1-1-1988

AUDITOR'S ASSISTANT: A knowledge engineering tool for audit decisions

Glenn Shafer

Prakash P. Shenoy

Rajendra P. Srivastava

4

AUDITOR'S ASSISTANT: A Knowledge Engineering Tool For Audit Decisions*

Glenn Shafer,
Prakash P. Shenoy,
Rajendra P. Srivastava
University of Kansas

1. Introduction

In recent years, there has been significant interest in developing expert systems for assistance in audit decisions [see e.g, Boritz and Wensley, 1988; Chandler, 1985; Hansen and Messier, 1986a, and 1986b; Leslie *et al.*, 1986]. It is believed that use of such systems will facilitate audit decisions and make audits more efficient and effective. This appears to be the reason that major accounting firms are committing increasingly greater resources to developing such systems [see e.g., Boritz and Brown, 1986; Kelly, 1987; Shpilberg and Graham, 1986].

Most of the expert systems being developed are rule-based. While such systems have many attractive features such as modularity of knowledge-base, ease of updating knowledge-base, etc., they are not well-suited for coherent reasoning under uncertainty. This is because in rule-based systems, the user has no control over the chain of inference whereas, coherent reasoning under uncertainty requires controlled firing of rules [Shafer, 1987]. Because of this difficulty, some developers of expert systems have avoided dealing with uncertainties altogether [Kelly *et al.*, 1986]. In domains where uncertain reasoning is unavoidable, heuristic approaches have been attempted with little success [Shortliffe and Buchanan, 1975; Duda *et al.*, 1976]. In recent years, considerable theoretical work has been done on the subject of coherent uncertain inference using Bayesian probabilities and belief-functions [see e.g., Pearl, 1986; Kong, 1986; Shenoy and Shafer, 1986; Mellouli, 1987; Shafer, Shenoy and Mellouli, 1987; Lauritzen and Spiegelhalter, 1988; Shafer and Shenoy, 1988]. The expert system described in this article represents one of the first practical applications of these new techniques.

The purpose of this paper is to describe an interactive tool called AUDITOR'S ASSISTANT (AA). The system, when fully developed, should

* This research has been supported in part by grants from the Peat Marwick Foundation, the National Science Foundation grant No. IST-8610293 and General Research Fund of the University of Kansas. The authors are grateful for discussions and assistance with programming from Yen-Teh Hsia, Debra Zarley and Ragu Srinivasan.

enable its users (auditors) to construct a network of variables and evidence. The system will automatically aggregate all evidence that is entered and display the resulting beliefs in all variables in the network. The system will have the capability of using both the Bayesian and the belief-function formalisms for managing uncertainties. It will provide a graphic interface for constructing a network of variables and evidence and it will automatically revise beliefs in all variables as new pieces of evidence are entered.

This paper is divided into four sections. Section 2 provides a detailed discussion of AUDITOR'S ASSISTANT. Section 3 discusses an example demonstrating the process of constructing a network of variables and evidence and aggregation of evidence using the belief-function calculus. The final section summarizes the results. A brief introduction to the theory of belief-functions is given in Appendix A.

2. Auditor's Assistant

AUDITOR'S ASSISTANT is an interactive system for assisting auditors in making audit decisions. AA's theoretical foundation is based on coherent management of uncertain inference. With this system, an auditor can graphically create a network of variables and evidence, input judgments about the degree of support provided by a piece of evidence to the variable it is linked to, and evaluate the resulting total belief in all variables in the network. An auditor can also use the system to decide which procedure or test to perform next and also to decide when sufficient evidence has been obtained to issue an opinion.

In auditing the financial statements of a firm, there are two major conceptual tasks. First, an *argument* needs to be constructed. This is the process of organizing different pieces of evidence and the variables which they support. One formal result of this process is a network of variables and evidence. We shall refer to this network as a *design* [see Shafer and Cohen, 1987]. The process of constructing a design cannot be easily automated. It has to be done by a human expert, i.e., an experienced auditor. However, we can assist the auditor in this process by providing examples in the form of templates and by checking certain technical conditions, e.g., the Markov property [Shafer, Shenoy and Mellouli, 1987; Shafer and Shenoy, 1988], that have to be satisfied.

Second, once an argument is in place, evidence has to be collected, judgments about the degree of support provided by such evidence to variables have to be made, and these judgments have to be aggregated and evaluated for all variables in the tree. The collection of evidence and judgments of degree of support are tasks that have to be done by the auditor. However, the aggregation and evaluation of evidence can be automated.

The process of collecting evidence, making judgments, and aggregating judgments is iterative. Items of evidence are evaluated as they are collected, and this evaluation influences what evidence is collected next. The decisions about what evidence to seek next is one aspect of *control* [Cohen, 1987; Shafer and Cohen, 1987]. Again, this is not easy to automate. The experienced auditor makes these decisions. However, an interactive system should assist the auditor in these decisions in two ways. First, the system should automatically aggregate evidence as it is obtained and entered into the system, and the system should display the net effect of all evidence on all variables in the network. Second, the system should allow a what-if analysis by allowing its user to enter a hypothetical piece of evidence and displaying its effect on all the variables. The user should then be able to retract this hypothetical evidence.

In general, as discussed in the professional standards [AICPA, 1987] and also in the academic literature [see, e.g., Graham 1985a-1985e], auditors gather three types of evidence. One type comes from reviews of the external and internal environments in which the business is operating. External environments include economic, social and political environments. Internal environments include management integrity, quality of management, structure of management, and the general business awareness of the management. A second type deals with the strength of internal accounting controls. A strong set of internal accounting controls may mean more reliable accounting data and, therefore, less need for substantive tests. The third type comes from performing substantive tests to determine directly whether account balances are fairly stated in accordance with generally accepted accounting principles. Such tests include analytical review procedures and direct tests of balances such as confirmations of receivables from customers.

There are several formalisms to aggregate uncertain evidence, including the Bayesian probability calculus [Pearl, 1986; Shenoy and Shafer, 1986; Lauritzen and Spiegelhalter, 1988; Shafer and Shenoy, 1988] and Shafer's theory of belief-functions [Shafer, 1976; Shenoy and Shafer, 1986; Kong, 1986; Shafer, Shenoy and Mellouli, 1987; Mellouli, 1987]. These calculi differ in their need for structure, inputs, flexibility and computational complexity. The Bayesian probability calculus demands structure in the form of conditional independence, and it demands numerous inputs in the form of priors and conditional probabilities, but it is relatively efficient computationally. The belief-function calculus offers more flexibility and demands less inputs, but it can be computationally more intensive than the Bayesian calculus.

AUDITOR'S ASSISTANT uses the belief-function calculus to represent and aggregate evidence. Shafer and Srivastava [1989] have demonstrated the importance and relevance of belief-functions for audit decisions based on the structure of audit evidence. Since the belief-function calculus reduces to the Bayesian calculus when all inputs demanded by the Bayesian calculus are available, AA can also work with probabilities.

