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Aspects and Abstracta

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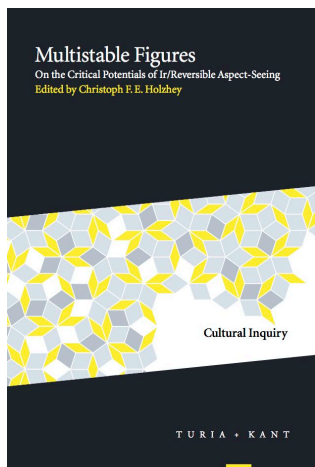
Multistable Figures: On the Critical Potential of Ir/Reversible Aspect-Seeing, ed. by Christoph F. E. Holzhey, Cultural Inquiry, 8 (Vienna: Turia + Kant, 2014), pp. 41–66

ASPECTS AND ABSTRACTA

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Philosophers of perception and psychologists first studied 'multistable' or 'reversible' figures, *Kippbilder*, in the nineteenth century. The earliest description of the phenomenon of a 'sudden and involuntary change in the apparent position' of a represented object occurred in a letter written by Louis Albert Necker in Geneva to Sir David Brewster on 24 May 1832 and published six months later in the *Philosophical Magazine*.[2] The picture in question would subsequently become known as the Necker cube (Fig. 1).
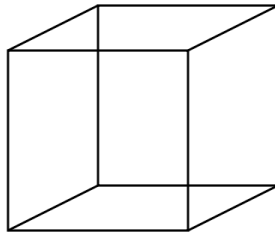

Fig. 1. Necker cube.

Necker's original remark was only a passing observation; the systematic investigation of multistability began in the second half of the nineteenth century. Franz Brentano, Alexius Meinong, and their students discussed multistable figures as part of their general study of optical illusions.[3] The most important philosophical treatment of multistable figures, however, appeared in 1953 in Wittgenstein's *Philosophical Investigations*.[4] Wittgenstein's chief example, due to the late nineteenth-century psychologist Joseph Jastrow, is the 'duck-rabbit' figure, 'which can be seen as a rabbit's head or as a duck's'.[5]

It is this 'seeing-as' structure that is central to Wittgenstein's discussion of the *Kippbild*. Seeing-as, for Wittgenstein, is not reducible to mere seeing:

> If I saw the duck-rabbit as a rabbit, then I saw: these shapes and colours (I give them in detail) – and I saw besides something like this: and here I point to a number of different pictures of rabbits. – This shews the difference between the concepts. 'Seeing as …' is not part of perception. And for that reason it is like seeing and again not like.[6]

But what it is in seeing-as that exceeds bare seeing, that does not form 'part of perception' in the same manner as merely receiving impressions of spatial relations among parts of the visual field, is unclear. The difference seems, to a first approximation, to lie in something like cognitive access to a special property of the perceived object – the property, in the case of the duck-rabbit seen qua rabbit, of *representing a rabbit*, a property rendered manifest to the perceiver by the image's resemblance to other rabbit-images.[7]

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This article takes its inspiration – and, to a certain extent, its form – from Wittgenstein's discussion. It consists of a set of observations about the 'seeing-as' structure, followed by sketches for a tentative research programme for investigating it.

Unlike Wittgenstein, however, I do not hold that the ultimate result of any investigation of our talk of seeing something as something must be 'to *accept* […] the everyday language game', if by this it is meant that we should abandon the attempt to describe the metaphysical structure of the world, the phenomenology of our mental attitudes, or the logical form of the propositions that lie at their interface.[8] Instead, I maintain that there is a general theory of seeing-as ascriptions to be discovered, and that it has an important, unexpected consequence: the structure '*S* sees *a* as *F*' generalizes beyond cases of concrete perception. There is a more comprehensive, unified class of intentional attitudes described by sentences of the form '*S* ϕ-s *a* as *F*', where substituends for ϕ are what I shall term *cognitive access verbs*: not only verbs of concrete perception – 'see', 'hear', 'touch', and the like – but verbs such as 'think about', 'cognize', and 'contemplate'.[9] To account fully for these intentional attitudes would far exceed the scope of this paper. My task here is merely to make a case for the existence of such attitudes by focusing on one particularly interesting class of examples: attitudes of contemplation of abstract objects under an aspect.

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For the purposes of this paper, I assume that the general contours of the abstract/concrete distinction can be made clear by example. Tables, elementary particles, minds, a particular red sense-datum, individual cognitive processes, space–time regions, a particular copy of *Madame Bovary*, a particular utterance of the English sentence 'What time is it?' – these are concrete. The number two, the empty set, redness, the property of being a space–time region, *Madame Bovary*, the English sentence 'What time is it?' – these are abstract.[10] Note that 'abstractness', in the sense at issue here, is in no way related to any psychological process: abstract objects are not concepts produced in the mind by 'generalizing' from concreta; they are not *concepts* at all. (There are such things as concepts of the number two and of *Madame Bovary*, but here we are not concerned with them; we are concerned instead with the number two and with *Madame Bovary*.) Some abstracta are *particulars*: √3, the empty set, *Jacob's Room*. Some are *universals*: greenness, the property of being a triangle, and so on. The mathematical abstracta that we shall consider are particulars – numbers, sets, and so on.

But, in addition to the abstracta discussed in mathematics and general metaphysics, there are *artefactual* abstracta: objects ontologically dependent on human activities and thus (in some sense) temporal, but nonetheless causally inert and nonspatial.[11] Among these are literary and musical works. At first, it may seem strange to speak of (some) artworks as abstract; however, as I shall argue, this position makes the best sense of their properties: literary and musical works are abstract *types* of which specific copies and specific performances, respectively, are concrete *tokens*.[12] We nonetheless normally gain cognitive access to these non-physical types through the corresponding tokens. Moreover, that access is usually aspect-relative.

The structure of this article is as follows: §3 discusses multistable figures in general; §4 offers a few observations about concrete instances of seeing-as; §5 gives examples of the aspect-relative cognition of mathematical objects; §6 gives analogous examples for artworks; and §7 offers a tentative conclusion and a programme for further research. An Appendix includes a few notes about the realist perspective towards mathematical and artefactual abstracta assumed in the paper. The discussions in §§5–6 and the Appendix are largely independent of each other; readers interested only in the mathematical or the aesthetic examples should feel free to skip the other sections, although – as I argue at

the end of §7 – there may be insights to be gained from considering the two types of cases together.

<h1 style="text-align:center">3</h1>

Nowhere in the *Philosophical Investigations* does Wittgenstein attempt to offer an analysis of what it is to be a multistable figure. This is not surprising, since his interest is in more general phenomena; the ultimate purchase of the notion lies in its connection to the theory of meaning, as the example of 'aspect-blindness' makes clear.[13] Moreover, Wittgenstein passes quite freely between what we may term 'strict' visual seeing-as, which occurs in cases of two-dimensional multistable images, and the broader phenomenon that takes place, for example, when a geometrical figure is used in a book to illustrate various physical objects of a common shape.[14] He seems to hold that these cases have a shared general structure, which is tied to the notion of an interpretation ('But how is it possible to *see* an object according to an interpretation?'[15]), although the border between interpreting and seeing is not easy to draw.[16]

I shall follow him in this regard: on my view, 'I see the picture as an image of a duck' not only displays the same structure as 'I see the tin rectangle as a street sign', but the same kind of intentional attitude is denoted by the 'see *a* as *F*' verb phrase in each case. Nonetheless, it is worthwhile to set out the core components of the paradigm cases – literal, two-dimensional visual *Kippbilder*. It will then be possible to determine to what extent these features carry over to perceptual structures – and non-perceptual cognitive structures – that are relevantly similar to the central cases. I propose that the following conditions are *sufficient* for something to be a multistable figure. I leave aside the question of whether they are necessary; at the very least, it seems that giving necessary conditions would require treating cases of multistable figures with more than two configurations. Nonetheless, these conditions pick out at least one paradigmatic class of *Kippbilder*.

