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THE INTRODUCTION OF "ARABIC" NUMERALS IN EUROPEAN ACCOUNTING

Abstract: The general adoption of "Arabic" numerals by European bookkeepers occurred at least five hundred years after their introduction to the scholarly world. The early availability yet late adoption of this numeration is shown to be due to several factors, not least to interplay between the culture and cultural conservatism of clerks and the educational and intellectual changes of the early Italian Renaissance.

THE MODERN VIEW OF THE RELATIONSHIP OF ROMAN NUMERALS TO MEDIEVAL ACCOUNTING[1]

A commonly held view of the heritage of Roman accounting in Medieval times is that of Chatfield [1974, p. 16]: "Taken as a whole, the Roman accounting legacy to the Middle Ages was tenacious but of doubtful value. The preference for Roman numerals continued among bookkeepers until the sixteenth cen-

[1] Terminology notes:

(1) In the body of this paper the term "Arabic" is applied to the numerals whose modern descendants are written 0, 1, 2, ... in European langauges. The origins of these numerals lie beyond the present topic; suffice it to say that the origins are complex, probably ultimately involving contributions by Mesopotamian, Indian, Greek, and Arabian cultures. The use of the term "Arabic" should not be taken as expressing a commitment to the dominance of one part of that heritage.

(2) The modern and Medieval meanings of many words are different. Even the term "abacus" had many meanings at all times. The device was not called *abacus* in ancient times; in fact, in the strict sense, it had no name at all. The normal terminology in Latin was *tabula (calculatoria)*, and the tokens used were called *calculi* (literally, "pebbles"). A user of the device was called *calculator.* In Greek, the corresponding term was τράπεζα, while the tokens were called ψῆφος and the operator was called ψήφιστης. We cannot be sure of the terms in Old English, the only other language of pre-Renaissance Europe with a surviving large scholarly literature, but they were probably *tæfel* (a loan from Latin), perhaps *tæfel-stan*, and *rimere*, respectively. Recognizing this terminology, the "abacus" is mentioned occasionally (which is all we should expect) in classical literature (*cf.* the references given by de Ste. Croix [1956, p. 60 n. 5]; also the notes of Macve [1985, p. 262]) and in Medieval literature before the late tenth century (a convenient sampling is in Latham [1975, *s.vv. calculatio, calculator,*

tury, hundreds of years after the introduction of Arabic numbers." The preference is well-known,[2] but its causes are not.

Chatfield [1974, p. 24] also reflects another commonly held view in saying of English Exchequer accounting, "The use of Roman numerals made arithmetic cumbersome and errors had to find. Worse, it perpetuated a narrative form account in which no real attempt was made to bring receipts and expenditures face to face in parallel columns."[3] This vision of a relationship between double-entry and Arabic numeration is apparently traceable to Sombart [1916], for whom both the notation and the bookkeeping practice were part of the relationship between rationality and capitalism.[4, 5] The notion of a relationship be-

calculatorius, calculo, calculus); other Medieval Latin dictionaries also provide convenient references). See also below, especially nn. 9 and 30, for the use of *abacus* to denote arithmetic in general. In the body of this paper, unless otherwise specified in context, "abacus" is used in the modern sense.

(3) The term *algorithm* is used in its modern sense, a well-specified set of computational instructions, while the cognate term *algorism* and its derivatives are used to designate arithmetic technique(s) supposedly based on the works of the Arab mathematician al-Khwarizmi. See also below, n. 24.

[2] The introduction of "Arabic" numerals in the West is a much more complicated subject than is usually presented. The standard presentation is still Smith and Karpinksi [1911]; see also Flegg, ed., [1989] and Ifrah [1981]. There is little doubt that "Arabic" numerals were well-known to academic circles by the eleventh century; the introduction of the new numeration system to bookkeeping seems to have occurred primarily in the late fifteenth century. The first examples are actually early fourteenth century, from Italian sources (*v. infra*). However, the introduction was slow — most fifteenth-century books of account use Roman numerals; "Arabic" numerals are not standard until the late sixteenth century (instructive is the selective but still useful survey of Arabic numeral usage and forms by Hill [1915]). Many modern presentations (*e.g.*, Parker [1989]) write as if the advent of the new numeration was instantaneous.

[3] Compare also Baxter [1983, p. 136-7] on the "cramping effect that they [*viz.*, Roman numerals] had on ways of thought." See also Littleton [1933, p. 20-21]. The notion that Roman numerals were an impediment to accounting is perpetuated even in Parker [1989], and Weis and Tinius [1991], evidently drawing only on secondary sources.

[4] A concise critical survey of Sombart's views is in Braudel [1982, p. 572 f]. See also Nussbaum [1933, p. 159 f]. However, Sombart's notion seems to have been based, among other sources, on a possible exaggeration of the impact of Leonardo of Pisa.

There is obviously no association between the oppositional form of accounts and "Arabic" numeration. Examples of double-entry accounting in Roman numeration are easily found — *e.g.*, Castellani [1952, *passim*]. Furthermore, columnar arrangements of many kinds employing Roman numeration are common throughout the Medieval period.

[5] The notion (see Chatfield [1974, p. 33]) that Arabic numerals were "within a generation after their exposition by Leonardo of Pisa (1202), widely used by

tween Arabic numeration and the rise of double-entry accounting was revised and advocated again by de Ste. Croix [1956], once more resting on the supposition that Arabic numerals, presumably largely due to the advocacy of Leonardo of Pisa ("Fibonacci") had a direct and immediate impact on European commercial record keeping.

Careful consideration of numeric notation and the associated computational system(s) used in Medieval accounting leads, however, to a picture different from the views just cited. Such a consideration must rest, above all, on direct examination of the primary sources. This evidence shows that the tenacity of Roman numeration[6] was not due to irrationality, ignorance, or the inferior performance of arithmetic. That tenacity, together with the circumstances of the ultimate triumph of a new numeric notation, was bound up, not just with the needs of merchants or bankers, but also with the intellectual history of late Medieval and early Renaissance Europe. In turn, this intellectual history had implications for the development of accounting after Roman numeration had been abandoned.

On the other hand, while the lack of documentation makes it impossible to establish with certainty the role played by Leonardo of Pisa, it will appear from the evidence presented here that the introduction of Arabic numerals into European bookkeeping was probably a Pisan innovation, datable at least to the early fourteenth century. Such a chronology does not, however, by itself confirm de Ste. Croix's views on the origins of double-entry. Confirmation of his hypothesis can come only from showing that some double-entry documents in Arabic notation predate all double-entry documents in Roman notation.

COMPUTATION IN MEDIEVAL ACCOUNTING: ROMAN NOTATION AND ITS RELATIONSHIP TO THE ABACUS

Whether in the Italian double-entry, English Exchequer, or in other systems, Western European commercial records of the

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Italian merchants" stems apparently from a speculation by Ball [1915, p. 168], picked up by Littleton [1933, p. 21]. At best it appears that Leonardo's impact was confined to Pisa, and even then is not clearly attested until 1305 — see below.

[6]Convenient and general introductions to Roman numeration are surprisingly hard to find. A good summary is Karpinski [1925, p. 19 f.]; in a specialized but important context, see Menninger [1958, tr. 1969, p. 279 f.]. On the origins of Roman numeration, seemingly in tally marks, see Keyser [1988].

Middle Ages almost uniformly use Roman numeric notation until the fifteenth century. The modern student knows this notation only from a few days' study, which was probably rather painful and seemingly impractical. It is today an occasional source of amusement for arithmetic teachers to threaten children with having to do "long division" in Roman numerals.[7]

Nevertheless, the notation was in its time both effective and efficient — indeed, as will be seen, superior in some respects to Arabic numeration. Roman notation was probably introduced to most Medieval schoolchildren early in their education.[8]

In understanding the role of Roman numerals, it is essential to understand that their effective usage was bound up with the use of the abacus.[9] The abacus[10] was the primary calculating

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[7] Relatively efficient techniques for direct manipulation of Roman numerals are possible; however, there is no evidence for them in the Middle Ages. See Detlefsen, *et al.* [1976].

