Accounting Historians Journal

Volume 35
Issue 1 *June 2008*

Article 6

2008

Market for Luca Pacioli's Summa Arithmetica

Alan Sangster

Gregory N. Stoner

Patricia McCarthy

Accounting Historians Journal
Vol. 35, No. 1
June 2008
pp. 111-134

Alan Sangster
MIDDLESEX UNIVERSITY
Gregory N. Stoner
UNIVERSITY OF GLASGOW
and
Patricia McCarthy
OPEN UNIVERSITY BUSINESS SCHOOL

THE MARKET FOR LUCA PACIOLI'S *SUMMA ARITHMETICA*

Abstract: This paper looks at an aspect of Luca Pacioli and his *Summa Arithmetica* that has not previously been explored in detail – the market for which he wrote the book. In order to do so, it follows a path identified by two clues in the bookkeeping treatise as to the nature of this market that modern eyes, unaware of how life was in late 15th century Italy, have missed. After discussing the curriculum taught in schools at that time, this paper considers a range of possible markets for which the book may have been written. The paper concludes that it was written primarily for, and sold mainly to, merchants who used the book as a reference text, as a source of pleasure from the mathematical puzzles it contained, and as an aid for the education of their sons.

INTRODUCTION

Luca Pacioli's mathematics compendium, *Summa de Arithmetica, Geometria, Proportioni et Proportionalita* (*SA*), was first printed and published in Venice in 1494. It included a 27-page treatise on bookkeeping, *Particularis de Computis et Scripturis*. For many years, most accounting researchers have focused upon it, virtually to the exclusion of the remaining 588 pages of the book. To some extent, this is understandable. It is the only significant part of the book that has ever been translated into English[1]; it is the only part that is specifically about accounting;

[1] Some of the introduction to *SA* has been translated into English by, for example, Taylor [1942]. The Index has been translated into English by Volmer [1994].

Acknowledgments: The authors would like to thank Dr. Esteban Hernández-Esteve for his advice on many aspects of this project, particularly in Nantes, France at the 11th World Congress of Accounting Historians in 2006, and also the two anonymous reviewers for their perceptive suggestions.

it represents the first known printed treatise on bookkeeping; and it is widely believed to be the forerunner of modern bookkeeping practice [Fogo, 1905; Littleton, 1928; Langer, 1958; Macve, 1996]. Non-English-speaking accounting researchers have also not deviated from this research focus; the bookkeeping treatise has been translated into at least 13 other languages.

The first section of the book, on arithmetic, was translated or, more accurately, used as the basis for a book written in Spanish in 1514 by Andrés de Saragossa. The arithmetic and algebra sections of *SA* were the source of two-thirds of an anonymous Catalan manuscript containing 160 folios on algebra and commercial arithmetic dating from the early 16th century [Rey, 2006]. In general, however, researchers from other disciplines[2] have failed to publish translations of significant parts of the book.[3] What has been translated and published includes some of the mathematical problems by, for example, Jayawardene [1976] and Rankin [1992] and extracts of some of the mathematical content [e.g., Fauvel and Gray, 1987, pp. 249-252].

Many researchers have suggested that the bookkeeping treatise was sourced from a teaching manual circulating at the time in Venice [see, Peragallo, 1938, p. 74, fn. 16]. Despite all the efforts of the renowned Italian accounting researcher, Fabio Besta (1845-1922) and his students in the Venetian archives [see Vianello, 1896, p. 116], no handwritten text on accounting has ever been found that predates Pacioli's bookkeeping treatise other than a five-page overview of bookkeeping in a manuscript written in Naples in 1458 by Benedetto Cotrugli [Melis, 1950, p. 597; Tucci, 1990; Jouanique, 1996].

There are virtually no worked examples in the treatise, and many have suggested that this creates a lack of clarity in the explanations of the method described. It has also been implied or concluded that the treatise was inadequate for those wishing to teach themselves double-entry bookkeeping (DEB) [Geijsbeek, 1914; Hernández-Esteve, 1994a, b; Yamey, 1978, 1994a, b, 2004;

---

[2] Non-accounting researchers have focused on different aspects of the book from those typically considered by accounting researchers, such as the range of mathematical puzzles it contains and their similarity to those contained in other texts; and explaining why different founts are to be found in different extant copies of *SA*.

[3] This is not to say that the remainder of the book was of little interest beyond the period of its publication. The second section, which is on algebra, was heavily cited by Renaissance algebraists in the years that followed its publication and is said to have been the enabling framework for the advances in algebra made in the 16th century [Rose, 1976, p. 145; Grendler, 2002, p. 427].

Nobes, 1995]. This conclusion appears entirely justified. Anyone using the bookkeeping treatise to learn how to do bookkeeping would need to have either been in business himself or, as suggested by Yamey [1978, p. 580], to have known someone he could ask for help in following it.

The lack of worked examples can itself be explained if Pacioli's source was a Venetian manual on bookkeeping. It is entirely conceivable that such a source might not have included worked examples as they could have been added by teachers when they went through the material with their students. Nevertheless, Pacioli was a renowned teacher who would have realized the benefits of including examples. Furthermore, he must have had some examples of his own that he could have used since he worked as a tutor to the sons of a Venetian merchant, was an assistant to that merchant for six years from 1464-1470, and was in business as a merchant in Naples for a few months in 1472 [Taylor, 1942, p. 170].

The First Clue to the Identity of the Intended Market for SA: The virtual absence of worked examples in the bookkeeping treatise must, therefore, have been the result of Pacioli believing that there was little benefit in incorporating examples within it, something apparently confirmed when he writes: "It is not possible to give here full examples for all these operations, but from those few that we give here you will be able to understand how to go ahead in other cases" [Pacioli, 1494, folio 203 recto, translated by Geijsbeek, 1914, p. 51].

This must have been, at least in part, because his intended audience for the treatise was merchants [see, Peragallo, 1938, p. 56]. This goes some way towards explaining the absence of worked examples. Even without them, merchants would have been able to understand and learn how to adopt the Venetian method of DEB as described by Pacioli. In addition, bookkeeping was one of the subjects taught in the Venetian *abbaco* schools attended by the sons of merchants [Grendler, 1989, p. 319]. As a result, for them and their merchant fathers, Pacioli's text would have been relatively easy to follow, but, surely, even merchants would have appreciated and benefited from the inclusion of worked examples in the bookkeeping treatise.