A two-dimensional figure *F* is multistable if

(a) in every standard perceptual condition, the figure represents only one thing;

(b) there is at least one standard perceptual condition under which the figure represents something with property *F* (we shall call any such condition an *F*-condition);

(c) there is at least one standard perceptual condition under which the figure represents something with property *G* (we shall call any such condition a *G*-condition);

(d) there is no standard perceptual condition under which the figure represents something with both properties *F* and *G*;

(e) there are perceptual experiences of the figure that involve instantaneous or near-instantaneous change from an *F*-condition to a *G*-condition or vice versa; and

(f) there are no perceptual experiences of the figure that involve a gradual or prolonged change from an *F*-condition to a *G*-condition or vice versa.

These criteria make use of the terms 'perceptual experience', (visual) 'representation', 'perceptual condition', and 'representation under a perceptual condition'.

Perceptual experiences are simply the occurrent experiences that one undergoes when one is perceiving; *visual* perceptual experiences, more specifically, are the occurrent experiences that one undergoes when one is seeing. Identity of perceptual content is necessary for identity of perceptual experiences (whether it is sufficient need not concern us). I shall also occasionally speak of a 'perceptual state': a perceiver's perceptual state at time *t* is individuated by the totality of perceptual experiences that he or she is undergoing at *t*. (I do not take a position here on the issue of *externalism* about mental content – the question of whether facts about extramental entities are ineliminably involved in the contents of perception.)

Although the notion of visual representation is far from unproblematic, we can provisionally separate the difficulties involved in the question of representation from the problem of multistability. The basic notion, in the case of two-dimensional figures, can be made clear by examples: a painting of Mont St Michel represents Mont St Michel, an appropriately shaded coloured circle represents a sphere, and so on. (In the circumscribed sense of 'represent' at issue here, a painting of Mont St Michel does not represent the power of the mediaeval Church, or the dogma of the Assumption, or the painter's personal attachment to Normandy, although it might evoke, convey, or allegorize these things.) What is crucial is that a figure can represent something with properties different from those of the figure itself: a drawing of the Taj Mahal represents something that is not flat, even though the drawing itself is flat.

Representation is fundamentally a property of objects, not of perceivers (although, to be sure, it is a property that objects have in virtue of their dispositions to induce certain perceptual experiences). More generally, we can speak of an object's representing something *for an observer* (or *for an observer at a time*). I shall suggest in §4 that such cases are at least sufficient, if not necessary, for seeing-as: if *a* represents an *F* for *S*, then *S* sees *a* as *F*.

A perceptual condition is a general set-up, containing a perceiver, a perceived object, and a set of surrounding circumstances, under which the object is disposed to produce particular perceptual experiences in the perceiver. Specifically, a system consisting of object *O*, environment *S*, and perceiver *P* is in perceptual condition *K* if and only if *O* is disposed, when in the conditions specified by *S*, to cause *P* to have perceptual experiences of kind *K*.[17] By 'perceptual condition', I mean a *type* of perceptual condition, which is correlated one-to-many to distinct triples of object, environment, and perceiver; for instance, a single drawing of the duck-rabbit image displays *duck-manifesting* and *rabbit-manifesting* perceptual conditions, each of which can be entered into by many different perceivers under different visual background circumstances.

In the cases we shall be interested in, the perceptual experiences in question are experiences in which a two-dimensional figure is perceived to represent something; in such cases, we say that the figure *represents* something (for an observer in a set-up) under a specified perceptual condition. More generally, *a* represents an *F* under perceptual condition *K* just in case, whenever *a* forms part of a system <*a, S, P*> that is in perceptual condition *K*, *a* is disposed to represent an *F* for *S*. A perceptual condition is 'standard', in the visual case, when it involves ordinary light, when the observer faces the observed object directly, and so on. There can be many different standard perceptual conditions for one object: my proposal is intended to allow that there can be ways of perceiving a figure that not everyone sees; a figure can be multistable even if some people see only one of its aspects. Perceptual conditions are time-relative: it is only at a particular time that a perceiver is in one perceptual condition or another, and for a perceiver's perceptual experience to change from an *F*-condition to a *G*-condition is simply for the perceiver to be in an *F*-condition at some time t_0 and to be in a *G*-condition at some time $t_1 > t_0$.

It is clear that conditions (a)–(f) divide into two groups: (a)–(d) relate to representational structure; (e)–(f) relate to aspect change over

time. We could certainly imagine aspect-relative attitudes that do not occur one at a time or do not shift; the objects of these attitudes would not be multistable figures, but they would nonetheless be objects that are encountered aspect-relatively. For example, it is by no means impossible to hear a musical work under different aspects simultaneously (although change in aspect can also occur with musical perception).

My focus in the remainder of the paper will be solely on the *structural* aspects of multistability and its analogues, rather than on the phenomenon of aspect shift. To the extent that my argument extends the notion of the literal multistable figure, it does so in a way that preserves the aspect-relativity of the core cases rather than their temporal structure.

It might be objected that aspect-relativity cannot be studied independently from aspect change. As Wittgenstein repeatedly points out, it is normally only through aspect change that one comes to understand that what one is seeing is aspect-relative at all:

> I may, then, have seen the duck-rabbit simply as a picture-rabbit from the first. That is to say, if asked, 'What's that?' or 'What do you see here?' I should have replied: 'A picture-rabbit'. [...] I should not have answered the question 'What do you see here?' by saying: 'Now I am seeing it as a picture-rabbit.'[18]

For Wittgenstein, to speak of seeing the figure *now* under an aspect, without being aware that there is another aspect under which one could see it, is senseless: 'It would have made as little sense for me to say "Now I am seeing it as …" as to say at the sight of a knife and fork "Now I am seeing this as a knife and fork."'[19] We should be wary, however, of following Wittgenstein in this argument. It is certainly the case that our evidence of aspect-relativity generally comes through aspect shift; perhaps it is empirically impossible to acquire the concept of aspect-seeing without having experienced aspect shift. In addition, it may well be pragmatically infelicitous to say 'now I am seeing it as a picture-rabbit' if I do not know that it is possible to see it as a duck. But neither of these facts makes the statement *false*: I might very well be seeing it now as a rabbit without being in a position to say so.[20]

Criteria (a)–(f) serve to pick out the general notion of a multistable figure, but the problem can be refined by focusing on the question of aspect-seeing. The most basic notion used in (a)–(d) was that of an object *a*'s representing an *F* under conditions *K*, which we explained in terms of a disposition to represent an *F* to observers forming part of systems under *K*-conditions. But it seems intuitively plausible that if *a* (visually) represents an *F* to *S*, then *S* sees *a* as an *F*. At the very least, it seems difficult to conceive of a situation in which one could assert: 'The drawing represents a house to me, but I don't see it as a house.'

A caveat is necessary here. To be sure, one might often assert: 'The drawing represents a house, but I don't see it as a house', but that is predicted by the analysis. In claiming that the drawing represents a house, one is (in normal contexts) claiming only that it represents a house under standard perceptual conceptions, which is to say that it is disposed to represent a house to perceivers who form part of systems that are themselves under standard conditions. If one is aware of this fact, but one is suffering perceptual disturbances, in an unusual environment, or otherwise not under standard conditions, or if the disposition to generate the relevant perceptions is for some reason unmanifested, then one will be in a position to assert: 'The drawing represents a house, but I don't see it as a house.'

