Which exchange rates are stable Paretian?

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1. Introduction

The distribution of price movements is important in data analysis and economic modeling. If a distribution is stable Paretian but not normal the sample variance is an inappropriate measure of variability and many statistical tools, such as least-squares, are inappropriate (Fama, 1963). Also, because the variability of price movements distributed according to a stable Paretian function is different from those distributed according to a normal function, risk avoidance behavior is different and, for example, standard option pricing models are no longer valid.

While weekly exchange rates, both under fixed and floating regimes, have been found to be stable Paretian (Westfield, 1977), daily exchange rates in general appear to be a mixture of normal distributions (Coppes, 1995). However, in contrast with other exchange rates the daily German Mark/Great Britain Pound rate seems to stable Paretian, what Coppes (1995) attributes to dependence between daily price changes that might be caused by the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS) limiting those price

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changes.

We tested several exchange rates and found that while those rates that are under some constraint (like those in the ERM or the Canadian dollar/US dollar) seem to be stable Paretian, those without apparent constraints seem to be a mixture of finite variance distributions (like the normal). However, we argue that it is not dependence induced by a constraint like the ERM that induces a distribution to be stable Paretian instead of Gaussian: whether the marginal distribution of a random variable is stable Paretian or Gaussian has nothing to do with the dependence structure of the process.

This argument proceeds in two fronts. On one hand, we present empirical evidence that both limited exchange rates (those that are under some institutional constraint) and non-limited exchange rates (those for whom that constraint is not apparent, for example, the Japanese Yen/German Mark) suffer from lack of independence and still exhibit different distributions. On the other hand, we show that the German Mark/Great Britain Pound exchange rate suffers from autocorrelation (and thus also from lack of independence) before entering the ERM but not after, and still the properties of the distributions before and after are very similar.

2. Stable Paretian distributions

The characteristic function of stable Paretian distributions is:

$$\phi(t)=\exp[i\delta t-\gamma|t|^{\alpha}\{1-i\beta(t/|t|)\omega(\alpha,\ t)\}] \tag{1}$$

with

$$\omega(\alpha,\ t)=\begin{cases}\tan(\alpha\pi/2), \text{ if } \alpha\neq 1\\ -2\log|t|/\pi, \text{ if } \alpha=1\end{cases}$$

where α is the characteristic exponent, β is a measure of skewness, γ and δ are the dispersion and location parameters and i is the imaginary unit[1].

To determine whether a symmetrical distribution ($\beta=0$) is better fitted by the normal or by other stable Paretian function we need to estimate the characteristic exponent. This exponent can take values in the interval $0<\alpha\leqq 2$. As α increases the leptokurtosis of the distribution decreases until it reaches its minimum with the normal distribution (when $\alpha=2$). To estimate α a three-step procedure may be used (Fama and Roll, 1968, 1971): first, $c=\gamma^{1/\alpha}$, the scale parameter, is estimated using:

$$\hat{c}=(1/2)(1/0.827)(x_{0.72}-x_{0.28}) \tag{2}$$

where $x_{0.72}$ and $x_{0.28}$ are the 72nd and 28th percentiles of the data distribution; second, z_f (for a large value of f, with $0<f<1$) is calculated with the aid of the following expression:

$$z_f=(x_f-x_{1-f})/2\hat{c} \tag{3}$$

where x_f is the $f(N+1)$th order statistic used to estimate the fth percentile of the data distribution with size N; finally, this z_f is referred to a table of percentiles of standardized symmetric stable distributions (Table 2 in Fama and Roll, 1968) to obtain the value of α whose fth percentile best matches z_f.

If a sample of observations is drawn from a stable Paretian distribution then every non-overlapping sum of observations of that sample will have the same characteristic exponent. On the other hand, if the distribution is not stable but has finite second moments, the estimates of α should increase towards 2 as the number of observations in each

1) For a complete treatment see Feller (1968) or Stuart and Ord (1987).

sum increases (Fama, 1963). This fact can be used to judge the stability of a distribution.