Once a network is in place, the auditor conducts procedures and, on the basis of the results, he or she provides numerical degrees of support for the variable the evidence is linked to. Then, AA aggregates the evidence and maintains a display of the degrees of support provided by all evidence collected so far to all variables in the network.

As it exists today, AA allows an auditor to construct only a tree of variables and evidence. No loops are allowed. However, AA is currently being updated to include arbitrary networks. The user creates the tree visually and interactively using a mouse as an input device. The nodes of the tree represent variables and the links between nodes represent relations between variables. The user has many options for manipulating the tree on the screen: moving a node by dragging it, collapsing a sub-tree into a node, etc.

3. An Example

In this section we will describe the use of AA in a simple audit engagement.[1]

Suppose ABC Hardware Co. is a small wholesale distributor of hardware located in the Midwest. Most of ABC's customers are retail hardware stores. Srifer & Co. has been asked to perform an annual audit of ABC's financial statements.

3.1. Constructing a Network of Variables and Evidence

Srifer & Co. has audited ABC Hardware's financial statements for the last four years. After reviewing the previous years' working papers and understanding the client's business environment, the audit team (consisting of a senior, manager and partner) constructs a network of variables and evidence related to accounts receivables (AR) and allowance for bad debts (ABD). For simplicity of exposition, we assume that the audit team has decided not to depend on the internal accounting controls in the sales and collection cycle. Thus, the audit team will depend only on the environmental factors, analytical review results, and some direct tests of balances. This network is shown in Figure 1.

The rounded rectangular nodes represent *variables* that are of interest to the auditor. For example, the main variable in Figure 1 is whether net accounts receivable is fairly stated. Associated with each variable is a collection of mutually exclusive and collectively exhaustive *values*. For example, the values associated with the net accounts receivable variable are nar (denoting that net accounts receivable is fairly stated) and ~nar (denoting that net accounts receivable is not fairly stated). All variables in Figure 1 are binary-valued. A brief description of each variable is indicated inside the node.

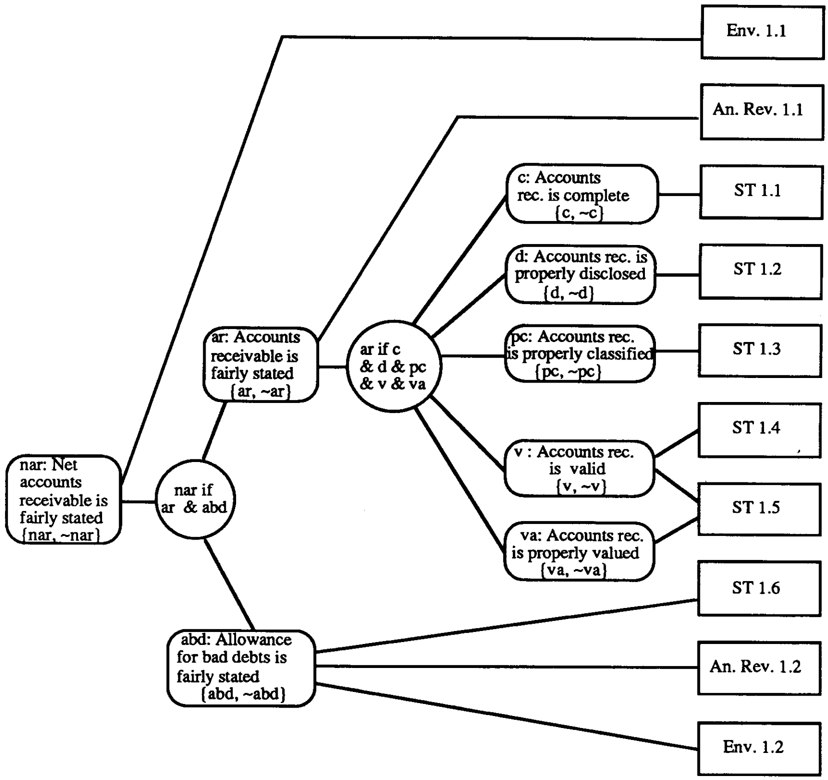
The circular nodes represent *relations* between the variables they are linked to. For example, net accounts receivable is fairly stated if and only if both accounts receivable and allowance for bad debts are fairly stated. Also, accounts receivable is fairly stated if and only the following objectives have been met: *completeness, ownership, adequate disclosure, proper classification, validity* and *valuation* (see, e.g., Arens and Loebbecke [1988], for further discussion of these objectives). Formally, a relation is modeled as a belief-function. For example, the relation between net accounts receivable, accounts receivable and allowance for bad debts can be represented in terms of a basic probability assignment function m as follows (see Appendix A for a definition of m):

$$m(\{(\text{nar},\text{ar},\text{abd}), (\sim\text{nar},\text{ar},\sim\text{abd}),(\sim\text{nar},\sim\text{ar},\text{abd}),(\sim\text{nar},\sim\text{ar},\sim\text{abd})\}) = 1.$$

The rectangular nodes represent *evidence.* A description of the procedures and tests leading to the evidence shown in Figure 1 is given in Table 1. The links between evidence nodes and variable nodes indicate that the evidence provides some support for the variables it is linked to. For example, in Figure 1

[1] The main purpose of this example is to illustrate the use of AA in planning and evaluation decisions. The numerical inputs used in the example are purely for illustration purposes.

Figure 1. A network of variables and evidence for ABC Hardware.



evidence Env. 1.1 provides support directly to the net accounts receivable variable.

Formally, each piece of evidence is modeled as a belief-function on the set of possible values of the variables it is linked to. For example, if the outcome of the Env. 1.1 procedure results in a 60% degree of support for nar, then this piece of evidence is represented in the system as follows:

$$m(\{nar\}) = 0.60, \ m(\{nar, \ \sim nar\}) = 0.40.$$

Functions of this type (where a certain degree is committed to one value of a variable and the rest is uncommitted) are called *simple support functions*. We expect most of the evidence to be of this type. Thus, in order to make a judgment about a piece of evidence, an auditor needs to decide whether the evidence supports the affirmative or negative value of a variable, and the degree (a number between 0 and 1) to which it does so.

At the outset of the engagement (before any tests or procedures have been performed), a network of variables and evidence, such as the one shown in Figure 1, serves as a plan for performing the audit. Before a procedure is

TABLE 1. Description of procedures and tests leading to evidence shown in Figure 1.