The Second Clue to the Identity of the Intended Market for SA: Even 50 years after the publication of *SA*, the fact that the intended readers were likely to have access to other material probably would not have resulted in the omission of worked

examples from the bookkeeping treatise but, in 1494, book publishing was very different and printers were still learning how to do things in the most efficient way. Even the printing of simple geometrical figures was a relatively new technique in 1494. Paper was expensive, half the cost of producing a book [Richardson, 1999, p. 26]. Including worked examples would have significantly increased the length of the bookkeeping treatise, perhaps by as much as 30% if modern texts are a guide. It would also have considerably increased the complexity, and therefore the cost, of the typesetting and required many costly wood blocks to be carved or metal plates to be cast. It is unlikely to have been an accident that the journal entries shown on the last page appear after all the text. For pragmatic economic reasons, if material was not considered essential in a printed book in the late 15th century, it was omitted. This observation is supported by Pacioli's own words: "For if we wanted to give you an example of all the ways in which merchants do business… this would make our treatise very long, which, on the contrary, I intend to make short" [Pacioli, 1494, folio 203 recto, translated by Geijsbeek, 1914, p. 51].

This reluctance to print unnecessary content raises another question, if nothing was included in books at that time that was not considered essential, why did Pacioli include a bookkeeping treatise for merchants in *SA*, a book on mathematics? Surely the last thing anyone interested in mathematics wanted to read was a treatise on bookkeeping. To modern eyes, this would almost certainly be the case, but was it also the case in 1494? Was there a group of people for whom the bookkeeping in *SA* was every bit as important as the rest? Could merchants, for whose benefit Pacioli explicitly included the bookkeeping treatise in *SA*, have been interested in a book on mathematics, or was it some other group altogether to which *SA* in its entirety was principally directed?

Yamey [2004, p.144] suggests that to comprehend the *SA* required the reader to possess a humanist education, which merchants typically did not, but he fails to justify that contention and then goes on to suggest that it was purchased by "mathematicians and other learned individuals rather than by merchants." Could this have been the case? Was there an alternative, more likely market for the book than mathematicians?

The remainder of this paper addresses these questions by seeking to identify the group for whom bookkeeping was of as much interest as mathematics. Far from being a strange choice of material to include in *SA*, the bookkeeping treatise was a vital

component that made *SA* a comprehensive reference book to its primary-intended readership.

It seeks to approach this issue from two directions. First, having established that bookkeeping was taught in some schools in Renaissance Italy, consideration is given to other subjects taught to whom in those schools and, in particular, whether the curriculum included mathematics of the type included in *SA*. Second, consideration is given to the content of *SA* in relation to the major occupations and groups who may have had an interest in at least some of its topics when published, including those suggested by Yamey [2004].

MATHEMATICS AND SCHOOLING IN RENAISSANCE ITALY

The developments of mathematics and accounting were intertwined during the Renaissance. Mathematics was in the midst of a period of significant development in the late 15th century. Hindu-Arabic numerals and algebra were introduced to Europe from Arab mathematics at the end of the 10th century by the Benedictine monk Herbert d'Aurillac [Hernández-Esteve, 2006]. But it was only after Leonardo Pisano (Fibonacci) put commercial arithmetic, Hindu-Arabic numerals, and the rules of algebra together in his *liber abaci* in 1202 that Hindu-Arabic numerals became widely used in Italy.

Algebra did develop slowly over the following 300 years [Rankin, 1992]. Even in the late 15th century, the notations used when writing mathematical computations and algebraic equations were not standardized and were far more cumbersome than today. There were no signs for plus, minus, divide, multiply, or equals; no use of superscripts for powers; no root symbol; and no use of letters to denote parameters/variables in algebra. In *SA*, Pacioli introduced the symbols p̃ (for piu, i.e. plus) and m̃ (for meno, i.e. minus) for the first time in a printed book, symbols that became standard notation in Italian Renaissance mathematics. *SA* was also the first known book printed in Italy and written in the vernacular (i.e. the spoken language of the day) to contain algebra.[4]

The manner in which mathematics developed in Renaissance Italy owed much to the commercial revolution following the Crusades and the resulting expansion of trade and the establishment of a system of agencies distant from the center

[4] For more information on the development of mathematics, see Rose [1976], Grendler [1989, 2002], and Rankin [1992].

of the business and long-term partnerships rather than one-off business ventures between two persons. Gras [1942, p. 28] describes this as the rise of the sedentary merchant: "...a merchant too wise, too occupied, too economical to travel. His distant connections were maintained by agents, traveling or resident.... The outstanding sedentary merchants were called merchant princes."

De Roover [1956, p. 115] suggests that three factors integral to the commercial revolution of the 13th century contributed most to the progress of accountancy – partnership, credit, and agency. Consequently, as described by De Roover [1942, p. 35], one necessary result of this commercial revolution of the 13th century was the need for more advanced systems of accounting:

> One innovation of major importance was the current account kept in bilateral form, that is, the personal account divided vertically into two columns, one for the debit and one for the credit. Later, double-entry bookkeeping was introduced by adding impersonal accounts to the existing personal accounts. Good methods of bookkeeping were essential in order to keep accounts straight when two persons, residing in different cities, had numerous business dealings with each other. Merchants had to know where they stood, and accounting served as a guide by revealing profits and losses.

Apart from the development of DEB, this expansion of trade gave rise in Italy to the development of mathematics useful for merchants based on Fibonacci's *liber abaci* and called *abbaco*[5] which meant solving practical, business-related mathematical problems on paper [Grendler, 1989, p. 308]. This term had little to do with the word "abacus" used for a counting frame other than sharing a common etymological root [Van Egmond, 1981, p. 5], a position reinforced by Pacioli himself who wrote on folio 19 (recto and verso) of *SA* that he thought that "*abaco*" was either a corrupted form of "*modo arabico*" or a Greek word. The emergence of *abbaco* led, in turn, to the creation of a new type of mathematician, the *abachist*, and to the founding of a new form of school, the *abbaco* (or *abbacus*[6]) schools [see Rankin, 1979; Burnett, 2005; Blume et al., 2007].

---

[5] sometimes spelt, *"abaco"*

[6] Many writers use the term "abbacus" or "abacus" instead of *"abbaco"* when referring to these vernacular schools, presumably because the abacus was used in these schools when they first began to appear in 13th century Italy. Use of the abacus in the schools soon ceased, replaced by pen, paper, and ink used to write numbers in columns ("place-value numerals") [see Pin, 1993, p. 168; Burnet, 2005].

Abachists[7]: An *abachist* was a school teacher who taught boys[8] commercial mathematics and elementary accounting (always in the vernacular) [Grendler, 2002, p. 420]. The accounting was of two main types, *quaderno* (the ledger) and *far conti* (bookkeeping). Pacioli's bookkeeping treatise is the first indication we have of how these topics, which were taught for many years before *SA* was published, were taught. [Grendler, 1989, p. 316] The majority of known *abachists* during the 14th and 15th centuries were Tuscan, mostly from Florence [Grendler, 1989, p. 308]. Domenico Manzoni,[9] the author of what is considered the first important book on DEB after Pacioli's bookkeeping treatise (on which it was directly based) [Peragallo, 1938, p. 60], first published in 1534, was an *abachist* [Grendler, 1989, p. 309].

