A Novel Generalized Scattering Through Heterostructure With A Time-Periodic Potential

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الملخص

إن التفاعل مع مجالات خارجية تعتمد على الزمن في منظومات صغيرة الأبعاد يؤدى في حالات كثيرة إلى ظواهر نقل الكترونية جديدة . إن ميكانيكا الكم لأنظمة مجبرة ، يمكن تحليله بصورة مقبولة بدلالة طيف فلوكت Floquet أو (طيف شبه الطاقة) . من خلال إطار حالات فلوكت Floquet states فقد عممنا مبدأ تغير الكتلة الفعالة لحساب خواص النقل وعلى وجه الخصوص معامل الانتقال باستخدام مصفوفة الانتشار Scattering matrix.

في هذه الدراسة نحن نوسع نموذج تغير الكتلة الفعالة لمواد مختلفة في تركيب heterostructure . و لهذه الحالة نحن استخدمنا طريقة عامة تسمح لنا بالتعامل مع جهد متغير مع الزمن يؤثر على تركيب بئر كمي . و فى هذا الصدد فقد طبقنا طريقة وينجن لي و لورا راشيل W.Li and L.E.Reichl [1] في حالة عدة طبقات من مواد مختلفة ، بافتراض أن الكتلة الفعالة هى على الترتيب μ_I^* ، μ_{II}^* ، μ_{III}^*

ABSTRACT

Interaction with external time-dependent fields in low-dimensional systems leads in many cases to completely new phenomena of electronic transport. Dealing with a driven system, its quantum dynamics is adequately analyzed in terms of the Floquet or quasienergy spectrum.

In this paper, we extend the effective mass model to the case of different effective masses for different materials in a heterostructure. For this case we present a general method which allows us to treat a time modulated potential acting upon a quantum well structure.

We extend the work of Wenjun Li and L. E. Reichl, [1], and consider the case of a multilayer composed of different materials, assuming the

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effective mass in the layers I, II and III have the values m_I^*, m_{II}^* and m_{III}^* respectively.

Within the framework of the Floquet states approach we have generalized the concept of the a position-dependent-effective-mass (PDEM) of a heterostructure. We present a method of calculating transport properties, in particular, transmission coefficient of nanostructures by using the scattering matrix in the framework of Floquet theory. The recursive Floquet S matrix technique is quite a simple but powerful way of calculating the transmission properties for mesoscopic systems

Keywords: quasienergy spectrum , effective mass, scattering matrix , Floquet states, transmission coefficient

INTRODUCTION:

In recent years, tunneling through time periodically driven potential barriers has been intensively investigated, both theoretically [2] and experimentally on quantum dots [3], and on superlattices [4-7]. These studies renewed the interest in photon–assisted transport in semiconductor nanostructures. The possibility to investigate experimentally time–dependent transport through mesoscopic systems has opened the way to a deeper understanding of new effects strongly relying on the spatiotemporal coherence of electronic states. Moreover [8], in most time–dependent experiments like electron pumps, photon–assisted–tunneling, and lasers require an analysis going beyond the linear response theory in the external frequency. Thus, many efforts have been devoted, in last years, to the theoretical investigation of nonlinearities in semiconductor nanostructures, electronic correlations, and screening of ac fields.

There has been attracted attention of physicists in the analytical solutions of the one dimensional Schrödinger equation with a time-dependent linear potential [9-12]. In order to calculate the transmission, many methods have been developed over the past few years. On the one hand, tight-binding-like techniques [13], have been applied satisfactorily in many problems, especially, in those including disorder [14]. On the other hand, mode-matching methods have been used in problems like those of disorder-free electron waveguides. One-dimensional periodic structures have been also studied with this technique [15]. Similar approaches have been used to study photo-assisted effects in the tunneling through quantum point contacts [16] and driven quantum wells [17]. Basically, the frequency of the driving field has been shown to produce sidebands due to the non-linearity of the current-voltage characteristics.

Shirley [18] also used the Floquet's theorem to develop a general formalism for treating periodically driven quantum systems. Using this formalism, which replaces the solution of the time-dependent Schrödinger equation with the solution of a time-independent Schrödinger equation represented by an infinite matrix, he obtained closed expressions for time-average resonance transition probabilities of a strongly-driven two-level system. More recently, a variety of approaches have been proposed to deal analytically with strongly driven two-level systems [19-25]. Three- and four-level systems driven by intense laser fields has also been treated analytically [26,27]. In all of these works, interaction between the electrons has been neglected.

Electrons interacting with a time-dependent potential [28] can gain or loss energy and thus the electron system has no stationary states and, in particular, there is no stationary ground state. However if the external potential is periodic in time we can describe the state of a system using the Floquet function [1,18,28,29] which is a superposition of wave functions with energies shifted by $n\hbar\omega$ (here n is an integer; ω is the frequency of the driving potential). The existence of many components (side bands) of a wave function has a strong effect on the properties of a mesoscopic system. For instance, side bands open up additional channels for transmission through the mesoscopic system - photon assisted transmission [1,29]. The existence of side bands is also a necessary condition for pumping charge through an unbiased mesoscopic sample [31].

One of the interesting features of localized time-periodic potentials is the presence of resonances or quasi-bound "states", which could be thought of as electrons dynamically trapped by the oscillating potential. This is also a feature common to all multi-channel quantum scattering problems [31,32].

