Schematic Representation for Illustrating the Procedure of Optical Noise Figure in Erbium-doped Fiber Amplifier (EDFA) and Praseodymium-doped Fiber Amplifier (PDFA)

Abdel Hakeim M. Husein and Fady I. EL-Nahal

***Abstract*—The erbium-doped fiber amplifier (EDFA) operating at 1530 nm wavelength and the praseodymium-doped fiber amplifier (PDFA) at 1300 nm wavelength are promising components for optical fiber communications due to their high gain, high saturation powers, low noise and low crosstalk. In this paper, using comprehensive models which take into account the spectroscopic properties of the fiber amplifiers, we have analyzed the temperature-dependent noise figure effects on both EDFAs and PDFAs.**

***Index Terms*—EDFA, PDFA, gain, noise, noise figure.**

I. Introduction

RARE earth doped fiber amplifiers and lasers are important tools in understanding and designing new optical devices. EDFAs are attractive devices for single-mode fibers in optical communication systems in the 1530 nm wavelength band which is known as a third window for fiber optic communication. EDFAs have many advantages such as high gain and low noise in optical communication networks.

EDFAs are characterized by gain which depends on temperature and this feature is very interesting in the modern optical transmission systems which use wavelength division multiplexing (WDM) [1]. Various temperature values in published works have then been used to experimentally find the gain at optimum amplifier lengths by analytical solution of the rate equation derivation [2–5]. The temperature-dependent gain and noise figure (NF) in EDAFs have been studied theoretically and experimentally for 980 nm and 1480 nm with the EDFAs of different lengths. These results have shown that the NF is temperature dependent [6].

Praseodymium Pr^{3+}-doped fiber amplifiers (PDFAs) are now attracting more interest, because they expected to play a vital role in upgrading 1300 nm optical systems that are used in almost all terrestrial optical telecommunication networks [7]. PDFAs have the quantum efficiency of the $^1G_4 \rightarrow {}^3H_5$ of the transition in the 1300 nm wavelength region [8–11]. For this point, the low-phonon-energy glass hosts are needed to examine the amplifier span and all optical link-capacity.

Recently, some efforts have been methodically made to develop these types of optical amplifiers for utilizing over a wide range of temperatures. Especially, one major issue of PDFA research is to develop the gain efficiency with low NF of PDFA using ZBLAN (ZrF_4, BaF_2, LaF_3, AlF_3, NaF) fluoride, sulfide (GeGaS) and some borate-based glasses as a host material [12–15]. In addition, the temperature dependence of the gain characteristics of PDFAs is critical for these systems.

In this work, we have compared the noise figure of EDFA and PDFA amplifiers after solving the rate equation by including the temperature effect observed on the transitions. The numerical results are given for both EDFA and PDFA amplifiers, operating at the 1530 nm and 1300 nm signal wavelengths respectively. EDFAs are widely used in the third window so a wider band WDM that would cover the band from 1300 to 1550 nm can be achieved [8]. The great interest in PDFA amplifiers results from the fact that an important part of the fiber optic network worldwide uses the 1300 nm second communication window. The variation of NF over the temperature range -20 °C to +60 °C was studied for different amplifier lengths.

II. Schematic Representation of NF

This work will give understanding of the physical concept of two and four level fiber amplifiers and obtaining the noise figure of the signals at different temperature range by using rate equation technique. The energy levels amplification mechanism for erbium Er^{3+} and praseodymium Pr^{3+} ions doped in glass hosts are shown in Fig. 1 and Fig. 2 respectively.

The single-mode fiber doped Er^{3+} ions can be considered homogeneously broadened in a two-level amplification system. Fig. 1 illustrates the energy diagram of erbium ions in glass hosts, R_S denotes the stimulated absorption and emission rates

A. H. M. Husein is with the Al-Aqsa University, Physics Department, P.O. Box 4051, Gaza, Gaza Strip, Occupied Palestinian Territories (phone: +970(59)9165678; fax: 970(8) 286 5309; e-mail: hakeim00@yahoo.com).

F. I. El-Nahal is with the Islamic University of Gaza, Electrical Engineering Department, Gaza, Gaza Strip, Occupied Palestinian Territories (e-mail: fnahal@iugaza.edu.ps).