3. Results

From the FOREX data base (made available by the *Nihon Keizai Shinposha*) the daily returns of exchange rates of the Belgium franc (BFR), the Canada dollar (CAN), the French franc (FFR), the Netherlands's Guilder (DGL), United Kingdom pound (GBP) and the Japanese Yen (JPY) against US dollar (USD) and the German mark (DEM), and also of the DEM/USD rate were computed. The mean, standard deviation, skewness, kurtosis, Ljung–Box Q-statistic (for p=12, 24)[2], and estimated α (for $f=0.95$) for the original series of daily returns, and also for the sum of five, twenty and forty consecutive non-overlapping daily returns for each of the above exchange rates for the period from March 1, 1973 to July 30, 1993 are presented in Table 1. We also present in this table the results for the returns of the GBP/DM rate for the period from March 1, 1973 to October 5, 1990 (thus excluding the period after Great Britain became full member of the EMS) and for the period from October 9, 1990 to September 14, 1992 (the period when it was full member). The Jarque–Bera statistic (not shown) was also computed and allows us to reject the hypothesis of normality for every level of significance for all daily returns series both for the entire sample as well for several portions of it.

Observation of Table 1 shows that:

(1) —the sample means are approximately zero (but their signs are not always the expected ones: for example, although the DM ap-

2) The choice of p is arbitrary: for large values of p some power of the test is lost, but for small values highly significant correlations at relatively high lags is not captured.

Table 1 Descriptive statistics, Ljung–Box Q–statistics and estimated values for the characteristic exponent (for the original series and for the sum of 5, 20 and 40 consecutive observations) of several daily exchange rate returns

Ex. Rate	Mean	Std. Dev.	Skewness	Kurtosis	Q(12)	Q(24)	$\alpha(1)$	$\alpha(5)$	$\alpha(20)$	$\alpha(40)$
BFR/USD	2.81E-05	0.008	0.29	30.3	125.13**	135.26**	1.41	1.51	1.72	2
CAN/USD	5.45E-05	0.003	0.66	70.4	22.80*	40.24*	1.49	1.60	1.77	1.69
DGL/USD	−3.05E-05	0.008	1.11	63.2	177.61**	204.39**	1.46	1.60	1.86	2
DM/USD(1)	1.15E-04	0.007	−0.10	7.2	27.45**	39.73*	1.48	1.62	1.81	1.87
USD/GBP(2)	−7.82E-05	0.007	−0.02	8.0	25.58*	41.16*	1.36	1.57	1.98	2
USD/GBP(3)	−3.46E-05	0.007	0.06	8.4	27.38**	42.40*	1.33	1.50	1.93	2
USD/GBP(4)	−5.00E-05	0.008	−0.17	4.3	11.88	23.80	1.69	1.91	2	2
FFR/USD	9.06E-05	0.007	0.32	11.5	35.24**	53.98**	1.40	1.50	1.87	2
JPY/USD	1.62E-04	0.007	−0.04	9.9	43.59**	58.16**	1.33	1.44	1.60	2
BFR/DM	9.58E-05	0.005	1.35	112.7	954.13**	975.66**	1.28	1.26	1.24	1.26
CAN/DM	1.65E-04	0.007	0.02	7.5	21.99*	34.32	1.52	1.61	2	2
DGL/DM	3.54E-05	0.005	2.95	406.7	1010.30**	1068.80**	1.24	1.17	1.09	1.12
DM/GBP(2)	−1.77E-04	0.006	−0.26	10.2	33.71**	47.15**	1.42	1.41	1.39	1.54
DM/GBP(3)	−1.72E-04	0.006	−0.26	9.0	27.75**	37.85*	1.45	1.42	1.42	1.56
DM/GBP(4)	−1.35E-04	0.003	0.02	9.5	10.15	17.28	1.56	1.57	1.43	1.39
FFR/DM	1.59E-04	0.004	1.06	63.4	424.70**	435.80**	1.25	1.21	1.01	1
JPY/DM	−7.79E-05	0.006	−0.16	6.8	33.98**	54.02**	1.56	1.80	1.72	2