An. Rev. 1.1	Review AR journal for unusual items and compare individual customer balances over a stated amount with previous years.
An. Rev. 1.2	a. Compare allowance for bad debt as a percentage of accounts receivable with previous years. b. Compare number of days accounts receivable outstanding with previous years. c. Compare bad debt expense as a percentage of gross sales with previous years.
Env. 1.1	Review the competence and trustworthiness of the accounting personnel working in sales transactions.
Env. 1.2	Review management's credit policy.
ST 1.1	Trace a sample of accounts from the subsidiary ledger to the aged trial balance.
ST 1.2	Review the minutes of the board of directors' meetings for any pledged or factored accounts receivable. Also inquire of management whether any receivables are pledged or factored.
ST 1.3	Review the receivables listed on the aged trial balance for notes and related party receivables.
ST 1.4	Trace a sample of accounts from the trial balance to the related subsidiary ledger.
ST 1.5	Confirm accounts receivables from customers.
ST 1.6	Discuss with credit manager the likelihood of collecting older accounts over 120 days and evaluate whether the receivables are collectible.

performed, it is represented in the system as a *vacuous* belief-function (see Appendix A for the definition of a *vacuous* belief-function). Propagating all these belief-functions results in zero belief for each value for all variables in the network. In other words, before collecting any evidence, the auditor is completely ignorant about whether the financial statements are fairly presented or contain a material error. However, once a test is performed, the auditor makes a numerical judgment about the degree of support provided by the test to the variable the evidence is linked to in the network. After this is entered into the system, the system propagates the evidence to all variables in the network and the revised beliefs for all variables are then displayed.

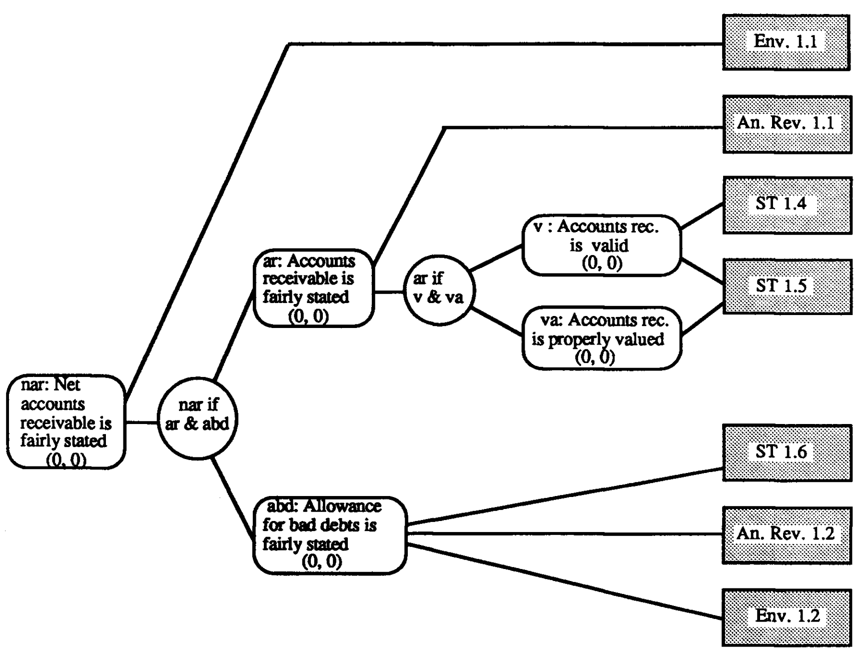
At any stage of the audit, the auditor has to decide which procedures he or she is going to perform next. Of course, at any stage of the audit, depending on the results of the tests already conducted, an auditor may decide that certain procedures are unnecessary. On the other hand, an auditor may need to change his or her plan to include more tests because the tests planned for do not provide the necessary evidence to issue an opinion.

3.2. Planning and Aggregation of Evidence

To illustrate the planning of the audit and the aggregation of evidence, we will further simplify the example. Assume that the audit team has concluded that the objectives of *completeness, ownership, adequate disclosure,* and *proper classification* have been met without any reservations. The objectives yet to be verified are *validity* and *valuation* for AR. The network relevant to this situation is shown in Figure 2. The rectangular nodes are shown with a dotted fill in Figure 2 to indicate that none of these procedures has been performed yet. Since no procedures have been performed yet, no support is available to any of the values of the variables as shown in Figure 2. For each variable in the network, there are two numbers shown inside the rectangular box at the bottom. The first of these two numbers indicates the total belief for the affirmative value of the variable. For example, for the NAR variable, Bel({nar}) = 0 in Figure 2. The second number indicates the total belief for the negative value of the variable. For example, for the NAR variable, Bel({ ~ nar}) = 0 in Figure 2.

We will consider two different scenarios and the resulting evaluations about the fairness of NAR.

Figure 2. The simplified network of Figure 1.

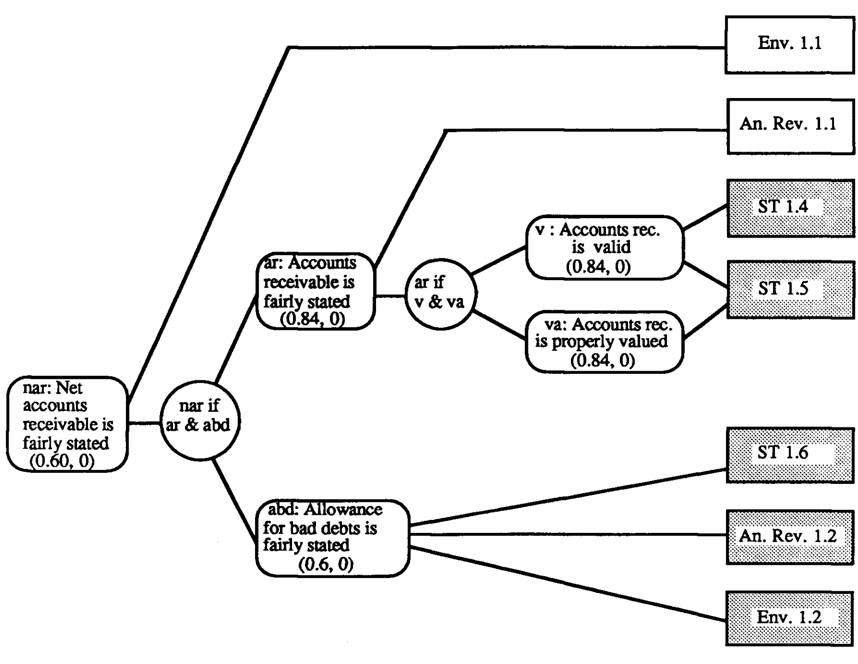


3.2.1. Scenario One

Suppose that the audit team finds the management and accounting personnel to be competent and trustworthy (Env. 1.1). The audit team decides that this evidence supports nar to degree 0.60. Also, the results of the analytical review procedures (An. Rev. 1.1) show no unusual items and no apparent problems in AR balance. The team makes a judgment that this supports 'ar' to degree 0.60. These judgments are propagated through the network resulting in the beliefs shown in Figure 3. Notice that there is now an overall support of 0.60 for the assertion that NAR is fairly presented and no support for the assertion that NAR is materially misstated (i.e., $\mathrm{Bel}(\{\mathrm{nar}\}) = 0.60$, $\mathrm{Bel}(\{\sim\mathrm{nar}\}) = 0$). Although there is no support for the assertion that NAR is materially misstated, there is a maximum 40% risk based on the two pieces of evidence that NAR could be materially misstated (i.e., $\mathrm{Pl}(\{\sim\mathrm{nar}\}) = 0.40$ where Pl is a plausibility function related to the belief-function Bel by the relation $\mathrm{Pl}(\{\sim\mathrm{nar}\}) = 1 - \mathrm{Bel}(\{\mathrm{nar}\})$.