Fibonacci's *liber abaci* formed the basis for hundreds of *abbaco* texts written by the *abachists* which were used in the private and municipal schools where the sons of merchants were taught [Allen, 2000]. These texts were more than simply textbooks; they were didactical supports for the teachers, that is, instructor manuals [Pin, 1993, pp. 169-170]. The oldest surviving example of these texts dates from the late 13th century.

Schools in Renaissance Italy: Trade and, therefore, merchants dominated Renaissance Italy, and one of the results of their dominance was the creation of private and municipal schools in which their sons and the sons of craftsmen were educated. All lessons were given in the vernacular [Grendler, 2002, p. 420] with a focus on the teaching of *abbaco*. The curriculum of the vernacular schools emerged from the merchant culture and was designed to prepare sons of merchants and craftsmen for their future working lives [Grendler, 1990]. There was another parallel set of schools, the Latin (either scholastic or humanist) schools, where the sons of the privileged were taught in Latin.

The two sets of schools taught very different subjects. The Latin schools sought to teach the future leaders of society and those that aided them, e.g., secretaries and lawyers [Grendler, 1989, p. 311]. They specialized in the trivium of grammar, rhetoric, and logic. *Abbaco* was never included "because it added nothing to the social status and goals of their students" [Grendler, 1989, p. 311]. On the rare occasions when mathe-

[7] sometimes spelt "*abacist*;" sometimes referred to as "abacus master"

[8] Very occasionally, girls were also taught by the *abachists*.

[9] Manzoni also published a self-teaching book in 1550 covering his vernacular school's entire primary and secondary level curricula [Grendler, 1989, p. 309].

matics was taught in these schools, it took the form of "classical or medieval Latin mathematics" [Grendler, 1989, p. 309]. In contrast to the vernacular schools, boys leaving the humanist schools often went to university.

Typically, the boys in the vernacular primary schools were aged between six and ten and were taught reading, writing, business correspondence, and notarial formulas. From 11, they moved to vernacular secondary schools, the *abbaco* schools, where they read books by the likes of Aesop and Dante and abridged vernacular versions of Fibonacci's *liber abaci*.[10] Pin [1993, p. 168] and Grendler [2002, p. 420] provide lists of the topics they covered, including elementary accounting and how to solve business-related problems, such as interest calculation, loans discount, money exchange, partnership divisions, measurement, currencies, weights, and distance problems. "The mathematics employed combined arithmetic, [computing with numbers, especially decimals], algebra, geometry, and what might be called ingenious reasoning" [Grendler, 2002, p. 420].

They were also taught some basic Latin grammar at some point of their education, often at the vernacular primary school, sometimes after the *abbaco* teaching was completed, but the vast majority of the vernacular secondary school teaching was in commercial mathematics for merchants, focused on two main elements, geometry and arithmetic. At the core of the arithmetic was the study of proportion. The main rule taught in this area was the "rule of three," a very simple method of finding an unknown from three known inter-related items still in use today [Baxandall, 1972, p. 94]. Boys in the vernacular schools learnt how to use the rule to solve problems involving many more variables by reducing such problems down to one involving three inter-related known items and one unknown.

An example of the rule of three is the following: if you want to know how much 500g of oats will cost when the price of 600g is $3, you multiply $3 by 500 and divide by 600 and get the answer, $2.50. The rule of three was used for all manner of problems during the Renaissance, including discount, barter, and currency exchange. Virtually half the content of all known books on arithmetic of that era focused on this one rule [Baxan-

[10] In some schools, different sequencing of topics was used. Some, for example, taught reading, writing, *abbaco*, and accounting to boys as young as six but, overall, the subjects studied across the vernacular schools were relatively similar. Accounting was taught more in the major commercial centers like Venice than elsewhere, and *abbaco* was not taught in a small number of the schools [see Grendler, 1989 for a detailed description of schooling in Renaissance Italy].

dall, 1972, p. 96]. In modern-day schooling, we learn the same technique through concepts such as distance equals speed times time and questions such as, if it takes one man a week to dig a ditch, how long would two men take to dig the same ditch?

As a result, all those educated in the vernacular schools were well-prepared for a career in commerce. They knew elementary accounting, how to read and write, and basic Latin grammar, which meant they could recognize, read, and understand some Latin. Although they would not have had sufficient fluency in Latin to attend university, they would not appear completely ignorant by comparison. They also had a good grounding in geometry, arithmetic, and proportions which, by virtue of their focus in the schools, was most relevant to merchants and customers of merchants as well as craftsmen such as artists, architects, engineers, and stonemasons.

Their knowledge of mathematics formed a major part of their set of personal skills. They used it daily, not just in their work. It was such a part of their overall knowledge that they joked about it, played games using it, bought books on it, and were extremely proud of their ability to apply it [Baxandall, 1972, p. 101].

SUMMA ARITHMETICA

Luca Pacioli was not simply a friar and a university teacher, he was also an *abachist*. He taught *abbaco* and bookkeeping to the sons of a merchant, Rompiasi, in Venice for six years from 1464 [Grendler, 1989, p. 320]. Camerota [2006, p. 327] wrote:

> Pacioli, one of the foremost abacus masters, was active not only in the abacus schools but also in the artists' workshops. Among his pupils were painters, architects and stonemasons, and the applications of Euclidean geometry are specifically identified by him as pertinent not only to the art of merchants and surveyors, but also to architecture, linear perspective, sculpture, wooden inlays, fortifications, the construction of machines and the arraying of armies.

Pacioli used the title "*Magister*" [Taylor, 1942, p.148], which indicated that he was a pre-university teacher[11] [Grendler, 1989, p.

[11] Others, including Taylor [1942, p. 149], take Pacioli's use of the title "*Magister*" to mean he had been awarded a higher university degree between 1480 and 1486, during which period his name is completely absent from any extant university roll. It does seem that the *abachist* sense of the word is more plausible in this case.

25]. Pacioli, however, was a rarity in his day. Very few *abachists* were also university teachers; Grendler [1989, p. 28] could only identify two others. To have been appointed as the first chair in mathematics at two different universities [Taylor, 1942], his skill in geometry must have been exceptional.

In *SA*, Pacioli claimed that he made no original contribution to mathematics. He did, however, present new approaches to arriving at solutions to old problems [see, Rankin, 1992, p. 131]. He simply sought to incorporate within one book all extant knowledge on arithmetic, algebra, geometry, and, although it was not separately recognized at the time, trigonometry in a manner that provided lucid explanations of the subjects.[12] These were virtually all the topics taught in the vernacular *abbaco* schools, and they were included in *SA* in a didactic style, mirroring that in which they were taught in those schools.

