1963

New mathematics -- a controversy

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Recommended Citation
Quarterly, Vol. 09, no. 4 (1963, December), p. 14-17

WRITING ABOUT the *new* mathematics these days is a safe and popular venture: everybody is talking about it, and no one really knows exactly what it is. Such erudite institutions as *The New York Times, The Saturday Review,* The American Mathematical Society, and The Mathematical Association of America have entered the foray with the result, surprisingly enough, of adding new dimensions to the confusion.[1]

The widespread self-consciousness about education in the United States, as Sputnik I orbited the Earth in 1958, focused national attention on the importance of mathematics. Though mathematicians were pleased to find that their traditionally "dull" subject had become the object of public concern and support and even catapulted to editorial pages, the sudden glare of the limelight took them by surprise. A sense of urgency surrounded the ever-present task of revising curricula and courses; and, for the past five years, mathematics teachers have found themselves beset by new proposals to do this and do that and by a vast array of definitions, sometimes contradictory, of just what new educational problems need to be solved. The controversy and confusion which now surrounds the *new* mathematics is due in part to the haste in which this reappraisal was undertaken.

There are, of course, a number of other reasons for the controversy; and it is hardly necessary to comment that any attempt to bring order out of the present confusion must trace the causes of it. As mathematics teachers are painfully aware, the definition of a problem is a major part of its solution.

As I see it, the sources of the arguments surrounding the *new* mathematics are the following; the problem of SEMANTICS, which, for example, finds different writers attaching to the same word quite different meanings; an honest difference of opinion as to what the CONTENT of the *new* mathematics should be; and a failure to give the *new* mathematics its proper HISTORICAL perspective (for example, when did it begin, how "new" is it really, and where is it going) with the result that its importance and innovations are often exaggerated.

Any attempt to treat these three extensive problems in an intensive manner is really quite difficult, since they have been widely misunderstood, but I shall comment on each of them briefly and in the order in which they are stated. Since almost all of the present debate has concerned itself with secondary school mathematics, my remarks shall be confined, for the most part, to mathematics at that level.

With regard to the problem of SEMANTICS, the first error here is in the use of the word *new* in connection with mathematics at the secondary school level. I presume that this use of the word would imply to the layman that the mathematics now being taught has just been discovered; whereas nothing could be farther from the truth. Professor W. W. Sawyer, a noted mathematics scholar and author, in a recent denunciation of the use of the word *new* in this connection, commented, after noting that most of the mathematics now being introduced into high school curricula was known by the nineteenth century at the latest, "We do not serve the cause of education, of mathematics, or of honesty by calling old things new, by making simple ideas appear imposing."[2] What is NEW is the emphasis being given to topics which were previously taught and the introduction of topics which were not previously treated; and I shall discuss these further in connection with the content of the present secondary school curriculum.

To press the semantic problem somewhat further, since it is the basis of so much of the present debate, I would like to cite several other instances of words in mathematics which have come to have widely divergent connotations:

—To high school students the word *algebra* denotes a subject (probably epitomized by quadratic equations) which is studied in the ninth grade and, more often than not, over again in eleventh grade; whereas to professional mathematicians algebra denotes an extensive area of higher mathematics which is presently alive with research and new results. Little wonder that freshmen become confused when a professor tells them that he has written his Ph.D. thesis in algebra.

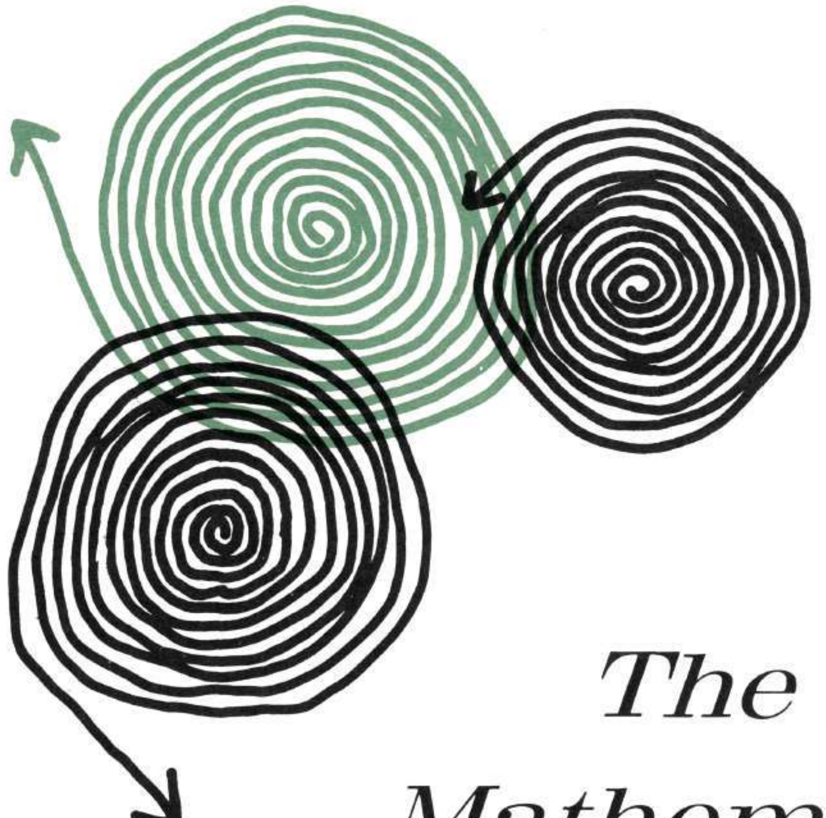
—The Pittsburgh Public Schools refer to their brand-new and sophisticated course for high school seniors as analysis, but to students at Ohio State University, for example, analysis means the first really high-powered graduate course in functions of a real variable. And now, just recently, our own Department of Mathematics at Carnegie has chosen to relabel its freshman and sophomore analytical geometry and calculus sequence simply analysis.

