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COLLISIONAL DRIFT WAVES OF A WEAKLY MAGNETIZED PLASMA MODIFIED BY TEMPERATURE VARIATION AND $\vec{E}\times\vec{B}$ ROTATION

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نموذج للموجات المنحرفة فى البلازما المحصورة بواسطة مجال مغناطيسى معتمداً على التذبذب فى طاقة الإكترون و الدوران الناشئ عن المجال الكهربي

ملخص: درس هذا البحث الموجات المنحرفة في البلازما المحصورة في أسطوانة بواسطة مجال مغناطيسي خطي، مع الأخذ في عين الإعتبار ظاهرتيين فيزيائيتين أساسيتين هما التذبذب في طاقة الإلكترون و الدوران الناشئ عن المجال الكهربائي. لقد وجدت هذه الدراسة أن وجود المجال الكهربي و الدوران الناشئ عنه أدى إلى حدوث تغير مهم في نظرية الموجات المنحرفة التي صيغت بواسطة أبشر و سياسوف (1988). النظرية التي تم دراستها طبقت على بلازما الهيليوم بطريقة رنجا – كوتا، و الحسابات الناتجة أوضحت أن الدوران الناتج عن المجال الكهربي و التغير في طاقة الإلكترون أدى إلى حدوث تغير في تردد الموجات المنحرفة وسعة الجهد الكهربي المتذبذب.

Abstract: The two-fluid equations are used to derive a model of collisional drift waves for cylindrical magnetized plasmas. Both the radial electron temperature variation and the sheared $\vec{E}\times\vec{B}$ rotation in the plasmas have been taken into account. It is found that the presence of the $\vec{E}\times\vec{B}$ rotation leads to an important modification of the theory of drift waves derived by Aebischer H.A. and Sayasov Yu.S. (1988). The theory is applied to an experimental data of helium plasma using Runge-Kutta integration method. Our calculation shows that the temperature variation and the $\vec{E}\times\vec{B}$ rotation are important in the predictions of drift wave frequency and radial position of the maximum wave amplitude.

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1. Inroduction

Universal instabilities occur as a result of gradients in density and temperature in a plasma. In laboratory conditions, any finite plasma confined by a static magnetic field possess these gradients, hence the resulting waves are called “universal instabilities”. These waves are low frequency oscillations $\omega << \omega_{ce}$ (where ω_{ce} is the electron cyclotron frequency) propagating azimuthaly mainly perpendicular to both the magnetic field and the gradients with the well-known electron diamagnetic drift velocity V_{de}, so it is termed “drift waves” or “drift instability”. Drift waves are chracterized by their azimuthal mode number m (number of cycles occuring azimuthally). Usually they are found to localize where the radial plasma density gradient is largest: a region which typically falls about midway between the center of the column and the edge [1, 2].

The drift instability has been observed in many devices both linear and toroidal, and in both the collisionless and collisional dominated regimes [2-10]. The instability amplitude can attain very high levels and in many cases drift waves lead to anomalous transport of plasma across magnetic field lines [5, 6, 11, 12]. The overall appearance of the drift mode and its harmfull effect on plasma confinement have made it a prime candidate for both theoretical and experimental study.

In the last few years, some theoretical work has been done in cylindrical plasma geometry. In these papers, which always use the two-fluid equations of motion [2, 4, 5, 7], some simplifying theoretical assumptions are made that are inconsistent with the real situation as,

(i) The electron- density distribution is usually taken into account, but the electron-temperature distribution is always assumed to be constant. In fact, however, in cylindrical plasmas, the electron temperature drops dramatically from the center to the edge [13, 14]. Therefore, the variation of the radial electron temperature should be considered.

(ii) The radial electric fields are often not considered. This does not make much physical sense, since in cylindrical plasma, the particles diffuse in a direction opposite the gradient in density. The step length is the magnitude of the Larmor radius r_L which is the radius of gyration. As a result, the ions move faster than electrons because of their higher Larmor radius, and hence a radial electric field is build up in the direction of density gradient as shown in Fig. 1 [1].

When drift waves occur in a plasma column which has a radial electric field, the drift-wave frequency is affected by plasma column rotation by an "$\vec{E}\times\vec{B}$ drift". The frequency of plasma column rotation ω_E caused by the $\vec{E}\times\vec{B}$ drift is given by [1, 2]

$$\omega_E = \frac{m}{r}\frac{E}{B_0} \tag{1}$$

In his work, Zhang [14] had considered this field, but he had been eliminated the electron temperature oscillation in the theory. He also had been considered a collisionless plasma.