Let us assume that the audit team plans to conduct the audit so that they obtain at least 90% overall support for nar, i.e., targeted $\mathrm{Bel}(\{\mathrm{nar}\})$ is 0.90. Note that the evidence from An. Rev. 1.1 provides no support yet to nar since no support for abd has yet been obtained from procedures, ST 1.6, An. Rev. 1.2, and Env. 1.2 (remember that NAR is fairly stated only when AR *and* ABD

Figure 3. The network of variables after performing Env. 1.1 and An. Rev. 1.1 in Scenario One.

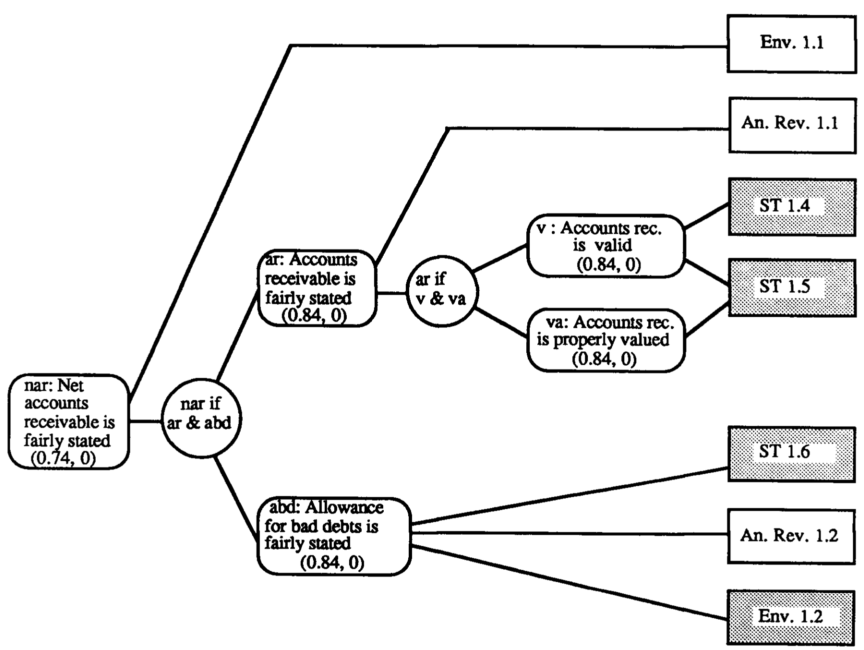


are fairly stated). It should be noted that ar and the two objectives of ar (validity and valuation) in Figure 4 have 84% support from Env. 1.1 and An. Rev. 1.1. The 0.60 support for nar is entirely due to Env. 1.1. Therefore, the team decides to perform analytical review procedures for allowance for bad debts (An. Rev. 1.2) next.

Suppose they find that the allowance is reasonable given the accounts receivable balance. Also, certain ratio analyses suggest that ABD is fairly presented. The team makes a conservative judgment that a 60% degree of support is obtained from this evidence for abd. The resulting network is shown in Figure 4. Thus, propagating the three judgments through the network results in an overall support for nar of 0.74 and no support for ~nar (i.e., Bel({nar}) = 0.74, Bel({~nar}) = 0).

Next, since not enough support is available yet for nar, the team decides to perform substantive test procedures for validity and valuation of AR. (Of course, the team recognizes that certain substantive test procedures are required by the AICPA. For example, confirmations of AR from the customers is a requirement [AICPA, 1987, AU331]). The extent of testing would depend on the level of support desired by the team. Let us assume that they plan on achieving 80% support for validity of AR by tracing a sample of accounts from the aged trial balance to the related subsidiary ledger. The senior performs the

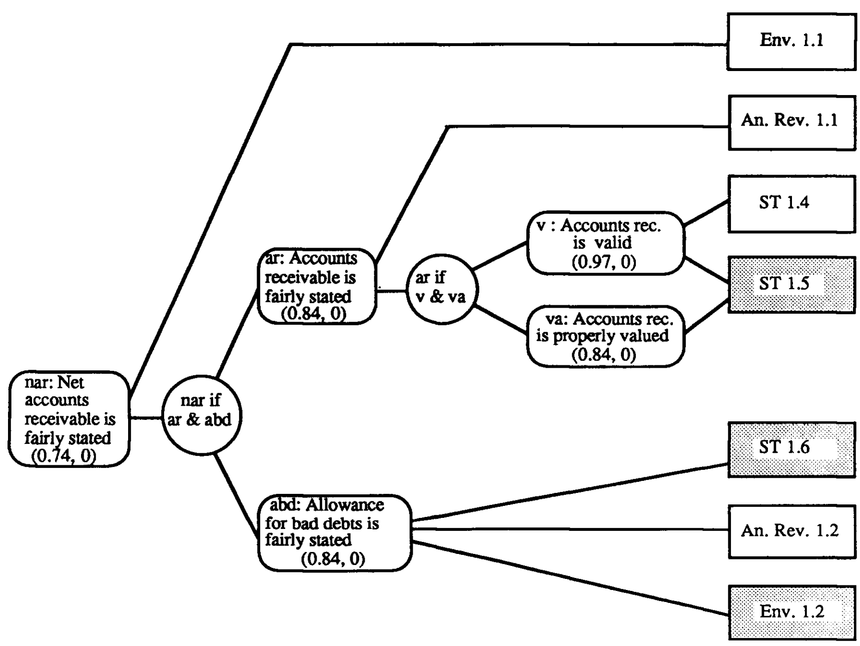
Figure 4. The network after performing Env. 1.1, An. Rev. 1.1 and An. Rev. 1.2 in Scenario One.



test and finds no exceptions. The team makes a judgment that an 80% degree of support is obtained by the evidence for the validity objective. This evidence is entered into the system and the resulting network is shown in Figure 5. The overall support for nar is still 0.74. The reason for no change in the overall support for nar is that the evidence from ST 1.4 supports only the validity objective. There is no direct support yet for the valuation objective. Since both objectives have to be met for AR to be fairly presented, ST 1.4 provides no support by itself to the fair presentation of AR. However, the level of support shown in Figure 5 represents the overall support when all the items of evidence have been aggregated.

As discussed earlier, AUDITOR'S ASSISTANT would have the capability of performing a what-if analysis for deciding the nature, timing, and extent of tests. In principle, the auditor can assume a certain level of support that he or she plans to obtain from a test procedure and see its impact on the overall support for the main assertion of interest. Of course, the decision about what test to perform next, and the extent of the test, depends on the auditor. The cost of performing a test has to be balanced with the level of support desired. Usually, analytical review procedures do not provide a high level of support unless the test involves statistical analyses. Similarly, making inquiries of the

Figure 5. The network after performing Env. 1.1, An. Rev. 1.1, An. Rev. 1.2 and ST 1.4 in Scenario One.

