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# ROTARY OSCILLATION OF A RIGID SPHERE IN A COUPLE STRESS FLUID

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# ROTARY OSCILLATION OF A RIGID SPHERE IN A COUPLE STRESS FLUID


## Abstract

In this paper, the rotary oscillation of a rigid sphere in an incompressible couple stress fluid is studied. The classical no slip boundary conditions are imposed on spherical boundary. Moreover, it is assumed that the couple stresses on the boundary of the sphere vanish. In the present study, the motion is generated by a sudden rotary oscillation of the rigid sphere about an axe passing through its center with a time-dependent angular velocity. Stokasian assumption is taken into consideration so that the non-linear terms are neglected in the equation of motion. The torque experienced by the couple stress fluid on the spherical body is obtained using an integral formula. Exact solutions are obtained and results are illustrated through graphs..



## Keywords

Couple stress fluid, spherical particle, rotary oscillation

# 1. INTRODUCTION

The study of non-Newtonian fluids is an attractive subject for scientists interested in studying complex fluids due to its wide area of applications. Such type of fluids is widely used in many industrial areas such as liquid crystals, animal blood and polymer thickened oils. The well known classical model of Navier-Stokes do not describe adequitly the real behavior of such fluids. Different models have been introduced to treat such non adequacy. One of these models is called micropolar fluids model. This model was introduced by Eringen in 1964 as one class of more general theory named microfluids theory (Eringen, 1998). The other model is called couple stress fluids model which has been introduced by Stokes (Stokes, 1966). This model differs from the classical Navier-Stokes model in containing a non-symmetric stress tensor, body couples and a couple stress tensor. In his introduced theory of couple stress fluids, Stokes considered the classical Cauchy stress tensor along with couple stresses (Stokes, 1966 and Stokes, 1984). Stokes introduced a fourth order differential equation which describes mathematically the motion of a couple stress fluid together with two constitutive equations consisting of stress and couple stress tensors. Different studies have been done on this model of fluids. Devakar et al studied analytically the solutions of some fundamental problems of couple stress fluid flows namely Couette, Poiseuille and generalized Couette flows (Devakar, Sreenivasu and Shankar, 2014).

The rotary motion of a spherical object attracted the attention of many researchers to study due to its applications different fields such as drug delivery and tissue engineering. Chadwich and Liao investigated the high frequency oscillation of a rigid sphere moving in an incompressible viscous fluid normal to a rigid plane (Chadwick and Liao, 2008). Faltas et al investigated the problem of the interaction of two spherical particles rotating in a micropolar fluid (Faltas, Sherief and Ashmawy, 2012). Aparna and Murthy investigated the solution for rotary oscillation of a permeable sphere in a micropolar liquid (Aparna and Murthy, 2012). Sherief et al considered problems with similar geometries in the theory of micropolar fluids (e.g. Faltas, Sherief and El-Sapa, 2019 and Sherief, Faltas and El-Sapa, 2019). Ashmawy investigated the rotary oscillation of a composite sphere in a concentric spherical cavity with slip on the surface of the spherical cavity and stress jump on the porous fluid interface using Brinkman model (Ashmawy, 2015). The same author studied the unsteady Stokesian flow of an incompressible couple stress fluid around a rotating sphere (Ashmawy, 2016 a). He also obtained an analytical formula for the drag force acting on a slip spherical object moving in a couple stress fluid (Ashmawy, 2016 b). Many other recent research papers treating different problems in the theory of couple stress fluids are available in the literature such as (Ashmawy, 2018, Shehadeh and Ashmawy, 2019 and Ashmawy, 2019). The present work discusses the oscillatory flow of an incompressible couple stress fluid due to the rotary oscillation of a rigid sphere about its diameter. The no slip boundary condition is applied on the surface of the spherical object. In addition, the torque exerted by the fluid flow on the rigid sphere is deduced and discussed numerically through graphs.

# 2. FORMULATION OF THE PROBLEM

The fluid flow of an incompressible couple stress fluid with time dependence, Assuming that the body forces and body couples are absent, is described by the following equations (Stokes, 1984)

Equation of conservation of mass

Eq. (2.1) $$\nabla.\vec{q} = 0.$$

Equation of conservation of momentum

Eq. (2.2) $$\rho\frac{\partial\vec{q}}{\partial t} + \eta\nabla\times\nabla\times\nabla\times\nabla\times\vec{q} + \mu\nabla\times\nabla\times\vec{q} - \nabla\mathrm{p} = 0.$$

where $\vec{q}$ denotes the velocity and p represents the fluid pressure. The material constant $\mu$ denotes the classical viscosity parameter of the fluid, $\eta$ is the new viscosity coefficient which characterizes the couple stress effect and $\rho$ represents the fluid density. These coefficents have the dimensions respectively $M/LT$ and $ML/T$.

The stress and couple stress constitutive equations are, respectively, given by

Eq. (2.3) $$t_{ij} = -p\delta_{ij} + 2\mu d_{ij} - \frac{1}{2}\varepsilon_{ijk}m_{sk,s},$$

Eq. (2.4) $$m_{ij} = m\delta_{i,j} + 4\eta w_{i,j} + 4\eta'\ w_{j,i}.$$

The deformation rate tensor $d_{ij}$ is defined by

Eq. (2.5) $$d_{ij} = \frac{1}{2}(q_{i,j} + q_{j,i}).$$

The tensor $w_{j,i}$ represents the spin tensor and the vorticity vector is defined by