In the present paper, we show that the inclusion of temperature gradients and $\vec{E}\times\vec{B}$ rotations is essential and important in the predictions of drift wave frequency and radial amplitude distribution. We first formulate our theory independently of any given laboratory plasma, using the full non-viscous electron-energy equation and considering the radial dependent of the collision frequencies, making assumptions that are usually well satisfied in the plasma used to study drift waves, so that the theory is valid for a variety of plasmas. We then apply it to specific experimental data from [4] and demonstrate its usefulness.

The outline of the paper is as follows. In sec. 2, we formulate our theory based on the inclusion of the electron-temperature variations and the radial electric field $\vec{E}$ (and hence $\vec{E}\times\vec{B}$ rotation). In sec. 2.1, the basic two-fluid hydrodynamical equations are given. In sec. 2.2, we apply these equations to drift waves and derive the final differential equation for the oscillating potential, the eigenvalue of which is the complex drift-wave frequency. From the eigenfunction, representing the radial distribution of the oscillating potential, the radial distributions of all other oscillating quantities can be calculated with the formula given. Section 3 is a description of the numerical method used to solve the complex-eigenvalue problem. Section 4 presents the numerical results of the application of our theory to experimental data from [4] and the comparison of the theoretical and experimental results. Section 5 gives our conclusions.

2. Formulation of The Theory

2.1 Basic Equations

We shall consider a weakly ionized cylindrical low-β plasma (coordinates r,θ,z) in which collisions between charged particles and neutrals are important but coulomb collisions can be neglected [13]. This plasma is immersed in a strong, constant and homogeneous magnetic field pointing in the axial direction $\hat{z}$:

$$\vec{B}_0 = B_0 \hat{z} \tag{2}$$

The two-fluid equations (α =e, i) are as follows;

(i) The equation of motions [15] are;

$$n_\alpha m_\alpha \left[\frac{\partial \vec{V}_\alpha}{\partial t} + \left(\vec{V}_\alpha \cdot \vec{\nabla}\right)\vec{V}_\alpha \right] = -\vec{\nabla} P_\alpha + e_\alpha n_\alpha \left(\vec{E} + \vec{V}_\alpha \times \vec{B}_0\right) - m_\alpha n_\alpha \nu_\alpha \vec{V}_\alpha \tag{3}$$

where collisions are represented by the drag term $m_\alpha n_\alpha \nu_\alpha \vec{V}_\alpha$.

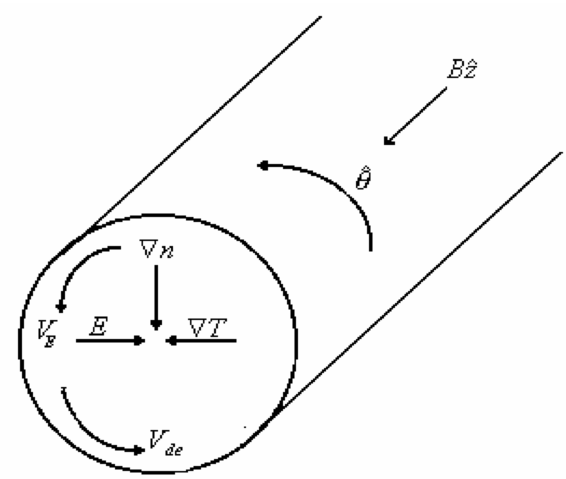

FIG. 1 CYLINDRICAL PLASMA COLUMN. GRADIENT IN DENSITY AND TEMPERATURE GIVE RISE TO THE ELECTRON (ION) DIAMAGNETIC VELOCITY (AFTER CHEN, 1984).

(ii) The equation of continuity [16] is;

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla} \cdot \left(n_\alpha \vec{V}_\alpha\right) = 0 \tag{4}$$

(iii) The equations of state as we take the ideal gas law [17] are;

$$P_\alpha = n_\alpha K_B T_\alpha \tag{5}$$

(iv) The non-viscous electron-energy equation in its reduced form, i.e. the kinetic energy and the Ohmic dissipation terms have been eliminated with the aid of the electron equations of motion and continuity [18]:

$$\frac{3}{2} n_e \left(\frac{\partial}{\partial t} + \vec{V}_e \cdot \vec{\nabla} \right) K_B T_e = -\vec{\nabla} \cdot \vec{q}_e - n_e K_B T_e \vec{\nabla} \cdot \vec{V}_e \tag{6}$$

$\vec{q}_e$ is the electron thermal flux vector.