In order to do so, he "based *Summa Arithmetica* on several manuscripts from the [*abbaco*] tradition" [Heefer, 2005, p. 16]. Field [1999, p. 301] states that, "*Summa*… was the first[13] printed treatise to deal with the kind of mathematics associated with the [vernacular *abbaco* schools]." Rowland [1995, p. 702] claims that "Pacioli's *Summa de Arithmetica*…may be the most elegant and compendious of all vernacular manuals," while Høyrup [2004, p. 2] writes that "it was…obvious to those…who did work on *abbaco* material that it belonged within a current leading from Fibonacci to Luca Pacioli,[14] Tartaglia and Cardano." In other words, Pacioli picked-up what Fibonacci wrote and incorporated additions to date. Others then followed him.

As Maccagni and Giusti [1994, p. 18] wrote: "[*SA*] was an all encompassing work which summarised and rendered obsolete everything previously written about *abbaco*." This is why it includes 150 pages devoted to matters directly related to commerce, including a treatise on DEB, material on barter, bills of exchange, weights and measures, and exchange rates. It is

[12] Pacioli [1509b] was famed for his ability to provide clear explanations in both lectures and in his texts, for example, in relation to his translation of Euclid into Latin. A contemporary, Daniele Caetani of Cremona wrote, "Lucas Paciolus has penetrated between the Symplegades and into the many swirling Charybdis of error, the road has been made even, the passage safe, the route unencumbered through the dark byways, and the true Euclid has been brought back accessible to everyone. This has been done by the very skilful genius, the careful revision, and the unwavering judgement of our Master Lucas" [Taylor, 1942, p. 320].

[13] Field was incorrect as at least three others are known to have been printed in Italy earlier [Van Egmond, 1981], but it was certainly one of the first and certainly the most comprehensive.

[14] See, also, Franci and Rigatelli [1985].

why the bookkeeping treatise contains advice on how to write business letters. It is one of the reasons why it is written in the vernacular. It is likely to be why some of the factual details in the bookkeeping treatise were out-of-date in 1494,[15] especially if it was the bookkeeping manual that he himself used or wrote while tutoring the sons of Rompiasi 30 years earlier. It is also why, in several places throughout the rest of *SA*, Pacioli "makes detailed analysis of Venetian mercantile transactions" [Peragallo, 1938, p. 74, fn. 16, paraphrasing Pendorf, 1933].

SA's identification as a *liber abaci*, an *abbaco* text, leaves little doubt that Pacioli wrote *SA*, at least in part, for merchants and for their sons, a point that will be developed more fully in the next section. However, it was also a true mathematical "*summa*," a compendium of extant mathematical knowledge, and the first *abbaco* text to widen its audience beyond the merchant class [Jayawardene, 1973, p. 512] There were others who would have been interested in at least some of the content of *SA*, including those suggested by Yamey [2004]. Could their potential interest have meant that the primary intended market for *SA* was not the merchant class? Before considering this point further, it would be appropriate to examine first the actual content of *SA*.

The Structure and Contents of SA: At 615 pages, *SA* was a very large book, the equivalent of a 1,500 page textbook if typeset today [Pin, 1993, p. 165]. Its content clearly went beyond the level of the other *abbaco* texts, particularly the algebra. It was the complete technical manual for merchants and, for its day, contained the most comprehensive range of material available to meet their needs. After 16 introductory pages, the material in *SA* is presented in ten primary chapters, printed and separately paginated into two volumes: [16]

Volume 1	
1 to 7	on arithmetic (222 pages)
8	on algebra (78 pages)
9	on business (150 pages) divided into 12 sections, the first ten on various items relevant to business (including barter and bills of exchange), the eleventh on bookkeeping (27 pages), and the twelfth on weights and measures and exchange rates
Volume 2	
10	on geometry and trigonometry (151 pages)

[15] See, for example, Nobes [1995, p. 383].

[16] These paginations are taken from the 1494 edition.

Taking into account the overall content and sequencing of the material within it, the price at which it sold, the curricula of the schools and universities, the demographics of those attending them and working in them, and societal values of late 15th century Italy, various conclusions concerning the probable significance of possible markets for *SA* can be drawn.

POTENTIAL MARKETS FOR *SA*

Students of Mathematics in Universities: The initial impression to the modern eye is usually that *SA* was probably intended for use in universities. However, had Pacioli intended it primarily for that market, he would have concentrated on the geometry, trigonometry, and algebra. The business material would have constituted a separate volume, if included at all. Commercial arithmetic, the first seven chapters of *SA*, and business were not taught in universities at that time. Instead, the geometry and trigonometry were paginated separately from the rest of the book, clearly packaged as a separate volume, while the algebra is packaged with the commercial arithmetic and business material.

In addition, university teaching in late 15th century Italy was conducted in Latin, which was also the language of the textbooks used [Grendler, 1989]. *SA* is written in the vernacular with occasional phrases in Latin [see Lee, 1989; Marinoni, 1997, p. x; Field, 1999, p. 301]. Its selling price of 119 soldi [Dunlop, 1985, p. 153], just under 1 ducat, or about $150 in today's terms, would also have reduced any possible demand from these students.

Finally, there were only a maximum of 5,400 students and less than 500 university faculty in all Italy in the late 15th century of whom no more than 26 were teaching mathematics [Grendler, 2002, pp. 415, 515]. Clearly, only a small number of the students were studying mathematics. Thus, despite how it appears to modern eyes, it is unlikely that students of mathematics in universities would have been considered a significant potential market for *SA*.[17]

---

[17] An earlier manuscript written by Pacioli in 1477-1478 [see Vianello, 1896 and Cavazzoni, 1994 for a description of the manuscript] is thought by some accounting researchers to have been written for Pacioli's university students. One reviewer suggested that, if so, *SA* may also have been written for university students. However, this would contradict what is known about the language of university teaching, the language of the textbooks used, and the subjects taught at that time.

University Teachers of Mathematics: Yamey [2004] suggests that mathematicians, presumably both university teachers of mathematics and non-university mathematicians, would have been among the purchasers of the book. Let us examine university and non-university mathematicians in turn.

There were, on average, only one or two mathematicians per Italian university until the 17th century and only 13 universities in 1494 [Grendler, 2002, pp. 2, 415]. This maximum of 26 university mathematicians in Italy[18] would have had access to other sources for the same material in a form and style that better fitted their own humanist education and the need to teach in Latin, such as the 1482 printing of the Latin translation of Euclid's Elements by Campanus (1220-1296).

