sequential Diagnosis by Computer," *Journal of the American Medical Association,* 1968, Vol. 205, and E. S. Epstein, "A Bayesian Approach to Decision Making in Applied Meteorology," *Journal of Applied Meteorology,* 1962, Vol. 1.

10. See Ward Edwards and Amos Treversky, *Decision Making,* Penguin Books, 1967, pp. 1-95 for a summary of the literature on the distinction between a utility-based and a monetary payoff function.

Appendix

The following tables show the computation of EVSI for a sample of 5 where the sampling distribution under S_1 is a binomial distribution with $p = .2$ and under S_2, $p = .05$.

(1) Sample results	(2) Prior	(3) Likelihood of Sample Result*	(4) Product of (2) · (3)	(5) Posterior
1. 0 error, 5 correct	error .1	.3277	.03277	.045
	correct .9	.7738	.69642	.955
			.72919	
2. 1 error, 4 correct	error .1	.4096	.04096	.183
	correct .9	.2036	.18324	.817
			.22420	
3. 2 error, 3 correct	error .1	.2048	.02048	.515
	correct .9	.0214	.01926	.485
			.03974	
4. 3 error, 2 correct	error .1	.0512	.00512	.838
	correct .9	.0011	.00099	.162
			.00611	
5. 4 error, 1 correct	error .1	.0064	.00064	1.000
	correct .9	.0000	.00000	.000
			.00064	
6. 5 error, 0 correct	error .1	.0003	.00003	1.000
	correct .9	.0000	.00000	.000
			.00003	

Sample Outcome	Action	Expected Payoff	Change in Decision?	Value of Sample Info
1	a_1	$-20(.045) + 7(.955) = 5.785$		
	a_2	$3(.045) + (-1)(.955) = -.82$	no	0
2	a_1	$-20(.183) + 7(.817) = 2.059$		
	a_2	$3(.183) + (-1)(.817) = -.268$	no	0
3	a_1	$-20(.515 + 7(.485) = -6.905$		$1.060 - (-6.905) = 7.965$
	a_2	$3(.515) + (-1)(.485) = 1.060$	yes	
4	a_1	$-20(.838) + 7(.162) = -15.626$	yes	$2.352 - (-15.626) = 17.978$
	a_2	$3(.838) + (-1)(.162) = 2.352$		
5	a_1	$-20(1) + 7(0) = -20$	yes	$3 - (-20) = 23$
	a_2	$3(1) + (-1)(0) = 3$		
6	a_1	$-20(1) + 7(0) = -20$	yes	$3 - (-20) = 23$
	a_2	$3(1) + (-1)(0) = 3$		

$EVSI = 7.965(.03974) + 17.978(.00611) + 23(.00064) + 23(.00003) = .443$

* The likelihood of the sample result is the probability of the sample result occurring given that the sample was from state S_1, where the error rate is .2 or state S_2 where the error rate is .05. The probabilities are from a binomial table.