Eq. (2.6) $$\vec{w} = \frac{1}{2}\nabla \times \vec{q}.$$

The material constant $\eta'$ is a second viscosity parameter which appears in the couple stress tensor. The couple stress viscosity coefficients , $\eta$ and $\eta'$ .are satisfying the following inequalities, $\eta \geq 0$ and , $\eta \geq \eta'$ .The scalar quantiy $m$ represents one third of the trace of the couple stress tensor, $\delta_{i,j}$ and $\varepsilon_{ijk}$ are respectively the Kronecher delta and the alternating tensor defined by

$$\delta_{i,j} = \begin{cases} 0 \ if\ i \neq j \\ 1 \ if\ i = j \end{cases},$$

$$\varepsilon_{ijk} = \begin{cases} 1, & if\ (i,j,k) is\ a\ cyclic\ \ permutation\ of\ (1,2,3) \\ -1, & if\ (i,j,k)\ is\ a\ non\ cyclic\ permutation\ of (1,2,3) \\ 0, & if\ at\ least\ two\ of\ the\ indices\ are\ equal \end{cases}.$$

Now we consider the rotary oscillation of a couple stress fluid about a rigid sphere of raduis $a$. The motion is generated by the rotary oscillation of the sphere about a vertical diameter with time-dependent angular velocity $\Omega e^{i\sigma t}$, where $\Omega$ is a constant with the dimension of angular velocity and $i = \sqrt{-1}$ . We take a spherical polar coordinate system $(r, \theta, \phi)$ with the center of the sphere as origin and a z-axis along the axis of rotation with base vectors $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$. Since the flow is axisymmetric, therefore all physical quantities are independent of $\phi$.

The fluid velocity vector is assumed to take the form

Eq. (2.7) $$\vec{q} = (0,0, q_\phi\ (r,\theta,\mathrm{t})),$$

The following condition is applied on the spherical surface

Eq. (2.8) $$q_\phi(r,\theta,t) = a\Omega sin\theta e^{i\sigma t} \quad \text{on } r = a.$$

The remaining condition satisfied on the surface of the sphere, $r = a$, is the vanishing couple stress

Eq. (2.9) $$m_{r\theta} = 0 \ \text{ on } \ r = a.$$

## 3. SOLUTION OF THE PROBLEM

In view of Eq. (2.8), we can assume that $q_\phi(r,\theta,t) = f(r,\theta)e^{i\sigma t}$

Therefore, Eq. (2.2) reduces to

Eq. (3.1) $$(E^4 - \lambda^2 E^2 + \lambda^2 l^2) r sin\theta\, f(r,\theta)e^{i\sigma t} = 0,$$

where $E^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial \theta}$, $\lambda^2 = \frac{\mu}{\eta}$ and $l^2 = \frac{i\sigma\rho}{\mu}$

Assume that $f(r,\theta) = \frac{1}{r}g(r)sin\theta$, and let $D^2 = \frac{\partial^2}{\partial r^2} - \frac{2}{r^2}$, so the above equation reduces to

Eq. (3.2) $$(D^4 - \lambda^2 D^2 + \lambda^2 l^2) g(r) = 0.$$

This equation will be factorized to:

Eq. (3.3) $$(D^2 - \xi_1^2)(D^2 - \xi_2^2)g(r) = 0.$$

where $\xi_1, \xi_2$ are the positive roots of the characteristic equation

Eq. (3.4) $$\xi^4 - \lambda^2\xi^2 + \lambda^2 l^2 = 0.$$

Solving the complex polynomial of degree 4 in Eq. (3.4) we get

Eq. (3.5) $$\xi_1 = \lambda\frac{\sqrt{1-\sqrt{\lambda^2-4l^2}}}{2} \text{ and } \xi_2 = \lambda\frac{\sqrt{1+\sqrt{\lambda^2-4l^2}}}{2}.$$

The solution of this differential equation (3.2) is found to be

Eq. (3.6) $$g(r) = \sqrt{r}\sum_{i=1}^{2} A_i K_{\frac{3}{2}}(\xi_i r),$$

where $K_n(.)$ denotes the modified Bessel function of the second kind of order $n$.

Therefore, the velocity component of the fluid flow is obtained as

Eq. (3.7) $$q_\phi(r,\theta,t) = e^{i\sigma t}\frac{sin\theta}{\sqrt{r}}\sum_{i=1}^{2} A_i K_{\frac{3}{2}}(\xi_i r).$$

After computing the vorticity vector, the non-vanishing components are

Eq. (3.8) $$w_r = \frac{e^{i\sigma t}cos\theta}{r\sqrt{r}}\sum_{i=1}^{2} A_i K_{\frac{3}{2}}(\xi_i r).$$

Eq. (3.9) $$w_\theta = \frac{e^{i\sigma t}sin\theta}{2r\sqrt{r}}\sum_{i=1}^{2} A_i \left\{K_{\frac{3}{2}}(\xi_i r) + r\xi_i K_{\frac{1}{2}}(\xi_i r)\right\}.$$

Using equation (2.3), the tangential stress component is

Eq. (3.10) $$t_{r\phi} = \mu\left(\frac{\partial q_\phi}{\partial r} - \frac{q_\phi}{r}\right) + \frac{1}{2}\left(\frac{\partial m_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial m_{\theta\theta}}{\partial\theta} + \frac{2m_{r\theta}+m_{\theta r}}{r} + \frac{cot\theta}{r}\{m_{\theta\theta} - m_{\phi\phi}\}\right).$$

Then by using equation (2.4) of the couple stress tensor we get the following equations

Eq. (3.11) $$m_{rr} = m + 4(\eta + \eta')\frac{\partial w_r}{\partial r}.$$

Eq. (3.12) $$m_{r\theta} = 4\left\{\eta\frac{\partial w_\theta}{\partial r} + \eta'\left(\frac{1}{r}\frac{\partial w_r}{\partial\theta} - \frac{w_\theta}{r}\right)\right\}.$$

Eq. (3.13) $$m_{\theta r} = 4\left\{\eta'\frac{\partial w_\theta}{\partial r} + \eta\left(\frac{1}{r}\frac{\partial w_r}{\partial\theta} - \frac{w_\theta}{r}\right)\right\}.$$

Eq. (3.14) $$m_{\theta\theta} = m + 4(\eta + \eta')\left(\frac{1}{r}\frac{\partial w_\theta}{\partial\theta} - \frac{w_r}{r}\right).$$