[8] For a general account of Medieval educational practices, see, *e.g.*, Wagner, [ed., 1983]; Medieval education for the pupil's early years was primarily concerned with the trivium, particularly its first segment, "grammar". But the term "grammar" as actually applied in schools must be understood very broadly, connoting something like modern "literacy" (except in pre-Conquest England, almost exclusively in Latin). It is instructive that essentially every figure up through the tenth century today thought of as a major figure in the history of Medieval mathematics (*e.g.*, Boethius, Bede, Gerbert) was in his time also famous in connection with the trivium. This is nowhere more evident than in the works of Gerbert, always strongly influenced by rhetoric and logic ("dialectic").

No Medieval text in which Roman numeric notation is used makes a special effort to introduce and explain it; its usage seems to have been presumed as a normal part of writing. Other numeric notation systems were also common, such as the runic system (in many ways a scholarly system — for an introduction to runic writing and the associated numeric and pseudo-numeric notations, see, *e.g.*, Klingenberg [1973], Düwel [1983]) and the tally system (see, *e.g.*, Menninger [1958 tr. 1969, p. 223 f.]; also now Baxter [1989]).

[9] The vocabulary of the abacus and late Medieval arithmetic is difficult. In the Medieval period up to the last quarter of the tenth century, the device known today as the abacus was known as the *tabula*, but most references to it use the more distinctive term *calculus* and its derivatives. In the last quarter of the tenth century the term *abacus* appears, but refers to the scholarly device; from the eleventh century onwards, this term is also used by extension to denote the subject of calculation in general. This usage of *abacus* continues until at least the sixteenth century. Until about 1400 the term *arithmetica* and its derivatives refer not to arithmetic but to a discipline which today would be called "numerology."

[10] The most recent discussion of the abacus in an accounting context is Baxter [1989], but the discussion is not helpful. On the nature and use of the abacus in general, see Pullan [1969], particularly valuable for an account of the usage of the classical and Medieval abacus. See also Menninger [1958, tr. 1969,

device of the Middle Ages on Western Europe.[11] The Medieval commercial abacus had several variants, ranging from the large

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p. 295 f.]; Barnard [1916] also gives presentations of most of the published techniques for using the Medieval abacus or counting-table. Several types of abacus should be distinguished, including the classical and Medieval forms (implemented primarily by tokens placed on or between lines drawn on a flat surface, such as a table), the Medieval monastic or academic form (a large device, with a distinctive arch-topped columnar layout), and the modern forms employing beads on rods. As the illustrations in Pullan [1969, p. 22, 29, 37, 49, 50, 53] show, the Medieval commercial abacus was essentially unchanged from the form known in classical antiquity. This seems unthinkable unless the device had continued to be used essentially without interruption. There are other reasons for expecting that the abacus continued in use as needed: it was widely known, inexpensive, and performed a function otherwise difficult to duplicate in societies where writing materials were comparatively expensive. Moreover, it is likely that at least the elementary use of the abacus was taught not in the quadrivium (the part of the Medieval education curriculum concerned primarily with mathematics), but rather either informally as part of occupational training or in the trivium (the part of the curriculum concerned with language skills). A list of books owned by an English grammar master in the tenth century shows "reckoning" (*gerim*) among a set of books which is otherwise purely part of the trivium (see Robertson [1956, p. 250-251] — very interesting is that the books are all evidently in Latin except the arithmetic, which seems to have been in Old English. This could have been simply a calendar — so Robertson, ed. — or a copy of Bede. But perhaps it was a translation of Boethius? Or of Victorius?) Note, too, that the summary of the usual early Medieval curriculum (Rabanus Maurus, *De Institutione Clericum*) clearly indicates that "arithmetic" was to be understood in the sense of Boethius — *i.e.*, numerology, not computation.

[11] Occasionally modern writers still write as if the abacus had disappeared from Western Europe during the centuries following the fall of the Roman Empire. Yeldham [1926] held that "manual" reckoning (*cf.* Bede, in a scholarly context) was the normal way in which arithmetic was done during the "Dark Ages"; the view still obtains in Murray's otherwise extremely perceptive history [1978, p. 163 f.]. But Murray's interpretation of early references to the abacus is doubtful. In particular, the late tenth-century English reference (see Murray [1978, p. 454 n. 11]) as a *tabula pictoria* is possibly a mistake for *tabula pictagoria*, "Pythagorean table". The earlier references to the abacus in connection with geometry (seemingly not arithmetic) are probably highly significant — scholarly knowledge of the abacus was probably distinct from popular usage and may well have been associated with mathematical literature called "geometry". However, it must be born in mind that the term *geometria* did not designate a discipline precisely comparable to the modern "geometry;" see Shelby [1983]. It is probably not merely faulty editing that resulted in the attachment of an account of the "Arabic" numeration system in the *Geometry* of (pseudo-?) Boethius.

De Ste. Croix [1956, esp. p. 60] questions the primacy of the abacus, particularly for ancient Greek alphabetic notation. However, there is no direct or indirect reference to algorithmic calculation or anything like it anywhere in classical literature or archaeology. Moreover, de Ste. Croix ignores the evidence

surface[12] used in the English Exchequer to the more common "lines" form apparently derived directly from the abaci of classical antiquity.[13]

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which plainly connects the abacus and Roman (and Greek acrophonic) notation to the abacus. Perhaps the Greek alphabetic numeral system (and its Semitic precursors) were connected primarily with computation by tables (see particularly Menninger [1958, tr. 1969, p. 272 f.]), but this need not preclude the abacus and by no means implies algorithmic calculation.

[12] Whether the surface was ruled off to resemble a modern checker-board is questionable. Holmes [1952, p. 271] argues that in the twelfth century the term *scaccarius* was applied in general to gaming or convenient flat tables, such as were used for chess.

The relevant passage in Richard Fitz Nigel's *de Scaccario* specifically states that the exchequer was named according to its resemblance to a gaming table (perhaps hence, by implication, not like the usual abacus table); see also Baxter [1989], but the *de Scaccario* explicitly mentions lines, not squares. On the other hand, Fitz Nigel extends the metaphor in a fashion which makes a checker-board-like appearance by no means impossible. In any event, the Exchequer was not the usual commercial abacus, but was intelligible to those who knew such abaci. It is certainly an interesting subject for further research that the only modern game commonly played on "lines" (such as — in fact certainly the same as — the lines of the abacus) is backgammon — whose boards are commonly still printed on the back of checkerboards. On the early history of sectioned game-boards, see in general Murray [1913].

[13] No abaci seem to survive from the early Middle Ages. This is, however, hardly evidence that the abacus ceased to exist; in fact, it is possible that they are still in existence but unrecognized. The resurgence of interest in arithmetic in the late tenth century and the publicity accorded to "abacists" is sometimes taken as the rediscovery of the abacus — but the first writer, Gerbert (c. 980) says not that the knowledge of the abacus had been lost, but merely that there had been no writing on the subject for a long time — see Bubnov, ed. [1899, p. 6]; *Itaque cum aliquot lustra jam transierint, ex quo nec librum, nec exercitium harum rerum habuerimus, quaedam repetita memoria, eisdem verbis proferimus, quaedam eisdem sententiis*, "And so, since several long periods have passed during which we have had neither a book nor [even] an exercise [manual relating to] these matters, we offer up certain things repeated verbatim from memory, and certain things [not verbatim but] the same in concept." This passage has in the past (see Bubnov's note *ad locum*) been taken as referring to an earlier work of Gerbert's, but seems at least equally likely to refer to some work(s) or lore (now lost) on which Gerbert drew. Indeed, the entire substance of the introduction to Gerbert's *Regulae de Numerorum Abaci Rationibus* makes clear that he is drawing on a tradition which was not new. The real question is more likely to be whether this work of Gerbert actually refers to the abacus (in the modern sense) at all.