For this reason, I propose to follow Wittgenstein and take the mental attitudes expressed by '*S* sees *a* as an *F*-figure' as the basic *explananda* from which a generalized theory of aspect-relative phenomena should start. But how is the structure of 'seeing *a* as an *F*-figure' to be understood? It is worthwhile to begin, as Wittgenstein does in the relevant chapter of the *Philosophical Investigations*, with a preliminary observation about two different intentional attitudes that both go under the name 'seeing'. Wittgenstein states the point as follows:

> Two uses of the word 'see.' The one: 'What do you see there?' – 'I see this' (and then a description, a drawing, a copy). The other: 'I see a likeness between these two faces' – let the man I tell this to be seeing the faces as clearly as I do myself. The importance of this is the difference of category between the two 'objects' of sight.[21]

The passage is not entirely clear, but on the most straightforward reading, the distinction is one between an intentional attitude directed

towards a proposition, normally expressed by a sentence of the form 'I see that *p*' (where *p* is itself a sentence), and an intentional attitude directed towards a non-propositional individual object, normally expressed by a sentence of the form 'I see *a*' (where *a* is a proper name or definite description). We shall term attitudes of the first type (and their analogues for relations other than seeing) *propositional attitudes*; and those of the second, *objectual attitudes*. Wittgenstein slightly complicates the issue by using the form 'I see a likeness', which might be taken to represent an objectual attitude toward a dyadic universal or individual property-instance, a 'likeness'. However, it is more likely that Wittgenstein places little weight on this aspect of surface grammar, which is apt to mislead, as more mundane examples of the same form suggest: 'I see the arrival of the train' is most plausibly a paraphrase, expressing the same proposition as 'I see that the train arrives', rather than a objectual-attitude ascription parallel to 'I see the postman'. Objectual attitudes have been neglected in the philosophy of mind; indeed, it is often simply assumed that objectual attitudes are reducible to propositional attitudes.[22] However, as Michelle Montague has pointed out in an incisive article, this assumption is quite unwarranted.[23]

How do we get from '*S* sees *a*' to '*S* sees *a* as *F*'? In order to have a concrete example, let us use the labelled Necker cube in *Tractatus* 5.5423 (Fig. 2). It is clear that my perceptual state when I see the four A vertices in the foreground differs from my perceptual state when I see the B vertices in the foreground; call the former state *A* and the latter *B*. In each case, one component of being in the relevant perceptual state is having a specific objectual attitude: whether I am in state *A* or state *B*, I undoubtedly see *something*, not just *that the world is thus and so*. I see the drawing; I do not merely see *that* the drawing is arranged in a particular fashion. In state *A*, we wish to say that I see the drawing *as* having its A vertices facing forward; in state *B*, we wish to say that I see the drawing *as* having its B vertices facing forward.
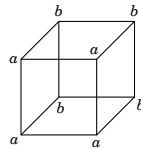

Fig. 2. Labelled Necker Cube.

How are we to interpret this? It is natural to start by attempting a reductive analysis: a biconditional of the form '*S* sees *a* as *F* if and only if …' where the problematic 'as' formulation does not appear on the right-hand side. One simple reduction suggests itself: perhaps to have a *seeing-as* attitude is simply to have *both* an objectual attitude and a related propositional attitude. More specifically, perhaps *S* sees *a* as *F* at time *t* if and only if *S* sees *a* at time *t* and *S* sees at time *t* that *a* is *F*. So, for example, my being in perceptual state *A* would amount to my seeing the drawing as A-foregrounded, and my seeing the drawing as A-foregrounded would amount to the conjunction of my (objectually) seeing the drawing and my (propositionally) seeing that it is A-foregrounded. (Here the relevant sense of 'see that' will have to be non-factive, by analogy to the non-veridical objectual sense of 'see' that allows for error and illusion.)

This reductive analysis, however, is unsuccessful. The conjunction of seeing *a* and seeing that *a* is *F* may be necessary for seeing *a* as *F*, but it is not sufficient. In seeing the drawing, I also see that it is composed of lines less than a millimetre wide, but I do not see it *as* composed of small lines in the same way in which, at any particular time, I see it *as* A-vertex-foregrounded or *as* B-vertex-foregrounded. My perception of the thickness of the lines that make up the figure does not enter – even when I am directly, occurrently aware of it – into the content of my visual experience in the same way as my perception of the figure's apparent orientation. The orientation of the figure as to foreground and background plays a fundamental role in determining the *kind* of thing that I take the figure to represent; line width, in contrast, is incidental.

This counterexample refutes only the simplest reduction of '*S* sees *a* as *F*'. I am not prepared to say that *no* analysis can succeed, although I think the prospects are poor. I think it is more fruitful, however, to take '*S* sees *a* as *F*' on its own terms – at least provisionally – rather than to expect any reduction that would eliminate the 'as' component. This does not resolve all questions about its logical form; we might ask, for instance, whether 'as *F*' is an adverbial modification (and thus, on the standard analysis, denotes a property of events of seeing)[24] or part of a compound singular term '*a* as *F*'.[25] The crucial consideration, however, is the following: to see *a* as *F* implies, but is not implied by, seeing *a* and seeing (at least in the non-factive sense of 'see') that *a* is *F*.

Over and above these components, there is, at least in most cases, an additional element involved – a kind of relevance of *F*-ness to the

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sort of thing that *a* is taken to be. This is the roughest of characterizations, but it will suffice to motivate the search for non-perceptual analogues falling under the form '*S* ϕ-s *a* as *F*'.

5

My first mathematical example comes from group theory. It seems intuitively compelling that I can simply *think* about, for instance, the set {1, 2} and, in thinking about it, have access to it as an object of reflection, independently of any beliefs that I may be forming about it. It may be, as a matter of practical fact, impossible for me to think *about* {1, 2} without also thinking certain propositions about it – for example, the proposition that it has two members. Yet the two are distinct acts. In other words, there are objectual attitudes of contemplation of mathematical objects as well as propositional attitudes. But are there *contemplating-as* attitudes – aspect-relative attitudes that are linked with, but not reducible to, objectual and propositional attitudes?

Let us consider a somewhat more complicated example – the set *G* consisting of the three cube roots of unity: $G = \{1, -\frac{1}{2}(1+i\sqrt{3}), -\frac{1}{2}(1-i\sqrt{3})\}$. I can conceive of the elements of *G* simply as complex numbers, or of *G* itself as one of the elements of the power set of the complex numbers. But I can also think of *G* as the underlying set of the structure $\Gamma = \langle G, \cdot \rangle$, where '·' denotes ordinary multiplication of complex numbers; in so doing, I attend to the algebraic properties of the elements of *G* under that operation. Γ is a cyclic group of order three – that is to say, an algebraic structure consisting of a three-element set {*a, b, c*} and an associated binary operation, ∘, which obeys a specific transformation table (Fig. 3). (It is easy to verify both that Γ does obey this transformation table, and that any algebraic structure obeying this transformation table is in fact a group.[26]) Because every structure that is a cyclic group of order three is isomorphic to every other such structure, it is customary to speak simply of *the* cyclic group of order three, Z_3. The axioms of group theory are among the simplest examples in mathematics of *formal* rather than *assertory* axioms: they do not state that anything exists; they merely set conditions for something to be a structure of a certain kind, and for convenience, when doing group theory, we ignore those features of cyclic-group-of-order-three-structured objects that are not given in the transformation table (in other words, we treat them as equivalent 'up to isomorphism').[27] So *G* under the operation of

complex multiplication, the set {0, 1, 2} under the operation of addition modulo 3, and the set of rotations in a plane {0°, 120°, 240°} under the operation of composition are all instances of the structure for which the term 'Z_3' goes proxy.

∘	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Fig. 3. Transformation Table for Z_3.