The most common method of fabricating low-dimensional structures is by "growing" compositionally graded semiconductor alloys in high-vacuum molecular beam epitaxy (MBE) machines. Structures that confine electrons are made by changing the aluminum fraction c during crystal growth, leading to a compositionally graded alloy of the form $Al_{c(r)}Ga_{1-c(r)}As$, where c varies spatially. The resulting band structure variation produces a spatially varying conduction band minimum. Hence, an electron added to the conduction band through doping, optical excitation, or electrical injection, sees a position-dependent potential. By varying c appropriately, one can engineer confining potentials that restrict electron motion to fewer

than three dimensions. In practice, however, it is possible to vary *c* in one direction only, resulting in, at best, a two-dimensional system.
In recent times, the concept of a position-dependent-effective-mass (PDEM) quantum Hamiltonian is rapidly gaining acceptance because of its increasing relevance in describing the motion of electrons in problems of compositionally graded crystals [33], following the ability to fabricate semiconductor nanostructures), quantum dots [34], liquid crystals [35], etc.. The appearance of PDEM is also well known in the energy density functional approach to the nuclear many-body problem [36] and its applications [37,38] in the context of nonlocal terms of the accompanying potential. Other theoretical considerations where PDEM have been exploited include the derivation [39] of the underlying electron Hamiltonian from instantaneous Galilean invariance and implementation of the path integral techniques [40] to calculate the Green's function [41] for step and rectangular-barrier potentials and masses. Further, PDEM has proved to be appealing in the construction of acceptable quantum mechanical systems by seeking exact solutions of the Schrödinger equation [42-50] by extending the already existing methods of spectrum generating or potential algebras [51] and those of supersymmetric quantum mechanics [52-55]. more recently, in BEC's, in quantum computation and spintronics.

In this paper, we extend the effective mass model to the case of different effective masses for different materials in a finite multilayer. In the case of a multilayer composed of different materials, the effective mass $m^*(x)$ can be a step function assuming the values m_I^*, m_{II}^* and m_{III}^* respectively in the layer *I, II* and *III*.

The proposed Hamiltonian is periodic in time with period $2\pi/\omega$. Hence, a Floquet approach can be used. Any solution of the time-dependent Schrödinger equation can be expanded as a linear combination of time-periodic states - called Floquet states of the system - with coefficients oscillating in time as $exp(-iE_it)$ where E_i is called the quasi-energy of the Floquet state.

Floquet Scattering :

We regard the situation as sketched in FIG. 1 but we consider the regions I, II & III have the effective masses μ_I^*, μ_{II}^* and μ_{III}^* respectively. The height of the potential barrier, extending from –L/2 to L/2 is V_0 subject to the harmonic driving force $V_1\cos(\omega t)$ as given by Eq.(1):

$$V(x,t)=\begin{cases}0, & x<-\frac{L}{2} \text{ and } x>\frac{L}{2}\\ V_0+V_1\cos\omega t, & -\frac{L}{2}\le x\le\frac{L}{2}\end{cases} \tag{1}$$

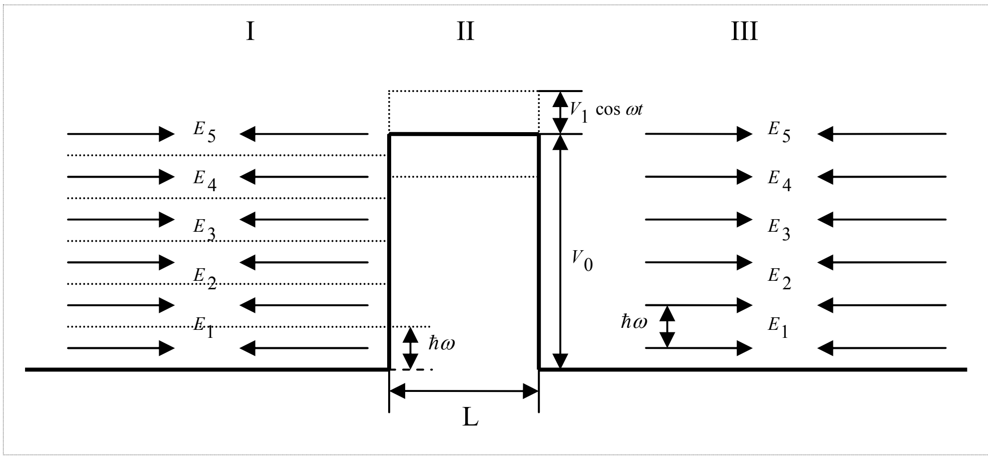

FIG.1. Floquet scattering model. Incoming and outgoing channels have energy spacing of $\hbar\omega$ and are divided into Floquet zones according to $E_n = E_0 + n\hbar\omega, \left[n \in [0,\infty)\right]$.

The Schrödinger equation can be written as:

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m^*}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t), \tag{2}$$

By the Floquet theorem the solution of the time-dependent Schrödinger equation could be converted into a time-independent eigenvalue problem. Accordingly the solution of Eq.(2) takes the form:

$$\psi_F(x,t) = e^{-iE_F t/\hbar}\,\phi(x,t), \tag{3}$$

where E_F is the Floquet eigenenergy and $\phi(x,t)$ is a periodic function , i.e. $\phi(x,t) = \phi(x,t+T)$, with period $T = 2\pi/\omega$. Differentiating Eq.(3) once and twice with respect to time and x respectively, from which we substitute into the Schrödinger equation, finally we get:

$$E_F f(x,t) = -\frac{\hbar^2}{2m^*}\frac{\partial^2 f(x,t)}{\partial x^2} - i\hbar\frac{\partial f(x,t)}{\partial t} + V(x,t)f(x,t), \tag{4}$$

In order to solve the above equation we have to match the wave functions at the boundaries of our system which consists of three regions denoted by I, II, and III respectively.