—For the first time in the public schools, children are taught about sets, sometimes as early as in the fourth grade, and they are told that the word *sets* refers to such collections of things as the children in their classroom or the states of some union. But to graduate students in mathematics the word *sets* suggests such things as, perhaps, a geometric manifold with strange topological properties.

—To mathematicians the word *topology* refers to a fairly abstract area of higher mathematics which students are really not mature enough to deal with until after they have finished their undergraduate work. And yet several months ago, as a judge at an essay contest for high school students, I heard a tenth-grader read a ten minute paper on topology, in which he felt that he had dealt with the whole subject quite adequately.

It is really very paradoxical, and indeed quite embarrassing, that mathematics, which presumably is THE academic discipline in which words and concepts are precisely and unambiguously defined, should find itself for the moment in such a semantic spin. And there is little wonder that various writers and editors find themselves sparring with each other as to what the *new* mathematics is all about. The University of Illinois Committee on School Mathematics (UICSM), in addition to its many notable achievements, is trying to straighten out the semantic problem, at least at the high school level, by substituting brand-new words for ones whose meanings have become clouded; but I think that the question as to whether this dispels the confusion or compounds it is also a matter for debate.

With regard to the CONTENT of the *new* mathematics, I have already commented that the first thing that can be said it that it is not *new!* What, then, does distinguish it from the mathematics which was taught almost everywhere until five years ago? The answer lies in the emphasis and selectivity of the topics in the various new



The New Mathematics – a Controversy

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We are indebted to the *Carnegie Tech Quarterly* for this reprint from their January '63 issue.

curricula, and while there are differences here from program to program, I think that certain common features are beginning to emerge which are fairly easy to delineate. In my own best judgment these characterizing features of the *new* mathematics are the following:

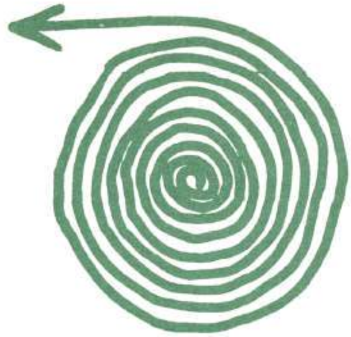
1. *It eliminates those topics which are relatively unimportant.* Probably the two best examples of topics which are, or were, overworked in high school are trigonometry and solid geometry. In trigonometry, which previously occupied a full semester, the students spent much of their time computing the widths of imaginary rivers which they could not cross, or computing the height of some flagpole at different times of day — correct to more and more decimal places — using logarithms. This work is now being trimmed down to one-half or two-thirds of a semester, and the emphasis is being placed on the analytic aspects of the subject. In this connection it is interesting to note that in our own course for Margaret Morrison women, S-204 Fundamentals of Mathematics, we treat all of trigonometry in four weeks, and we have found that there is no significant difference between the performance of those students who have had a whole semester's previous work in trigonometry and those who have had none. Similarly, much of solid geometry had consisted of theorems which were of no abiding interest even to professional mathematicians. For this reason, Carnegie Tech in 1957 dropped the subject from its entrance requirements and thus became one of the first of many colleges to do so.

2. *It integrates those topics which are important.* What remains of solid geometry is being combined into a one-year course with plane geometry. This has been found to be a reasonable combination time-wise, and there are obvious pedagogical advantages to treating a given problem in two dimensions and three dimensions simultaneously. Futhermore, plane geometry had been taught in such a sterotyped manner that students all over the country, from Bangor to Berkeley, arrived at the same theorem at Christmas time and then at another thorem at Easter. One purpose of high school geometry is to teach deductive reasoning, and the School Mathematics Study Group (SMSG), in one of its new experimental text books, feels that it has achieved this purpose by introducing a shorter "deductive chain" which takes only ten weeks of study. (I must give credit here to my colleague Professor Borden Hoover who, in several talks before high school teachers in 1956, foresaw that plane and solid geometry could be combined and shortened in this way.)