Even if the university teachers of mathematics preferred to have all the material they needed in one book, *SA* was not a book that they would have bought for its whole; they would not have been interested in the 150 pages on business which was not taught in universities. For the same reason, they would not have been very interested in the 222 pages on commercial arithmetic. "[They] taught (in Latin) mathematics, astronomy, and astrology…to men who would become physicians, philosophers, legists, and humanists" [Grendler, 2002, p. 420]. They would, therefore, have had little motivation to spend a week's salary on a copy of the book.[19]

Nevertheless, some of the more enlightened among them did purchase it or, at the very least, read it and used it in the development of their own books. Included was the renowned mathematician and physician, Girolamo Cardano of Milan [Rose, 1976, p. 146], though he had switched to become a professor of medicine long before doing so.

The market for *SA* to university teachers of mathematics would clearly not have been large enough to justify printing extra copies of the book for that market. Even if every one of them had purchased it, no more than 20 sales would have been generated. Given an estimate that as many as 2,000 copies of *SA* were printed in 1494 [Sangster, 2007], it was not a book primarily intended as a reference text or source of teaching material for university mathematicians.

---

[18] If *SA* was intended for use in universities outside Italy, it would have been written in Latin rather than in the vernacular.

[19] The annual salary of a university teacher of mathematics in the late 15th century was between 40 and 60 ducats.

Non-University Mathematicians: Renowned mathematicians were not only to be found in universities in Renaissance Italy. Leaving aside most of the *abachists*, who were primarily teachers rather than theoretical mathematicians,[20] the foremost Renaissance mathematicians included Piero della Francesca, an artist and author of three scribal books on mathematics, including an *abbaco* text; Federico Commandino, a court mathematician and physician; Bernardino Baldi, a poet, court historian, abbot, architectural historian, orientalist, and polyglot (also the first biographer of Pacioli); Rafael Bombelli, an engineer and architect; Giovanni Battista Benedetti, a court mathematician; and Guidobaldo de Monte, a nobleman.

Many non-university mathematicians were educated in the vernacular schools and would therefore have had far more interest in and empathy for the contents of *SA* than university mathematicians who could not use most of it in their teaching. Yamey [2004] believes mathematicians would have been among the purchasers of *SA*. Indeed, some non-university mathematicians are known to have done so, including Leonardo da Vinci [Reti, 1968, p. 86, note 93] and Fillipo Calandri [Dunlop, 1995, p. 152]. Others, including Commandino, Baldi, and Bombelli [Rose, 1976, pp. 146, 264], are all known to have read *SA* and, in the latter two cases at least, to have made use of it in writing books of their own. However, as with university teachers of mathematics, there simply were not enough non-university mathematicians to create a significant market for the book. *SA* would have been useful to them but was not written primarily for them.

Abbaco *School Teachers*: The *abachists* would have been able to use all the material in the book in their work. However, estimates suggest that there were only approximately 58 *abachists* in Venice around the time *SA* was published.[21] Clearly this was far more than the number of university teachers of mathemat-

[20] A few of the *abachists* were mathematicians rather than simply teachers of mathematics. These included Niccolò Tartaglia, one of the foremost mathematicians of the 16th century.

[21] Census data for 1587-1588 shows that there were 258 pre-university teachers in Venice, of whom 75 taught in the vernacular schools. There were 2,160 pupils in the humanist schools and 2,465 in the vernacular schools [Grendler, 1989, pp. 48-49]. The population of Venice in 1587-1588 was 149,000 [Grendler, 1989, p. 31] and 115,000 in 1509 [www.wga.hu/database/glossary/cities/venice.html]. Assuming linearity between population, teachers, and pupils, there may have been approximately 58 vernacular school teachers in Venice around the time *SA* was published.

ics, but it does not represent a very significant market. Even if extrapolated to all of Italy, it is unlikely to have amounted to more than one thousand individuals. In addition, they typically earned less than university teachers, so only the more successful would have been able to afford a copy easily. Nevertheless, some consideration must have been given to this potential market when the decision to print 1,000 to 2,000 copies of *SA* was made [Sangster, 2007].

Craftsmen, such as Architects, Artists, Engineers, and Stonemasons: As an *abbaco* text, the content of *SA* is aligned with the *abbaco* instruction that is likely to have formed the basis of this group's education [see Camerota, 2006]. The advanced material in Volume 2 of *SA* was highly suitable for those craftsmen who might use it in their work [Olschki, 1919], a market targeted directly when Pacioli published *De Divina Proportione* [1509a]. However, most craftsmen would have had much less work-related use for the contents of Volume 1.

Therefore, the book in its entirety was not likely to have appealed to craftsmen as an aid to their work unless they ran their own businesses, although the positioning of geometry in a separate volume was probably intended to enable that volume to be sold separately to this group should they wish to purchase it for that purpose.

Evidence from library catalogue entries relating to extant copies suggests that they did not do so as virtually all extant copies contain both volumes. This is unsurprising since craftsmen often worked for wealthy patrons and were therefore likely to have been able to consult the book from the library of their patron should they have felt the need to do so. They could even have requested a patron to purchase a copy for that purpose.

It is, however, possible that a craftsman whose son was attending a vernacular school would have purchased the book. It is also possible that a more successful craftsman may have purchased the book for the entertainment he would have derived from the problems it contained in the arithmetic and algebra sections [see Baxandall, 1972, p. 101]. While this was not a large market compared to the much more affluent merchant market, it is possible that the two volume division of *SA* was created so as to increase the appeal of the book to this group.

The Upper Classes: Pacioli used Latin in the Introduction, probably to impress the Italian upper classes and, in particular, his patron, the Duke of Urbino, to whom *SA* is dedicated. In addi-

tion, the initial letter of each section throughout *SA* is always represented by a decorative woodcut initial except the first two, which were left blank, to be hand-painted. This would have helped give the book the feel of high quality in the eyes of bibliophiles as it would have stood out to anyone opening the book and glancing at the first few pages. The fact that *SA* contained many more pages than a typical book of the period would also have been a major selling point in this market for large books were considered to be "important books" during the Renaissance, irrespective of their content [Harris, 2006]. Simply having a book of this size in a personal library would impress anyone who saw it.

However, despite Pacioli's clearly having done what he could to convince this market of the merits of the book, the content is unlikely to have motivated its purchase. Apart from any who enjoyed mathematics as a hobby, which would have been relatively few as they were almost without exception educated in Latin schools where *abbaco* was not taught, those who did purchase it would generally have done so for its appearance rather than its contents. Some of the more enlightened humanists would have warmed to the use of the vernacular, but many may have been alienated by both the use of a gothic typeface [Grendler, 1989, pp. 323-324] and by the "modern" focus and sources of much of the content.

Consequently, apart from the Introduction and the two blank initial letter sections, the content of *SA* was not affected by this potential but small market. Nevertheless, it was a market into which Pacioli, the sponsor of the book, or the printer of the book could have presented "free" copies and gained the rewards in return that were typical of the era, such as money or property.[22] *SA* was neither written primarily for this market nor for anticipated small sales into this market, contrary to the view expressed by Yamey [2004].