Floquet Solution Inside The Oscillting Potential

Solving Eq.(4) by the separation of variables method, so we can write $f(x,t) = g(x)f(t)$. The potential takes the form given by Eq.(1) inside the oscillating region. Substitute into Eq.(4), dividing by $g(x)f(t)$ and collecting the same variables together ,we get:

$$E_F = \left[-\frac{\hbar^2}{2m^*}\frac{1}{g(x)}\frac{\partial^2 g(x)}{\partial x^2} + V_0\right] - i\hbar\frac{1}{f(t)}\frac{\partial f(t)}{\partial t} + V_1\cos\omega t$$

Since each term has only a one variable dependent, then it will be equal to a constant E. Where we get an equation for $g(x)$:

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2 g(x)}{\partial x^2} + V_0 g(x) = Eg(x), \tag{5}$$

Another equation for $f(t)$, leads to:

$$i\hbar\frac{\partial f(t)}{\partial t} - V_1\cos\omega t\, f(t) = (E - E_F)f(t), \tag{6}$$

To integrate Eq.(6), we follow the same procedure as before in the preceding chapter, then we have:

$$f(t) = e^{-i(E-E_F)t/\hbar}\exp\left(-\frac{iV_1}{\hbar\omega}\sin\omega t\right), \tag{7'}$$

Now since $e^{\pm iz\sin\theta} = \sum_{n=-\infty}^{\infty} J_n(z)e^{\pm in\theta}$, then the last equation can be written as:

$$f(t) = e^{-i(E-E_F)t/\hbar} \sum_{n=-\infty}^{\infty} J_n\left(\frac{V_1}{\hbar\omega}\right) e^{-in\omega t}, \tag{7}$$

Since $f(t) = f(t+T)$, Eq.(7) requires that $E = E_F + m\hbar\omega$, where m is an integer. Eq.(5) can be written in a more compact form as:

$$g''(x) + q_m^2\, g(x) = 0$$

Which has the solution:

$$g(x) = \sum_{m=-\infty}^{\infty} (a_m e^{iq_m x} + b_m e^{-iq_m x}), \tag{8}$$

Where a_m and b_m are constant coefficients can be determined by the boundary conditions as well as E_F, and if we use the fact that $E = E_F + m\hbar\omega$, the q_m equation will be given by:

$$q_m = \sqrt{\frac{2m_{II}^*}{\hbar^2}(E_F + m\hbar\omega - V_0)}, \tag{9}$$

Combining the solutions for $f(t)$ and $g(x)$, to get the expression for the Floquet state inside the oscillating region $\psi^{II}(x,t)$:

$$\psi^{II}(x,t) = e^{-iE_F t/\hbar} \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \left(a_m e^{iq_m x} + b_m e^{-iq_m x}\right) J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) e^{-in\omega t}, \tag{10}$$

Floquet Solution Outside The Oscillting Potential:

The wave function outside the barrier consists of many Floquet sidebands with energy spacing $\hbar\omega$. The wave function in these regions is a superposition of an infinite number of these sidebands, and can be written in regions I and II respectively as:

$$\psi^I(x,t) = \sum_{n=-\infty}^{\infty} \left(A_n^i e^{ik_n^I x - iE_n t/\hbar} + A_n^o e^{-ik_n^I x - iE_n t/\hbar}\right) \tag{11}$$

$$\psi^{III}(x,t)=\sum_{n=-\infty}^{\infty}\left(B_n^i e^{-ik_n^{III}x-iE_nt/\hbar}+B_n^o e^{ik_n^{III}x-iE_nt/\hbar}\right) \qquad (12)$$

Where A_n^i and B_n^i are the probability amplitudes of the incoming waves from the left and right, respectively, while A_n^o and B_n^o are those of the outgoing waves and:

$$k_n^I=\sqrt{\frac{2m_I^*E_n}{\hbar^2}}\ ,\ k_n^{III}=\sqrt{\frac{2m_{III}^*E_n}{\hbar^2}}$$

The Floquet S Matrix:

The wave function $\psi(x,t)$ and its derivative must be continuous at the boundaries $x=\pm L/2$. Firstly at $x=-L/2$, $\psi^I(x=-L/2,t)=\psi^{II}(x=-L/2,t)$

$$\sum_{n=-\infty}^{\infty}\left(A_n^i e^{-ik_n^IL/2-iE_nt/\hbar}+A_n^o e^{ik_n^IL/2-iE_nt/\hbar}\right)=$$

$$e^{-iE_Ft/\hbar}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left(a_m e^{-iq_mL/2}+b_m e^{iq_mL/2}\right)J_{n-m}\left(\frac{V_1}{\hbar\omega}\right)e^{-in\omega t}$$

Equating the terms under the summation over n in both sides. On the RHS the t exponent term could be collected as $e^{-i(E_F+n\hbar\omega)t/\hbar}=e^{-iE_nt/\hbar}$, which will be cancelled with the same term on the LHS, and finally we get:

$$A_n^i e^{-ik_n^IL/2}+A_n^o e^{ik_n^IL/2}=\sum_{m=-\infty}^{\infty}\left(a_m e^{-iq_mL/2}+b_m e^{iq_mL/2}\right)J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) \qquad (13)$$

And for the first derivative

$$\frac{1}{m_I^*}\frac{\partial\psi^I(x=-L/2,t)}{\partial x}=\frac{1}{m_{II}^*}\frac{\partial\psi^{II}(x=-L/2,t)}{\partial x}$$

$$\frac{1}{m_I^*}\sum_{n=-\infty}^{\infty}\left[ik_n^I A_n^i e^{-ik_n^I L/2-iE_n t/\hbar}+(-ik_n^I)A_n^o e^{ik_n^I L/2-iE_n t/\hbar}\right]=$$

$$\frac{e^{-iE_F t/\hbar}}{m_{II}^*}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left[iq_m a_m e^{-iq_m L/2}-iq_m b_m e^{iq_m L/2}\right]J_{n-m}\left(\frac{V_1}{\hbar\omega}\right)e^{-in\omega t}$$

The t exponent term could be collected and cancelled with the same term on the LHS to have finally:

$$\frac{k_n^I}{m_I^*}\left[A_n^i e^{-ik_n^I L/2}-A_n^o e^{ik_n^I L/2}\right]=$$
$$\frac{1}{m_{II}^*}\sum_{m=-\infty}^{\infty}q_m\left[a_m e^{-iq_m L/2}-b_m e^{iq_m L/2}\right]J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \tag{14}$$