As to physical evidence, few abaci survive from antiquity, when they surely were common. The problem lies in the fact that the abacus was more of a concept than an artifact — any flat surface and a few tokens could be used to make an abacus. The nature of the abacus is shown in its name: the classical word (Greek ἄβαξ, Latin *abacus*) means 'table' or simply 'flat surface' (compare

The essential principle of the abacus (in any form) is that of place-value notation. Since, as will be shown below, Roman numerals are closely bound to the abacus, this means that the claim that Roman numeration "knows nothing of place-value" (de Ste. Croix, [1956, p. 52]) is at least much too simple. In the abacus, a token (bead, counter, or even a simple impression in a sand-covered table) derives its value from its location in a column or on a line. Arithmetic is made possible by two place-value rules:

(1) numbers are represented by the number of tokens in a place-value location;
(2) a place-value location can only contain a specified maximum number of tokens.

These two rules allow operations such as addition to be accomplished by the obvious placement of stones. When a place-value location fills up, the location is cleared and a stone is placed in the next adjacent (higher-valued) location, an act essentially the same as "carrying" in modern arithmetic with paper and pencil. To indicate the place value attached to a location, ancient and Medieval abaci used lines, usually marked at one end with a numeric symbol which indicated the upper limit on the number of stones allowed in the location. To facilitate rapid interpretation of the numbers shown on the board, stones were typically placed both on and between the lines.

The examples shown by Barnard [1916, p. 254 f.] and Pullan [1969, especially p. 62 f.] show one convention (described originally by Recorde, 1542), by which the Roman numeral constituents *I, X, C,* and *M* correspond to lines, with *V, L,* and *D* used between the lines. This abacus could be read quickly and required relatively few counters to function. The positions

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its usage to describe part of a column in Vitruvius); despite the frivolous etymology frequently suggested — still even in Baxter [1989] — the word is unlikely to be associated with a Semitic word for "dirt" (not "sand" or "dust" in the sense of a medium for drawing figures). Similarly Greek τράπεζα and Latin *tabula*.

For use on rough-hewn tables, a heavy table-cloth with lines or squares on it allowed the rapid movement of counters or tokens. The device used in Medieval English treasury computations, the *scaccarius* 'exchequer', is named from the resemblance of its cloth covering to the checker- or chess-board. This may be an English or Norman innovation of the early twelfth century — see Richard Fitz Nigel, *Dialogus de Scaccario*, giving both the "modern" (*i.e.*, twelfth-century) name and the older name for the treasury (ed. Johnson, rev. Carter and Greenway [1983, p. 7]).

of the counters (when correctly placed) corresponded directly to the written representation of numbers in Roman notation.[14]

Thus Roman notation was tied closely to the abacus and to place value. This is reflected even in the ancient technical terminology of treatises on the abacus; *articulus* and *digitus* (in modern terms, "the digit of the next higher abacal position" and "the digit of the currently considered abacal position," respectively).[15]

Any common arithmetic operation could be readily performed with the abacus; addition and subtraction are well-known to be rapid and easy. Other operations, such as multiplication and division, were more difficult, but readily possible with a little training.[16] Modern examples (using the Oriental bead-and-rod abacus) are common in which skilled abacists

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[14] The one seeming exception is the subtractive notation — *e.g.*, XLIV for 44., but this is a secondary refinement which reduces the chance of confusion between expressions such as XXXIIII and XXXXIIII. Examples of numbers written without the subtractive convention are common throughout the whole history of Roman numerals, but more rarely are the symbols for 5, 50, or 500 not used. See also Smith [1925, v. 2, p. 58-59].

[15] On the antiquity of the terminology, see any good Latin dictionary for references under *articulus* and *digitus*. Despite the seemingly obvious interpretation as references to counting on fingers, the terminology may have another origin as references to lines. It is possible that *digitus* designated the lines of the abacus while *articulus* designated the spaces between the lines (the fundamental meaning of *articulus* seems to be "a little juncture" between two things).

That Roman notation has no special symbol for zero does not mean the absence of the concept of place value; the need for zero is a representational issue which would not arise with computation on an abacus. On the other hand, it arises naturally in tabular computation and is very convenient for algorithmic calculation.

[16] We do not know with certainty how multiplication and division were performed. Probably multiplication and division were performed with the aid of tables, such as those of Victorius (*v. infra*), whose design seems to presuppose some rules such as Gerbert's *Regulae* — which, conversely, seem to imply the availability of such tables. See Pullan [1969], for relatively modern techniques. Roman hand-held calculators (see particularly the illustrations in Menninger [1958, tr. 1969, p. 305]) also show symbols for the fractions 1/2, 1/4, and 1/3. Techniques for such operations as extractions of roots are also well-known to modern users of the abacus (see, *e.g.*, Crook [1958]).

In the accounts of authors such as Gerbert, the operations of multiplication and division (indeed, much of the work of the abacist) are frequently described as immensely difficult. See Murray [1978, p. 158]. What is unclear is the extent to which such descriptions are purely literary (it must be remembered that men such as Gerbert had a primarily rhetorical education). The origins of the imagery of the "sweating abacist" certainly bear further study.

compute faster and more accurately than persons using calculators (see, *e.g.*, Flegg, ed., [1989, p. 194]).

Besides being an efficient and rapid way to achieve arithmetic results, the abacus is also flexible. By changing the rule about the limit of tokens allowed in a place-value location, arithmetic can accommodate a very wide variety of notations. A good example is in older monetary systems, such as the English system before the currency reform of 1974. English school children of the twentieth century, who worked mostly without abaci, had a much more difficult task in computing monetary amounts than did their predecessors who had only to remember that the limit in one location of the abacus was 12 (pence in a shilling), in the next 20 (shillings in a pound). In fact, the abacus is still the only conveniently available device which permits computations with numbers in which each position is expressed in a different base.

In its power, convenience, and direct correspondence with Roman notation, the Medieval abacus not only supported this numeration; it made it efficient and easily understood. To modern eyes, it might seem that difficulties could arise in the case of very large or very small numbers. However, even such numbers, uncommon in Medieval commercial contexts, can be handled.[17] The power of the abacus as a general calculating device is seen in its academic extension, the large abaci constructed by Gerbert of Aurillac and others.[18] These were constructed appar-

[17] For large numbers, the problems are vocabulary and symbolism; workable symbolisms were ancient and fairly well-known (see Menninger [1958 tr. 1969] and Pullan [1969]). The vocabulary might have been another matter, particularly for the extremely large numbers treated by the scholarly abacus. It is likely that at least some of the interests of the Medieval abacists lay in the question of the existence of a "largest number" or its name (if it existed). Thus the development of a general system for naming very large numbers may well have had some philosophical importance. For small numbers, too, the vocabulary was also of importance; the techniques for manipulating such numbers rested primarily on reduction to a useful common denominator (usually twelfths or sixtieths). Tables such as those of Victorius frequently included results of operations on fractions. That interest in naming numbers was high is consistent with the generally rhetorical character of much early Medieval mathematics.

[18] For examples see particularly Menninger [1958 tr. 1969]. In somewhat the same way as modern supercomputers, these abaci developed a (semi-)popular fame. The scholarly abacus seems usually to have been implemented with columns at the top of which an arch was represented — hence the term *arcus pythagoreus*, encountered in many variations. The earliest clear references to the scholarly abacus are to the work of Gerbert of Aurillac (late tenth century), but it is not certain that such abaci originated with Gerbert.

ently for special computations involving very large numbers — at least as large as 10^{27} — but they were not commercially useful. Perhaps the scholarly abacus was involved with speculative numerology stimulated by the advent of the year AD 1000 (see esp. Murray [1978, p. 164]).

THE INTRODUCTION OF THE NEW NUMERATION IN ACCOUNTING

To answer the question of why Arabic numeration was adopted, the resources available to late Medieval accountants must be surveyed to determine how they were perceived at the time.

Background to the New Numeration: Other Systems of Notation and Computation

Despite the importance of the abacus in calculation, other methods were known in the Medieval period. Hand signals[19]

The arch appears to be an allusion to a traditional iconography of Wisdom, which extended well outside the realm of the abacus — see, *e.g.*, Masi [1983, p. 13 f.]. The iconography was of ancient derivation — *e.g.*, the calendar for AD 354 preserved in a Carolingian copy (*cf.* Schapiro [1940]; there are many Medieval scriptural canons which use the form also) — and was connected with celestial imagery, appropriate since much of the Pythagorean heritage of mathematics and wisdom was preserved in a peculiarly astronomical context (see still Cumont [1912]). This heritage, in turn, probably has its roots in the ancient Near East.