Eq. (3.15) $$m_{\phi\phi} = m + 4(\eta + \eta')\left(\frac{w_r}{r} + \frac{w_\theta}{r}cot\theta\right).$$

Inserting the vorticity expressions (3.8)-(3.9) into the above mentioned couple stress components, we get

Eq. (3.16) $$m_{rr} = m - 4(\eta + \eta')\frac{cos\theta}{r\sqrt{r}}e^{i\sigma t}\sum_{i=1}^{2}\xi_i A_i K_{\frac{5}{2}}(\xi_i r).$$

The vanishing of the couple stress component $m_{rr}$ on the spherical boundary $r = a$ results in

Eq. (3.17) $$m = 4(\eta + \eta')\frac{cos\theta}{a\sqrt{a}}e^{i\sigma t}\sum_{i=1}^{2}\xi_i A_i K_{\frac{5}{2}}(\xi_i r).$$

Eq. (3.18) $$m_{r\theta} = \frac{-2sin\theta}{r\sqrt{r}}e^{i\sigma t}\sum_{i=1}^{2}\xi_i A_i\left\{(\eta+\eta')K_{\frac{5}{2}}(\xi_i r) + r\eta\xi_i K_{\frac{3}{2}}(\xi_i r)\right\}.$$

Eq. (3.19) $$m_{\theta r} = \frac{-2sin\theta}{r\sqrt{r}}e^{i\sigma t}\sum_{i=1}^{2}\xi_i A_i\left\{(\eta+\eta')K_{\frac{5}{2}}(\xi_i r) + r\eta'\xi_i K_{\frac{3}{2}}(\xi_i r)\right\}.$$

Eq. (3.20) $$m_{\theta\theta} = m_{\phi\phi} = m + 2(\eta + \eta')\frac{cos\theta}{r^2\sqrt{r}} e^{i\sigma t} \sum_{i=1}^{2} A_i \left\{3K_{\frac{3}{2}}(\xi_i r) + r\xi_i K_{\frac{1}{2}}(\xi_i r)\right\}.$$

Inserting these results in the equation of tangential stress (3.10) we get

Eq. (3.21) $$t_{r\phi} = \frac{sin\theta}{r^2\sqrt{r}} e^{i\sigma t} \sum_{i=1}^{2} \xi_i A_i \left\{\left[r^2\left(\eta{\xi_i}^2 - \mu\right) - 2(\eta + \eta')\right]K_{\frac{5}{2}}(\xi_i r) - 2\eta\xi_i r K_{\frac{3}{2}}(\xi_i r)\right\}.$$

The remaining boundary conditions are used to give a system of two equations with two unknown $A_1$ and $A_2$ as follow

Eq. (3.22) $$\frac{-2sin\theta}{r\sqrt{r}}\xi_1\left\{(\eta+\eta')K_{\frac{5}{2}}(\xi_1 r) + r\eta\xi_1 K_{\frac{3}{2}}(\xi_1 r)\right\}A_1 + \frac{-2sin\theta}{r\sqrt{r}}\xi_2\left\{(\eta+\eta')K_{\frac{5}{2}}(\xi_2 r) + \quad r\eta\xi_1 K_{\frac{3}{2}}(\xi_2 r)\right\}A_2 = 0.$$

Eq. (3.23) $$\frac{1}{\sqrt{r}}K_{\frac{3}{2}}(\xi_1 r)A_1 + \frac{1}{\sqrt{r}}K_{\frac{3}{2}}(\xi_1 r)A_2 = a\Omega.$$

Solving the system of linear equations in (3.22) and (3.23) we get

Eq. (3.24) $$A_2 = \frac{-\Omega}{\Delta_0}\left\{a\sqrt{a}\xi_1\left(a\eta\xi_1 K_{\frac{3}{2}}(\xi_1 a) + (\eta+\eta')K_{\frac{5}{2}}(\xi_1 a)\right)\right\}.$$

Then putting $A_2$ in one of the system of equations we can deduce that

Eq. (3.25) $$A_1 = \frac{-\Omega}{\Delta_0}\left\{a\sqrt{a}\xi_2\left(a\eta\xi_2 K_{\frac{3}{2}}(\xi_2 a) + (\eta+\eta')K_{\frac{5}{2}}(\xi_2 a)\right)\right\}.$$

where $\Delta_0 = K_{\frac{3}{2}}(\xi_1 a)\xi_2\left(a\eta\xi_2 K_{\frac{3}{2}}(\xi_2 a) + (\eta+\eta')K_{\frac{5}{2}}(\xi_2 a)\right) - K_{\frac{3}{2}}(\xi_2 a)\xi_1\left(a\eta\xi_1 K_{\frac{3}{2}}(\xi_1 a) + (\eta+\eta')K_{\frac{5}{2}}(\xi_1 a)\right).$

## 4. TORQUE ON THE RIGID SPHERE

The torque exerted by the couple stress fluid on the surface of the sphere can be obtained by employing the following integral formula

Eq. (4.1) $$T_z = 2\pi a^3 \int_0^{\pi} t_{r\phi}|_{r=a} sin^2\theta \, d\theta.$$

Inserting the expression (3.21) evaluated at $r = a$ into the above-mentioned integral formula, we obtain

Eq. (4.2) $$T_z = \frac{8\pi}{3}\sqrt{a} e^{i\sigma t} \sum_{i=1}^{2} \xi_i A_i \left\{\left[a^2(\eta{\xi_i}^2 - \mu) - 2(\eta + \eta')\right]K_{\frac{5}{2}}(\xi_i r) - 2\eta\xi_i a K_{\frac{3}{2}}(\xi_i r)\right\}.$$

## 5. NUMERICAL RESULTS

Here, we represent graphically the non dimensional torque $T_z^* = T_z/T_0$, where $T_0 = 8\pi\mu a^3\Omega$, acting on a rigid sphere oscillating rotationally in a couple stress liquid. The torque is illustrated against different parameters such as $t, \sigma, \eta,$ and $'$ .