Similarly for the second boundary condition leads to:

at $x=L/2$, $\psi^{III}(x=L/2,t)=\psi^{II}(x=L/2,t)$

$$\sum_{n=-\infty}^{\infty}\left[B_n^i e^{-ik_n^{III}L/2-iE_n t/\hbar}+B_n^o e^{ik_n^{III}L/2-iE_n t/\hbar}\right]=$$

$$e^{-iE_F t/\hbar}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left[a_m e^{iq_m L/2}+b_m e^{-iq_m L/2}\right]J_{n-m}\left(\frac{V_1}{\hbar\omega}\right)e^{-in\omega t}$$

Giving:

$$B_n^i e^{-ik_n^{III}L/2}+B_n^o e^{ik_n^{III}L/2}=\sum_{m=-\infty}^{\infty}\left[a_m e^{iq_m L/2}+b_m e^{-iq_m L/2}\right]J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \tag{15}$$

And for the other first derivative, we have:

$$\frac{1}{m_{III}^*}\frac{\partial\psi^{III}(x=L/2,t)}{\partial x}=\frac{1}{m_{II}^*}\frac{\partial\psi^{II}(x=L/2,t)}{\partial x}$$

$$\frac{1}{m^*_{III}}\sum_{n=-\infty}^{\infty}\left(-ik_n^{III}B_n^i e^{-ik_n^{III}L/2-iE_nt/\hbar}+ik_n^{III}B_n^o e^{ik_n^{III}L/2-iE_nt/\hbar}\right)=$$

$$\frac{e^{-iE_Ft/\hbar}}{m^*_{II}}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left(iq_m a_m e^{iq_mL/2}-iq_m b_m e^{-iq_mL/2}\right)J_{n-m}\left(\frac{V_1}{\hbar\omega}\right)e^{-in\omega t}$$

Equating the terms under the summation over n in both sides and canceling the exponent terms on both sides leads to:

$$\frac{k_n^{III}}{m^*_{III}}\left[B_n^i e^{-ik_n^{III}L/2}+B_n^o e^{ik_n^{III}L/2}\right]=$$

$$\frac{1}{m^*_{II}}\sum_{m=-\infty}^{\infty}q_m\left(a_m e^{iq_mL/2}-b_m e^{-iq_mL/2}\right)J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \tag{16}$$

Multiply Eq.(13) by $\frac{k_n^I}{m_I^*}$ and add it to Eq.(14), the result is:

$$\frac{k_n^I}{m_I^*}2A_n^i e^{-ik_n^IL/2}=\sum_{m=-\infty}^{\infty}\left[a_m e^{-iq_mL/2}\left(\frac{k_n^I}{m_I^*}+\frac{q_m}{m_{II}^*}\right)+b_m e^{iq_mL/2}\left(\frac{k_n^I}{m_I^*}-\frac{q_m}{m_{II}^*}\right)\right]J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \tag{17}$$

And multiply Eq.(15) by $\frac{k_n^{III}}{m^*_{III}}$ and subtract Eq.(16) from the result, yields:

$$\frac{k_n^{III}}{m^*_{III}}2B_n^i e^{-ik_n^{III}L/2}=\sum_{m=-\infty}^{\infty}\left[\begin{array}{l}a_m e^{iq_mL/2}\left(\frac{k_n^{III}}{m^*_{III}}-\frac{q_m}{m^*_{II}}\right)\\ +b_m e^{-iq_mL/2}\left(\frac{k_n^{III}}{m^*_{III}}+\frac{q_m}{m^*_{II}}\right)\end{array}\right]J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \tag{18}$$

Combine Eq.(17) and Eq.(18) by adding and subtracting, we get the following matrix equation:

$$2\left(g_I k_n^I A_n^i e^{-ik_n^I L/2} \pm g_{III} k_n^{III} B_n^i e^{-ik_n^{III} L/2}\right) = \tag{19}$$

$$\sum_{m=-\infty}^{\infty} \left[\begin{array}{l} \left\{ \left(g_I k_n^I + q_m\right) a_m \pm \left(g_{III} k_n^{III} + q_m\right) b_m \right\} e^{-iq_m L/2} \\ \pm \left\{ \left(g_{III} k_n^{III} - q_m\right) a_m \pm \left(g_I k_n^I - q_m\right) b_m \right\} e^{iq_m L/2} \end{array} \right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right),$$

$$2\left(g'_I A_n^i e^{-ik_n^I L/2} \pm g'_{III} B_n^i e^{-ik_n^{III} L/2}\right) =$$

$$\sum_{m=-\infty}^{\infty} \left[\begin{array}{l} \left\{ \left(g'_I + q_m\right) a_m \pm \left(g'_{III} + q_m\right) b_m \right\} e^{-iq_m L/2} \\ \pm \left\{ \left(g'_{III} - q_m\right) a_m \pm \left(g'_I - q_m\right) b_m \right\} e^{iq_m L/2} \end{array} \right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \tag{19'}$$

Where $g'_I = g_I k_n^I$ and $g'_{III} = g_{III} k_n^{III}$.