The scholarly abaci seem always to have used special counters, called *apices*, "letters." On each counter was written a symbol for the number of tokens which it represented. Apparently Gerbert's academic abacus, like its successors, used 9 numeric symbols which are to be identified with the "Arabic" numerals 1-9. Some importance seems to have been attached to knowing the names of the numerals and the abacal columns in which the tokens were placed; in the case of the names of the columns, the reason lies perhaps in the attention given to developing a vocabulary for very large numbers. In the case of the names of the 9 different *apices*, the names were believed to be "Chaldean" — and indeed they are (with one certain exception and a conceivable second) Late Babylonian Akkadian).

[19] Described by Bede, *de Temporum Ratione* (early eighth century). For a convenient English translation, see Yeldham [1926, p. 30 f.]; see also Menninger [1958 tr. 1969, p. 201 f.]. The method(s) is (are) older than the Middle Ages. Note the ancient counters shown by Yeldham, [1926, p. 31], also by Menninger [1958 tr. 1969, p. 211 f.]. The counters show the same hand-symbolism as in Bede. What was the use of these counters? Menninger thinks the tokens were game counters, which is at least believable. But no known ancient game involves the kind of arithmetic which seems to be implied by the tokens. Could they have been abacal tokens like Gerbert's *apices*?

were more a way of representing numbers and counting than a calculation technique. Also known (in academic circles, at least) was the use of tables for arithmetic.[20] This technique, dependent on the availability of writing materials,[21] was respectable, old, and powerful.[22] Tabular computation, however, seems to have been primarily restricted to scholarly contexts because no Medieval popular reference to or evidence for it have been found.

Besides Roman numerals, many other systems of numeric notation were known to the Ancient world (see Menninger [1958 tr. 1969]). Of particular importance to the historian is the cuneiform notation, which even in its early forms used a place-value notation (albeit somewhat inconsistent — see Neugebauer and Sachs [1945, p. 2f.]) By its latest period, even a symbol for zero was consistently used in certain scholarly contexts (see Neugebauer [1955]). This system seems to have been the stimulus for developments (apparently novel) in India[23] leading to an

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[20] The canonical set of tables was the *Calculus* (the name is revealing, although one must remember that titles of ancient and Medieval books are not purely the authors' creations in the same way as are modern titles) of Victorius of Aquitaine. The only modern edition is by Friedlein [1871]. The *Calculus* seems to have been associated with astronomical (and hence calendrical) computation — see Friedlein [1869, p. 43]. Tables were, from ancient times onwards, the normal way in which specialized and advanced computation was achieved; for example, Ptolemy's *Almagest* is full of them. But whether tabular computation was truly a separate stream of computational lore is unclear — commercial arithmetic then (as now) required multiplication and division only rarely.

[21] And certainly facilitated by, but perhaps not requiring the presence of an abacus. In fact, Gerbert's *Regulae de Numerorum Abaci Rationibus* gives only rules for placement of results, not the results themselves, which seems to imply the availability of a set of tables such as that of Victorius.

[22] Tabular computation originated in ancient Mesopotamia. Neugebauer [1945 with Sachs, 1957] presents a comprehensive view of the Old Bablyonian tabular system. It was the stimulus (although perhaps not directly the source) of the calculational system employed by classical mathematicians and astronomers, particularly in an astronomical context (*e.g.*, Ptolemy). This tradition was well-known throughout Medieval times.

It is a mistake to treat tabular computation as evidence for the ignorance of arithmetic in the Middle Ages. The technique was used for advanced calculations, although it may occasionally (perhaps in some periods often) have entered the commercial world for problems such as the computation of interest (as in Babylonia — see Neugebauer and Sachs [1945, p. 36 and note 96d]).

[23] The impact of Babylonian mathematical scholarship in India seems to be particularly associated with astronomy. See Pingree [1978, 1987]; this is hardly a coincidence.

algorithmic[24] method for arithmetic. The technique, unfortunately, depended on the convenience of writing materials, which were expensive in the Medieval West.

Aside from their importance in the history of mathematics, these other systems of computation were significant because of their implications for the use of writing materials. Algorithmic arithmetic contributed an interest in convenient, inexpensive writing materials; tabular computation carried with it the habit of tabular (including columnar) arrangement of documents.

The Arrivals of Algorism and Arabic Notation in the West

Essential to understanding the ultimate bookkeeping adoption of Arabic numerals is the manner of their introduction to Western Europe. The new arithmetic was seen in the West as having received a supreme expression in the works of al-Khwarizmi (early ninth century).[25] This work was communicated fairly rapidly to the Latin-speaking West, where it attracted great attention.[26] The new arithmetic technique, "algorism", was at least fairly well-known in scholarly circles in Western Europe by the second half of the eleventh century.[27]

The arrival of algorism occurred about the same time[28] as that of the system of arithmetic notation today popularly known

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[24] The word derives from the name of al-Kawarizmi, the great Arab mathematician of the early ninth century AD. A convenient summary of the life and works of al-Khwarizmi is Sezgin [1974, p. 288 f].

[25] The Arabic text(s) of this work (or works?) is now lost, and the work is known through Latin. For the Latin, see Vogel [1963]; an English translation with important comments is given by Crossley and Henry [1990]. Despite the great importance of al-Khwarizmi's work on arithmetic, he left much to be done in the development of efficient algorithms.

[26] Perhaps the attention is due not to its stimulation of mathematical thinking but rather because it arrived when such thought was already vigorous. It is significant that al-Khwarizmi's work is expressed in terms which, although overlaid with Islamic conventions, evoke Pythagorean numerology — the same language in which western works were expressed.

[27] The relationship (if any) of the work of Gerbert of Aurillac (late tenth century) to the methods of al-Khwarizmi has not been studied. Gerbert's methods appear (at first glance) oriented towards the abacus, but also could be interpreted in terms of tabular calculation or even algorism. Gerbert's vocabulary (particulary the terms *articulus* and *digitus*), which was already old, was at the very least not simply an imitation of al-Khwarizmi.

[28] Perhaps the advent of the "Arabic" numerals was more in the nature of a revival. Medieval tradition, dismissed by modern scholars, associated the Arabic numerals with Boethius (late 5th-early 6th century AD); the association was particularly in the *Geometry* attributed to him, which includes a curious and

as "Arabic". This system soon came to be inextricably bound up with algorism, where its notation was more convenient than Roman numerals because of its greater compactness, generality, and (perhaps most important) its association with the East, from which wisdom was believed to come. Works on algorism show a knowledge of (and often, but not always, use) this notation.[29]

Despite a strong association, Arabic numerals were not confined to algorism. Academic works on the abacus (such as those of Gerbert's followers) show a knowledge of the notation in the context of "abacal" arithmetic.[30] However, whether associated with abacal or algorismic calculations,[31] Arabic numeration was confined to academic usage for a long time. Specifically, it was used extensively in calendrical and astronomical calculations. Outside of works on mathematics and astronomy, the notation first appeared in dates.[32]

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seemingly irrelevant excursus using the numerals. Boethius, in turn, was following in the ancient tradition of Pythagorean numerology and geometry, which seems to have its roots in the ancient Near East. See Masi [1983] on Boethius' mathematical agenda; on the background of Pythagorean mathematical lore there is no useful recent work, but some hint of its nature and scope can be found in Heath [1921, ch. III and V]. The "Arabic" numerals seem to be mentioned in a Syriac scholarly context (astronomical) dating from AD 662 [Nau, 1910, p. 225-226].

[29] Interestingly, the manuscript of the Latin translation of al-Khwarizmi's work (see Crossley and Henry [1990]) uses mostly Roman symbols, but the Roman numerals are clearly an adaptation of an underlying "Arabic" notation. This is frequently the case in other works on algorism — and serves as a caution. Algorism is not inherently tied to "Arabic" numerals, despite their convenience.

[30] Still by far the best survey of the development of both abacal and algorismic arithmetic in the early second millennium is that of Cantor [1894]. Murray [1978] is also convenient but gives fewer details.