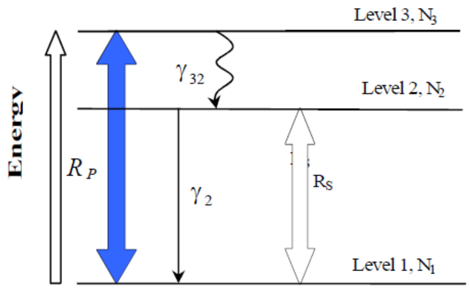

Fig. 1. The energy diagram of erbium ions in glass hosts such as silica tellurite and fluorozirconate.

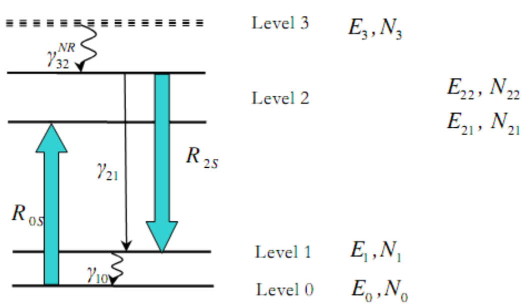

Fig. 2. The energy levels and amplification mechanism for Pr^{3+} ions in glass hosts.

associated with the signal between the population inversion densities N_1 and N_2 of the lower (ground) and upper (metastable) levels, respectively, the first and the second (metastable) levels respectively and R_p refers to both the pumping rate from the first to the third level and the stimulated emission rate between them. N_1, N_2 and N_3 are population of Er^{3+}-ions in the first, second and third level, respectively. γ_2 is the transition probability, which is the sum of the spontaneous radiative and nonradiative transition probabilities from the metastable to the ground level. γ_{32} is the transition probability per unit time for an Er^{3+} ion and represents the thermal relaxation from the upper state (third level) to the lower state (second level) [16], then the rate equation can be written as

$$\frac{dN_1}{dt} = -R_p\left(N_1 - N_3\right) + \gamma_2 N_2 + R_S(N_2 - N_1) \quad (1)$$

$$\frac{dN_2}{dt} = -R_S(N_2 - N_1) - \gamma_2 N_2 + \gamma_{32} N_3 \quad (2)$$

$$\frac{dN_3}{dt} = R_p\left(N_1 - N_3\right) - \gamma_{32} N_3 \quad (3)$$

The rate equations are solved the conditions of steady state, where all of the level populations are time invariant, i.e. $dN_i / dt = 0$, $(i = 1,2,3)$.

$$N_1 = N\frac{R_P\beta + R_S + \gamma_{21}}{R_P(1+2\beta) + R_s(2+\beta) + \gamma_{21}} \quad (4)$$

$$N_1 = N\frac{R_P\beta + R_S + \gamma_{21}}{R_P(1+2\beta) + R_s(2+\beta) + \gamma_{21}} \quad (5)$$

where $\beta = \frac{N_m}{N_{m-1}} = \exp(-\Delta E_m / k_B T)$ [19] is the Boltzmann's population relation where $\Delta E_m = E_m - E_{m-1}$ is the energy difference between the third and the second level, k_B is the Boltzmann's constant ($k_B = 1.38x10^{-23} J/K$), and T is the temperature in degrees Kelvin [20].

The gain of EDFA amplifier is given by [21]

$$G = \exp\left[\Gamma_s \sigma_s^a (\eta N_2 - N_1) L\right] \quad (6)$$

where σ_s^a is the signal absorption cross sections, L is the length of the fiber, η is the ratio between the signal emission and absorption cross sections and Γ_S is the signal mode overlap factor of the fiber with erbium distribution and dimensionless integral overlap. The amplified spontaneous emission (ASE) is taken into consideration, but the excited state absorption (ESA) effect is neglected for simplicity. The noise figure as a function of the signal gain, input and output ASE spectral densities, in decibel (dB) is given by

$$NF(dB) = 10\ \log_{10}\left(\frac{1}{G} + \frac{P_{ASE}^o(\lambda_s)}{Gh\upsilon_s} - \frac{P_{ASE}^i(\lambda_s)}{h\upsilon_s}\right) \quad (7)$$

where *1/G* is the beat noise, P_{ASE}^i and P_{ASE}^o are the amplified input and the amplified output ASE spectral density respectively, υ_s is the frequency of the signal wavelength and $h = 6.626 \cdot 10^{-34}$ Js is the Planck constant.