Figs. 1 and 2 illustrate, respectively, the variation of the real and imaginary parts of the torque acting on the spherical object versus the time $t$ for different values of the first couple stress viscosity parameter $\eta$. It is noticed that the increase in the value of $\eta$ results in a decrease in the amplitudes of the real and imaginary parts of the torque. Figs. 3 and 4 represents, respectively, the variation of the real and imaginary parts of the torque acting on the spherical object versus the time $t$ for different values of the second couple stress viscosity parameter $\eta$'. It is observed that the increase of the coefficient $\eta'$ results in a slight increase in the amplitudes of the real and imaginary parts of the torque. Figs. 5 and 6 discuss the influence of $\sigma$ versus the time $t$ for the real and imaginary parts of the non dimensional torque. It is found that the wavelength of the real and imaginary parts of the torque decreases with the increase in the value of $\sigma$. Figs. 7 and 8 show the variation of the real and imaginary parts of the torque versus $\eta$ for different values of $\eta'$. It is shown that the increase in the values of $\eta'$ decreases the values of the torque very slightly. Figs. 9 and 10 represent the behaviours of the real and imaginary parts of the velocity of the fluid flow versus the time $t$ for different values of $\eta'$. Again it is observed that the effect of the viscosity parameter $\eta'$ is slight. It is found that the amplitudes of the real and imaginary parts of the torque increases slightly with the increase in the value of $\eta$' . Figs. 11 and 12 discuss the influence of $\sigma$ on both real and imaginary parts of the fluid velocity. It is shown that as the values of $\eta$ increase, the wavelength of the real and imaginary parts decrease. Finally, figs. 13 and 14 show the variation of the real and imaginary parts of the velocity versus $r$ for different values of $\sigma$. It is noticed that as the value of $\sigma$ increases, the value of the real part of the velocity decreases.

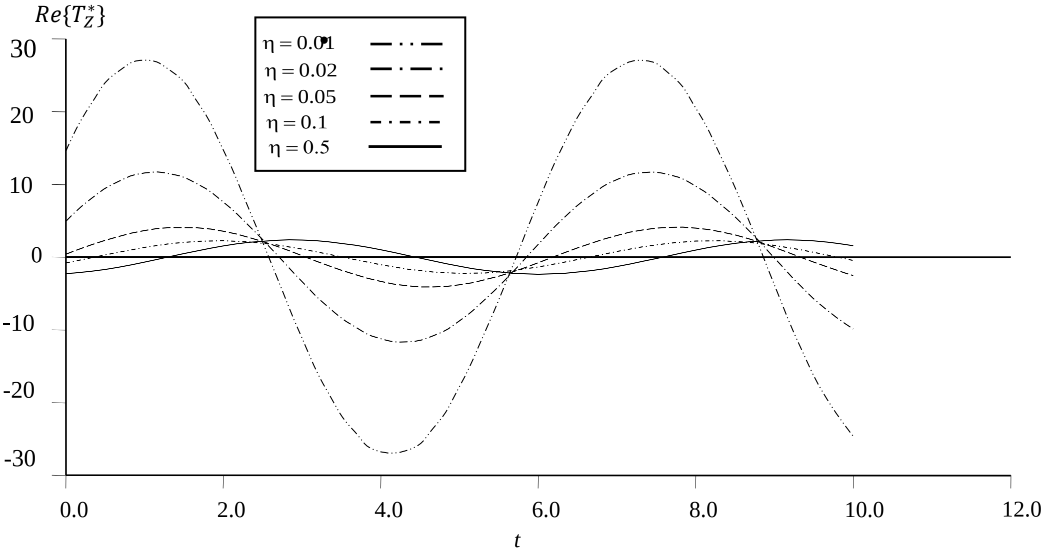


Fig.1: Variation of the real part of the normalized torque when $\sigma = 1.0$ and $\eta' = 0.0$

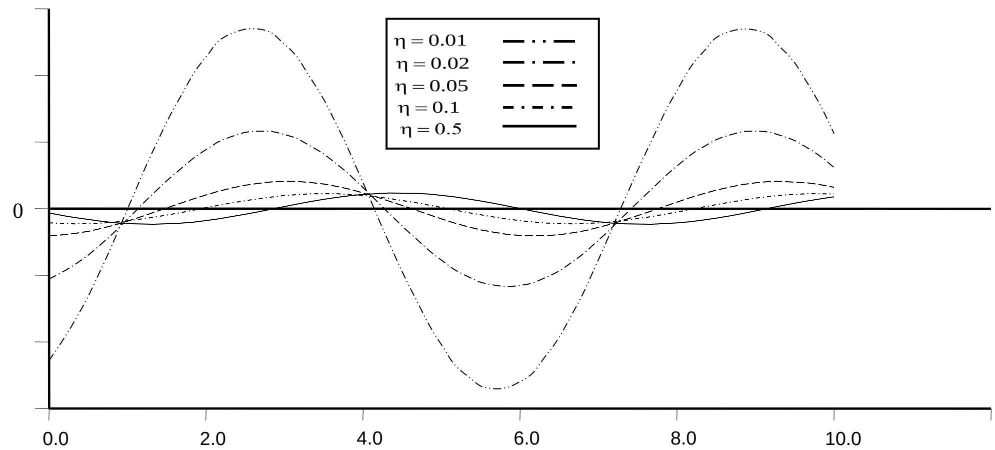


Fig.2: Variation of the imaginary part of the normalized torque when $\sigma = 1.0$ and $\eta' = 0.0$