[31] Problems such as the terminology make the borderline between the abacal and algorismic techniques less than certain. Thus the work of Leonardo of Pisa had the title *Liber Abbaci*, but was a work which strenuously advocated the superiority of algorism. The picture is further confused by the famous sentence near the beginning of the *Liber Abbaci* in which Leonardo says that he traveled widely and learned many methods of calculating, *sed hoc totum et algorismum atque arcus pictagore quasi errorem computavi respectu modi indorum*. This seems to be translatable as "but I counted all of this, along with algorism and the Pythagorean arch, as almost an error in comparison with the method of the Indians." (The "Pythagorean arch" is probably a reference to the scholarly abacus.) Probably Leonardo meant here the algorism of his day, which he regarded as deficient in comparison to his own knowledge.

[32] See Murray [1978, ch. 7 f.]; Smith and Karpinski [1911, ch. VIII]; Hill [1915, *passim*]; Menninger [1958, tr. 1969, p. 438 f.].

Evidence for the Use of Arabic Notation in Commercial Records

Before the fifteenth century, there are traces of Arabic numerals in commercial usage — but these traces stand out in contrast to the general, indeed, almost complete, dominance of Roman numeration. Wherever Arabic numerals are found in commercial documents before 1400, the usage has the atmosphere of a somewhat disreputable innovation, except (perhaps) in Pisa. This would be natural, for one of the most important attributes of commercial records is, necessarily, credibility, which is always risked by innovation.

Instances in Commercial Documents. Arabic notation for money amounts and quantities first appears (in published material) sporadically in the journals of some branches of the Gallerani firm as early as 1305 (see Bigwood, [1961, vol. 1, p. 6f.])[33] and in the trial balances of the delle Brache firm in Pisa (1326, see Antoni [1967, esp. p. 9]); it was more or less standard in those records for at least a quarter of a century, after which Roman numeration again dominates. The notation reappeared late in the century (again in trial balances, but this time only in the bookkeeper's notes) in the records of the Datini Company's Pisa branch.[34] It is reasonable to speculate that other Pisan records of the fourteenth century also sometimes used Arabic numerals.

[33] The Gallerani material thus seems to be the earliest example published to date. Most of the instances of "Arabic" numerals are dates or folio references (hence essentially outside the commercial sphere), but monetary amounts are found (*e.g.*, September 13, October 6, etc.). Without access to the originals, it is impossible to confirm a general impression that "Arabic" numerals were preferred by the clerk when running out of space. It is also possible that the clerk may have been copying the numerals from memoranda or that the numerals show that algorism (rather than an abacus) was being used for computation.

Unfortunately, the identity of the clerk who used "Arabic" numerals in the Gallerani records is unknown. It is not inconceivable that he was from Pisa (although the records were written elsewhere, such as in London), which would be consistent with the practices of the delle Brache.

[34] See Zerbi [1952, p. 134]; unfortunately, Zerbi's transcription does not distinguish Roman numerals (used in the entries proper) from "Arabic" (used in the notations and folio references). A photographic reproduction of a similar document from the Barcelona branch (1399) is in de Roover [1956, pl. V]. Caution must be exercised in reading modern editions of Medieval accounts; for example, Sapori's otherwise careful edition of the books of the Alberti del Giudice [1952] uniformly employs "Arabic" interpretations of Roman numerals.

Struik (1948 p. 105, relying on a private communication from Edler de Roover) had already called attention to the appearance of "Arabic" numerals in the Medici books as early as 1406. The usage of the Medici books is identical to

There are two points which suggest perspective on these Pisan Arabic numbers: first, Pisa was, of course, the home of Leonardo of Pisa, whose attempt to popularize the Arabic notation as early as 1202 has already been mentioned. However, these early commercial examples seem (probably) confined to Pisa and unparalleled elsewhere. This suggests a local, Pisan tradition, perhaps ultimately traceable to Leonardo himself (although that is speculation — there is no published evidence for the period from 1202 to 1305).[35] Second, the consistent use of the numerals appears only in the trial balances. As described (admittedly somewhat later), *e.g.*, by Pacioli [1494, ch. 34], the trial balance is basically an internal document, not truly part of the accounting records themselves. The appearance of Arabic numeration in amounts in the Gallerani journals is so sporadic that conclusions are difficult — but even there Roman numerals clearly predominate, which confirms the view that Roman numeration was considered correct, even by a clerk who knew Arabic notation.

With the possible exception of the Gallerani material, Arabic numerals do not appear outside of Pisan contexts in any commercial documents prior to the fifteenth century when they appear in the amount columns in the records of the Medici bank (1439, see Struik [1948]). Thereafter the usage of Arabic numeration becomes steadily more common, first in Italy, spreading to the rest of Europe during the sixteenth century.

The fact that the introduction of Arabic numeration to commercial records seems to be Italian is probably significant — one might have expected, for example, a Catalonian connection arising from the well-known presence of Arabic numerals in early scholarly manuscripts from the region (see, *e.g.*, Hill [1915]). However, documents such as those published by Bisson [1984] show no trace of Arabic numeration, nor does the notation appear anywhere in published French, German or English commercial records from before the fifteenth century.

Knowledge of and Need for Arabic Numerals. The relatively late commercial acceptance of Arabic numerals contrasts with

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that of the Datini records until 1439, when the Medici books begin to use "Arabic" numeration in the entries proper.

[35] An anonymous reviewer of an earlier version of this article indicates awareness of the appearance of "Arabic" numerals in *postings* in Pisan records of the late thirteenth century. Apparently the documents are not yet published. In any case the chronological gap between the appearance of the *Liber Abbaci* and the appearance of "Arabic" numerals in Pisa is still considerable.

knowledge of such materials among educated men. The numerals, along with algorism, although not necessarily known in detail or used practically, were nevertheless known to many educated Europeans from the eleventh century onwards.[36] Individuals familiar with accounting practice knew and mentioned these numbers (*e.g.*, Chaucer and Langland).[37] Both algorism and the number symbols were apparently taught in at least some schools.[38] The reasons for the failure of accounting to adopt the new system did not lie in ignorance; knowledge could spread rapidly even in Medieval times — compare the rapid spread[39] of the "abacist" literature from Gerbert's time, or the development of the Exchequer accounting system.[40] Similarly, there is plenty of evidence to show international communication of accounts (the records of the Gallerani, delle Brache, and Datini firms alone show this).[41]

Conversely, the introduction of Arabic numeration was not due to the demands of merchants. Although it is an *argumentum de silentio*, one does not hear complaints from Medieval

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[36] Menninger [1958 tr. 1969], Murray [1978], and Flegg [1989] all give examples. For literary and related examples, see, *e.g.*, Yeldham [1926, ch. V, ch. VIII, *et passim*].

[37] Chaucer, at least a large part of whose career was spent in the context of accounting, knew "Arabic" numeration (as *noumbres of augrim, i.e.*, of algorism — in the treatise on the astrolabe, I.7). But Roman numerals are universal in the English accounting of Chaucer's day. Chaucer also knew the scholarly abacus — see "Miller's Tale", I. 24, along with astronomical or astrological accouterments; for Langland, Gower, and other citations, see the *Oxford English Dictionary*.

[38] See particularly Smith and Karpinski [1911, ch. VIII]. By "schools" is meant here institutions below the university level. But the evidence is presently lacking to permit an assessment as to the geographical or chronological extent of such teaching, or the level(s) at which it might have occurred. The only concrete detail we have is the statement of Villani (1345) that as many as 1,000 pupils were studying the abacus and algorism in Florence. This is possible, but Florence is not necessarily typical (note its close, if not always amicable, relationship with Pisa); moreover it is hard to tell just what kind of teaching Villani means. (See particulary Murray, 1978, p. 172).

[39] See Murray [1978, p. 163 f.]. The scholarly abacus was certainly known in most of Europe by at the latest a century after Gerbert's death, probably in most places within twenty years.

[40] Or, perhaps more comparably, the virtually complete replacement of Old English in administrative and literary use in England following the conquest of 1066. Within 30 years, Old English was rare in any written context, although it had a large written literature which continued to be consulted.