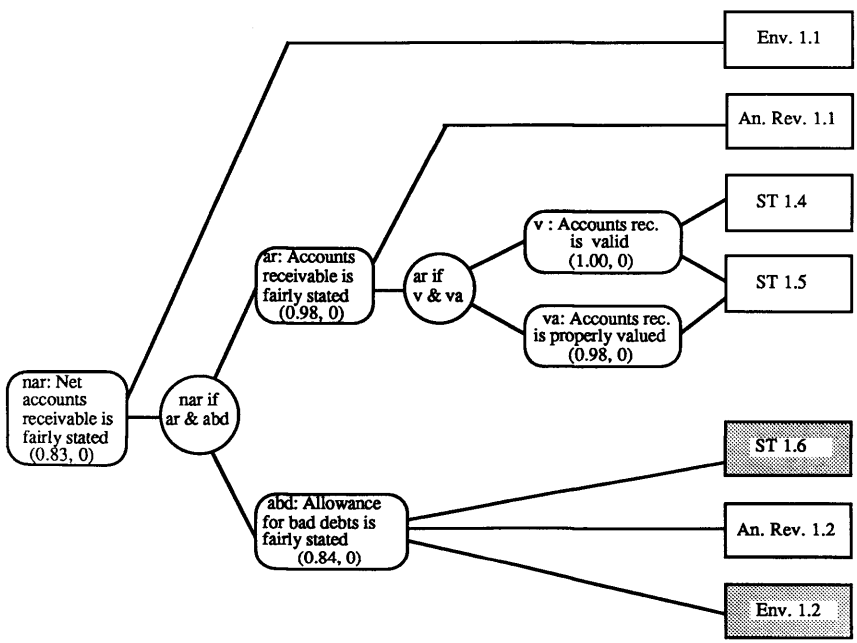


client provides a lower level of support. However, confirmation from third parties is considered to be reliable and it provides a higher level of support.

Now suppose that the team decides to send a sample of positive confirmations to the client's customers in order to achieve a 90% degree of support for the validity of AR. The confirmation test also provides support for the valuation objective to a great extent because the customer usually checks the account balance for accuracy. The audit staff analyzes the returned confirmations and finds no exceptions. The team, having reviewed the staff's work, makes a judgment that the confirmation test provides a 90% degree of support for the validity objective and an 85% degree of support for the valuation objective. For simplicity of exposition, we will assume that the above two judgments are independent. The resulting beliefs of all variables are shown in Figure 6. The overall support for nar is now 0.83 and there is still no support for ~nar (i.e., Bel({nar}) = 0.83, Bel({~nar}) = 0).

Since the overall support for nar is still below the target level of 0.90, the team plans to perform some further tests. Since the support for ar is already quite high (Bel({ar}) = 0.98), they conclude that there is no need for further evidence that supports ar. However, support for abd is still low (Bel({abd}) = 0.84). Thus, they decide to meet with the credit manager to discuss whether the firm has any collectibility problems with their accounts (ST

Figure 6. The network after performing Env. 1.1, An. Rev. 1.1, An. Rev. 1.2, ST 1.4, and ST 1.5 in Scenario One.



1.6). They find that there is no account that is more than 120 days overdue. Furthermore, all accounts seem to be quite good. The team makes a judgment that this evidence supports abd to degree 0.60. The resulting beliefs for the variables are shown in Figure 7. The overall support for nar is now 0.92 and there is no evidence to support ~nar (i.e., Bel({nar}) = 0.92, Bel({~nar}) = 0).

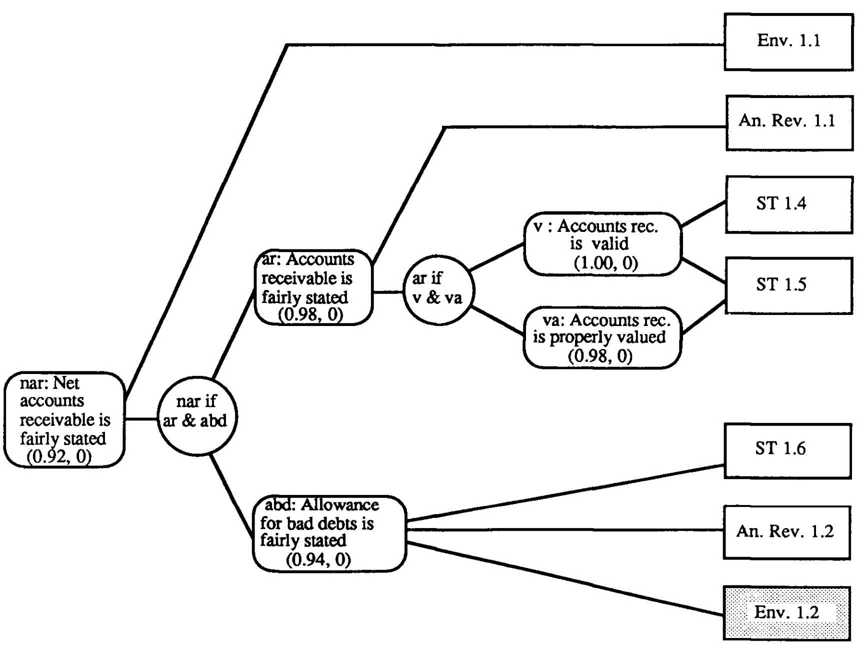
At this stage, the audit team decides to conclude the audit since they have sufficient evidence to issue an opinion about the fairness of NAR. The audit team also knows that given the evidence, the maximum risk that NAR is materially misstated is only 8%.

Although the audit team had initially planned to review ABC's credit policy (Env. 1.2), they do not perform this test since, on the basis of tests already conducted, they have a sufficiently high belief that NAR is fairly stated. Without a formal analysis of the type shown above, perhaps an audit team may end up doing more tests than necessary. AUDITOR'S ASSISTANT, when fully developed, should provide assistance to auditors in deciding when sufficient evidence has been collected to issue an opinion.

3.2.2. Scenario Two

In this case, assume that the results of Env. 1.1, An. Rev. 1.1, ST 1.4, and

Figure 7. The network after performing Env. 1.1, An. Rev. 1.1, An. Rev. 1.2, ST 1.4, ST 1.5, and ST 1.6 in Scenario One.