Expressions for the probability amplitudes of the outgoing waves are taken from Eq.(13) and Eq.(15), and are given by:

$$A_n^o = \sum_{m=-\infty}^{\infty} \left[a_m e^{iq_m L/2} + b_m e^{-iq_m L/2} \right] e^{-ik_n^I L/2} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) - A_n^i e^{-ik_n^I L}, \tag{20}$$

$$B_n^o = \sum_{m=-\infty}^{\infty} \left[a_m e^{iq_m L/2} + b_m e^{-iq_m L/2} \right] e^{-ik_n^{III} L/2} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) - B_n^i e^{-ik_n^{III} L}, \tag{21}$$

Now we will work on the last five numbered equations. First trying to put it in a more compact form and rewrite Eq.(17) & Eq.(18) as follow:

$$\left[\alpha^I a_m e^{-iq_m L/2} + \beta^I b_m e^{iq_m L/2} \right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = 2(gk_n)^I e^{-ik_n^I L/2} A_n^i, \tag{22}$$

$$\left[\beta^{III} a_m e^{iq_m L/2} + \alpha^{III} b_m e^{-iq_m L/2} \right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = 2(gk_n)^{III} e^{-ik_n^{III} L/2} B_n^i, \tag{23}$$

Multiply Eq.(22) by α^{III} & Eq.(23) by α^{I}, to have:

$$\left[\alpha^{III}\alpha^{I}\alpha_m e^{-i\theta_m L/2} + \alpha^{III}\beta^{I}\beta_m e^{i\theta_m L/2}\right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = 2\alpha^{III}(\gamma k_n)^{I} e^{-ik_n^{I}L/2} A_n^{i}$$

$$\left[\alpha^{I}\beta^{III}\alpha_m e^{i\theta_m L/2} + \alpha^{I}\alpha^{III}\beta_m e^{-i\theta_m L/2}\right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = 2\alpha^{I}(\gamma k_n)^{III} e^{-ik_n^{III}L/2} B_n^{i}$$

We will add these two equations and subtract them respectively. First by addition, after simplifications, we have:

$$\left\{\left[\alpha^{III}\alpha^{I} e^{-i\theta_m L/2} + \varepsilon e^{i\theta_m L/2}\right](\alpha_m + \beta_m) - \zeta e^{i\theta_m L/2}(\alpha_m - \beta_m)\right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \Gamma_{I,III}\left(\alpha^{III}A_n^{i}\delta_{I,III} + \alpha^{I}B_n^{i}\delta_{III,I}\right) \quad (24)$$

As we have assumed:

$$\Gamma_{I,III} = 2(\gamma k_n)^{I,III} e^{-ik_n^{I,III}L/2}$$

$$\Gamma_{I,III}\delta_{I,III} = 2(\gamma k_n)^{I} e^{-ik_n^{I}L/2}$$

$$\Gamma_{I,III}\delta_{III,I} = 2(\gamma k_n)^{III} e^{-ik_n^{III}L/2}$$

And

$$\varepsilon = \gamma^{I}k_n^{I}\gamma^{III}k_n^{III} - \theta_m^2 \quad , \quad \zeta = \theta_m(\gamma^{I}k_n^{I} - \gamma^{III}k_n^{III}) \quad (25)$$

$$\begin{aligned}\alpha^{III}\beta^{I} &= (\gamma'_{III} + \theta_m)(\gamma'_{I} - \theta_m) \quad , & \alpha^{I}\beta^{III} &= (\gamma'_{I} + \theta_m)(\gamma'_{III} - \theta_m)\\ &= (\gamma'_{I}\gamma'_{III} - \theta_m^2) + \theta_m(\gamma'_{I} - \gamma'_{III}), & &= (\gamma'_{I}\gamma'_{III} - \theta_m^2) - \theta_m(\gamma'_{I} - \gamma'_{III})\\ &= (\varepsilon + \zeta) \quad , & &= (\varepsilon - \zeta)\end{aligned}$$

Secondly by subtraction, we get:

$$\left\{ \alpha^{III}\alpha^{I} e^{-iq_mL/2}(a_m - b_m) + e^{iq_mL/2}\left(-\alpha^{I}\beta^{III} a_m + \alpha^{III}\beta^{I} b_m \right) \right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) =$$

$$\Gamma_{I,III}\left(\alpha^{III} A_n^i \delta_{I,III} - \alpha^{I} B_n^i \delta_{III,I} \right)$$

Finally yields:

$$\left\{ \left[\alpha^{III}\alpha^{I} e^{-iq_mL/2} - \left(e^{iq_mL/2} \right] (a_m - b_m) + \left(e^{iq_mL/2}(a_m + b_m) \right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \tag{26}$$

$$\Gamma_{I,III}\left(\alpha^{III} A_n^i \delta_{I,III} - \alpha^{I} B_n^i \delta_{III,I} \right)$$

Combining Eq.(24) & Eq.(26) in the following matrix equation, we have:

$$\left\{ \left[\alpha^{III}\alpha^{I} e^{-iq_mL/2} \pm \left(e^{iq_mL/2} \right] (a_m \pm b_m) \mp \left(e^{iq_mL/2}(a_m \mp b_m) \right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \tag{27}$$

$$\Gamma_{I,III}\left(\alpha^{III} A_n^i \delta_{I,III} \pm \alpha^{I} B_n^i \delta_{III,I} \right)$$

Similarly, multiplying Eq.(22) by β^{III} & Eq.(23) by β^{I} to obtain:

$$\left[\beta^{III}\alpha^{I} a_m e^{-iq_mL/2} + \beta^{III}\beta^{I} b_m e^{iq_mL/2} \right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = 2\beta^{III}(\gamma k_n)^{I} e^{-ik_n^{I}L/2} A_n^i$$

$$\left[\beta^{I}\beta^{III} a_m e^{iq_mL/2} + \beta^{I}\alpha^{III} b_m e^{-iq_mL/2} \right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = 2\beta^{I}(\gamma k_n)^{III} e^{-ik_n^{III}L/2} B_n^i$$

And make the same operations as before, by addition, we have:

$$\left\{ \left[\left(e^{-iq_mL/2} + \beta^{I}\beta^{III} e^{iq_mL/2} \right] (a_m + b_m) - \left(e^{-iq_mL/2}(a_m - b_m) \right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \tag{28}$$

$$\Gamma_{I,III}\left(\beta^{III} A_n^i \delta_{I,III} + \beta^{I} B_n^i \delta_{III,I} \right)$$