Merchants: The discussion in earlier parts of this paper concluded that merchants were probably the primary market for *SA*. All but the most experienced merchants would have benefited from instructional material in the book relevant to their trade, such as estimating the contents of a container of any size or proportion, a crucial aspect of most merchants' trade and

[22] See, for example, Richardson [1999, pp. 51-56] for a discussion of this form of income generation, which almost certainly explains the content of the Introduction to the book.

one of the principal subjects covered in the geometry volume; how to calculate complex numerical problems quickly and accurately; how to calculate exchange rates when multiple currencies and/or barter were involved, both common occurrences in 1494; the steps to take to ensure that debts were honored; and how to ensure that the accounting records of the business would be accepted as legitimate.

Pacioli emphasizes the usefulness to merchants of the material preceding the bookkeeping treatise, which includes 300 pages on arithmetic and algebra, by stating at the start of the treatise (Folio 198 verso) that all that preceding material is essential for an understanding of the content of the treatise.[23] A merchant would have seen even more use for this earlier material than Pacioli suggests for the first two sections on commercial arithmetic and algebra contain numerous examples applied to business. Other elements of the book, the manner in which it would have been marketed and the background of the merchants themselves, increased the likelihood of purchase by this group.

Geometry and Proportions: To merchants, all the material after the bookkeeping treatise would have appeared no less important than the rest of *SA*. Knowledge of geometry was a fundamental attribute of being a merchant in the late 15th century, and the use of the words, "*Proportioni et Proportionalita*" in the title of the book would have been a clear indicator to merchants that the book contained material relevant to them on that topic, recalling their time spent studying gauging (geometry) and proportions in their *abbaco* schools.

The Influence of the Bookseller: It would not have taken more than a quick glance inside the book for merchants to have confirmed that *SA* could be an ideal reference text for them to use in their business, especially as the bookseller would swiftly have guided them to what Pacioli wrote about the rest of the book in both the introductions to the bookkeeping treatise and to the book itself.

First Generation Merchants: Just as is the case today, merchants were not necessarily themselves sons of merchants. They may

[23] The text states that the second (of three) things necessary in business is to be a good bookkeeper and ready reckoner, and, for this reason, all the basic canons and rules required for any operation are included in the preceding sections of the book.

well have had the benefit of *abbaco* schooling, but they would have been unable to obtain advice on business from their fathers. *SA* would have provided much of the advice they may have otherwise received from that source. The bookkeeping treatise, for example, has many recommendations for merchants, such as only keeping one set of books, avoiding taking advice from the ignorant, and working hard to be successful.

*Mathematics as a Hobby (*Abbaco *School Attendees)*: Having attended an *abbaco* school, many of the merchants would have been devotees of recreational mathematics and would, as with craftsmen, have been attracted to the book by the large number of problems it contained in the arithmetic and algebra sections. Merchants would generally have been more financially equipped than the craftsmen to purchase it.

Merchants with Sons Attending an Abbaco *School or Being Privately Tutored*: Merchants with sons attending an *abbaco* school would have had an incentive to purchase the book so that their sons could use it in their studies. Usual practice was for pupils of those schools to take whatever relevant texts their family possessed to school [Grendler, 1989]. Even Venice alone was a large potential market. Around the time *SA* was printed, there were probably some 1,900 pupils in the vernacular schools of Venice of whom approximately 80% were at the secondary *abbaco* level, the majority the sons of merchants [Grendler, 1989, p. 49].

In addition, many merchants who did not send their sons to vernacular schools employed private tutors to educate them. *SA* would have been extremely useful to those tutors in providing them with the material they needed to carry out their task. Merchants in either situation would have had the double benefit from owning a copy; a reference text for their own use and one that could be used to educate their sons.

The Bookkeeping Treatise: The bookkeeping treatise itself would have been invaluable to many merchants. While bookkeeping was one of the subjects taught in the *abbaco* schools and by the tutors of merchant sons, the method taught depended upon the knowledge of the teacher and any text used by that teacher. *SA* contained what Pacioli described as instructions in how to use the superior[24] Venetian method. Consequently, merchants could

[24] Yamey [1994b] uses the term, "the best"; Nobes [1995, p. 383] suggests that Pacioli used a term meaning, "much recommended."

follow its advice on how better to maintain their own accounting records. Further, those merchants who employed a bookkeeper could instruct them to use it in order to switch to the Venetian method or improve its application. The merchant could also use the bookkeeping treatise as a guide to ensure that his bookkeeper was actually recording inventory and transactions in the appropriate manner. This would have been particularly useful to merchants lacking either the benefit of *abbaco* schooling or a bookkeeping tutor for they would have been open to fraud by their bookkeeper, a theme raised by Pacioli in the treatise. *SA* effectively gave merchants the capability to audit their bookkeepers through standardizing the DEB method in use.

At 119 soldi, *SA* was a very expensive book in comparison to *Aesop's Fables*, an extremely popular book of the period used in the primary years of the vernacular school curriculum, which sold for approximately 2 soldi [Richardson, 1999, p. 115]. Merchants were the new middle class, many of whom were extremely wealthy. Even those who were not would have understood the advantages of possessing a text like *SA* and view it as a worthwhile investment.

It is said that copies of *SA* flew off the shelves of booksellers as merchants from all over Europe rushed to obtain a copy [Favier, 1998]. While this was probably an exaggeration, *SA* sold steadily over a long period [Sangster, 2007], presumably as merchants came to hear of its existence and appreciate its relevance to them. This longevity of sales plus the number of extant copies, at least 120 of the 1494 edition [Sangster, 2007] support the view that it was used as a reference text [Harris, 2006] to be consulted when needs arose. The high survival rates are consistent with a book's use for reference.

CONCLUSION

Analysis of previous criticism of Pacioli's bookkeeping treatise in the context of life in 15th century Italy identified two clues as to the intended primary market for *SA*. These clues lead to an explanation for the inclusion of material on bookkeeping and may also account for the presence of other material in the book. The first clue was the criticism that the virtual absence of any worked examples in the bookkeeping treatise meant that only merchants, or those with access to merchants' records or to suitable advice, would be able to follow fully the Venetian method of double entry. This is consistent with Pacioli's own statement in the Introduction that it was intended for merchants and

written to promote the adoption of the superior Venetian method of DEB. The lack of worked examples meant that anyone else would have found it difficult to understand or learn DEB from the treatise. This then raised the question as to why include it in *SA* if the people who could follow and learn from it were not also intended to read the remainder of the book. Did this mean that the rest of the book was also intended for merchants?

The second clue was the fact that printing was in its infancy in 1494, and that nothing was included in a printed book unless considered essential. Pacioli would easily have been able to provide worked examples to include in the text but chose not to.