(Should we consider these various mathematical structures as *tokens* of the Z_3-type? This is a difficult question, which we need not answer here. First, because group-theoretical axioms are not assertory, it is unclear that we should take existential commitment to such a thing as 'the' Z_3 group at face value. To do so is not required by our interpretation of the intentional state: in 'S ϕ-s a as F', 'F' stands in predicate, not in singular term, position, and even if the relevant logical form were 'S ϕ-s a as the F', there would be no need to assume that the truth of such a statement entails the existence of a unique F such that S ϕ-s a as it. Secondly, the token/type relationship is paradigmatically a relation of concreta to abstracta. Here, however, structures such as Γ are abstract, and 'the cyclic group of order three' denotes an abstract object, if it denotes at all, so any token/type relationship would be one of abstracta to other abstracta.)

However, being a cyclic group of order three is not just any mathematical property. It is a property sufficiently important and natural to mark out a particular kind of cognitive access to mathematical objects; it is, in other words, a property that can give rise to a possible mode of cognition under an aspect. To think about G as the base set of a Z_3 group is not the same thing as thinking about it as a subset of the complex numbers; the first is a natural way to think about the set when doing group theory, the second, perhaps, a way one might approach it in complex analysis. Nor is it sufficient, in order to think about G under its Z_3 aspect, to think about G and to believe that it is the base set of a cyclic group of order three; one might well know this whilst thinking of G in another way entirely. Thus, it seems that thinking about G as the base set of a Z_3 group is an example of a cognitive attitude that has an irreducible 'as'-structure analogous to that involved in

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cases of 'seeing-as'. Much as seeing the Necker cube *as* forward-facing is something over and above seeing it, and seeing *that* it is forward-facing, thinking about G as the base set of a Z_3 group – thinking about it in the way in which one thinks about it when one considers it as a group-theoretical object – is something more than merely thinking about it and thinking *that* it is the base set of a Z_3 group.

My second example derives from a famous thought experiment offered by Paul Benacerraf in his essay 'What Numbers Could Not Be'. Imagine two children, Ernie and Johnny, who were taught mathematics starting from standard set theory (in other words, the Zermelo-Fraenkel axioms and the axiom of choice). Each was then taught arithmetic in terms of the set theory that he already knew; each was taught that there was a set of natural numbers, whose members were sets he was already familiar with, and each was told how to define certain operations on those sets – the operations called addition, multiplication, and so on. Having learned to use the vernacular – to call the first natural number 'zero' and the second 'one', as abbreviations for their proper names – they could then go on to talk about arithmetic to each other (and to the rest of us, who learned arithmetic the ordinary way). We shall allow Benacerraf to tell the rest of the story:

> Delighted with what they had learned, they started proving theorems about numbers. Comparing notes, they soon became aware that something was wrong, for a dispute immediately ensued about whether or not 3 belonged to 17. Ernie said that it did, Johnny that it did not. Attempts to settle this by asking ordinary folk (who had been dealing with numbers *as* numbers for a long time) understandably brought only blank stares. In support of his view, Ernie pointed to his theorem that for any two numbers, x and y, x is less than y if and only if x belongs to y and x is a proper subset of y. Since by common admission 3 is less than 17 it followed that 3 belonged to 17. Johnny, on the other hand, countered that Ernie's 'theorem' was mistaken, for given two numbers, x and y, x belongs to y if and only if y is the successor of x. These were clearly incompatible 'theorems.' Excluding the possibility of the inconsistency of their common set theory, the incompatibility must reside in the definitions.[28]

The incompatibility lay, it turned out, in the fact that Johnny and Ernie had learned different set-theoretical definitions of the numbers. Johnny had learned that the numbers were the Zermelo ordinals,

$$0 = \varnothing, 1 = \{\varnothing\}, 2 = \{\{\varnothing\}\}, 3 = \{\{\{\varnothing\}\}\} \dots,$$

(where 'Ø' denotes the empty set), but Ernie had learned that the numbers were the von Neumann ordinals,

0 = Ø, 1 = {Ø}, 2 = {Ø, {Ø}}, 3 = {Ø, {Ø}, {Ø, {Ø}}} ….

Who is right? There is no *arithmetical* difference between the two systems; each one proves the same theorems of arithmetic. Both Ernie and Johnny can prove, for instance, that there are five primes between 10 and 24 or that $a \times (b + c) = (a \times b) + (a \times c)$. (The statement that started the controversy, '3 ∈ 17', is a statement of set theory, not of arithmetic.) And it seems that only arithmetical properties should count for deciding between Ernie and Johnny; we might have reasons of convenience or caprice for preferring one account or the other for some purpose, but these reasons cannot be grounds for thinking one right and the other wrong. Thus, both are right or neither is; and obviously both cannot be right, for if 2 = {{Ø}} and 2 = {Ø, {Ø}}, then {{Ø}} = {Ø, {Ø}}, and that is certainly not the case. But if both Johnny and Ernie are wrong, deeply wrong, about the fundamental question of what it is to be a number, then why do both of them seem to do arithmetic perfectly well?

Benacerraf suggests, tentatively, that the lesson is that numbers are not objects. I suggest that, instead, we should conclude that numbers are not sets – they are rather *sui generis* objects. Their properties are not reducible to those found in the Peano axioms; what it is to be a natural number is also determined, in part, through the role natural numbers play as finite cardinals.[29] But then how is it that Johnny and Ernie could talk about numbers using sets, if numbers are not sets? Here the notion of a proxy-structure is essential: it is characteristic of any series isomorphic to the natural numbers – in technical terms, any ω-sequence – that it can go proxy for the numbers. My thesis is that one can learn to think about the natural numbers by thinking of the elements of a proxy-structure under an appropriate aspect. (This should not be surprising: most of us were introduced to the numbers through a particular proxy-series: the numerals.[30]) In learning to use {{Ø}} as 2, Johnny did not merely learn isolated mathematical facts; he learned to use a particular series as a proxy-structure and thereby learned to conceive of {{Ø}} under the 2-aspect, rather than simply as a particular pure set. Likewise, Ernie learned to conceive of {Ø,{Ø}} under the 2-aspect. In so doing, however, they gained indirect knowledge of 2 itself – they learned, at the very least, *what kind of thing* 2 is – through an aspect-relative cognition that goes deeper than a merely functional determination.

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My examples of aspect-relative contemplation of artefactual abstracta involve literary works. The first work is a poem from *Cent mille milliards de poèmes (A Hundred Million Million Poems)* by Raymond Queneau. Queneau, along with the mathematician François Le Lionnais, founded the experimental literary group OULIPO (Ouvroir de Littérature Potentielle) devoted to the study and production of constrained writing. The poem is an elegantly constructed, if somewhat irreverent, sonnet:

> Du jeune avantageux la nymphe était éprise
> snob un peu sur les bords des bords fondamentaux
> une toge il portait qui n'était pas de mise
> des narcisses on cueille ou bien on est de veaux
>
> Quand on prend des photos de cette tour de Pise
> d'où Galilée jadis jeta ses petits pots
> d'une étrusque inscription la pierre était incise
> les Grecs et les Romains en vain cherchent leurs mots
>
> L'esprit souffle et resouffle au dessus de la botte
> le touriste à Florence ignoble charibotte
> l'autocar écrabouille un peu d'esprit latin
>
> Les rapports transalpins sont-ils biunivoques?
> les banquiers d'Avignon changent-ils les baïoques?
> le Beaune et le Chianti sont-ils le même vin?[31]

The text occupies a single page in the collected edition of Queneau's poetry, with horizontal rules separating the lines; it is the sixth of ten sonnets thus presented. Each of the ten printed sonnets follows an *abababababccdeed* rhyme scheme, and each line in each sonnet rhymes with the corresponding line in every other sonnet. More impressively, each line in each sonnet is also syntactically isomorphic to the corresponding line in every other sonnet. Thus, by simple permutation, there are 10^{14} possible sonnets that can be generated from the ten-poem matrix; in the original edition, instead of horizontal rules there are cuts in the paper between the individual lines, and the permutation can be accomplished by hand.