And by subtraction, we get:

$$\left\{\left[\left(\alpha e^{-iq_mL/2} - \beta^I\beta^{III}e^{iq_mL/2}\right)(a_m-b_m) - \zeta e^{-iq_mL/2}(a_m+b_m)\right]\right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \Gamma_{I,III}\left(\beta^{III}A_n^i\delta_{I,III} - \beta^I B_n^i\delta_{III,I}\right) \tag{29}$$

Combining Eq.(28) & Eq.(29) in the following matrix equation as:

$$\left\{\left[\left(\alpha e^{-iq_mL/2} \pm \beta^I\beta^{III}e^{iq_mL/2}\right)(a_m\pm b_m) - \zeta e^{-iq_mL/2}(a_m\mp b_m)\right]\right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \Gamma_{I,III}\left(\beta^{III}A_n^i\delta_{I,III} \pm \beta^I B_n^i\delta_{III,I}\right) \tag{30}$$

The four equations represented by Eq.(27) & Eq.(30) can be combined into two expressions and finally to one general formula. First adding the upper halves of Eq.(27) & Eq.(30), and then adding the lower halves separately, we get the two following expressions as:

$$\left\{\begin{array}{l}\left[(\alpha^{III}\alpha^I+\varepsilon)e^{-iq_mL/2} + (\beta^I\beta^{III}+\varepsilon)e^{iq_mL/2}\right](a_m+b_m) \\ -\zeta\left(e^{iq_mL/2}+e^{-iq_mL/2}\right)(a_m-b_m)\end{array}\right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \Gamma_{I,III}\left[(\alpha^{III}+\beta^{III})A_n^i\delta_{I,III} + (\alpha^I+\beta^I)B_n^i\delta_{III,I}\right], \tag{31}$$

and

$$\left\{\begin{array}{l}\left[(\alpha^{III}\alpha^I+\varepsilon)e^{-iq_mL/2} - (\beta^I\beta^{III}+\varepsilon)e^{iq_mL/2}\right](a_m-b_m) \\ +\zeta\left(e^{iq_mL/2}-e^{-iq_mL/2}\right)(a_m+b_m)\end{array}\right\} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) = \Gamma_{I,III}\left[(\alpha^{III}+\beta^{III})A_n^i\delta_{I,III} - (\alpha^I+\beta^I)B_n^i\delta_{III,I}\right], \tag{32}$$

Finally combining the last two equations we get the following general matrix formula:

$$\left[\begin{array}{l} \left[(\alpha^{III}\alpha^{I}+\varepsilon)e^{-i\theta_m L/2} \pm (\beta^{I}\beta^{III}+\varepsilon)e^{i\theta_m L/2}\right]\chi_m^{\pm} \\ \mp\zeta\left(e^{i\theta_m L/2} \pm e^{-i\theta_m L/2}\right)\chi_m^{\mp} \end{array}\right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right) =$$

$$\Gamma_{I,III}\left[(\alpha^{III}+\beta^{III})A_n^i\delta_{I,III} \pm (\alpha^{I}+\beta^{I})B_n^i\delta_{III,I}\right] \quad (33)$$

By the matrix format we introduce the matrices whose matrix elements are defined as follows, thereafter the Floquet S matrix can be constructed.

$$(M_\alpha^{\pm})_{nm} = \left[(\alpha^{I}\alpha^{III}+\varepsilon)e^{-i\theta_m L/2} \pm (\beta^{I}\beta^{III}+\varepsilon)e^{i\theta_m L/2}\right] J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \quad (34)$$

$$(M_r^{I})_{nm} = (2\gamma^{I}k_n^{I})(2\gamma^{III}k_n^{III})e^{-ik_n^{I}L/2}\delta_{n,m}, \quad (35)$$

$$(M_r^{III})_{nm} = (2\gamma^{I}k_n^{I})(2\gamma^{III}k_n^{III})e^{-ik_n^{III}L/2}\delta_{n,m}, \quad (36)$$

$$(M_{CI}^{\pm})_{nm} = e^{-i(k_n^{I}\pm\theta_m)L/2} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \quad (37)$$

$$(M_{CIII}^{\pm})_{nm} = e^{-i(k_n^{III}\pm\theta_m)L/2} J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \quad (38)$$

$$(M_i^{I})_{nm} = e^{-ik_n^{I}L}\delta_{n,m}, \quad (39)$$

$$(M_i^{III})_{nm} = e^{-ik_n^{III}L}\delta_{n,m}, \tag{40}$$

$$(\xi^{\pm})_{nm} = \zeta\left[e^{iq_mL/2} \pm e^{-iq_mL/2}\right]J_{n-m}\left(\frac{V_1}{\hbar\omega}\right), \tag{41}$$

Eq. (33)can be written as:

$$M_a^{\pm}.C^{\pm} \mp \xi^{\pm}.C^{\mp} = M_r^I.A_n^i \pm M_r^{III}.B_n^i, \tag{42}$$

From the above equation we will determine the coefficient vector $C^{\mp}$ by some algebra. First eliminating the coefficient C^{-} by multiply Eq.(43) by $(\xi^{+})^{-1}$ and Eq.(44) by $(M_a^{-})^{-1}$ and then adding the outturn:

$$M_a^{+}.C^{+} - \xi^{+}.C^{-} = M_r^I.A_n^i + M_r^{III}.B_n^i, \tag{43}$$

$$M_a^{-}.C^{-} + \xi^{-}.C^{+} = M_r^I.A_n^i - M_r^{III}.B_n^i, \tag{44}$$

$$\left[(\xi^{+})^{-1}.M_a^{+} + (M_a^{-})^{-1}\xi^{-}\right]C^{+} = \left[(\xi^{+})^{-1}.M_r^{I} + (M_a^{-})^{-1}.M_r^{I}\right]A^i +$$

$$\left[(\xi^{+})^{-1}.M_r^{III} - (M_a^{-})^{-1}.M_r^{III}\right]B^i$$

Taking M_r^I and M_r^{III} outside the brackets we get:

$$\left[(\xi^{+})^{-1}.M_a^{+} + (M_a^{-})^{-1}\xi^{-}\right]C^{+} = \left[(\xi^{+})^{-1} + (M_a^{-})^{-1}\right]M_r^{I}.A^i +$$

$$\left[(\xi^{+})^{-1} - (M_a^{-})^{-1}\right]M_r^{III}.B^i$$

And in a more compact form:

$$\infty_{+}C^{+} = \Lambda_{+}^{+}.M_r^I.A^i + \Lambda_{+}^{-}.M_r^{III}.B^i, \tag{45}$$