Putting both these clues together leads to the conclusion that the bookkeeping treatise was not only intended to be read and used by merchants and their sons, it was designed specifically for them. Further analysis of the content and sequencing of *SA* indicates that the entire book was written primarily as a reference text for merchants and as a school text for their sons. It was sourced mainly from *abbaco* texts and mirrored much of the curriculum of the *abbaco* schools attended by the sons of merchants. It was an extended text that included all extant material known to Pacioli to be directly relevant to merchants. No *abbaco* school or tutor would previously have had access to such a wide range of relevant material in a single source.

Following a discussion of the nature of schooling in Renaissance Italy, the importance of a number of other potential markets for *SA* was considered. As a result, it has been possible to discount the possibility that the book was written primarily for university students or university teachers of mathematics, principally because of its having been written in the vernacular and its lack of relevance to the university curriculum. A similar conclusion was reached concerning the upper classes.

Small markets were identified for non-university mathematicians, particularly because of Pacioli's treatment of algebra, and craftsmen (e.g., architects, artists, engineers, and stonemasons), principally because of the geometry section of the book and the problems in the arithmetic and algebra sections. A potentially larger market was identified in the form of *abbaco* school teachers and those who acted as private *abbaco* tutors to sons of merchants and craftsmen because of the alignment of the entire book to the *abbaco* curriculum. But this market was seen as minimal compared to the most likely principal market for the book – merchants.

Just as the *abbaco* schools taught all that was considered important for a merchant to know, so *SA* presented merchants with

all the generic (i.e., non-trade specific) information they needed to run a business, some of which they would already have known by virtue of their schooling and experience, and some of which they may never have known or may have forgotten. Apart from its use to them in their business, many merchants would also have been motivated to purchase it because of the large number of arithmetical puzzles it contains. Some would have been motivated to buy it because they had sons attending *abbaco* schools or under tutelage. It therefore seems reasonable to conclude that *SA* was intended primarily as a reference text for merchants and as a school text for their sons, and that the large majority of sales of the book were to the mercantile classes.

REFERENCES

Allen, G.D. (2000), *The Renaissance: Algebra in the Renaissance*. Available at www.math.tamu.edu/~dallen/masters/renaissance/renainsc.pdf. Accessed September 24, 2006.

Andrés de Saragossa, J. (1515), *Sumario breue d[e]la pratica dela arithmetica d[e] todo el curso de larte marca[n]tiuol bien declarado: el qual se llama maestro de cuento* (Valencia: Iuan Ioffre).

Baxandall, M. (1972), *Painting and Experience in Fifteenth Century Italy* (Oxford: Oxford University Press).

Blume, D., Bondioli, M., Falchetta, P., Franci, R., Long, P. O., McGee, D., Rossi, F., Stahl, A.M., and Wallis, F. (2007), *The Michael of Rhodes Project*. Available at http:// brunelleschi.imss.fi.it/michaelofrhodes/math_abacus.html. Accessed July 5, 2006.

Burnett, C. (2005), *Leonard of Pisa (Fibonacci) and Arabic Arithmetic*. Available at www.muslimheritage.com/topics/default.cfm?ArticleID=472. Accessed September 30, 2006.

Camerota, F. (2006), "Teaching Euclid in a Practical Context: Linear Perspective and Practical Geometry," *Science & Education,* Vol. 15, Nos. 2-4: 323-334.

Campanus of Nevara (1482), *Euclid Elementa Geometriae* (Venice: Erhard Ratdolt).

Cavazzoni, G. (1994), "Luca Pacioli ed il Trattato dei Computi e delle Scritture," in Riera, A. (ed.), *Tractatus de Computis et Scripturis di Luca Pacioli* (Bari: Cacucci Editore): ix–xxi.

De Roover, R. (1942), "The Commercial Revolution of the Thirteenth Century," *Bulletin of the Business Historical Society*, Vol. 16, No. 2: 34-42.

De Roover, R. (1956), "The Development of Accounting prior to Luca Pacioli according to the Account-Books of Medieval Merchants," in Littleton, A.C. and Yamey, B.S. (eds.), *Studies in the History of Accounting* (London: Sweet and Maxwell Limited): 114-174.

Dunlop, A.G. (1985), "Biographical Notes on Five Copies of Pacioli's *Summa Arithmetica* (1494) in Scotland," *Abacus,* Vol. 21, No. 2: 149-173.

Fauvel, J. and Gray, J. (1987), *The History of Mathematics: A Reader* (London: Open University Press).

Favier, J. (1998), *Gold and Spices: The Rise of Commerce in the Middle Ages* (New York: Holmes & Meier).

Field J.V. (1999), "Luca Pacioli e le matematica del Rinascimento. Atti del convegno internazionale di studi. Sansepolcro 13-16 aprile 1994," *Burlington Magazine*, Vol. 141, No. 1154: 300-301.

Fogo, J.R. (1905), "History of Book-Keeping," in Brown, R. (ed.), *A History of Accounting and Accountants* (New York: Cosimo Books Inc.), 2004 reprint: 93-170.

Franci, R. and Rigatelli, L.T. (1985), "Towards a History of Algebra from Leonardo of Pisa to Luca Pacioli," *Janus*, Vol. 72, Nos. 1-3: 17-82.

Geijsbeek, J.B. (1914), *Ancient Double-Entry Bookkeeping* (Denver: John B. Geijsbeek).

Gras, N.S.B. (1942), "Capitalism – Concepts and History," *Bulletin of the Business Historical Society*, Vol. 16, No. 2: 21-34.

Grendler, P.F. (1989), *Schooling in Renaissance Italy: Literacy and Learning, 1300-1600* (Baltimore: The Johns Hopkins University Press).

Grendler, P.F. (1990), "Schooling in Western Europe," *Renaissance Quarterly*, Vol. 43, No. 4: 775-787.

Grendler, P.F. (2002), *The Universities of the Italian Renaissance* (Baltimore: The Johns Hopkins University Press).

Harris, N. (2006), correspondence with the authors.

Heefer, A. (2005), *The Rhetoric of Problems in Algebra Textbooks from Pacioli to Euler*. Available at http://logica.ugent.be/albrecht/thesis/AlgebraRhetoric.pdf. Accessed September 25, 2006.

Hernández-Esteve, E. (1994a), "Luca Pacioli's Treatise *De Computis et Scripturis*: A Composite or a Unified Work?" *Accounting, Business & Financial History*, Vol. 4, No. 1: 67-82.

Hernández-Esteve, E. (1994b), "Comments on Some Obscure or Ambiguous Points of the Treatise *De Computis et Scripturis* by Luca Pacioli," *Accounting Historians Journal,* Vol. 21, No. 1: 17-80.