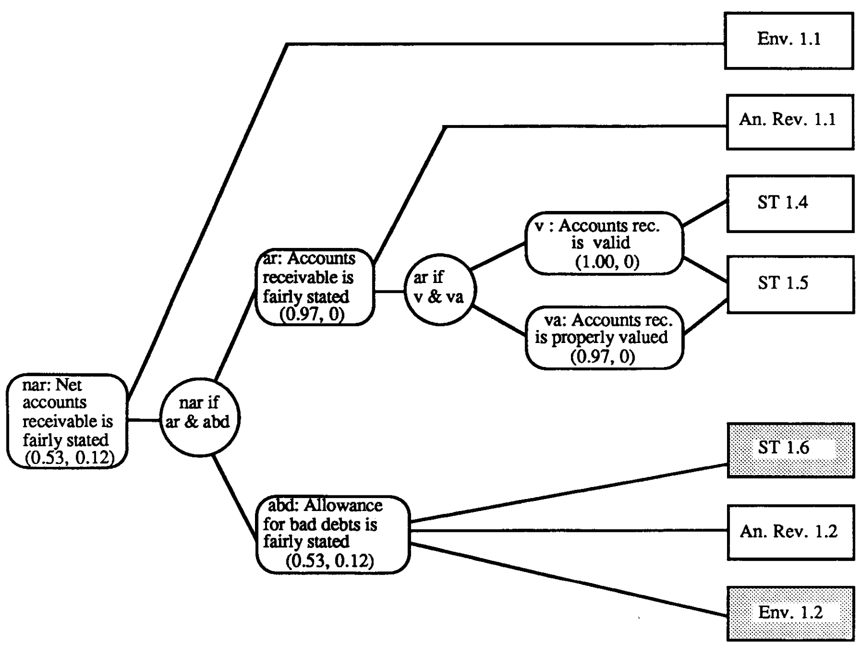
ST 1.5 are the same as in Scenario One. The results of An. Rev. 1.2 and ST 1.6 are different from the ones described above.

Suppose that the analytical review procedure An. Rev. 1.2 performed by the senior has revealed that the allowance for bad debts may be understated in relation to this year's accounts receivable balance. Also the AR balance has increased significantly compared to the credit sales, implying that a more liberal credit policy has been adopted this year, compared to the past. Furthermore, the collection of receivables is slow. Based on this evidence, the audit team makes a judgment that ABD is understated to degree 0.25. The aggregate beliefs in all variables are now shown in Figure 8. The overall beliefs in nar and ~nar are 0.53 and 0.12, respectively. The maximum risk of NAR being materially misstated is 0.47 (i.e., $Pl(\{\sim nar\}) = 1 - Bel(\{nar\}) = 0.47$).

The audit team now decides to review the client's credit policy (Env. 1.2). The senior performs the review and finds that this year, the client has been quite liberal in granting credit. He attributes the increase in AR balance this year to the firm's liberal credit policy. The team makes a judgment that the evidence supports ~abd to degree 0.40.

The senior also meets with the credit manger to discuss the firm's credit policy (ST 1.6). The credit manager agrees with the senior's assessment that allowance for bad debts may be understated. Based on this evidence, the audit

Figure 8. The network after performing Env. 1.1, An. Rev. 1.1, ST 1.4, ST 1.5, and An. Rev. 1.2 in Scenario Two.

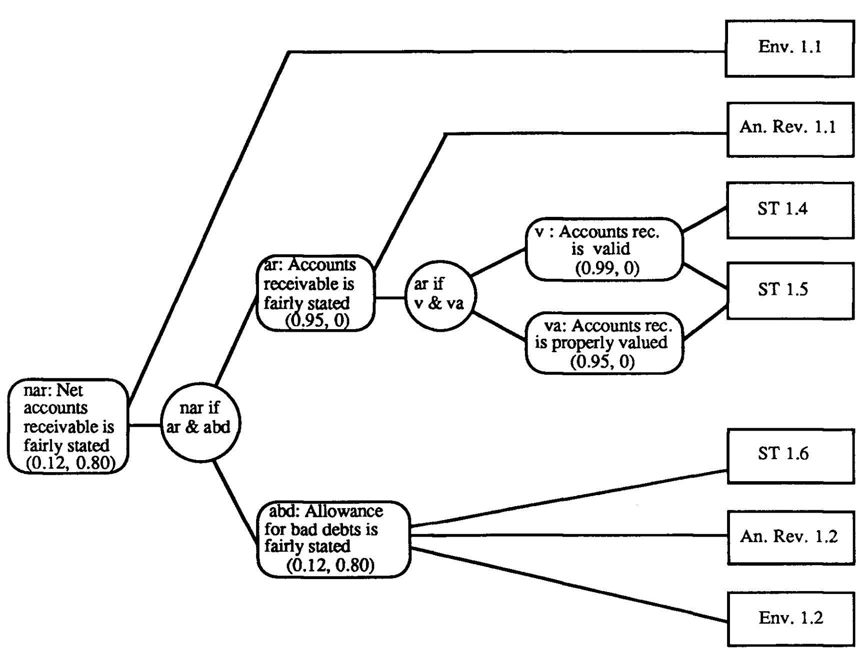


team makes a judgment that supports ~abd to degree 0.80. This judgment, when combined with the previous findings, yields an overall support of 0.80 for ~abd (see Figure 9). Therefore, the audit team decides at this point to propose an adjustment for ABD. No adjustment need be proposed for AR since the overall support for ar is 0.95, which is above their target level.

4. Summary

AUDITOR'S ASSISTANT is not a rule-based system. The knowledge-base of AA is a network of variables and evidence. Since each auditing engagement is unique, a network of variables and evidence has to be constructed by the user. There are several ways in which the system assists the user with this task. First, the graphics user interface of AA is designed to make the task of constructing a network as easy and intuitive as possible. Second, the user does not have to start from scratch. Instead, (s)he can start with a template and modify it to fit the engagement at hand. The system automatically handles technical aspects of network construction such as ensuring that the network satisfies the Markov property. Also, the system (when fully developed) should automatically reduce a non-tree network to a tree by clustering variables and using the resulting clustered tree to propagate the evidence. At this time, the

Figure 9. The network after performing Env. 1.1, An. Rev. 1.1, ST 1.4, ST 1.5, An. Rev. 1.2, Env. 1.2 and ST 1.6 in Scenario Two.



system is only capable of propagating belief-functions in networks that are already trees.

The user can use the network of variables and evidence as a planning device. At each stage, AA will display the beliefs for each variable in the network as a function of the evidence that has been collected and entered into the system. At each stage, the user needs to decide what test or procedure to perform next. AA can assist in this decision by performing a what-if analysis and indicating the degree of belief provided to the main variable of interest as a function of the test results. The auditor can then choose between different tests and sample sizes based on cost of test and increase in degree of belief for the main variable of interest.

When there is sufficient belief for the main variable of interest, the auditor can issue an appropriate opinion.

In summary, it is useful to think of AUDITOR'S ASSISTANT as a *knowledge engineering* tool instead of as an expert system. Coherent reasoning under uncertainty requires construction of an argument. Once an argument is in place, aggregation of evidence is easily automated.

References

American Institute of Certified Public Accountants, *AICPA Professional Standards,* Vol. A (Commerce Clearing House, 1987).

Arens, A. A., and J. K. Loebbecke, *Auditing: An Integrated Approach,* 4th Edition (Prentice Hall, 1988).

Boritz, J. E., and D. A. Brown, "The Expert-Ease at Expert Systems," *CA MAGAZINE* (August 1986), pp. 50-57.

Boritz, J. E., and A. K. P. Wensley, "Structuring the Assessment of Audit Evidence—An Expert Systems Approach," presented at the 1987 University of Waterloo Auditing Symposium, also to appear in the *Auditing: A Journal of Practice and Theory* (1989).