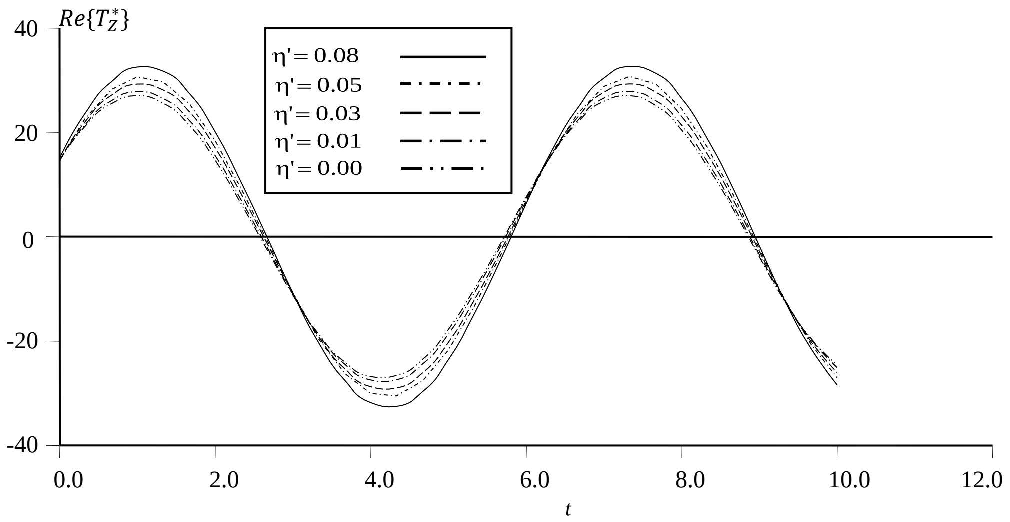


Fig.3: Variation of the real part of the normalized torque when $\sigma = 1.0$ and $\eta = 0.01$.

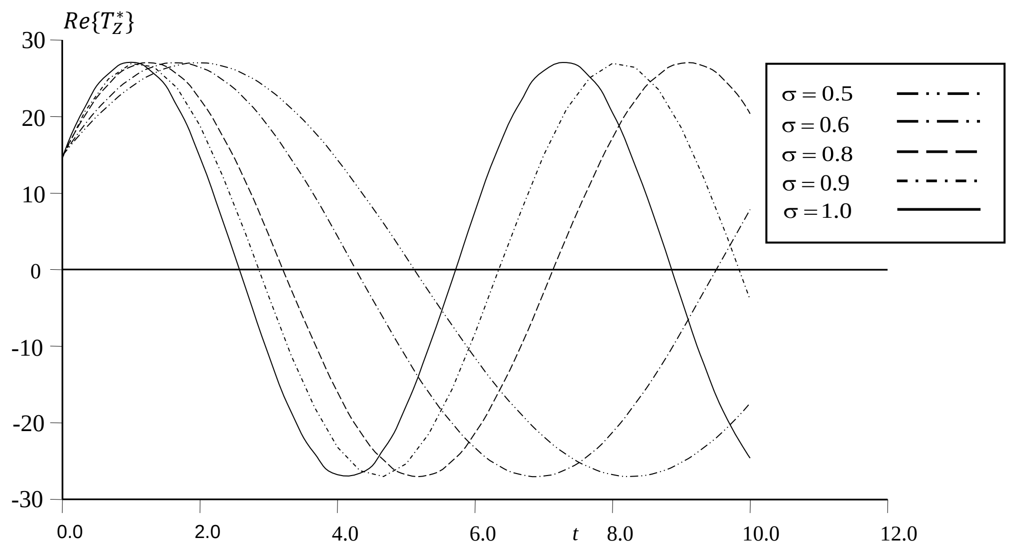


Fig.5: Variation of the real part of the normalized torque when $\eta'$=0.0 and $\eta = 0.01$

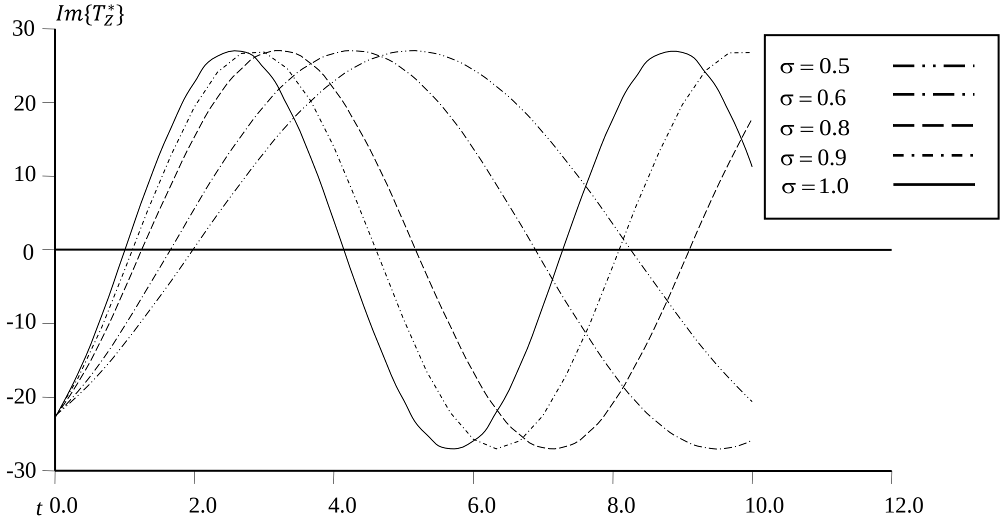


Fig.6: Variation of the imaginary part of the normalized torque when $\eta' = 0.0$ and $\eta = 0.01$