Hernández-Esteve, E. (2006), "Comentario sobre el libro de Alain Schärlig: Compter du Bout des Doigts. Cailloux, Jetons et Bouliers de Périclès à nos jours," *De Computis: Revista Española de Historia de la Contabilidad*, No. 4: 211-222.

Høyrup, J. (2004), "Leonardo Fibonacci and *Abbaco* Culture: A Proposal to Invert the Roles." Accessed September 26, 2006. Available at http://roskilde-university.com/upload/application/pdf/f51d6748/3_2004-1_Jens.pdf.

Jayawardene, S.A. (1973), "The Influence of Practical Arithmetics on the Algebra of Rafael Bombelli," *Isis*, Vol. 64, No. 4: 510-523.

Jayawardene, S.A. (1976), "The '*Trattato d'Abaco*' of Piero della Francesca," in Clough, C.H. (ed.), *Cultural Aspects of the Italian Renaissance: Essays in Honour of Paul Oskar Kristeller* (Manchester: Manchester University Press): 229-243.

Jouanique, P. (1996), "Three Medieval Merchants: Francesco di Marco Datini, Jacques Coeur and Benedetto Cotrugli," *Accounting, Business & Financial History,* Vol. 6, No. 3: 261-275.

Langer, C. (1958), "Paciolo – Patriarch of Accounting," *Accounting Review*: Vol. 33: No. 3: 482-484.

Lee, G.A. (1989), "Manuscript Additions to the Edinburgh University Copy of Luca Pacioli's *Summa de Arithmetica*," *Abacus,* Vol. 25, No. 2: 125-134.

Littleton, A.C. (1928), "Paciolo and Modern Accounting," *Accounting Review*, Vol. 3, No. 3: 131-140.

Maccagni, C. and Giusti, E. (1994), *Luca Pacioli e la matematica del Rinasciment* (Firenze: Giunti).

Macve, R.H. (1996), "Pacioli's Legacy," in Lee, T.A., Bishop, A., and Parker, R.H. (eds.), *Accounting History from the Renaissance to the Present: A Remembrance of Luca Pacioli* (New York: Garland Publishing, Inc.): 3-30.

Marinoni, A. (1997), "Introduction" in Marinoni, A. (ed.), *Luca Pacioli*'s *De Viribus Quantitatis* (Milan: Ente Raccolta Vinciana).

Melis, F. (1950), *Storia della Ragioneria* (Bologna: Dott. Cesare Zuffi Editore).

Nobes, C. (1995), "Pacioli's *Tractatus*: A Note on a Mystery and a Review of Some Commentaries," *Accounting, Business & Financial History,* Vol. 5, No. 3: 370-384.

Olschki, L. (1919), *Geschichte der Neusprachlichen Wissenschaftlichen Literatur* (History of the new linguistic scientific literature), (Heidelberg), cited in Taylor [1942, p. 197].

Pacioli, L. (1494), *Sŭma de Arithmetica, Geometria, Proportioni et Proportionalita* (Venice: Paganino de Paganini da Brescia).

Pacioli, L. (1509a), *De Divina Proportione* (Venice: Paganinus de Paganinis).

Pacioli, L. (1509b), *Euclidis megarensis philosophi acutissimi mathematico* (Venice: A. Paganius Paganinus).

Pendorf, B. (1933), *Luca Pacioli abhandlung über die buchhaltung 1494* (Stuttgart: C.E. Poeschel Verlag).

Peragallo, E. (1938), *Origin and Evolution of Double Entry Bookkeeping* (New York: American Institute Publishing Company).

Pin, A. (1993), "The Contribution of Luca Pacioli to the Development of Business Accounting," *Economic Notes by Monte dei Paschi di Siena*, Vol. 22, No. 2: 161-177.

Rankin, F.K.C. (1979), "Review of Vogel, K.: *Ein italienisches Rechenbuch aus dem 14. Jahrhundert: Columbia*," *Isis*, Vol. 70, No. 4: 612-613.

Rankin, F.K.C. (1992), *The Arithmetic and Algebra of Luca Pacioli (c. 1445-1517)*, Ph.D. thesis, Warburg Institute, University of London: London.

Reti, L. (1968), "The Two Unpublished Manuscripts of Leonardo da Vinci in the Biblioteca Nacional of Madrid —II," *The Burlington Magazine*, Vol. 110, Issue 779: 81-82, 85-86, 89, 91.

Rey, J.D. (2006), "Reading Luca Pacioli's *Summa* in Catalonia: An Early 16th-Century Catalan Manuscript on Algebra and Arithmetic," *Historia Mathematica*, Vol. 33: 43-62.

Richardson, B. (1999), *Printing, Writers and Readers in Renaissance Italy* (Cambridge: Cambridge University Press).

Rose, P.L. (1976), *The Italian Renaissance of Mathematics* (Geneve: Librarie Droz).

Rowland, I.D. (1995), "Abacus and Humanism," *Renaissance Quarterly*, Vol. 48, No. 4: 695-727.

Sangster, A. (2007), "The Printing of Luca Pacioli's *Summa Arithmetica* in 1494," *Accounting Historians Journal*, Vol. 34, No. 1: 125-146.

Taylor, R.E. (1942), *No Royal Road* (New York: Arno Press), 1980 reprint.

Tucci, U. (1990), *Benedetto Cotrugli, Raguseo: Il Libro dell' arte di Mercatura* (Venezia: Arsenale Editrice).

Van Egmond, W. (1981), *Practical Mathematics in the Italian Renaissance: A Catalog of Italian Abbacus Manuscripts and Printed Books to 1600* (Firenze: Editrice Giunti Barbèra).

Vianello, V. (1896), *Luca Paciolo nella Storia della Ragioneria* (Messina: Libreria Internazionale).

Volmer, F.G. (1994), "The Index of Pacioli's *Summa de Arithmetica*" in *Convegno internazionale straordinario per celebrare Fra' Luca Pacioli, Venezia, aprile* (Corsico (MI): Ipsoa Scuola D'Impressa: 78-95.

Yamey, B.S. (1978), "Pacioli's Pioneering Exposition of Double-Entry Bookkeeping," in Yamey, B.S. (ed.), *Studi in Memoria di Federigo Melis* (Napoli: Giannini Editore): 569-580.

Yamey, B.S. (1994a), "Notes on Pacioli's First Chapter," *Accounting, Business & Financial History*, Vol. 4, No. 1: 51-66.

Yamey, B.S. (1994b), "Luca Pacioli, the '*Summa*' and '*De Scripturis*'," in Gebsattel, A. and Yamey, B.S. (eds.), *Luca Pacioli: Exposition of Double Entry Bookkeeping* (Venice: Albrizzi Editore): 11-33.

Yamey, B.S. (2004), "Pacioli's *De Scripturis* in the Context of the Spread of Double Entry Bookkeeping," *De Computis: Revista Española de Historia de la Contabilidad,* No. 1: 142-154.