The system of equations will be closed with the quasi-electrostatic approximation as $\vec{E} = -\vec{\nabla}\phi$ and the assumption of quasi-neutrality as $n_e = n_i$.

2.2 Second Order Differential Equation For Drift Waves

Equations (3)-(6) can easily be solved by the procedure of linearization. By this we assume that the amplitude of oscillation is small. Thus, terms with higher power of amplitude factors can be neglected [1]. The most important features in identifying the drift instability are the oscillation (fluctuation) level in plasma density, electron temperature and electrostatic potential. We separate the dependent variables, namely the particle density n_α, the fluid velocities $\vec{V}_\alpha$, the electric potential ϕ, and the electron temperature T_e, into two parts: an "equilibrium" part, indicated by a subscript 0, and a comparatively small oscillating perturbation part indicated by a subscript 1. Considering the propagation properties of drift waves mentioned above, we can represent $n_\alpha, \vec{V}_\alpha, \phi$ and T_e in the form [13]

$$\Psi_\alpha = \Psi_{\alpha^0} + \Psi_{\alpha^1}(r)e^{i(m\theta + k_z z - \omega t)} \tag{7}$$

where ω is the complex drift wave frequency given by

$$\omega = \omega_R + i\omega_I \tag{8}$$

Where ω_R is the real part of ω, and imaginary part ω_I represents the growth rate of the corresponding drift wave mode m.

Applying the general set of equations (3)-(6) to drift waves, and adopting the following assumptions as described in different literatures [2, 9, 14], we have

(i)The ions are relatively cold: $T_{i^0} << T_{e^0}$. This allows us to neglect the pressure gradient $-\vec{\nabla}P_i$, and the ion drift velocity $\vec{V}_{i^0}$ in the ion equation of motion and the continuity equation.

[13] indicated that T_{i^0} is usually 5-6 times smaller than T_{e^0} in plasmas used to study drift waves, so the ions are considered to be cold $\left(T_{i^0} \approx 0\right)$.

(ii) The applied magnetic field is strong enough such that the total electron collision frequency is much lower than the electron cyclotron frequency: $\nu_e << \omega_{ce}$. This allows us to neglect the collision term in the electron motion perpendicular to $\vec{B}_0$. This assumption further allows us to assume that the electron diamagnetic velocity $\vec{V}_{e^0}$ has a component in the θ-direction only. It follows from the equilibrium electron equation of motion that,

$$\vec{V}_{e^0} = \vec{V}_{de} = -\frac{k_B T_{e^0}}{eB_0}\left(\frac{1}{T_{e^0}}\frac{dT_{e^0}}{dr} + \frac{1}{n_{e^0}}\frac{dn_{e^0}}{dr}\right)\hat{\theta} \tag{9}$$

The temperature-gradient term enters naturally when T_{e^0} is allowed to vary radially in a magnetized plasma. For simplicity, let us introduce the following shorthand notation:

$$\vec{K} = \frac{1}{n_{e^0}}\frac{dn_{e^0}}{dr}\hat{r}, \qquad \vec{K}' = \frac{1}{T_{e^0}}\frac{dT_{e^0}}{dr}\hat{r}$$

So equation (9) becomes:

$$\vec{V}_{e^0} = -\frac{k_B T_{e^0}}{eB_0}(K' + K)\hat{\theta} \tag{10}$$

The thermal flux vector $\vec{q}_e$ can also be assumed to be parallel to the magnetic field,
and is given by [18]

$$\vec{q}_e = -\lambda_z \frac{\partial T_e}{\partial z}\hat{z} \tag{11}$$

where λ_z is the thermal conductivity along the magnetic field B_0.

(iii) We assume a sheared radial electric potential $\phi_0(r)$ i.e. an electric field $\vec{E}$ in the direction of $\vec{\nabla} n$, this field produces an $\vec{E}\times\vec{B}$ drift velocity $\vec{V}_E$,

$$\vec{V}_E = \frac{\vec{E}\times\vec{B}_0}{B_0^{\,2}} = \frac{E}{B_0}\hat{\theta} \tag{12}$$

(iv)The plasma may carry a current along $\vec{B}_0$, which can be represented by a drift of the electrons at speed u_0 in the z-direction.