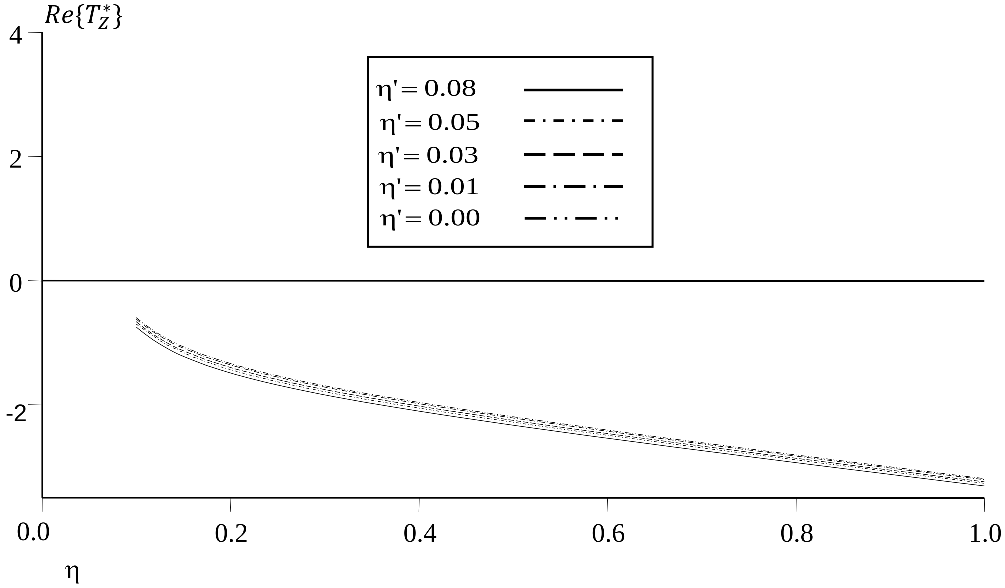


Fig.7: Variation of the real part of the normalized torque when $\sigma$=1.0 and $t = 0.1$.

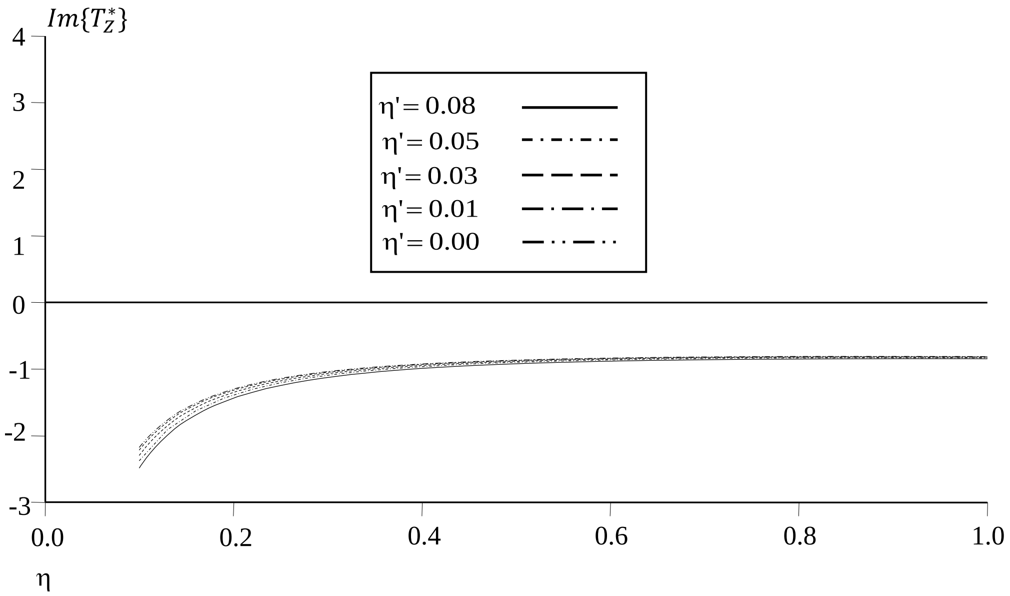


Fig.8: Variation of the imaginary part of the normalized torque when $\sigma$=1.0 and $t = 0.1$.

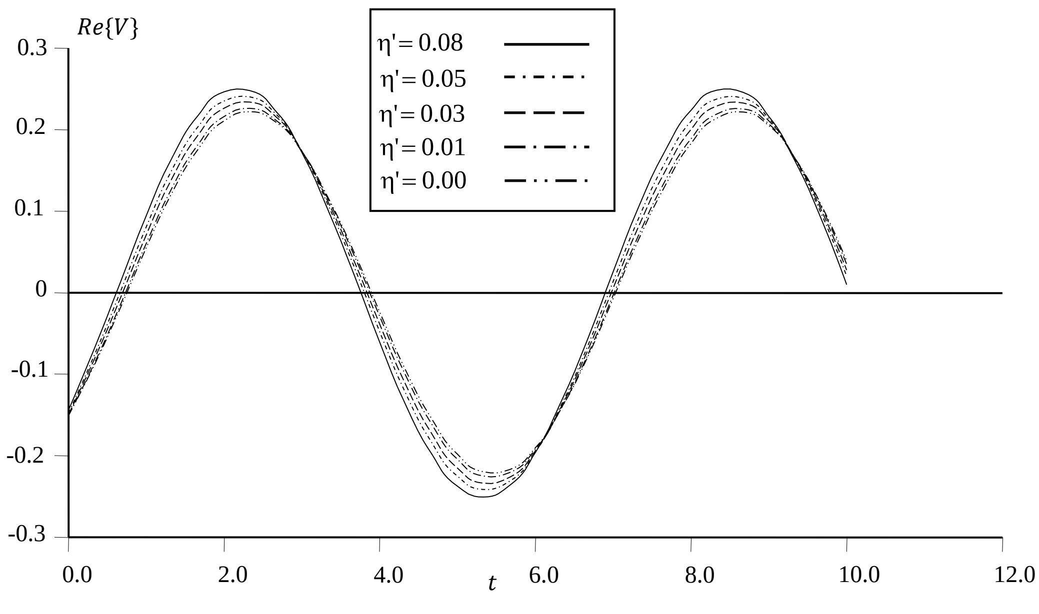


Fig.9: Variation of the real part of the velocity when $\sigma$=1.0, $\eta = 0.01$ and $r = 2.0$.