Where

$$\infty_{+} = (\xi^{+})^{-1}M_a^{+} + (M_a^{-})^{-1}\xi^{-}, \tag{46}$$

And

$$L^{\pm}_{+} = (x^{+})^{-1} \pm (M^{-}_{a})^{-1}, \qquad (47)$$

Multiply both sides of equation Eq.(45) by the inverse of the matrix N_{+} to get:

$$C^{+} = ¥^{-1}_{+}.L^{+}_{+}.M^{I}_{r}.A^{i} + ¥^{-1}_{+}.L^{-}_{+}.M^{III}_{r}.B^{i}$$

And finally the reduced equation to the simple form is:

$$C^{+} = H^{+}_{+}.M^{I}_{r}.A^{i} + H^{-}_{+}.M^{III}_{r}.B^{i}, \qquad (48)$$

where

$$H^{\pm}_{+} = ¥^{-1}_{+}L^{\pm}_{+}, \qquad (49)$$

Similarly, we will eliminate C^{+} and calculate C^{-}. Multiply Eq.(43) by $(M^{+}_{a})^{-1}$ and Eq.(44) by $(x^{-})^{-1}$ and subtracting the emergents to get:

$$\left[(x^{-})^{-1}.M^{-}_{a} + (M^{+}_{a})^{-1}x^{+}\right]C^{-} = \left[(x^{-})^{-1}.M^{I}_{r} - (M^{+}_{a})^{-1}.M^{I}_{r}\right]A^{i} - \left[(x^{-})^{-1}.M^{III}_{r} + (M^{+}_{a})^{-1}.M^{III}_{r}\right]B^{i}$$

Taking M^{I}_{r} and M^{III}_{r} outside the brackets we get:

$$\left[(x^{-})^{-1}.M^{-}_{a} + (M^{+}_{a})^{-1}x^{+}\right]C^{-} = \left[(x^{-})^{-1} - (M^{+}_{a})^{-1}\right].M^{I}_{r}.A^{i} - \left[(x^{-})^{-1} + (M^{+}_{a})^{-1}\right].M^{III}_{r}.B^{i}$$

And in a more compact form as:

$$¥_{-}C^{-} = L^{-}_{-}.M^{I}_{r}.A^{i} - L^{+}_{-}.M^{III}_{r}.B^{i}, \qquad (50)$$

Where

$$¥_{-} = (x^{-})^{-1}M^{-}_{a} + (M^{+}_{a})^{-1}x^{+}, \qquad (51)$$

$$L^{\pm}_{-} = (x^{-})^{-1} \pm (M^{+}_{a})^{-1}, \qquad (52)$$

Multiply both sides of equation (50) by the inverse of the matrix N_- to have:

$$C^- = \text{¥}_-^{-1}.L^-_-.M^I_r.A^i - \text{¥}_-^{-1}.L^+_-.M^{III}_r.B^i$$

And finally it takes the form:

$$C^- = H^-_-.M^I_r.A^i - H^+_-.M^{III}_r.B^i, \qquad (53)$$

Where

$$H^{\pm}_- = \text{¥}_-^{-1}L^{\pm}_-, \qquad (54)$$

Combining Eq.(48) and Eq.(53) in a one general formula as:

$$C^{\pm} = H^{\pm}_{\pm}.M^I_r.A^i \pm H^{\mp}_{\pm}.M^{III}_r.B^i, \qquad (55)$$

Where

$$\left.\begin{aligned} H^{\pm}_{\pm} &= \text{¥}_{\pm}^{-1}L^{\pm}_{\pm} \\ \text{¥}_{\pm} &= (x^{\pm})^{-1}M^{\pm}_a + (M^{\mp}_a)^{-1}x^{\mp} \\ L^{\pm}_{\pm} &= (x^{\pm})^{-1} \pm (M^{\mp}_a)^{-1} \end{aligned}\right\} \qquad (56)$$

The Floquet sideband coefficient vectors are then given by:

$$a = \frac{1}{2}\left(C^+ + C^-\right) = \frac{1}{2}\left[H^+_+.M^I_r.A^i + H^-_+.M^{III}_r.B^i + H^-_-.M^I_r.A^i - H^+_-.M^{III}_r.B^i\right]$$

$$a = \left[\frac{H^+_+ + H^-_-}{2}\right].M^I_r.A^i + \left[\frac{H^-_+ - H^+_-}{2}\right].M^{III}_r.B^i, \qquad (57)$$

And

$$b = \frac{1}{2}\left(C^+ - C^-\right) = \frac{1}{2}\left[H^+_+.M^I_r.A^i + H^-_+.M^{III}_r.B^i - H^-_-.M^I_r.A^i + H^+_-.M^{III}_r.B^i\right]$$

$$b = \left[\frac{H_+^+ - H_-^-}{2}\right].M_r^I.A^i + \left[\frac{H_+^- + H_-^+}{2}\right].M_r^{III}.B^i, \qquad (58)$$