[41] On international trade and the consequences for bookkeeping in Medieval times see, *e.g.*, Braudel [1979 *passim*], de Roover [1963], Pounds [1974, esp. ch. 8 and 9].

merchants or bankers concerning the quality or speed[42] of the arithmetic available to them, nor is there dissatisfaction with the Roman notation. In fact, arithmetic is rarely mentioned, although counting and counters are occasionally mentioned. Large, important transactions (such as international debts of kings) were handled without difficulty,[43] and the systems could handle high transaction volumes (by the standards of the times — for example, the English Exchequer system). One might at first blush think that perhaps merchants and bankers demanded more powerful arithmetic notation because of the expansion of trade — yet the late fourteenth and early fifteenth centuries, when the first introduction of Arabic numerals occurred, were hardly a time of general economic expansion.[44]

The Role of Leonardo of Pisa. Of the evidence which seems to suggest the early (*i.e.*, thirteenth- and fourteenth-century) use of Arabic notation in commerce, probably the most important is Leonardo of Pisa's *Liber Abbaci* (1202).[45] Despite its name, this

[42] The reference cited by Murray [1978, p. 166 and note *ad loc.*] to Smith [1925, v. 2 p. 188], quoting John Palsgrave is relatively late, to the latter's textbook, *Lesclarcissement de la Langue Francoyse (1530)* "I shall reken it syxe tymes by aulgorisme or you can caste it ones by counters." This is tendentious, although intended to explicate the term "algorism." A decade later, Palsgrave, translating *Acolastus* (from Latin), explains *rationes omneis concinnabo ad calculos* as "I wyll trymme al my reasons to counters. i. [alternatively] I wyll caste al my smalle parcelles together in order, into a great somme, or I wyl cast my counters, or with counters, make all my reckenynges." See Carver [1937, p. 40]. This wording could imply that Palsgrave saw counters as the normal way to get a sum. See also Barnard [1916, p. 255] and, for background, Ward [1899, vol. 1 p. 253 f.].

[43] Compare, for example, the long list of loans from Italian bankers to the English monarchy, all accounted (from the English kings' viewpoint) in Roman numerals using Exchequer methods (see Rhodes [1902]).

[44] The picture of late Medieval/early Renaissance times is, of course, complex but there is solid evidence for at least substantial declines in many, probably most, areas and aspects of the economy. See, *e.g.*, Pounds [1974, ch. 10]. This is not to say that a company such as Datini's might not have obtained success within the generally unfavorable conditions, with that success attributed to superior arithmetic and bookkeeping; but there is no obvious evidence for such a speculation.

[45] There is no convenient, complete modern translation. The work was edited as part of Leonardo's works by Boncompagni [1857]. A good account of the *Liber Abbaci* is in Yushkevich [1964, p. 371 f.].

It is sometimes casually asserted that Leonardo acquired his knowledge of arithmetic in a commercial context; but the *Liber Abbaci* says no such thing — the reasons for Leonardo's presence in various Near Eastern places are not necessarily the same as the manner in which he learned mathematics. See particularly Murray [1978, p. 192].

work is not primarily concerned with the abacus in the narrow sense of a computational device — it was, rather, a programmatic work on numeric methods.[46] Leonardo advocated algorism and displays a clear knowledge of the Arabic numerals.

The main reason for thinking that Leonardo's work should be associated with mercantile practice is that it is profusely illustrated with commercial examples, drawn from the author's own experience (and that of his father, as a Pisan commercial legate in North Africa).[47]

The evaluation of the commercial examples of the *Liber Abbaci* is problematic. Most of the examples employ Arabic numeration in a fashion which could be interpreted as didactic in

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[46] Leonardo was by no means the first Medieval writer with this agenda; quite a few other authors wrote on the use of algorism during the twelfth century. See particularly Beaujouan [1982], advocating the view that Leonardo's great contribution was written algorithms which could be used without erasures. This is very possible, but documentation of the details is difficult. Certainly the techniques evident in such documents as those of the Datini firm are more like Leonardo's than those of the twelfth-century treatises on algorism.

[47] Murray [1978, p. 192], discusses and dismisses the misunderstanding that Leonardo learned algorism from Arab merchants. Leonardo learned at least some of his mathematics directly from non-European sources. The misunderstanding was still present in de Ste. Croix [1956, p. 65-66], even quoting in Latin the relevant passage, which should be translated "Since my parent was appointed by his country as public scribe in the *duana* of Bougie for the Pisan merchants who met there, (and) caused me to join him in my youth, ... there he had me learn and attend studies in the abacus for a while. When I had been introduced to the technique of the nine Indian numerals by a marvelous teaching, I liked and understood the knowledge of the technique so much more than any others that wherever afterwards I went on business I learned with great interest and (despite) conflicting accounts whatever was studied of it in Egypt, Syria, Greece, Sicily, and Provence in all its various fashions." The passage does not say that he learned his mathematics in the course of commercial activity, but rather that the commercial activity brought him to countries where he learned mathematics. It is by no means clear that Leonardo encountered algorism in a commercial context. Similarly, the passage does not say that Leonardo learned his arithmetic from a "Moslem" master (*cf.* Parker [1989, p. 110]), which is only a plausible speculation.

Possible Arabic influence on European commercial arithmetic and accounting in the period roughly bounded by AD 1000-1500 is a subject for further study; the influence on mathematics is beginning to become clearer, but the details of interaction in the sphere of accounting are at present essentially unknown. One well-known fact is, however, pregnant with suggestiveness: Thomas Brown, considered such an expert that he was given a special position in the procedure of the English Exchequer, had previously been prominent in the Norman government of Sicily, where his name appears even in Arabic documents. Connections such as this offer a promising field for future research.

intent, not necessarily reflecting the actual practice of commercial arithmetic, but rather belonging to pure mathematics in a surprisingly modern sense; moreover, the *Liber Abbaci* was not widely read in its own time (see Murray [1978, p. 173-174]), even in academic circles. One example which may point towards a more secure contact with actual practice is from folio 10 (ed. Boncompagni), in which Leonardo advocates what is in effect a summary journal — this is written in Roman notation, but in the manuscript (next to the version in Roman) appear the same amounts in Arabic notation.[48] This is part of Leonardo's advocacy of the superiority of his version of algorism, intended to show how convenient the Arabic version was. However, the setup and implied procedure could be what underlies the appearance of Arabic numerals in the (apparently mostly or entirely Pisan) fourteenth-century examples cited previously.

Thus, aside from the sporadic and (mostly, if not entirely) Pisan instances of the fourteenth century, whose connection to Leonardo's methods remains unknown in detail, no surviving Medieval examples of bookkeeping appear to show influence by Leonardo's methods.

Secondary References to Arabic Numbers. There are occasional other instances of commercial reference to (not occurrence of) Arabic numerals. Most famous is probably the rule adopted in 1299 (and subsequently reaffirmed several times) by the Florentine Arte del Cambio (the Exchange Guild). The relevant text can be translated as follows:

> (Article CII)
>
> That No Member of the Guild May Write in His Book By Abacus
>
> Item, it is established and ordained that no one from this guild shall dare or permit through himself or through someone else to write or have written in his book [of account] or memorandum-book or in any other part of his, in which or in the [several items of] which he writes disbursements and receipts anything

[48] A photocopy of a manuscript of this passage kindly provided by an anonymous reviewer appears to show that the numerals in Leonardo are paleographically different from those found in the fourteenth century documents discussed earlier. A careful and detailed study of the paleography of the numerals may now be possible for a scholar with access to the manuscripts and would be most desirable. Some features of the picture are now clear, including the intrusion of characters of fundamentally Latin origin — see Lemay [1977] and, more briefly, Beaujouan [1982, p. 469-470].

> which is to be interpreted in the manner or letter[s] of the abacus, but [rather] he shall write openly and fully by letter[s].[49]

This legislation has customarily been interpreted as forbidding the use of Arabic numerals, with the consequent interpretation that the practice must have been sufficiently widespread to require suppression. Such an interpretation is, although possible, not necessitated by textual evidence.[50] A more likely interpretation is that the Florentine legislation was supposed to prevent a practice accepted in Pisa,[51] but even that is speculative without further evidence.