The population of Pr^{3+} levels is labeled as N_0, N_1, N_2, N_3, N_4 and N_5 respectively, and also the total population density N is taken as $N = N_0 + N_1 + N_2 (=N_{21}+N_{22}) + N_3 + N_4 + N_5$. In this point, the special names of N_2 and N_0 are the population densities of metastable and ground levels, respectively. The amplified spontaneous emission (ASE) is also neglected, as the signal input power was significantly above the equivalent ASE noise power. Also it includes the effect of the signal photon (ESA) to have better accuracy. To calculate all of the population of Pr^{+3} at steady state conditions, the effects of pump ESA and the cooperative upconversion are not taken into consideration. On the basis of the energy level diagram shown in Fig. 2, and using upper conditions the rate equation for Pr^{3+} population density can be written as follows

$$\frac{dN_3}{dt} = R_P N_0 - R_P N_3 - \gamma_{32} N_3 \quad (8)$$

$$\frac{dN_{22}}{dt} + \frac{dN_{21}}{dT} = R_{0S} N_0 - R_{2S} N_{22} - N_{22}\gamma_{21} + \gamma_{32} N_3 \quad (9)$$

$$\frac{dN_1}{dt} = R_{2S} N_{22} + \gamma_{21} N_{22} - \gamma_{10} N_1 \quad (10)$$

$$\frac{dN_0}{dt} = R_P N_3 - R_P N_0 - R_{0s} N_0 + \gamma_{10} N_1 \quad (11)$$

The rate equations are solved under the conditions of steady state, where all of the level populations are time invariant, i.e. $dN_i / dt = 0$, $(i = 1,2,3)$.

$$N_{22} = N\frac{R_P}{\gamma + R_P(1 + 1/\beta) + R_{2S}} \quad (12)$$

The gain of PDFA amplifier is given by [20]

$$G = \exp\left[\Gamma_S \sigma_{2S} N_{22} L\right] \quad (13)$$

where L represents the length of Pr^{3+}-doped amplifier and G denotes the signal gain and σ_{2S} is the stimulated emission cross section of transitions. The NF of the amplifier is defined as then degradation in signal-to-noise ratio from input to output of the amplifier, NF in decibel (dB) is given by:

$$NF(dB) = 10\ \log_{10}\left[\frac{1}{G}\left(1+\frac{P_{ase}\Delta\upsilon}{GP_{in}}\right)+\frac{1}{G}\frac{P_{ase}}{h\upsilon_s}\left(1+\frac{P_{ase}\Delta\upsilon}{2GP_{in}}\right)\right] \quad (14)$$

where ($1/G$) is the signal-spontaneous beat noise, P_{ase} is the ASE power density, P_{in} is the signal power density, υ_s is the frequency of the input signal, and $\Delta\upsilon$ is the optical linewidth of the amplifier.

III. Results and Discussion

The numerical calculations of the approximate expression of the noise figure of EDFA amplifier has been carried out. For simplicity the energy difference between levels of metastable is considered as 300 cm^{-1} in the room temperature. The relevant fiber parameters values for an Al/P-silica erbium-doped fiber amplifier Symbols Definitions are shown in Table 1 [21].

For numerical calculation, the fiber parameters for Pr^{3+} doped sulfide and ZBLAN amplifiers are taken from [22, 23]. The parameters and their values used in calculation are shown in Table 2. As can be noted from Table 2, the stimulated emission cross section of GeGa-sulfide glass is larger than the cross section of ZBLAN one. The lifetime of the transition τ is an important quantity to evaluate the amplifier performance since, if it is short, it then becomes necessary to pump very hard to maintain a population inversion. Here σ_{03} is the stimulated absorption of Pr^{3+} transitions υ_p and υ_s are the pump and signal frequencies respectively, P_p and P_s are the pump and signal powers respectively, Γ_p and Γ_s are the overlap factors; A_p and A_s are the effective doped areas of the core corresponding to pump and signal powers respectively, and h is Planck's constant. At room temperature for simplicity, the energy interval between N_{22} and N_{21} levels of GeGa-sulfide and ZBLAN-based Pr^{3+} ions are assumed to be close to 500 and 400 cm^{-1}, respectively. The relevant fiber parameters for both EDFA and PDFA models depend on the temperature range -20 °C to +60 °C.