(v)The phase velocity of the drift wave parallel to $\vec{B}_0$ is much higher than the ion thermal velocity $\frac{\omega}{k_z} >> \left(\frac{k_B T_{i^0}}{m_i}\right)^{\frac{1}{2}}$. This allows us to neglect ion motion parallel to $\vec{B}_0$.

(vi)The plasma is quasi-neutral at all times: $n_e = n_i$. The condition required for this approximation to be valid is that the Debye length $\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{n_0 e^2}\right)^{\frac{1}{2}}$ of the plasma is small compared with the physical size of the plasma L ($\lambda_D << L$) [10]. This assumption implies that $n_{e0} = n_{i0} = n_0$ and $n_{e1} = n_{i1} = n_1$ at all times.

(vii)The drift wave frequencies are low: $\omega << \omega_{ci} << \omega_{ce}$ [5].

With the seven assumptions stated above, and neglecting electron inertia, we obtain the following set of linearized equations from the general hydrodynamical equations (3)-(6)

$$-i\omega n_{i0} m_i \vec{V}_{i1\perp} = -en_{i0}\vec{\nabla}_\perp\phi_1 - en_{i1}\vec{\nabla}_\perp\phi_0 + en_{i0}B_0\vec{V}_{i1\perp}\times\hat{z} + en_{i1}V_E B_0\hat{r} - m_i n_{i0}\nu_i\vec{V}_{i1\perp} \tag{13}$$

$$-i\omega n_{i1} + \frac{1}{r}\frac{\partial n_{i1}}{\partial\theta}V_E + \vec{\nabla}n_{i0}\cdot\vec{V}_{i1\perp} + n_{i0}\vec{\nabla}_\perp\cdot\vec{V}_{i1\perp} = 0 \tag{14}$$

$$\begin{aligned} 0 = &-k_B n_{e1}\vec{\nabla}_\perp T_{e0} - k_B T_{e0}\vec{\nabla}_\perp n_{e1} - k_B\vec{\nabla}_\perp\left(T_{e1}n_{e0}\right) + en_{e1}\vec{\nabla}_\perp\phi_0 + en_{e0}\vec{\nabla}_\perp\phi_1 \\ &- eB_0 n_{e0}\vec{V}_{e1\perp}\times\hat{z} - eB_0 n_{e1}\left(V_{de}+V_E\right)\hat{r} \end{aligned} \tag{15}$$

$$0 = ik_z\left(-k_B T_{e1}n_{e0} - k_B n_{e1}T_{e0} + n_{e0}e\phi_1\right) - m_e\nu_e n_{e0}V_{e1z} \tag{16}$$

$$-i(\omega-\omega_1)n_{e1} + \frac{V_{de}+V_E}{r}\frac{\partial n_{e1}}{\partial\theta} + V_{e1r}\frac{dn_{e0}}{dr} + n_{e0}\vec{\nabla}\cdot\vec{V}_{e1} = 0 \tag{17}$$

$$\begin{aligned} \frac{3}{2}n_{e0}\left[k_BT_{e0}V_{e1r}K' + i\left(V_{de}\frac{m}{r} - (\omega-\omega_1-\omega_E)\right)k_BT_{e1}\right] = -\lambda_z k_z^2 k_B T_{e1} \\ -ik_BT_{e0}(\omega-\omega_1-\omega_E)n_{e1} + k_BT_{e0}\left(iV_{de}\frac{m}{r}n_{e1} + n_{e0}V_{e1r}K\right) \end{aligned} \tag{18}$$

where

$$\omega_1 = k_z u_0 \tag{19}$$

and

$$\omega_E = k_\theta V_E = \frac{m}{r}\frac{E}{B_0} = \frac{m}{r}\frac{1}{B_0}\frac{d\phi_0}{dr} \tag{20}$$

represent the drift wave frequency due to the electron motion parallel B_0 and $\vec{E}\times\vec{B}$ rotation respectively. Equation (13) is the ion equation of motion, (14) is the ion equation of continuity, (15) is the electron equation of motion perpendicular to the magnetic field lines, (16) is the electron parallel equation of motion, (17) is the electron equation of continuity, and (18) is the full non-viscous electron-energy equation.

It is possible to reduce the above system of equations to one single ordinary differential equation for the oscillating potential ϕ_1 by the procedure described below. The intermediate formulas that will be obtained are useful to describe the drift wave phenomena and to reveal important relations between the physical quantities.