Eq.(20) and Eq.(21) can now be rewritten in the matrix form:

$$A^o = M_{CI}^+.a + M_{CI}^-.b - M_i^I.A^i$$

$$= M_{CI}^+.\left\{\left[\frac{H_+^+ + H_-^-}{2}\right].M_r^I.A^i + \left[\frac{H_+^- - H_-^+}{2}\right].M_r^{III}.B^i\right\} + M_{CI}^-.\left\{\left[\frac{H_+^+ - H_-^-}{2}\right].M_r^I.A^i + \left[\frac{H_+^- + H_-^+}{2}\right].M_r^{III}.B^i\right\} - M_i^I.A^i$$

$$A^o = \frac{1}{2}\left\{M_{CI}^+.\left[H_+^+ + H_-^-\right]M_r^I + M_{CI}^-.\left[H_+^+ - H_-^-\right]M_r^I - 2M_i^I\right\}.A^i + \frac{1}{2}\left\{M_{CI}^+.\left[H_+^- - H_-^+\right]M_r^{III} + M_{CI}^-.\left[H_+^- + H_-^+\right]M_r^{III}\right\}.B^i$$

$$A^o = M_{AA}.A^i + M_{AB}.B^i, \qquad (59)$$

and

$$B^o = M_{CIII}^-.a + M_{CIII}^+.b - M_i^{III}.B^i$$

$$B^o = M_{CIII}^-.\left\{\left[\frac{H_+^+ + H_-^-}{2}\right].M_r^I.A^i + \left[\frac{H_+^- - H_-^+}{2}\right].M_r^{III}.B^i\right\} + M_{CIII}^+.\left\{\left[\frac{H_+^+ - H_-^-}{2}\right].M_r^I.A^i + \left[\frac{H_+^- + H_-^+}{2}\right].M_r^{III}.B^i\right\} - M_i^{III}.B^i$$

$$B^{o} = \frac{1}{2}\begin{bmatrix} M_{CIII}^{-}\left[H_{+}^{-} + H_{-}^{-}\right]M_{r}^{I} + \\ \\ M_{CIII}^{+}\left[H_{+}^{+} - H_{-}^{-}\right]M_{r}^{I} \end{bmatrix}.A^{i} + \frac{1}{2}\begin{bmatrix} M_{CIII}^{-}\left[H_{+}^{-} - H_{-}^{+}\right]M_{r}^{III} + \\ \\ M_{CIII}^{+}\left[H_{+}^{-} + H_{-}^{+}\right]M_{r}^{III} - 2M_{i}^{III} \end{bmatrix}.B^{i}$$

$$B^{o} = M_{BA}.A^{i} + M_{BB}.B^{i}, \tag{60}$$

Combining Eq.(59) and Eq.(60), we obtain:

$$\begin{pmatrix} A^{o} \\ B^{o} \end{pmatrix} = \begin{pmatrix} M_{AA} & M_{AB} \\ M_{BA} & M_{BB} \end{pmatrix}\begin{pmatrix} A^{i} \\ B^{i} \end{pmatrix}, \tag{61}$$

$$\begin{pmatrix} A^{o} \\ B^{o} \end{pmatrix} = \mathrm{S}\begin{pmatrix} A^{i} \\ B^{i} \end{pmatrix}, \tag{62}$$

Where A^{i}, B^{i} and A^{o}, B^{o} are the incoming and outgoing amplitude vectors, respectively. The matrix S consists of all the probability amplitudes which connect the coefficients A^{i}, B^{i} to coefficients A^{o}, B^{o}. Each element S_{nm} of the matrix S gives the probability amplitude that the electron is scattered from Floquet sideband m to sideband n $\left[n, m \in (-\infty, \infty)\right]$. If we only keep the propagating modes $\left[n, m \in (0, \infty)\right]$, then we obtain the scattering matrix $\overline{\mathrm{S}}$, which satisfies the equation

$$\begin{pmatrix} \overline{A}^{o} \\ \overline{B}^{o} \end{pmatrix} = \overline{\mathrm{S}}\begin{pmatrix} \overline{A}^{i} \\ \overline{B}^{i} \end{pmatrix}, \tag{63}$$

The transmission and reflection amplitudes for the propagating modes t_{nm} and r_{nm} which determine the scattering matrix $\overline{\mathrm{S}}$ are as follows:

$$\bar{\mathrm{S}} = \begin{pmatrix} \bar{R} & \bar{T}' \\ \bar{T} & \bar{R}' \end{pmatrix} = \begin{pmatrix} r_{00} & r_{01} & . & . & t'_{00} & t'_{01} & . & . \\ r_{10} & r_{11} & . & . & t'_{10} & t'_{11} & . & . \\ . & . & & & . & . & & \\ . & . & & & . & . & & \\ t_{00} & t_{01} & . & . & r'_{00} & r'_{01} & . & . \\ t_{10} & t_{11} & . & . & r'_{10} & r'_{11} & . & . \\ . & . & & & . & . & & \\ . & . & & & . & . & & \end{pmatrix} \tag{64}$$

Where t_{nm} and r_{nm} are for propagating modes incident from the left; t'_{nm} and r'_{nm} are similar quantities for modes incident from the right.
The total transmission coefficient can be obtained from the scattering matrix by the formula:

$$T = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{k_n}{k_m}\left|t_{nm}\right|^2 \tag{65}$$

The S-matrix is the most fundamental tool for analyzing quantum scattering phenomena in various fields of physics, providing with the most complete scattering data [6,8,37].

CONCLUSIONS:

In this paper, we extend the work of Wenjun Li and L. E. Reichl, reference [1] and by allowing a time modulated potential to act upon the quantum well structure with different effective masses on the regions beside the square well.

In conclusion, within the framework of the Floquet states approach we have extended the concept of the a position-dependent-effective-mass (PDEM) of a heterostructure and present a method of calculating transport properties, in particular, transmission coefficient of nanostructures using the scattering matrix in the framework of Floquet theory. The recursive Floquet S matrix technique is quite a simple but powerful way of calculating the transmission properties for mesoscopic systems.

Since the finite parabolic quantum well has wide applications the present work is valuable for the understanding of the designing for the cases where the structure is used, especially, for the optical density spectrum in electro - optical modulators.

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