TABLE I
The Relevant Fiber Parameters Values for an Al/P-Silica Erbium-Doped Fiber Symbols Definitions [21]

Symbol	Definition	Value
σ_s^e	Signal emission cross- section	$5.7\text{x}10^{-25}$ m^2
σ_s^a	Signal absorption cross- section	$6.6\text{x}10^{-25}$ m^2
σ_p^e	Pump emission cross- section	$6.6\text{x}10^{-25}$ m^2
σ_p^a	Pump absorption cross- section	$2.44\text{x}10^{-25}$ m^2
τ	Life time	10.8 ms
N	Erbium concentration	$3.86\text{x}10^{24}$ m^{-3}
λ_s	Signal wavelength	1530nm
λ_p	Pump wavelength	1480nm
ν_s	Signal frequency	$1.96x\ 10^{14}$ Hz
ν_p	Pump frequency	$2.027x\ 10^{14}$ Hz
$P_{ASE}^{+}(L)$	Copropagating ASE power	0.15mW
α_s	Signal abosrption constant	0.5m^{-1}
L	Fiber length	0 to 45m
P_p^i	Input pump power	30mW

TABLE II
Parameters and Their Values Used in Calculations for the Pr^{3+}-Doped Sulfide and ZBLAN Fiber Amplifiers

Parameters	GeGa-sulfide Ref.[23]	ZBLAN Ref.[24]
σ_{03}	9.7×10^{-26} m^2	4.24×10^{-26} m^2
σ_{2S}	–	1.2×10^{-26} m^2
τ	360 μs	110 μs
N	7.82×10^{25} m^{-3}	4.80×10^{25} m^{-3}
λ_s	1310 nm	1300 nm
λ_p	1028 nm	1017 nm
A_s, A_p	15.40 μm^2, 12.30 μm^2	8.04 μm^2
Γ_s, Γ_p	0.38, 0.40	0.6
L	8 m	36 m

A. *Noise Figure versus Length for Er^{3+}*

It was noted from Fig. 3 that the noise figure swiftly increases for 0–6 m length and there is no important effect of temperature dependency at different temperatures, while it is still low at lengths greater than 6 m. There is little noise effect at a variety of temperatures in this region (<6 m) because the gain and noise figure is low enough due to the length of the EDFA, as a result, the population inversion ratio is very low. If the EDFA length is low enough, the population inversion is very low so that the absorption and emission rates are low but we can find the temperature-dependent gain (temperature-dependent absorption and emission) and noise effect. It was noted that the variation of noise figure via the EDFA length in spectral contribution of copropagating ASE power. For different temperature the NF is nearly the same with some variation for different length up to 45 m.

B. *Noise Figure versus Length for Pr^{3+}*

For both GeGa-sulfide-based and ZBLAN-based Pr^{3+}-doped fiber amplifiers over the temperature range from -20 ºC to 60 ºC, the variations of noise figure with the length of the amplifier are considered. The results for the GeGa-sulfide based amplifier are shown in Fig. 4 and Fig. 5 respectively. It is obvious from the results that the signal noise figure rises with decreases in the length, at the same time the NF declines when the temperature decreases. However the NF decreases with increasing the length. Furthermore, the NF increases when the temperature rises.

We have analyzed the noise performance in EDFAs and PDFAs under different operating conditions. When comparing the two models, It is clear that the NF for the both the EDFA and PDFA increases with temperature. However, for the

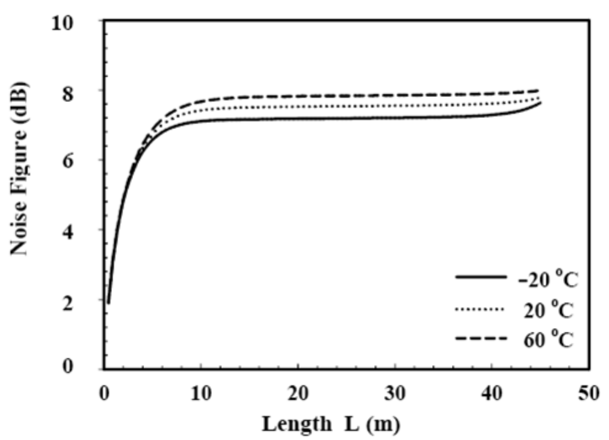

Fig. 3. Temperature-dependent noise figure analysis (the change of noise figure via erbium-doped fiber length in the spectral contribution of copropagating ASE power of 0.15 mW and when $P_p^i(0)$ = 30 mW and $P_s(0)$ = 10 µW).