The linearized ion equation of motion (13), is giving us the expression of the oscillating ion velocity $\vec{V}_{i1\perp}$ in terms of the oscillating potential ϕ_1:

$$\vec{V}_{i1\perp} = \frac{1}{B_0}\left[i\frac{\omega + i\nu_i}{\omega_{ci}}\left(\vec{\nabla}_\perp \phi_1 + \frac{n_{i1}}{n_{i0}}\vec{\nabla}_\perp \phi_0 - B_0\frac{n_{i1}}{n_{i0}}V_E \hat{r}\right) + \hat{z}\times\vec{\nabla}_\perp\phi_1 + \frac{n_{i1}}{n_{i0}}\hat{z}\times\vec{\nabla}\phi_0 - B_0\frac{n_{i1}}{n_{i0}}V_E\hat{\theta}\right] \tag{21}$$

Substituting this expression into the ion equation of continuity (14), yields the ion-density oscillation n_{i1} in terms of ϕ_1:

$$\frac{n_{i1}}{n_{i0}} = \frac{e}{k_B T_{e0}}\left[\frac{\omega + i\nu_i}{(\omega - \omega_E)}\rho^2\left(\frac{\partial^2\phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_1}{\partial r} - \frac{m^2}{r^2}\phi_1 + K\frac{\partial\phi_1}{\partial r}\right) + \frac{\omega_{de}}{(\omega - \omega_E)}\phi_1\right] \tag{22}$$

where ρ is the ion Larmor radius, but with the electron temperature T_{e0} in the numerator instead of the ion temperature T_{i0} [13]:

$$\rho^2 = \frac{k_B T_{e0}}{m_i \omega^2_{ci}} \tag{23}$$

The procedure for the electron equations is more involved. First, the electron perpendicular equation of motion (15) can be solved for the components V_{e1r} and $V_{e1\theta}$. The electron parallel equation of motion (16) can also be solved for the component V_{e1z} of the oscillating electron velocity V_{e1}. This yields

$$V_{e1r} = -i\frac{k_B T_{e0}}{eB_0}\frac{m}{r}\left(\frac{e\phi_1}{k_B T_{e0}} - \frac{n_{e1}}{n_{e0}} - \frac{T_{e1}}{T_{e0}}\right) \tag{24}$$

$$V_{e^{1\theta}} = \frac{1}{eB_0 n_{e^0}}\left[\begin{array}{l} k_B\left(-T_{e^0}\frac{\partial n_{e^1}}{\partial r} - n_{e^1}\frac{dT_{e^0}}{dr} - T_{e^1}\frac{dn_{e^0}}{dr} - n_{e^0}\frac{\partial T_{e^1}}{\partial r}\right) + en_{e^1}\frac{d\phi_0}{dr} + en_{e^0}\frac{\partial \phi_1}{\partial r} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad - eB_0 n_{e^1}(V_{de} + V_E) \end{array}\right] \quad (25)$$

$$V_{e^1 z} = -i\frac{k_z}{m_e \nu_e}\left(k_B T_{e^0}\frac{n_{e^1}}{n_{e^0}} + k_B T_{e^1} - e\phi_1\right) \quad (26)$$

The above equations can then be substituted into the electron equation of continuity (17). After mathematical treatments, we have obtained a simple form for the electron equation of continuity in terms of ϕ_1, n_{e^1}, and the electron-temperature oscillation T_{e^1}:

$$-i(\omega - \omega_1 - \omega_E + i\nu_{//})n_{e^1} + \frac{\nu_{//} n_{e^0}}{k_B T_{e^0}}\left(k_B T_{e^1} - e\phi_1\right) - i\frac{m}{r}\frac{n_{e^0}}{B_0}K\phi_1 = 0 \quad (27)$$

where $\nu_{//}$ is a shorthand notation for the expression

$$\nu_{//} = \frac{k_z^{\,2} k_B T_{e^0}}{m_e \nu_e} \quad (28)$$

We then continue by substituting the expression (24) for $V_{e^1 r}$ into the electron energy equation (18) and solving for the electron-temperature oscillation T_{e^1}. We get;

$$T_{e^1} = \frac{iT_{e^0}\left[\left(\omega_{de} - \frac{3}{2}\omega'_{de}\right)\frac{e\phi_1}{k_B T_{e^0}} + \left(\frac{5}{2}\omega'_{de} - \omega_s\right)\frac{n_{e^1}}{n_{e^0}}\right]}{\omega_z + \frac{i}{2}(5\omega_{de} - 3\omega_s)} \quad (29)$$

where

$$\omega_s = \omega - \omega_1 - \omega_E \quad (30)$$

$$\omega_z = \frac{1}{n_0}\lambda_z k_z^{\,2} \quad (31)$$

and

$$\omega_{de} = -\frac{k_B T_{e^0}}{eB_0}\frac{m}{r}K, \qquad \omega'_{de} = -\frac{k_B T_{e^0}}{eB_0}\frac{m}{r}K' \quad (32)$$