(1) The value of $\alpha(60)$ is 2
(2) Sample from 3–1–73 to 7–30–93
(3) Sample from 3–1–73 to 10–5–90
(4) Sample from 10–9–90 to 9–14–92
* Significant at 1%
**Significant at 5%

preciated against the USD from 2.7586 to 1.7425, or about 37%, during the sample period, the sample mean of the daily returns is positive; the same happens with the BFR and JPY against the USD);

(2) —the sample standard deviations for European currencies and the JPY is lower against the DM than against the USD; for the CAN the opposite is true. However, because stable Paretian distributions have not second moments, the standard deviation is not an unbiased measure of dispersion, and thus care should be taken in interpreting these results as indicating that, for example, the BFR is 1.6 more volatile against the USD than against the DM; using another variability measure, the absolute mean deviation, we get that the BFR is 4 times more volatile against the USD than against the DM, what is economically very significant;

(3) —skewness is higher against the DM than against the USD for the European currencies and the JPY, and the opposite holds for the CAN. Because the procedure used to estimate the characteristic exponent holds only for symmetric distributions it was considered that a further check of the symmetry of the sample values was necessary for the BFR, DGL and FFR against the DEM; thus we tested the null that half of the observations are below the mean using the statistic $s=2\{(N^-/N)-0.5\}\sqrt{N}$, where N^- is number of observations below the mean and N is the total number of observations, with $s \sim N(0, 1)$; while the null can not be rejected for the DGL and FFR, it can be rejected for the BFR/DM at 5% (but not at 2%) significance. Thus the estimated α's for BFR/DEM should be interpreted with reservation;

(4) —all rates are leptokurtic and some very much so; kurtosis is specially high for the CAN/USD and the BFR/DEM, DGL/DEM and FFR/DEM;

(5) —the Ljung-Box Q-statistic shows that there is autocorrelation

(and thus lack of independence) in all series, except for the daily DEM/GBP and USD/GBP return rates during the period when Great Britain belonged to the ERM of the EMS (October 9, 1990 to September 14, 1992);

(6) — the rates of return of all exchange rates against the USD (except for the CAN) and the JPY/DEM rate seem to be a mixture distributions with finite variance, because the estimated α's converge to 2 as the number of non-overlapping observations summed increases;

(7) — the rates of return of all exchange rates against the DEM (except for the CAN and JPY) and the CAN/USD and DEM/USD rates seem to be stable Paretian distributions, as the estimated α's appear to converge or hover around a value smaller than 2 as the number of non-overlapping observations summed increases.

4. Conclusions

We thus conclude that:

(1) — where exists an explicit or implicit arrangement to peg one currency to another it seems that the distribution of the daily returns of that exchange rate are stable Paretian; where such an arrangement does not exist the distribution seems to be a mixture of distributions with finite variance.

(2) — the GBP/DEM constitutes an exception, because even for the sample period when it did not belong to the ERM it seems to be stable Paretian; should this be taken as evidence that even before October 1990, he GBP was in some way linked to the DEM?

(3) — concerning Coppes' (1995) hypothesis that it is dependence induced by the EMS that causes the GBP/DM rate to be stable Paretian, it can be seen that it is not supported by the evidence because:

(a) — the European and Japanese exchange rates against the USD (and also the JPY/DEM) exhibit an high degree of autocorrelation (and thus lack of independence) and still are not stable Paretian;

(b) —the results for the sub-sample before Great Britain became member of the ERM are very similar to the results for the entire sample and for the sample when it belonged to the ERM. If it was the participation of the GBP in the ERM that caused dependence, the exclusion of the period of participation in the ERM from the sample would eliminate that dependence and thus it would be expected that the estimated α's of the sum of larger numbers of non-overlapping observations would converge to 2, but this is not the case.

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