These fourteen lines – by which I mean the fourteen lines as a poem-type, not any particular instance of them, whether printed in a

copy of the original incised edition or the collected works, or reprinted in this article, or even read aloud – are thus simultaneously constituents of two distinct artworks: the ten-sonnet matrix sequence, tokens of which are printed under the title *Cent mille milliards de poèmes*, and the physically unprintable 10^{14} poem sonnet sequence comprising all the permissible combinations of the 140 lines making up the matrix sonnets. Let us call the former work C_1 and the latter work C_2. There are two corresponding aspects under which one can understand the individual sonnet: let us call them D_1 and D_2. It is essential to understanding 'Du jeune avantageux la nymphe était éprise' that one be able to see it under these two different aspects. The aesthetic properties of the sonnet-under-aspect-D_1 and the sonnet-under-aspect-D_2 differ. For example, since Queneau composed C_1 directly and C_2 only indirectly, the link between the stylistic features of the sonnet-under-D_1 and those of Queneau's other compositions is far more direct than is the case for the sonnet-under-D_2.

Something similar is involved in homophonic translations – texts written, more or less grammatically, using words of one language chosen on the basis of their phonetic similarity to the words in a (usually famous) text in a different language. Queneau and the OULIPO group were also interested in homophonic translation; the second OULIPO manifesto, for instance, includes François Le Lionnais's version of the opening line of Keats's *Endymion*, uttered before the monkey cage at the Jardin des Plantes: 'Un singe de beauté est un jouet pour l'hiver.'[32] My example, however, comes from *Mots d'heures, gousses, rames: The D'Antin Manuscript*, an elaborate piece of mock-scholarship published in 1967 by the actor Luis D'Antin van Rooten. The volume purports to be an edition of an anonymous eighteenth-century manuscript; in the preface, we are informed that the poems 'assume a strangely familiar, almost nostalgic, homely quality'[33] when read aloud. To an Anglophone reader, they are indeed familiar, for Mother Goose hides behind *Mots d'heures, gousses*, and the refrain of Humpty Dumpty is to be found in the following lines:

> Un petit d'un petit
> S'étonne aux Halles
> Un petit d'un petit
> Ah! degrés te fallent
> Indolent qui ne sort cesse
> Indolent qui ne se mène

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Qu'importe un petit d'un petit
Tout Gai de Reguennes.[34]

But an exclusively Anglophone reader will miss the best part of the joke, for *Mots d'heures* purports to be an *annotated* edition. The extraordinarily convoluted glosses in the footnotes result from the constraint of taking the French, *as French*, seriously: 'un petit d'un petit', we are informed, is '[t]he inevitable result of child marriage', while obviously anyone who would 's'étonne aux Halles' must be a visitor from the provinces. The rustic hero will not end well, however, as is shown by the comparison with Gai de Reguennes, 'a young squire [...] who died at the tender age of twelve of a surfeit of Saracen arrows before the walls of Acre in 1191'.[35]

Here the aspect-relativity in question is relativity to a particular language: the same base text is interpretable as French and as English (or some near approximations to those languages), and its proper understanding requires that both aspects be available to cognition. In this case, there is an interesting disanalogy with literal multistable figures: it seems possible, at least after the first reading, to think about *both* the poem-under-the-English-aspect and the poem-under-the-French-aspect simultaneously.

7

These four examples, as I noted in §2, are intended only to motivate the claim that there can be aspect-relative cognition of abstracta, not to develop the consequences of that thesis. Nonetheless, it is worth at least adverting to one reason that the existence of such phenomena is important. Where the abstracta in question are mathematical objects, the structural kinship between seeing-as and contemplating-as or cognizing-as helps to account for the intuition, famously expressed by Kurt Gödel, that we have 'something like a perception' of at least some mathematical objects.[36]

Explaining this intuition is important for realists about mathematical objects. A common objection raised by nominalists against mathematical realists is that realism requires mysterious causal connections between abstract entities and minds in order for us to have knowledge of the truth of mathematical statements.[37] As a general anti-realist argument, this objection rests on causal requirements for knowledge that are

independently implausible.[38] But it might be thought to retain some force with respect to objectual cognitive attitudes, such as contemplation of the set *G*. Gödelian intuitions suggest that, if such attitudes exist, they are perception-like. Causal linkage may not be required for all knowledge, but it certainly is required for literal perception. If the only way for a cognitive attitude to be perception-like is for it to be grounded in a causal connection with its object akin to that involved in literal perception, and there are no causal connections between minds and abstracta, then we must reject Gödelian intuitions about our objectual attitudes to mathematical objects: either we do not genuinely have such attitudes, or they are not perception-like. Realists will not find this an appealing option, however, for the Gödelian thesis is *prima facie* quite plausible. But if a cognitive attitude's perception-like character can be explained through its potentially being subject to aspect-relativity, rather than through any causal link between mind and object, then we can maintain the Gödelian thesis without attributing mysterious causal powers to mathematical abstracta.

The importance of the aesthetic cases is less novel; outside the philosophy of perception, the 'seeing-as' structure has been most fully studied in aesthetics (although usually with concrete artworks as examples).[39] However, the connection between the occurrence of this structure in thought about artworks and its occurrence in thought about mathematicalia does point toward a number of new issues. To some degree, it may furnish an *indirect* argument against nominalism: to the extent that nominalists find mathematical cognition mysterious, they should also find ordinary aesthetic contemplation mysterious, since that contemplation has the same sort of non-causal intentional structure; but if the problems involved in explaining this structure afford us little reason to doubt the existence of symphonies and novels, they should also pose no epistemological threat to belief in numbers and sets.

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My mathematical examples, involving sets and numbers, will of course be rejected by those who reject the existence of sets and numbers. In general, for some domain of objects X, those who deny that Xs exist are called nominalists about Xs; those who accept that Xs exist, realists about Xs. I do not propose to give an extended defence of my version of realism – or, as it is often called, platonism – about mathematical objects.[40] However, because mathematical platonism is sometimes considered very implausible, it is worth giving a brief example of the sort of considerations that can be marshalled in support of it.[41] The simplest argument can be summed up in a sentence: 'There are four prime numbers less than ten; therefore, there are numbers.' And numbers are as uncontroversially abstract as can be imagined; they are abstract if anything is. Therefore, abstract objects exist. How would the nominalist respond to this argument? To deny that there are four prime numbers less than ten is absurd. Thus the nominalist must challenge the inference from 'there are four prime numbers less than ten' to 'there are numbers'. But the following inference, of the same apparent form, seems obviously valid: 'There are four lions in Berlin zoos; therefore, there are lions.' To reject the inference in the mathematical case and accept it in the leonine one requires that the meaning of 'there are' in the premise of the argument about numbers must be different from that of 'there are' in its conclusion, and, moreover, that it must be different in a way that is not the case in the corresponding statements about ordinary objects. The nominalist, in short, requires that mathematical quantifier expressions, such as 'there are' and 'there exist', be ambiguous in a way that other quantifier expressions are not. Yet the fact that we can pass seamlessly from mathematical to non-mathematical statements by rules of inference demonstrates that this is not the case; for instance, we can infer from 'there are four prime numbers less than ten' and 'there are four lions in Berlin zoos' to 'there are as many lions in Berlin zoos as prime numbers less than ten'; this would not be the case were mathematical quantification semantically deviant. Mathematical language is not an esoteric code, but rather a part of our common language, governed by the same general principles as the rest; we should take its claims at face value.[42]