On the other hand, the reference in the 1305 statutes of the University of Padua to the new numbers is ambiguous: the book-seller "shall also put the name of the seller together with his cognomen and the price of the book on the trade book on the outside and in an obvious place and in plain letters, not by means of ciphers."[52] But this is also an isolated situation —

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[49] A description of the penalties follows. The text reads *QUOD NULLUS DE ARTE SCRIBAT IN SUO LIBRO PER ABACUM. Item statutum et ordinatum est quod nullus de hac arte audeat vel permictat per se vel per alium scribere vel scribi facere in suo libro vel quaterno vel in aliqua parte eius, in quo vel quibus scribat data et accepta, aliquid quo per modum vel licteram abbachi intelligatur, set aperte et extense scribat per licteram.* [ed. Marri, 1955, p. 72-73]. The legislation immediately follows prohibition of usury and precedes a requirement for notaries.

[50] The text does not necessarily even outlaw the use of "Arabic" numerals. As noted earlier, "abacus" in late Medieval times is a general term for computation by any method. Moreover, there is a plausible alternative interpretation of the phrasing — that members of the guild were not to record amounts by writing down the positions of the tokens on the abacus, a practice which was perhaps common. (Pictures of token layouts are common in Medieval arithmetic and geometry books, especially when Pythagorean numerology is concerned. See also Pullan [1969, p. 43 f.] for later examples).

The context makes clear that the legislation is concerned primarily with fraud. In fact, the "Arabic" numerals were probably no easier to falisfy than were Roman numerals — but dots recorded from the abacus could have been easily falsified. (The later Venetian example given by Menninger [1958 tr. 1969, p. 426-427], unfortunately uncited, does not refer to the numerals known in 1299, which would have been difficult to change.) Nor do we find here the phrasing characteristically applied to "Arabic" notation, making reference to the nine (or ten) symbols.

[51] And, as kindly indicated by an anonymous reviewer of an earlier version of this article, explicitly permitted by the Pisan *arte*. Unfortunately, the Pisan legislation seems to remain unpublished.

[52] [ed. Denifle, 1892]; see also Murray [1978, p. 171-2 and p. 455 n. 39]. The Latin text reads *Ponat eciam in libro venali extrinsecus et in evidenti loco et claris litteris non per zyphras nomen venditoris cum ipsius congnomine et precium libri.*

book-sellers in a university environment knew the new notation and used it often to number folios (it was more compact).[53] Additionally, book-sellers were notoriously crafty, and this is an academic community, not the usual commercial businesses. The legislation is that of a university, not a civil authority.

Thus, aside from two references (one uncertain and one in a special context), there is little evidence of the use of Arabic notation in commerce outside Pisa before the fifteenth century. And even that evidence is purely Italian. The new numeration does not spread significantly in European accounting contexts until the latter part of the fifteenth century. That this event occurred roughly contemporaneously with the development of printing and the spread of the usage of paper[54] is surely not a coincidence. However, other forces were also at work.

WHY WAS ARABIC NOTATION EVER ADOPTED?

The causes of the ultimate triumph of Arabic notation in European bookkeeping are complex. From the discussion thus far, the one conclusion which emerges inescapably is that it was not mere superiority as notation nor association with algorism which caused the change. Had these considerations been sufficient, Roman numerals would have been replaced even before Leonardo of Pisa. Leonardo's own prestige and works may have helped spread the notation (especially in Pisa), but that is questionable in light of the evidence and remains to be explored.

To be sure, the intrinsic qualities of Arabic numeration must have contributed to the eventual displacement of Roman notation. So, in all likelihood, did the increasing convenience of algorism as inexpensive writing materials became available. However, the first instances of commercial usage of the new system are on older, comparatively expensive materials, and do not use conveniences such as the pencil, a sixteenth-century innovation

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[53] On the use of "Arabic" numerals by book-sellers see Bischoff [1967, p. 67 f.]. For an outline of the history of folio numeration, see above all Rouse and Rouse [1979] p. 32-34.

[54] On the early history of paper, see still Blum [1932]. One should, however, be cautious about the impact of paper. By no means all the writing of the Middle Ages survives, partly because items not intended for long-term preservation were written on cheap, perishable media, such as wood or bark (see in general Clanchy [1979, esp. ch. 3-5]; for Medieval examples, *e.g.*, Liestøl [1968]; an excellent discussion of the technical issues with references to ancient examples is Bowman and Thomas [1983, ch. 2]).

(and even then apparently rarely used by bookkeepers). To complete the explanation of the triumph of Arabic notation, it is necessary to turn to Renaissance culture and education.

Accountancy and auditing were not directly subjects of university curriculum during the late Middle Ages, but were associated with universities, particularly (it seems from the published materials) in England.[55] The connection was with grammar and associated legalistic studies, especially the *ars dictaminis*, or art of business communication, and the institution which delivered the instruction was the grammar school associated with a university. In England, it seems that Oxford University was the center of this study (see particularly Richardson [1941]); in Europe, the older Italian universities (such as Bologna and Padua - see Rashdall [1936] and Bowen [1975, p. 134]) were the centers. It is important to understand the association of accounting with the legal and linguistic curriculum, not with mathematics or theology; thus Leonardo of Pisa's works never stood much chance of adoption by the commercial bookkeeper, who was rarely exposed to them (at least directly — except, again, perhaps in Pisa; this, of course, implies little about the spread of Arabic numerals, which is another issue).

Medieval accounting literature (as known above all from England) does not show a linear growth and development. It flourishes in the thirteenth and early fourteenth centuries, but by the end of the fourteenth century the literature is clearly in decline.[56] This decline appears immediately to precede the introduction of Arabic notation. The coincidence points toward events in the educational practices of the times, which in turn were expressive of profound cultural currents.

In the context of the early Renaissance (especially in Italy), two processes took place in education which affected numeration in accounting — the gradual dissociation of accounting from universities, and a revision of the university curriculum itself. The first of these processes was a change in the character of university education during the fifteenth century to reduce

[55] See above all Oschinsky [1971] and the literature there cited. See also Bennett [1974], Richardson [1939, 1941], and Baldwin [1976].

[56] See Oschinsky [1971, particularly p. 56 and 61-62]. That our knowledge of the timing applies to English sources is a matter for further research; however, since it is in a university context, the timing is probably not too badly in error. If anything, we should expect the change to have occurred somewhat earlier in Italy.

the teaching of bookkeeping and auditing in the grammar schools which were appendages of the universities. This change resulted in a weakening of the educational tradition which maintained the old connection among accounting, law, and "grammar".[57] By the sixteenth century, commercial arithmetic was taught entirely outside the university environment.[58]

However, it was not mere loss of old subjects of study that affected fifteenth-century accounting practice. The content of education, particularly at advanced levels, was changing. In a revolution instigated by such fourteenth-century figures as Petrarch, a new scholarship deliberately cast aside much Medieval learning and sought to return to the supposed purity and nobility of the Greek and Roman classics. On the one hand, this new vision of learning carried with it the risk of exposure to the attitudes of classical antiquity, distinctly contemptuous of practical affairs such as bookkeeping (scarcely mentioned in classical literature);[59] while on the other hand, it resulted in a distinct reworking of previous scholarship. Of particular importance to bookkeeping was a significant popularizing of algorism (see Smith [1908] and Karpinski [1925, ch. III]). Arabic numerals and algorism were seen as, on the one hand, new and distinct from Medieval learning,[60] and, on the other hand, as imbued with their own authenticity by virtue of having come from the East, the fount of wisdom.

Nowhere was the change more in evidence than in Pisa's

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[57] The change in European higher education in the fifteenth century has long been noted. See, *e.g.*, Graves [1923, particularly p. 106 f.], Butts [1973, ch. VI], Bowman [1975, ch. 8] — but Bowen probably dates the changes in curriculum too late. For details [particularly in fourteenth-century Italy] see Rashdall [1936, vol. 2]. Note also Rashdall's insightful comments in vol. 3 [p. 456-458]. Some caution is, however, in order; the details of changes in European education during the fifteenth century have not been put into a comprehensive framework. When this is done, it may well emerge that in at least many contexts the older, more "Medieval" methods and curricula survived or changed in their own distinctive ways.