If we then substitute equation (30) for T_{e^1} into the equation of continuity (27), the electron density oscillation in terms of ϕ_1 can be produced, as which have done for the ions in (22):

$$\frac{n_{e^1}}{n_{e^0}} = \frac{\left(\nu_{//} - i\omega_{de}\right)\left[\omega_z + i\frac{3}{2}\left(\omega_{de}{}^* - \omega_s\right)\right] + \omega_{de}\left(\omega_{de} - \frac{3}{2}\omega'_{de}\right)}{\left(\nu_{//} - i\omega_s\right)\left[\omega_z + i\frac{3}{2}\left(\omega_{de}{}^* - \omega_s\right)\right] + \omega_s\left(\omega_{de} - \frac{3}{2}\omega'_{de}\right) + i\nu_{//}\left(\omega_{de}{}^* - \omega_s\right)} \frac{e\phi_1}{k_B T_{e^0}} \quad (33)$$

where

$$\omega_{de}{}^* = \omega_{de} + \omega'_{de} \quad (34)$$

From the quasi-neutrality condition, it then follows that (22) and (33) can be put equal to each other. This leads to the desired differential equation for the radial distribution of the oscillating potential ϕ_1:

$$\frac{d^2\phi_1(r)}{dr^2} + \left[\frac{1}{r} + K(r)\right]\frac{d\phi_1(r)}{dr} + \left[Q(r,\omega) - \frac{m^2}{r^2}\right]\phi_1(r) = 0 \quad (35)$$

where

$$Q(r,\omega) = \frac{\omega - \omega_E}{\rho^2(\omega + i\nu_i)} \times \left\{\frac{\omega_{de}}{\omega - \omega_E} - \frac{\left(\nu_{//} - i\omega_{de}\right)\left[\omega_z + i\frac{3}{2}\left(\omega_{de}{}^* - \omega_s\right)\right] + \omega_{de}\left(\omega_{de} - \frac{3}{2}\omega'_{de}\right)}{\left(\nu_{//} - i\omega_s\right)\left[\omega_z + i\frac{3}{2}\left(\omega_{de}{}^* - \omega_s\right)\right] + \omega_s\left(\omega_{de} - \frac{3}{2}\omega'_{de}\right) + i\nu_{//}\left(\omega_{de}{}^* - \omega_s\right)}\right\} \quad (36)$$

with boundary conditions at the plasma beam center r = 0, and the plasma beam radius $r = r_0$ [2,13]

$$\phi_1(0) = 0, \qquad \phi_1(r_0) = 0 \quad (37)$$

The difference between equation (35) and the equation obtained by [13] is the $Q(r,\omega)$ value. We have noticed that when the plasma column rotation is not considered, and the electron motion parallel to $\vec{B}_0$ is neglected, then $Q(r,\omega)$ in equation (36) becomes:

$$Q(r,\omega) = \frac{\omega}{\rho^2(\omega + i\nu_i)} \times \left\{\frac{\omega_{de}}{\omega} - \frac{\left(\nu_{//} - i\omega_{de}\right)\left[\omega_z + i\frac{3}{2}\left(\omega_{de}{}^* - \omega\right)\right] + \omega_{de}\left(\omega_{de} - \frac{3}{2}\omega'_{de}\right)}{\left(\nu_{//} - i\omega\right)\left[\omega_z + i\frac{3}{2}\left(\omega_{de}{}^* - \omega\right)\right] + \omega\left(\omega_{de} - \frac{3}{2}\omega'_{de}\right) + i\nu_{//}\left(\omega_{de}{}^* - \omega\right)}\right\} \quad (38)$$

which is the expression obtained by [13]. On other hand, if the radial electron temperature is also considered constant, i.e.

$$\frac{dT_{e^0}}{dr} = 0, \text{ and } \omega_{de} = \omega_{de}{}^* \quad (39)$$

then, equation (36) becomes similar to the expression derived by [5] .