With respect to artefactual abstracta, the argumentative stakes are reversed. Many people doubt that numbers exist, but few will deny that

numbers, if they exist, are abstracta. All will concede that *Bleak House* exists, but many will doubt that it is abstract, or conceding its abstractness, will doubt that it is artefactual. Here again I shall only give some rough arguments for motivation. *Bleak House* is not identical to any particular copy of it, for any particular copy – even the author's autograph manuscript – could be destroyed without damaging *Bleak House* itself in the least. *Bleak House* is surely linked to some of Charles Dickens's thoughts, but it is not itself and never was a thought in Dickens's mind: I have read *Bleak House*, but I cannot read minds. Is *Bleak House* the sum of all its individual copies – that is to say, is it the concrete whole that has them as its parts? Surely not: it might well be true that the concrete whole made up of all the copies of *Bleak House* in the world weighs two tonnes, but *Bleak House* itself does not weigh anything. Is it the set of all the individual copies? If so, it would presumably be abstract, because sets are abstract; but in any case, it is not. It is essential to a set that it have as members those things that it in fact has: it is not possible for the set {1, 3, 7} to have any members other than the numbers one, three, and seven. But clearly there could have been more or fewer copies of *Bleak House* than there in fact are. The best answer, in the face of all these objections, is the answer that we have been giving all along: *Bleak House* is a type, its individual copies tokens.

Bleak House, the type, is composed of sentence-types; nonetheless, it cannot simply be equated with the sequence of sentence types that composes it. Some causal facts about its composition are essential to it. This lesson is familiar to anyone who knows Borges's 'Pierre Menard, Author of the *Quixote*':

> He did not want to compose another *Don Quixote* – which would be easy – but *the Don Quixote*. It is unnecessary to add that his aim was never to produce a mechanical transcription of the original; he did not propose to copy it. His admirable ambition was to produce pages which would coincide – word for word and line for line – with those of Miguel de Cervantes. [...] The text of Cervantes and that of Menard are verbally identical, but the second is almost infinitely richer.[43]

Of course, Pierre Menard is himself fictional, and his *Quixote* does not really exist. Yet it would not do to reject the insight because of that. The story is, among other things, a thought experiment, and thought experiments are real sources of knowledge. (Needless to say, Borges's great text is not *merely* a thought experiment.) If every literary work

were identical to the sequence of sentences that composed it, then Pierre Menard's *Quixote* and Cervantes's *Quixote* would be identical, for each is comprised of the same sequence of sentences. But they are not. Moreover, it is wholly implausible that *one of the two* should be identical to the sequence of sentences that composes it, or more generally that *some* literary works should be identical to the sentences of which they are comprised and others not. Thus, we should say that no literary work is just a sequence of sentences, although every work is *constituted from* a sequence of sentences. An analogous argument applies to musical works: as Jerrold Levinson points out, '[a] work identical in sound structure with Schoenberg's *Pierrot Lunaire* (1912) but composed by Richard Strauss in 1897 would be aesthetically different from Schoenberg's work.'[44] Sound structure alone does not make a musical work, just as verbal sequence alone does not make a literary work.

NOTES

1 This article is the result of work done as a research fellow at the ICI Berlin Institute for Cultural Inquiry during the 2010–11 academic year. I am greatly indebted to all members of the ICI colloquium for their comments and suggestions. I also wish to thank Nick Stang for several extremely fruitful discussions about the topics discussed herein.

2 Louis Albert Necker, 'Observations on Some Remarkable Optical Phaenomena Seen in Switzerland, and on an Optical Phaenomenon which Occurs on Viewing a Figure of a Crystal or Geometrical Solid', *London and Edinburgh Philosophical Magazine*, 3rd ser., 1.5 (1832), pp. 329–37 (p. 336).

3 See Franz Brentano, 'Über ein optisches Paradoxon', *Zeitschrift für Psychologie*, 3 (1892), pp. 349–58, and 'Zur Lehre von den optischen Täuschungen', *Zeitschrift für Psychologie*, 6 (1893), pp. 1–7; Vittorio Benussi, 'Zur Psychologie des Gestalterfassens', in *Untersuchungen zur Gegenstandstheorie und Psychologie*, ed. by Alexius Meinong (Leipzig: Johann Ambrosius Barth, 1904), pp. 303–448; and Stephen Witasek, 'Über die Natur der geometrisch-optischen Täuschungen', *Zeitschrift für Psychologie*, 19 (1899), pp. 81–175. For more on Meinong, Benussi, and Witasek, see David F. Lindenfeld, *The Transformation of Positivism: Alexius Meinong and European Thought, 1880–1920* (Berkeley: University of California Press, 1980), pp. 220–42; for a general conspectus of early work, see J. E. W. Wallin, *Optical Illusions of Reversible Perspective: A Volume of Historical and Experimental Researches* (Princeton: Princeton University Press, 1905).

4 Ludwig Wittgenstein, *Philosophical Investigations (Philosophische Untersuchungen)*, trans. by G. E. M. Anscombe, reissued 2nd edn (Oxford: Blackwell, 1997), part II, subpart xi, pp. 193–229; *Werkausgabe* (Frankfurt am Main: Suhrkamp,

1984), I, pp. 518–77. For citations from the second half of *Philosophical Investigations* (in Anscombe's edition), a letter after the page reference indicates the paragraph; all other citations to Wittgenstein's works include, in addition to the pagination of the *Werkausgabe*, the sequential paragraph or proposition number indicated by Wittgenstein or his editors. In some cases, translations have been silently modified; where no English edition listed, the translation is my own. My discussion of aspect-relativity does not draw directly on any particular previous treatment in the literature, but I have benefited from consulting Richard Wollheim, *Art and Its Objects*, 2nd edn (Cambridge: Cambridge University Press, 1980), pp. 205–26; G. N. A. Vesey, 'Seeing and Seeing-As', *Proceedings of the Aristotelian Society*, 56 (1956), pp. 109–24; Hidé Ishiguro, 'Imagination', *Proceedings of the Aristotelian Society*, Supp. 41 (1967), pp. 37–56; Noel Fleming, 'Recognizing and Seeing As', *Philosophical Review*, 66 (1957), pp. 161–79; Charles E. M. Dunlop, 'Wittgenstein on Sensation and Seeing-As', *Synthese*, 60 (1984), pp. 349–67; and Malcolm Budd, 'Wittgenstein on Seeing Aspects', *Mind*, 96 (1987), pp. 1–17. For a very interesting contemporary treatment of multi-stable figures, see Fiona Macpherson, 'Ambiguous Figures and the Content of Experience', *Noûs*, 40 (2006), pp. 82–117.

5   Wittgenstein, *Philosophical Investigations*, p. 194b; *Werkausgabe*, I, p. 519: 'Man kann ihn als Hasenkopf, oder als Entenkopf sehen'. Wittgenstein's source here is Joseph Jastrow's 'The Mind's Eye', *Popular Science Monthly*, 54 (1899), pp. 299–312, reprinted in his *Fact and Fable in Psychology* (Boston: Houghton Mifflin, 1900), pp. 275–95. The image originated in *Fliegende Blätter* (München), 97.2465 (23 Oct. 1892), p. 147, and was reprinted in *Harper's Weekly*, 36.1874 (19 Nov. 1892), p. 1114, where Jastrow found it. For more on the history of the figure, see Peter Brugger, 'One Hundred Years of an Ambiguous Figure: Happy Birthday, Duck/Rabbit!', *Perceptual and Motor Skills*, 89 (1999), pp. 973–77. For a reproduction of Wittgenstein's duck-rabbit figure, see Fig. 4 of Christine Hentschel's chapter in this volume, and for one of the original duck-rabbit figure, see Fig. 1 of Luca Di Blasi's chapter.