Chandler, J. S., "Expert Systems in Auditing: The State of the Art," *The Auditor's Report,* Vol. 8, No. 3 (Summer 1985), pp. 1-4.

Cohen, P. R., "The Control of Reasoning under Uncertainty: A Discussion of Some Programs," *The Knowledge Engineering Review,* Vol. 2 (March 1987), pp. 5-25.

Duda, R. O., P. E. Hart, and N. J. Nilsson, "Subjective Bayesian Methods for Rule-Based Inference Systems," *AFIPS Conference Proceedings,* Vol. 45 (1976), pp. 1075-1082.

Graham, L. E., "Audit risk—Part I," *The CPA Journal* (August 1985a), pp. 12-21.

Graham, L. E., "Audit risk—Part II," *The CPA Journal* (September 1985b), pp. 34-40.

Graham, L. E., "Audit risk—Part III," *The CPA Journal* (October 1985c), pp. 36-43.

Graham, L. E., "Audit risk—Part IV," *The CPA Journal* (November 1985d), pp. 38-45.

Graham, L. E., "Audit risk—Part V," *The CPA Journal* (December 1985e), pp. 26-33.

Hansen, J. V., and W. F. Messier, "Expert Systems for Decision Support in EDP Auditing," *International Journal of Computer and Information Sciences,* Vol. 11, No. 5 (1982), pp. 357-379.

Hansen, J. V., and W. F. Messier, "A Preliminary Investigation of EDP-XPERT," *Auditing: A Journal of Practice and Theory,* Vol. 6, No. 1 (1986a), pp. 109-123.

Hansen, J. V., and W. F. Messier, "A Knowledge Based Expert System for Auditing Advanced Computer Systems," *European Journal of Operations Research* (September 1986b), pp. 371-379.

Kelly, P. K., "Modeling the Audit Planning Process," *Expert Systems Review For Business and Accounting* (September-October 1987), pp. 3-8.

Kelly, K. P., G. S. Ribar, and J. J. Willingham, "Interim Report on the Development of an Expert System for the Auditor's Loan Loss Evaluation," in *Proceedings of the 1986 Touche Ross/ University of Kansas Symposium on Auditing Problems* (1986), pp. 167-188.

Kong, A., "Multivariate Belief Functions and Graphical Models," Doctoral dissertation, Department of Statistics, Harvard University (1986).

Lauritzen, S. L., and D. J. Spiegelhalter, "Local Computations with Probabilities on Graphical

Structures and Their Application to Expert Systems," *Journal of the Royal Statistical Society,* Vol. 50(B) (1988), pp. 157-224.
Leslie D. A., S. J. Aldersley, D. J. Cockburn, and C. J. Reiter, "An Assertion Based Approach to Auditing," in *Proceedings of the 1986 Touche Ross/University of Kansas Symposium on Auditing Problems* (1986), pp. 31-68.
Mellouli, K., "On the Propagation of Beliefs in Networks Using the Dempster-Shafer Theory of Evidence," Doctoral dissertation, School of Business, University of Kansas (1987).
Pearl, J., "Fusion, Propagation, and Structuring in Belief Networks," *Artificial Intelligence,* Vol. 29 (1986), pp. 241-288.
Shafer, G., *A Mathematical Theory of Evidence,* Princeton University Press (1976).
Shafer, G., "Probability Judgment in Artificial Intelligence and Expert Systems," *Statistical Science,* Vol. 2, No. 3 (1987), pp. 3-44.
Shafer, G., and P. Cohen, "Management of Uncertainty in Expert Systems," Tutorial offered at the *Sixth National Conference on Artificial Intelligence,* AAAI-87, Seattle, WA (1987).
Shafer, G., P. P. Shenoy, and K. Mellouli, "Propagating Belief Functions in Qualitative Markov Trees," *International Journal of Approximate Reasoning,* Vol. 1 (1987), pp. 349-400.
Shafer, G., and P. P. Shenoy, "Local Computation in Hypertrees," School of Business Working Paper No. 201, University of Kansas (1988).
Shafer, G., and R. P. Srivastava, "The Bayesian and Belief Function Formalisms I: A General Perspective for Auditing," presented at the 1987 University of Waterloo Auditing Symposium, also to appear in *Auditing: A Journal of Practice and Theory* (1989).
Shenoy, P. P., and G. Shafer, "Propagating Belief Functions with Local Propagation," *IEEE Expert,* Vol. 1 (1986), pp. 43-52.
Shpilberg, D., and L. Graham, "Developing ExperTAX℠: An Expert System for Corporate Tax Accrual and Planning," *Auditing: A Journal of Practice and Theory* (Fall 1986), pp. 75-94.
Shortliffe, E. H., and B. G. Buchanan, "A Model of Inexact Reasoning in Medicine," *Mathematical Biosciences,* Vol. 23 (1975), pp. 351-379.

Appendix A

A Primer on The Theory of Belief Functions

Here we shall present the basics of the theory of belief-functions. See Shafer [1976] for details.

Let X denote a variable with possible values $x_1, \ldots, x_n$. We shall refer to the set of all possible values of a variable (exactly one of which is true) as a *frame of discernment*. A *basic probability assignment* (bpa) function on a frame Θ is a function $m: 2^\Theta \rightarrow [0,1]$ such that

$$m(A) \geqslant 0 \text{ for all } A \epsilon 2^\Theta,\ m(\phi) = 0, \text{ and } \Sigma\{m(A) | A \epsilon 2^\Theta\} = 1$$

Intuitively, m(A) represents the degree of belief assigned exactly to A (the proposition that the true value of X is in the set A). A basic probability assignment function corresponds to a probability mass function in Bayesian probability theory. Whereas a probability mass function is restricted to assigning probability masses only to singleton values of variables, a bpa function is allowed to assign masses to sets of values without assigning any mass to the individual values contained in the sets. For example, if we have absolutely no knowledge about the true value of a variable, we can represent this situation by a bpa function as follows:

$$m(\Theta) = 1,\ m(A) = 0 \text{ for all other } A \epsilon 2^\Theta.$$

Such a function is called a *vacuous* bpa function. Note that in Bayesian probability theory, the only way to express total ignorance is to assign a mass of 1/n to each value where n is the total number of possible values. Thus, in Bayesian probability theory we are unable to distinguish between equally likely values and total ignorance. The theory of belief-functions offers a richer semantics.