[58] See, *e.g.*, Smith [1925, v. 2 p. 186-192]; see also the accounts of early bookkeeping texts in the volume edited by Littleton and Yamey [1956, esp. p. 185-214].

[59] Note, for example, that one of the few passages from classical literature in which a bookkeeper is explicitly mentioned is in Petronius's *Satyricon* [ed. Heseltine, 1930, p. 92], a picaresque work and in a context deliberately designed to show contempt for bookkeeping.

[60] This despite their long-standing presence in Europe; see particularly Murray [1978, p. 167-174].

neighbor, Florence, a city ruled by a commercial oligarchy (see, *e.g.*, Hay [1962, p. 116 f.]); the change was evident especially in its university. Although never a great university such as Padua or Paris, this university had several peculiarities, such as a chair of poetry, a professorship of Greek, and other curricular features which betoken the agenda of the new, humanistic education advocated by figures such as Petrarch and Boccaccio (see Rashdall [1936, p. 50-1]). As the complex relations of Pisa and Florence developed during the Renaissance, the university at Florence failed; but its heir was to be Pisa [Rashdall, *ibid.*, and p. 36]. Little is known about the mathematical curriculum at these universities, but in the context it would not be surprising to find that these schools aggressively advocated the new mathematics and new symbolism.

The fifteenth century, particularly in its latter half, was a time when innovation was in the air. The changes were not merely in the university curriculum, nor in the spread of paper and printing, but above all in a reconstruction of the organization of knowledge. The old locus of mathematical education, the quadrivium, began to change[61] into the subject now known as mathematics, while commercial reckoning went its own way, outside the university environment. Grammatical studies turned away from the traditional Medieval trivium to the new, "humanistic" model,[62] even a new script was introduced.[63] More than any superiority for computational purposes, it was this complex of educational and cultural changes which led to the introduction of Arabic numeration in books of account. Temporarily,

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[61] See, *e.g.*, Bowen [1975, p. 227-231, ch. 8]. "Mathematics" tends somewhat to be replaced by "music" in the new curriculum; "grammar" by "literature" (in a modern sense). The trend in fifteenth century mathematical instruction seems away from practical studies (see Rashdall [1936, esp. ch. XIV]; see also Kristeller [1963], Sarton [1953]. But the critical change was, of course, the development of a mathematics "pure" in a new sense, freed from a responsibility towards numerology. See above all Struik [1948, p. 112 f.] and Woodward [1906, p. 240-1]. Underlying this was a general tendency for knowledge to become more compartmentalized and specialized.

[62] For a comprehensive study, see Grendler [1989]. Grendler's coverage of the mathematical curriculum (ch. 11) unfortunately blurs the distinctions among different times and places; it also rests on the traditional view of Leonardo of Pisa.

[63] See, *e.g.*, Wardrop [19963]. The relationship of numeric notation to the advent of the new humanistic script remains to be explored. However, the timing does not seem quite coincident (the script seems a little later), and the contexts are distinct (humanistic script first in a literary context).

accounting was cut adrift from its traditional base in grammar and law.

In other words, the intrinsic qualities of Arabic numeration were insufficient to guarantee its acceptance in the commercial world. The notation had to wait until its rival, identified with a time-honored way of doing things, became sufficiently discredited. This happened as the intellectual culture of the early Italian Renaissance was spread to businessmen and bookkeepers (see Grendler [1989, esp. p. 309-310]). The adoption of Arabic notation, available in Medieval times, was truly a Renaissance phenomenon, not only in its timing but also because of the nature of the Renaissance.

THE CONNECTION BETWEEN MATHEMATICS AND ACCOUNTING

The breakdown of old distinctions in the educational system was to be only temporary, but it permitted Pacioli to attempt a grand synthesis. The *Summa de Arithmetica Geometria Proportioni e Proportionalità* is today remembered by historians of accounting primarily as the first printed account of double-entry bookkeeping; but the work was much more than that, and the entirety is significant. The title proclaims this work as an attempt at a comprehensive treatment of the entire mathematical knowledge of the time,[64] an attempt to unify all knowledge which could be expressed in mathematical terms, and an attempt to define the scope of such knowledge.

Furthermore, the *Summa* appealed to both conservatives and innovators — it tried to bridge the gap between the old learning and the new. Thus the *Summa* contains a discussion of such "new" topics as algebra and accounting, while also containing a treatise on "arithmetic" which is an adaptation of Boethius' *de Institutione Arithmetica*. Typical of the comprehensive, yet innovative, spirit of the *Summa* is also that the version of the *de Institutione Arithmetica* is in Italian, not Latin. Typical, too, is the fact that Pacioli, fully aware that he was not the innovator of double-entry, nevertheless uses Arabic numeration for his entries (except in dates! It is as if Pacioli were attempting to demonstration the interchangeability of the two systems of notation).

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[64] A useful treatment of Pacioli from the mathematical viewpoint is Yushkevich [1964, p. 427 f.]. However see also Masi [1983].

That Pacioli did not introduce Arabic notation to books of account, yet used it in a book which had an ambitious program, attracts attention. Did this imply the final incorporation of accounting into the realm of mathematics?

Although Pacioli's work had a prescriptive character, his vision of the unification of things mathematical ultimately failed, perhaps because of the sheer size of the *Summa*. The introduction of the new notation was not a symptom of the domination of accounting by academic mathematics, but rather of the attempt to unify mathematics in the new cultural context. In arithmetic notation, Pacioli, a consummate scholar, reflects what he saw as the best practice of his day. The accounting part of the *Summa* soon became separated from the rest of the book, and took on a life of its own (which persists to this day),[65] even outside its native university context. Resting on both the authority derived from its integration with the rest of mathematics and on the fame of its author, the prestige of the *Summa*'s treatment of bookkeeping, in part, accounts for the separate life, as does the convenience of its clear, straightforward presentation. However, the separation also surely derives from the apparent irrelevance of most of the remainder of the *Summa* to the needs of working bookkeepers.

CONCLUDING COMMENTS

The survey of evidence given above shows that, on the one hand, Arabic numerals were widely known in Western Europe among scholars by at least the eleventh century and even among some bookkeepers by about 1300. Yet the general adoption of these numerals in commercial records is a phenomenon of the second half of the fifteenth century.

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[65] That life, of course, took above all the form of imitations. The view expressed here of Paciolo's purposes emphasizes the relations of the treatment of accounting to the rest of the work. Thompson [1991] argues that we should also emphasize the rhetorical and pedagogical character of the *Summa*, especially in its accounting material. Referring to Aho [1985] and Hoskin and Macve [1986], Thompson sees both double-entry in general and Paciolo's treatment in particular as responses to the cultural problem of assuring the credibility of accounting records, particularly in the light of hostility from certain quarters, such as the church. Thompson also points out that Paciolo's survey is profoundly allied to the works of Peter Ramus (early sixteenth century), particularly in the layout of the presentation. See the survey of English imitations in Gordon [1956]; more generally, see Melis [1950, p. 611 f.].

There can many explanations for a gulf between awareness and practice. In the case of the transition from Roman to Arabic numerals, a combination of cultural conservativism among bookkeepers and the comparative efficiency of Roman numerals and the arithmetic system associated with them were probably the main reasons for the slow adoption. Of these factors, cultural conservatism seems the more important, since demonstrations of the utility of Arabic numerals were readily available in at least a seemingly commercial context from the time of Leonardo of Pisa (1202) onwards.

The cultural conservatism of bookkeepers was maintained in part by the inherently greater credibility of well-understood methods and in part by an educational system which separated accounting from mathematics. Thus, in the West, academic mathematics has generally had little influence on the practice of accountancy. The segregation of the two disciplines has continued until the twentieth century. Pacioli, the first to attempt an explicit unification, was an imposing but exceptional figure. Shaped profoundly by its origins in grammar, rhetoric, and law, rather than in mathematics, Western accounting (like other traditions) has generally failed to employ sophisticated mathematical methods; the level of mathematics in even a mid-twentieth century textbook on the "mathematics of accounting" is no higher than second-year high-school algebra [Curtis and Cooper, rev. McCallion, 1961].

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