6   Wittgenstein, *Philosophical Investigations*, pp. 196j–197b; *Werkausgabe*, I, p. 524: 'Wenn ich den H-E-Kopf als H sah, so sah ich: diese Formen und Farben (ich gebe sie genau wieder) – und außerdem noch so etwas: dabei nun zeige ich auf eine Menge verschiedener Hasenbilder. – Dies zeigt die Verschiedenheit der Begriffe. Das "Sehen als …" gehört nicht zur Wahrnehmung. Und darum ist es wie Sehen und wieder nicht wie ein Sehen.'

7   It should be noted that representational properties are clearly not reducible to mere resemblance properties, and Wittgenstein nowhere claims that they are. For general discussions, see Nelson Goodman, 'Seven Strictures on Similarity', in *Problems and Projects* (Indianapolis: Hackett, 1982), pp. 437–46; see also Christopher Peacocke, 'Depiction', *Philosophical Review*, 96 (1987), pp. 383–410.

8   Wittgenstein, *Philosophical Investigations*, p. 200b; *Werkausgabe*, I, p. 529: 'Es ist vielmehr das alltägliche Sprachspiel *hinzunehmen* [...]'. For general discussions of the late Wittgenstein's methodology, see Cora Diamond, *The Realistic Spirit: Wittgenstein, Philosophy, and the Mind* (Cambridge, MA: MIT Press,

1995). For an alternative programme, with which I am broadly sympathetic, see Timothy Williamson, *The Philosophy of Philosophy* (Oxford: Blackwell, 2007).

9   In general, I have used names autonymously and been otherwise lax about use and mention except where precision is necessary. I have also eschewed quasi-quotation; the difference between object names and variables and schematic letters should be clear from the context.

10 For more on the abstract/concrete distinction, see David Lewis, *On the Plurality of Worlds* (Oxford: Blackwell, 1986), pp. 171–74; John Burgess and Gideon Rosen, *A Subject with No Object: Strategies for Nominalistic Interpretation of Mathematics* (Oxford: Oxford University Press, 1997), pp. 13–25; Wolfgang Künne, *Abstrakte Gegenstände: Semantik und Ontologie*, 2nd edn (Frankfurt am Main: Klostermann, 2007), pp. 48–95.

11 The view that there are such objects is hardly original; for a position similar to mine, see Amie Thomasson, *Fiction and Metaphysics* (Cambridge: Cambridge University Press, 1999). See also Stephen Schiffer, 'Language-Created Language-Independent Entities', *Philosophical Topics*, 24 (1996), pp. 149–67; Nicholas Wolterstorff, *Works and Worlds of Art* (Oxford: Clarendon Press, 1980); and particularly Jerrold Levinson, 'What a Musical Work Is', *Journal of Philosophy*, 77 (1980), pp. 5–28, rpt. in his *Music, Art, and Metaphysics: Essays in Philosophical Aesthetics*, 2nd edn (Oxford: Oxford University Press, 2011), pp. 63–88.

12 This terminology, due to Charles Sanders Peirce, is most easily explained by example. Consider a scrap of paper with the words 'THERE ARE TREES' written on it. How many letters are there on the paper? The question is ambiguous. In one sense, there are thirteen letters: five in 'THERE', three in 'ARE', and five in 'TREES'. In another sense, there are six letters on the paper: 'A', 'E', 'H', 'R', 'T', and 'S'. On the first reading, the question asks for the number of letter-tokens; on the second, for the number of letter-types. A type is the kind of thing that has tokens as its instances. Normally, a single type has multiple tokens as instances (although there can be singly instanced and even uninstanced types). See Charles Sanders Peirce, 'Prolegomena to an Apology for Pragmaticism' (1906), in *Collected Papers*, ed. by Charles Hartshorne and Paul Weiss (Cambridge, MA: Harvard University Press, 1931–58), IV, §§ 530–72 (§ 537). For further discussion, see Linda Wetzel, *Types and Tokens: On Abstract Objects* (Cambridge, MA: MIT Press, 2009).

13 Wittgenstein, *Philosophical Investigations*, p. 214d; *Werkausgabe*, I, p. 553.

14 See the example of the cube at *Philosophical Investigations*, p. 193h; *Werkausgabe*, I, p. 519. Note that this cube is not a Necker cube, as it is mistakenly represented in earlier printings of Anscombe's edition of the text.

15 Wittgenstein, *Philosophical Investigations*, p. 200e; *Werkausgabe*, I, p. 519: 'Wie ist es aber möglich, daß man ein Ding einer *Deutung* gemäß sieht?'

16 See Wittgenstein, *Philosophical Investigations*, p. 212d; *Werkausgabe*, I, p. 550.

17 The use of a disposition ascription is important. Characteristically, dispositions need not always become manifest, even under prescribed stimulus conditions; see David Lewis, 'Finkish Dispositions', *Philosophical Quarterly*, 47 (1997), pp.

143–58, and, for useful surveys, Michael Fara, 'Dispositions and Habituals', *Noûs*, 39 (2005), pp. 43–82, and Sungho Choi, 'Dispositional Properties and Counterfactual Conditionals', *Mind*, 117 (2008), pp. 795–841.

18 Wittgenstein, *Philosophical Investigations*, p. 194f–195a; *Werkausgabe*, I, p. 520–21: 'Ich konnte also den H-E-Kopf von vornherein einfach als Bildhasen sehen. D.h.: Gefragt, "Was ist das?", oder "Was siehst du da?", hätte ich geantwortet: "Einen Bildhasen". […] Ich hätte auf die Frage "Was siehst du da?" nicht geantwortet: "Ich sehe das jetzt als Bildhasen."'

19 Wittgenstein, *Philosophical Investigations*, p. 195c; *Werkausgabe*, I, p. 521: 'Zu sagen "Ich sehe das jetzt als …", hätte für mich so wenig Sinn gehabt, als beim Anblick von Messer und Gabel zu sagen: "Ich sehe das jetzt als Messer und Gabel."'

20 Wittgenstein, in fact, has the materials for an argument to this effect, but he fails to draw the proper conclusion. He notes that, in the represented situation, 'Nevertheless someone else could have said of me: "He is seeing the figure as a picture rabbit."' (*Philosophical Investigations*, p. 195b; *Werkausgabe*, I, p. 521: 'Dennoch hätte ein Anderer von mir sagen können: "Er sieht die Figur als Bild-H."') Indeed, the interlocutor would have been *correct* in doing so. Yet the meanings of third- and first-person pronouns are partly constituted by reference links. It cannot be the case that 'he ϕ-s' is true, where 'he' refers to me, unless I ϕ, and it cannot be the case that I ϕ unless 'I ϕ' is true. Similar issues about pronoun reference links are relevant to Wittgenstein's discussion of Moore's paradox; see *Philosophical Investigations*, part II, subpart i, pp. 190–92; *Werkausgabe*, I, pp. 513–17. The general point that truth conditions can diverge from assertability conditions is one of the central insights of post-Wittgensteinian philosophy of language: see generally Paul Grice, *Studies in the Way of Words* (Cambridge, MA: Harvard University Press, 1989). See also Saul Kripke's criticism of Wittgenstein's discussion of the standard metre (Wittgenstein, *Philosophical Investigations*, §50; *Werkausgabe*, I, p. 268) in *Naming and Necessity* (Cambridge, MA: Harvard University Press, 1980), p. 55.

21 Wittgenstein, *Philosophical Investigations*, p. 193a; *Werkausgabe*, I, p. 518: 'Zwei Verwendungen des Wortes "sehen". Die eine: "Was siehst du dort?" – "Ich sehe *dies*" (es folgt eine Beschreibung, eine Zeichnung, eine Kopie). Die andere: "Ich sehe eine Ähnlichkeit in diesen beiden Gesichtern" – der, dem ich dies mitteile, mag die Gesichter so deutlich sehen wie ich selbst. Die Wichtigkeit: Der kategorische Unterschied der beiden "Objekte" des Sehens.'