Associated with a bpa function are two related functions called belief and plausibility. A *belief-function* is a function $Bel: 2^{\Theta} \rightarrow [0,1]$ such that

$$Bel(A) = \Sigma\{m(B) | B \subseteq A\}.$$

Whereas m(A) represented the belief assigned exactly to A, Bel(A) represents the total belief assigned to A. Note that $Bel(\phi) = 0$ and $Bel(\Theta) = 1$ for any bpa function. For the vacuous bpa function m, the corresponding belief-function Bel is given by

$$Bel(\Theta) = 1, \text{ and } Bel(A) = 0 \text{ for all other } A \epsilon 2^{\Theta}.$$

A *plausibility function* is a function $Pl: 2^{\Theta} \rightarrow [0,1]$ such that

$$Pl(A) = \Sigma\{m(B) | B \cap A \neq \phi\}$$

Pl(A) represents the total degree of belief that could be assigned to A. Note that $Pl(A) = 1 - Bel(\sim A)$ where $\sim A$ represents the complement of A in Θ, i.e., $\sim A = \Theta - A$. Also note that $Pl(A) \geqslant Bel(A)$. For the vacuous bpa function, the corresponding plausibility function is

$$Pl(\phi) = 0, \text{ and } Pl(A) = 1 \text{ for all } A \epsilon \Theta.$$

If a bpa function m is also a probability mass function (i.e., all the probability masses are assigned only to singleton subsets), then $Bel(A) = Pl(A) = \Sigma\{m(\{x_i\} | x_i \epsilon A\}$ = probability of proposition A.

If m_1 and m_2 are bpa functions representing two independent pieces of evidence, then we can combine them using *Dempster's rule of combination* and obtain a new bpa function, denoted by $m_1 \oplus m_2$, representing the aggregated evidence as follows:

$$m_1 \oplus m_2(A) = K^{-1} \Sigma\{m_1(B_1) m_2(B_2) | B_1 \cap B_2 = A\} \text{ if } A \neq \phi, \text{ and } m_1 \oplus m_2(\phi) = 0$$

where $K = 1 - \Sigma\{m_1(B_1) m_2(B_2) | B_1 \cap B_2 = \phi\}$. The above definition assumes that $K \neq 0$. If $K = 0$, then the two pieces of evidence contradict each other completely, and it is not possible to combine such evidence.

Let us illustrate Dempster's rule of combination by means of two examples.

Example 1

Suppose that the variable under consideration is the validity of accounts receivable with frame $\{v, \sim v\}$. The results of substantive test 1.4 lead to the bpa function m_1 as follows:

$$m_1(\{v\}) = .8, \; m_1(\{v, \sim v\}) = .2$$

Furthermore, results of substantive test 1.5 lead to the bpa function m_2 as follows:

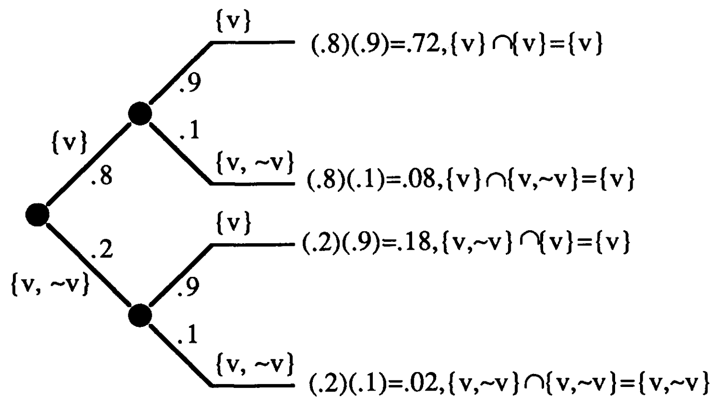
$$m_2(\{v\}) = 0.9, \; m_2(\{v, \sim v\}) = .1$$

Combining m_1 and m_2 by Dempster's rule leads to the bpa function $m_1 \oplus m_2$ as follows:

$$m_1 \oplus m_2(\{v\}) = .72 + .08 + .18 = .98,$$
$$m_1 \oplus m_2(\{v, \sim v\}) = .02.$$

The details of Dempster's rule are shown in Figure 10. In this example, there is no conflict between the two pieces of evidence, i.e., $K = 1$.

Figure 10. Dempster's rule for Example 1



Example 2

Suppose that the variable under consideration is the fairness of allowance for bad debts with frame $\{abd, \sim abd\}$. The results of an analytical review test lead to a bpa function m_1 as follows.

$$m_1(\{abd\}) = 0.8,\ m_1(\{abd,\ \sim abd\}) = 0.2$$

However, an environmental review uncovers the fact that one of the client's major customers has filed for Chapter 11 and may not be in a position to pay its bills. Let us represent this evidence as follows:

$$m_2(\{\sim abd\}) = 0.1,\ m_2(\{abd,\ \sim abd\}) = 0.9$$

Combining these two pieces of evidence leads to the aggregated bpa function:

$$m_1 \oplus m_2(\{abd\}) = .72/0.92 = .78$$
$$m_1 \oplus m_2(\{\sim abd\}) = .02/0.92 = .02$$
$$m_1 \oplus m_2(\{abd,\ \sim abd\}) = .18/0.92 = .20$$

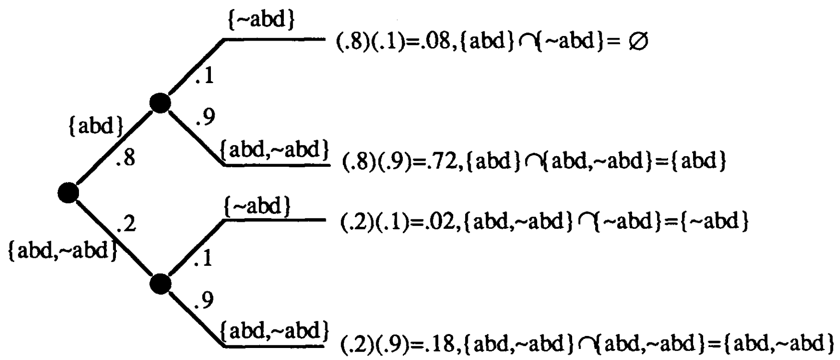
The details of Dempster's rule are shown in Figure 11. Note that in this case the evidence is conflicting $(K = 1 - .08 = 0.92)$ and so we end up renormalizing the bpa function so that the values add to 1.

In general, Dempster's rule of combination has the following properties:

(i) Commutativity: $m_1 \oplus m_2 = m_2 \oplus m_1$
(ii) Associativity: $(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)$
(iii) In general, $m_1 \oplus m_1 \neq m_1$. The bpa $m_1 \oplus m_1$ will favor the same subsets as m_1, but it will do so with twice the weight of evidence, as it were.
(iv) If m_1 is vacuous, then $m_1 \oplus m_2 = m_2$.

In Bayesian probability theory, evidence is aggregated using Bayes's rule. It is easy to show that Bayes's rule is a special case of Dempster's rule of combination.

Figure 11. Dempster's rule for Example 2



In general, Dempster rule of combination has the following properties:

(i) Commutativity: $m_1 \oplus m_2 = m_2 \oplus m_1$

(ii) Associativity: $(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)$

(iii) In general, $m_1 \oplus m_1 \neq m_1$. The bpa $m_1 \oplus m_1$ will favor the same subsets as m_1, but it will do so with twice the weight of evidence, as it were.

(iv) If m_1 is vacuous, then $m_1 \oplus m_2 = m_2$.

In Bayesian probability theory, evidence is aggregated using Bayes's rule. It is easy to show that Bayes's rule is a special case of Dempster's rule of combination.