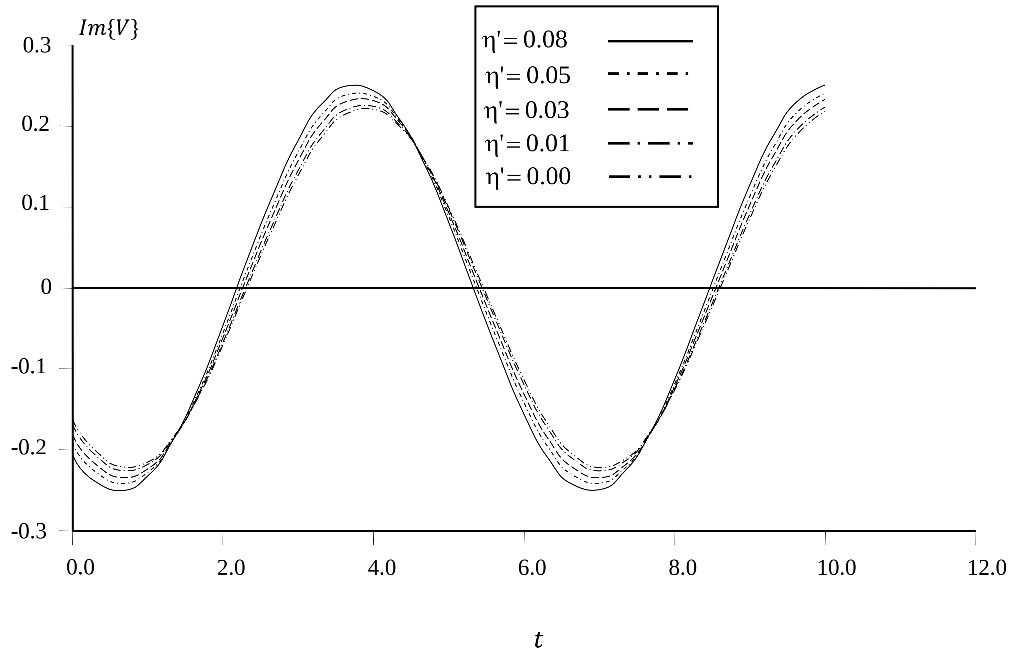


Fig.10: Variation of the imaginary part of the velocity when $\sigma$=1.0, $\eta = 0.01$ and $r = 2.0$.

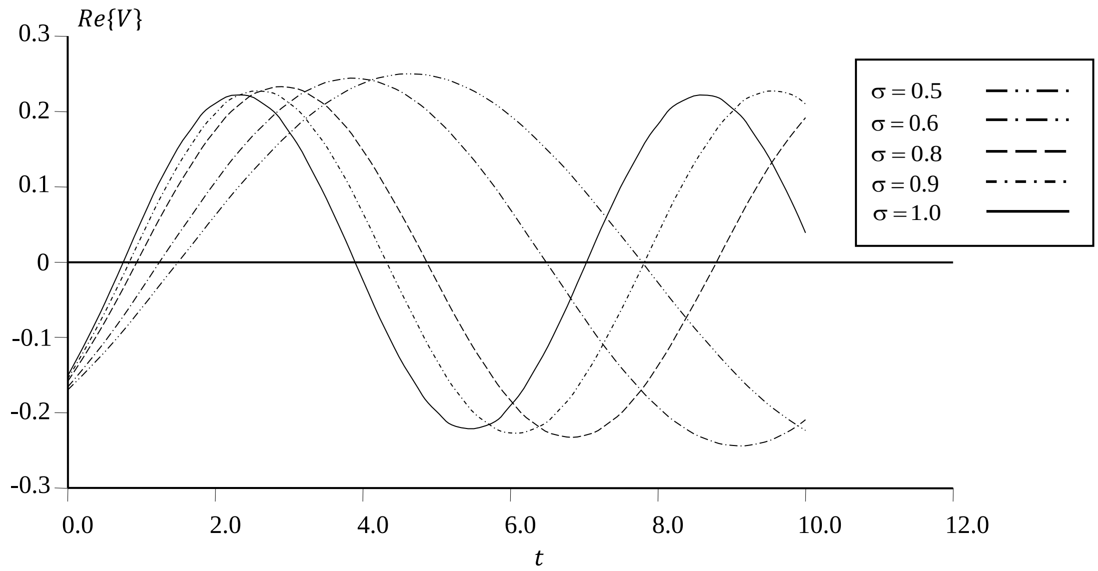


Fig.11: Variation of the real part of the velocity when $\eta'$=0.0, $\eta = 0.01$ and $r = 2.0$.

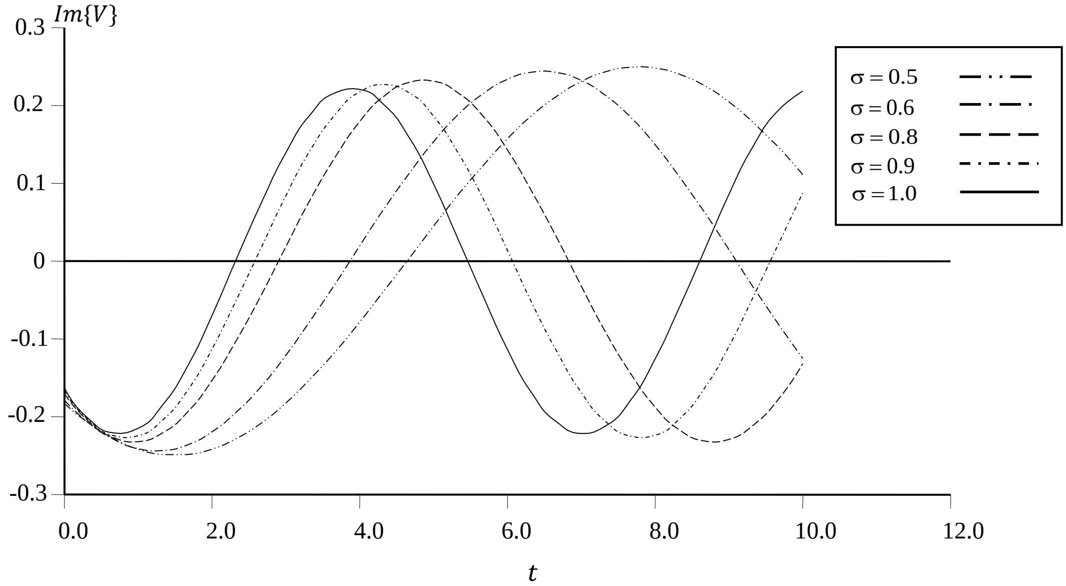


Fig.12: Variation of the imaginary part of the velocity when $\eta'$=0.0, $\eta = 0.01$ and $r = 2.0$.