3. *It introduces recent and important developments in mathematics.* For example, probability and statistics, with which almost everyone is confronted daily in the printed media and on television, was scarcely touched upon in the classroom until five years ago when the Commission on Mathematics of the College Entrance Examination Board came forth with its highly successful experimental textbook, *Probability and Statistical Inference for Secondary Schools.* Professor Frederick Mosteller of Harvard (an alumnus of Carnegie) was invited to deal with the same subject in his lecture series on NBC's Continental Classroom for the spring semester of 1960-61, the first year that a mathematics course was offered for college credit over network television.

4. *It emphasizes the structure of mathematics, rather than isolated topics.* For example, algebra and geometry had been treated as though they were disjoint disciplines, separated by a long summer, and taught by different teachers who had become specialists only within their own subjects. "Theorems" were unique to geometry, whereas "equations" were unique to algebra. Now students are led to understand that these two subjects have their counterparts in each other; that real numbers are useful in proving geometric theorems; and that such underlying principles as the associative and commutative laws, far from being limited to algebra, have important interpretations and applications in other branches of mathematics. Unfortunately, and unbelievably, these strong algebraic laws had often been dismissed by even algebra teachers as being unimportant or else too intricate to bother with, even though they had appeared in bold-face type in the textbooks.

5. *It introduces subject matter to students earlier than was previously thought possible.* For example, the notion of a set, which previously was not mentioned even in high school, is now sometimes introduced in grade school. Group theory which previously had its "corner" in the



junior year in college at the earliest is now being introduced in high school. And calculus, of all things, which the colleges previously guarded jealously, is being taught very successfully in some high schools which can supply the happy combination of able students and well-prepared calculus teachers. While these trends appear off-hand to hold nothing but advantages for student and teacher alike, I should hasten to add that some mathematicians have real fears about the dangers inherent in giving superficial treatment to profound and intricate mathematical concepts. They feel that an inevitable effect will be to train some youngsters to be simply pseudo-sophisticates in mathematics. The tenth-grader and his paper fell short of a real understanding of topology. Perhaps it is this development over the last five years, the introduction of concepts to students much sooner, that has led to the widespread and mistaken notion that sets, binary numbers, and group theory are *new*, in spite of the fact that all of them are at least a century old.

Finally, I would like to comment about the HISTORY of the *new* mathematics. There is no doubt that the *new* mathematics as it is perceived by the layman was swept into the public mind shortly after the successful launching of Sputnik I. A certain evolution in the pedagogy of mathematics was in progress slowly but unremittingly at that time, both on the secondary school and college levels, receiving its greatest impetus from The Commission on Mathematics which was established by the College Entrance Examination Board in 1955. But is was that historic satellite which blew the whole problem wide open. All of a sudden it made the competitive position of America clear, forced the country to re-examine its scientific resources, and focused attention on the fierce shortage of personnel, especially in mathematics, the discipline on which so many other sciences depend. I do not mean for a moment to underestimate the contribution which the vast interplay of political, social and technological events has forever made to the growth of mathematics. I am simply commenting on the explosive appearance of the *new* mathematics on the secondary school scene in 1958.

I recall attending a meeting of mathematicians in Washington, D.C., several months after the launching of Sputnik I, and the atmosphere of the conference was vibrant with the new importance which people everywhere were attaching to mathematics. A distinguished professor exclaimed to some of us in the hotel lobby one day. "Sputnik I has done more for the cause of mathematics and mathematicians in this country than we have been able to do for ourselves in the past two hundred years." It was as though we were poor cousins whose great aunt had just died and left us a fortune; or as though we had just been given membership cards to some elite club and did not yet have the proper formal clothes to wear.

One unfortunate consequence of that surprise attack, and the one which is still causing confusion as to what the *new* mathematics is, or what it should be, was the fact that it caught the forces of mathematicians in almost complete disarray, with the result that we didn't know which way to run and hence have found ourselves running in every direction at once. The *new* mathematics has become a controversy that has pitted old friends, and even old office mates, against each other; and I think that no one is really certain as to what the final outcome of this stimulating debate will be. Certainly to indicate that the matter is resolved, as some authors have chose to do, is an incorrect statement of fact.

What final form the *new* mathematics takes — and it is bound to take a more stable form, since the whole evolution is inevitable — is only a matter for time to decide. Or perhaps I should say for the mathematicians to decide — as they regroup their forces. At the moment, the *new* mathematics is essentially a renewed mathematics — renewed in the attention it has attracted from many interested participants and observers; in the searching reexamination which has been forced upon its pedagogical intricacies; and in its increased importance in and age and society deeply involved in technology — *Allen F. Strehler, Associate Professor, Department of Mathematics.*

[1] See, for example, "Teaching of 'New Math' Stirs Wide Debate Among Teachers," *The New York Times*, September 21, 1962; "The New Math," *The Saturday Review*, January 19, 1963; "On the Mathematics Curriculum in the High School," *American Mathematical Monthly*, March, 1962.

[2] "Debunker's-Eye View of 'New' Math," *Christian Science Monitor*, February 16, 1963.