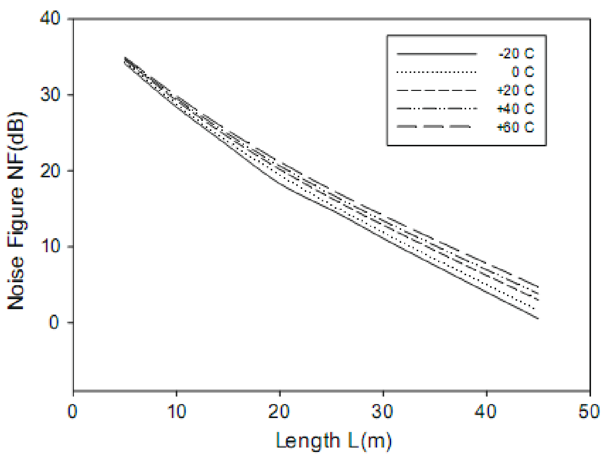

Fig. 4. The change of the noise figure with temperature and length of ZBLAN based Pr^{3+}-doped fiber amplifier for a signal input power of -30 dBm and pump power of 20 dBm.

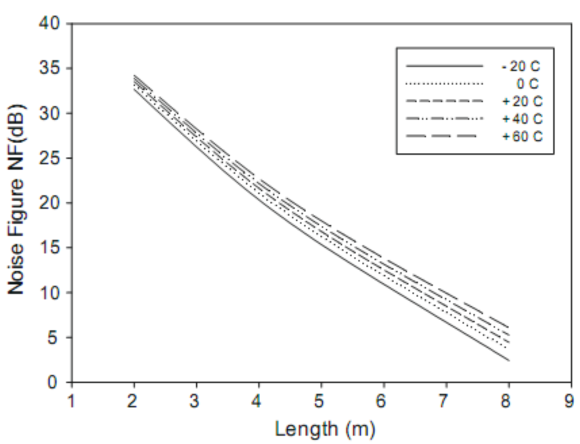

Fig. 5. The change of the noise figure with temperature and length of GeGa-sulfide based Pr^{3+}-doped fiber amplifier for a signal input power of -30 dBm and pump power of 20 dBm.

EDFA amplifier there is no variation of NF with temperature for lengths below 6 m. This can be explained by the fact that the population inversion ratio is very low. For any further increase in length it can be seen that NF increases with temperature, at the same time the NF stays almost constant with any further increase in the length. On the other hand, for the PDFA, the NF increases with increasing the temperature, at the same time, the NF decreases with increasing the length. This is due to the Boltzmann distribution change. PDFA amplifier, being a four-level system, the noise figure can be easily obtained compared with EDFA. The NF performance of the PDFA is comparable with that of EDFA. However the NF in EDFA is more sensitive to operating conditions at peak wavelengths, while PDFA it is not sensitive to operating conditions. We found that the NF is strongly signal wavelength dependent due to lower population inversion in EDFAs and due to ground state absorption (GSA) in PDFAs. We have confirmed that the NF is independent of numerical aperture (NA) for both amplifiers.

IV. Conclusion

The erbium-doped fiber amplifier is an attractive candidate for 1530 nm systems because of its very low noise and high gain, while praseodymium-doped fiber amplifier is for 1300 nm systems. The two models have been introduced as simple schematic representations for energy levels and amplification mechanisms (Two levels for Er^{3+} and four levels for Pr^{3+}). The basic rate equation models have been carried out, including the temperature effect to obtain the signal noise figure of the erbium and praseodymium-doped fiber amplifiers at research level. Moreover, we have shown the possibility of obtaining an analytical solution of the rate equations in some practical temperature ranges to comprehend the noise figure of both fiber amplifiers.

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