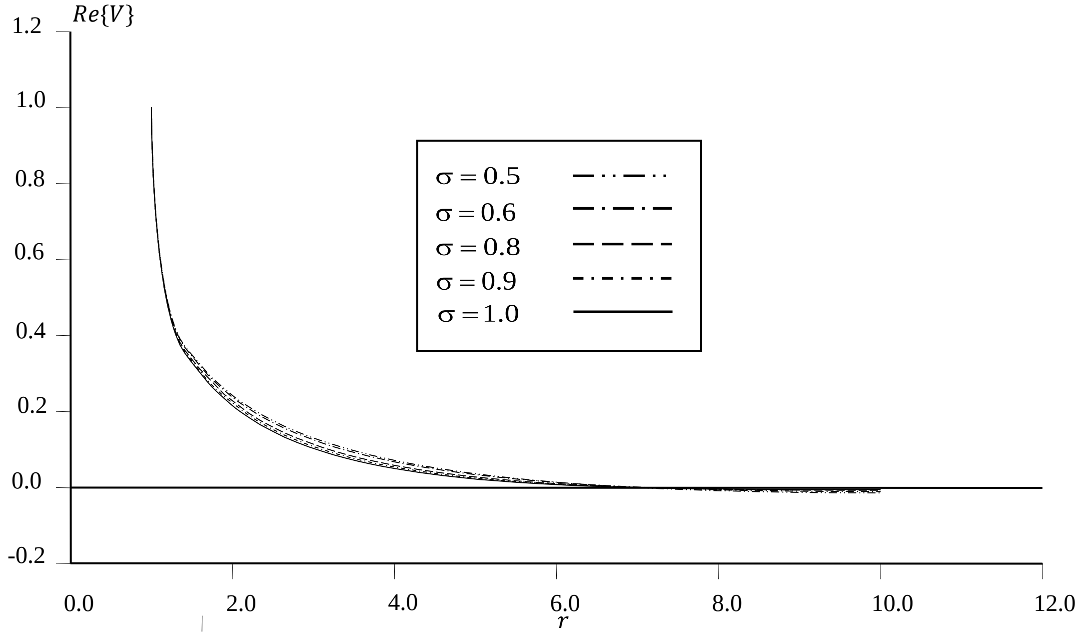


Fig.13: Variation of the real part of the velocity when $\eta'$=0.0, $\eta = 0.01$ and $t = 0.1$

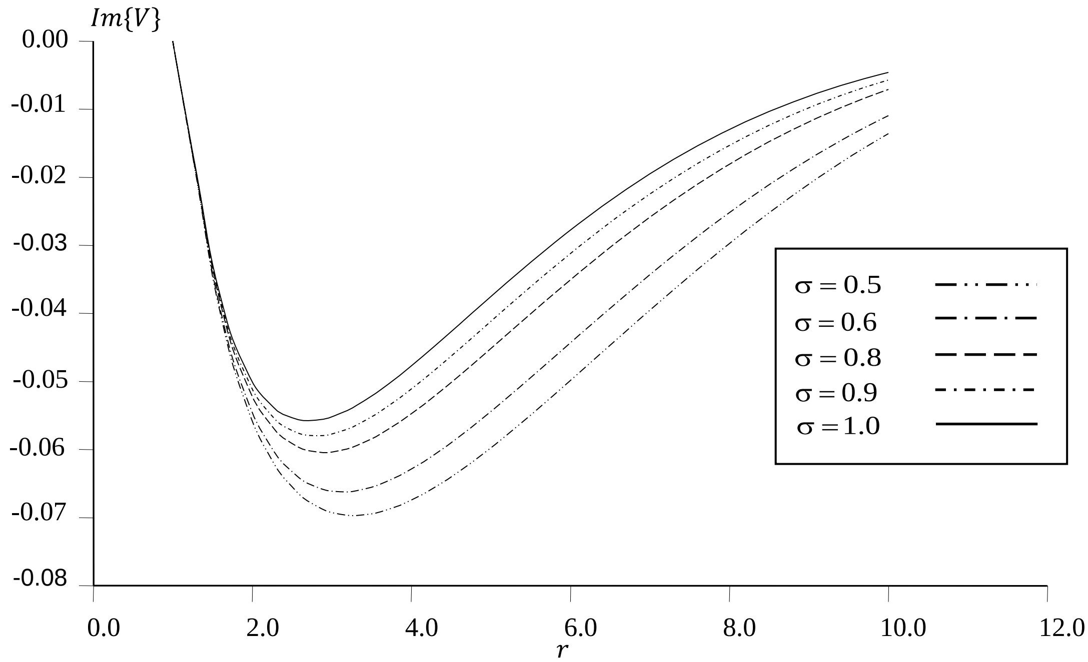


Fig.14: Variation of the imaginary part of the velocity when $\eta'$=0.0, $\eta = 0.01$ and $t = 0.1$

## 6. CONCLUSION

A. The problem of rotary oscillation of a rigid sphere in an incompressible couple stress fluid is solved.

B. The non-dimensional torque exerted by a couple stress fluid flow on the spherical surface of a rigid sphere oscillating rotationally in it is evaluated and discussed through graphs. It is concluded from the numerical results that the increase in the viscosity parameter $\eta$ results in an increase in each of the real and imaginary parts of the torque.

C. The numerical results indicate that the second couple stress viscosity coefficient, namely $\eta'$, has a small influence on the torque.

D. Taking the viscosity parameter $\eta$ to be zero, the classical case of Navier-Stokes theory can be recovered.

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