22 For an attempted treatment of seeing-as based solely on propositional attitudes, see Robert Howell, 'Seeing As', *Synthese*, 23 (1972), pp. 400–22.

23 Michelle Montague, 'Against Propositionalism', *Noûs*, 41 (2007), pp. 503–18.

24 See Donald Davidson, 'The Logical Form of Action Sentences', in *The Logic of Decision and Action*, ed. by Nicholas Rescher (Pittsburgh: University of Pittsburgh Press, 1967), pp. 81–95, rpt. with afterword in Davidson's *Essays on Actions and Events*, 2nd edn (Oxford: Clarendon Press, 2001), pp. 105–48.

25 One might argue that the objects of seeing-as are *qua* objects, such as '*a*-qua-*F*' in a sense similar to that developed in Kit Fine, 'Acts, Events, and Things', in

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Language and Ontology: Proceedings of the Sixth International Wittgenstein Symposium/Sprache und Ontologie: Akten des 6. Internationalen Wittgenstein-Symposiums, ed. by Werner Leinfellner, Eric Kramer, and Jeffrey Schank (Vienna: Holder-Pichler-Tempsky, 1982), pp. 97–105. For more general issues concerning the 'as' structure, see Allan Bäck, *On Reduplication: Logical Theories of Qualification* (Leiden: Brill, 1996).

26 The group axioms are as follows: (1) for all x and $y \in S$, $x \circ y \in S$; (2) for all x, y, and $z \in S$, $(x \circ y) \circ z = x \circ (y \circ z)$; (3) there exists some $e \in S$ such that for every $x \in S$, $e \circ x = x \circ e = x$; (4) for every $x \in S$, there exists some $y \in S$ such that $x \circ y = y \circ x = e$. These axioms are termed, respectively, *closure, associativity*, the *identity element axiom*, and the *inverse element axiom*.

27 On the difference between assertory and formal axioms, see Geoffrey Hellman, 'Category Theory and Mathematical Structuralism', *Philosophia Mathematica*, 3rd ser., 11 (2003), pp. 129–57 (pp. 134–35).

28 Paul Benacerraf, 'What Numbers Could Not Be', *Philosophical Review*, 74 (1965), pp. 47–73 (p. 54).

29 Here the fundamental principle is that which it is now customary to call Hume's Principle: the number of Xs = the number of Ys if and only if the Xs can be put into one-to-one correspondence with the Ys. It was Frege who emphasized the importance of the principle. One of his great accomplishments was to show that, given Hume's Principle and second-order logic, one can derive the Peano axioms for the finite numbers. This result is now known as Frege's Theorem; the neo-Fregean programme in philosophy of mathematics starts from it. See Crispin Wright, *Frege's Conception of Numbers as Objects* (Aberdeen: Aberdeen University Press, 1983), and the essays in George Boolos, *Logic, Logic, and Logic*, ed. by John Burgess and Richard Jeffrey (Cambridge, MA: Harvard University Press, 1999), pp. 135–341, and in *Frege's Philosophy of Mathematics*, ed. by William Demopoulos (Cambridge, MA: Harvard University Press, 1995).

30 See David Auerbach, 'Saying It with Numerals', *Notre Dame Journal of Formal Logic*, 35 (1994), pp. 130–46; Leon Horsten, 'Canonical Naming Systems', *Minds and Machines*, 15 (2005), pp. 229–57; Diane Ackerman, 'De Re Propositional Attitudes toward Integers', *Southwestern Journal of Philosophy*, 9 (1978), pp. 145–53; and the discussions in David Kaplan, 'Quantifying In', in *Words and Objections: Essays on the Work of W. V. Quine*, ed. by Donald Davidson and Jaakko Hintikka (Dordrecht: Reidel, 1969), pp. 178–214, rpt. in *Reference and Modality*, ed. by Leonard Linsky (Oxford: Oxford University Press, 1971), pp. 112–44 (particularly pp. 128–30), and Gareth Evans, *The Varieties of Reference* (Oxford: Clarendon, 1982), p. 98.

31 Raymond Queneau, 'Cent mille milliards de poèmes', in *Œuvres complètes*, ed. by Claude Debon, 3 vols to date (Paris: Gallimard, 1989–), I, pp. 331–47 (p. 339); for the original, see *Cent mille milliards de poèmes* (Paris: Gallimard, 1961).

32 François Le Lionnais, 'Le Second Manifeste', in OULIPO, *La Littérature potentielle: Créations, re-créations, récréations* (Paris: Gallimard, 1973), pp. 19–23 (p. 23).

33 Luis d'Antin van Rooten, *Mots d'heures, gousses, rames: The D'Antin Manuscript* (London: Penguin, 1980), n.p.

34 Ibid, p. 1.

35 Ibid.

36 Kurt Gödel, 'What Is Cantor's Continuum Problem?' (second version), in *Philosophy of Mathematics: Selected Readings*, ed. by Paul Benacerraf and Hilary Putnam (Englewood Cliffs, NJ: Prentice-Hall, 1964), pp. 258–73 (p. 271), rpt. in *Collected Works*, ed. by Solomon Feferman et al., 5 vols (Oxford: Oxford University Press, 1990–2003), II, pp. 245–70 (pp. 267–68): 'But, despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as seen from the fact that the axioms force themselves upon us as being true.' For an attempt, quite different from the one developed here, to work out the consequences of this idea, see Penelope Maddy, *Realism in Mathematics* (Oxford: Oxford University Press, 1990), pp. 36–80.

37 See Paul Benacerraf, 'Mathematical Truth', *Journal of Philosophy*, 70 (1973), pp. 661–79 (pp. 671–73); for subtler forms of the argument, see Hartry Field, *Realism, Mathematics, and Modality* (Oxford: Blackwell, 1989).

38 See Mark Steiner, 'Platonism and the Causal Theory of Knowledge', *Journal of Philosophy*, 70 (1973), pp. 57–66.

39 In addition to Wollheim's work cited in fn. 4, see Virgil C. Aldrich, 'Visual Metaphor', *Journal of Aesthetic Education*, 2 (1968), pp. 73–86; E. H. Gombrich, *Art and Illusion: A Study in the Psychology of Pictorial Representation*, 3rd edn (London: Phaidon, 1968), pp. 84–89; and William G. Lycan, 'Gombrich, Wittgenstein, and the Duck-Rabbit', *Journal of Aesthetics and Art Criticism*, 30 (1971), pp. 229–37.

40 I write 'platonism' in lower case for the modern doctrine, 'Platonism' with a capital letter for that held by the historical Plato and his followers.

41 For a much fuller discussion, see Burgess and Rosen, *A Subject with No Object* (cited in fn. 10).

42 The central idea behind this argument goes back to Frege's 1884 *Foundations of Arithmetic*; for a more direct source, see Benacerraf, 'Mathematical Truth', pp. 668–70.

43 Jorge Luis Borges, *Ficciones*, trans. by Anthony Kerrigan (New York: Grove Press, 1962), pp. 48–49, 52. For the original, see *Obras completas*, 4 vols (Buenos Aires: Emecé, 1989–96), I, pp. 446, 449: 'No quería componer otro Quijote – lo cual es fácil – sino el Quijote. Inútil agregar que no encaró nunca una transcripción mecánica del original; no se proponía copiarlo. Su admirable ambición era producir unas páginas que coincidieran palabra por palabra y línea por línea con las de Miguel de Cervantes […] El texto de Cervantes y el de Menard son verbalmente idénticos, pero el segundo es casi infinitamente más rico.'

44 Levinson, 'What a Musical Work Is', in *Music, Art, and Metaphysics* (cited above fn. 11), p. 70.

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