Equations (35)-(37) represent a complex-eigenvalue problem for the complex drift-wave frequency ω and the complex eigenfunction ϕ_1(r), the radial distribution of the oscillating electric potential. It can be solved numerically, especially for arbitrary given undistributed density and temperature profiles $n_0(r)$ and $T_{e^0}(r)$. Once $\phi_1(r)$ is known, the remaining oscillating quantities can be computed with the aid of (21)-(34). In addition, the maximum wave amplitude position of the drift wave can be determined. This would show where the drift wave localize in the plasma beam region.

3. Numerical Method

Equation (35) is a second order differential equation which predicts an eigenvalue problem for the complex drift-wave frequency ω and the eigenfunction $\phi_1(r)$, which represents the radial distribution of the oscillating electric potential. One suitable general strategy for numerical solution of an eigenvalue problem is an iterative one (this strategy is sometimes called the "shooting method"). We guess a trial eigenvalue and generate a solution by integrating the differential equation as an initial value problem. If the resulting solution does not satisfy the boundary conditions, we change the trial eigenvalue and integrate again, repeating the process until a trial eigenvalue is found for which the boundary conditions are satisfied, such that if one integrates the differential equation (35), starting at one boundary and considering the boundary condition there, the resulting solution $\phi_1(r)$ automatically satisfies the condition at the other boundary. The boundary value problem is thus transformed into an initial-value problem. This can be solved with the aid of the Runge-Kutta integration method if one transforms the original complex second-order equation (35) into a set of coupled first-order equations [19].

4. Numerical Results

We apply our theory to the weakly ionized helium plasma described by [4]. The main plasma parameters are B_0= .077T, r_o= 2.8cm, $k_B\overline{T}_{e^o}$= 3.5ev, $v_i = 2.1\times10^5$, $v_{//} = 7.7\times10^5$, $\omega_z = 1.7v_{//}$. The drift wave frequency for m = 6 mode is $\omega_R = 3.5\times10^5\, s^{-1}$ and the growth rate $\omega_I = 1\times10^4\, s^{-1}$. The measured radial number density profile can be approximated by the relation $n(r) = n_0 \big/ \left(1 + r^2 \big/ a^2\right)$, where a is constant [14]. The fitting curve for the measured electron temperature is also approximated as $T_e(r) = \frac{T_{eo}}{\left(1 + \frac{r^2}{c^2}\right)}$, where c is similar to, but smaller than a [14] (a = 7mm, c = 4mm). In such circumstances, equation (35) becomes:

$$\frac{d^2\phi_1(u)}{du^2} + \left[\frac{1}{u} - \frac{2u}{1+u^2}\right]\frac{d\phi_1(u)}{du} + \left[Q(u,\omega) - \frac{m^2}{u^2}\right]\phi_1(u) = 0 \qquad 40)$$

where

$$u = \frac{r}{a} \qquad (41)$$

and $Q(u,\omega)$ is the same as in equation (36), but r is replaced by u and $\frac{1}{\rho^2}$ by A, where A is:

$$A = \frac{a^2}{\rho^2} \qquad (42)$$

and

$$\rho^2 = 4\times10^{-8}\left(\frac{k_B T_{e^0}}{eB_0{}^2}\right) \qquad (43)$$

To see how the variation of the electron temperature and $\vec{E}\times\vec{B}$ rotation affect the drift-wave characteristics, we solved the radial wave equation (40) for theoretical plasma which has a radial density variation, but a constant electron temperature in radius and no $\vec{E}\times\vec{B}$ rotation terms (i.e. the equation that obtained by [5]). In this process the eigen-frequency for m = 6 mode is $3.3\times10^5 s^{-1}$ is purely real. To demonstrate that the radial temperature profile $T_e(r)$ influences the radial drift-wave amplitude distribution, the radial wave equation was also solved for a real laboratory plasma which has both radial variations of electron density and temperature (i.e. the equation that obtained by [13]). For this case we find that the eigen-frequency for m = 6 mode is: $\omega_R = 4.3\times10^5 s^{-1}$, $\omega_I = 0.86\times10^4 s^{-1}$ i.e. $\omega_I > 0$ This proves the importance of the effect of the electron-temperature variation and the usefulness of the theory of [13]. The two eigen-functions for the theoretical and real laboratory plasmas are shown in Fig. 2. Clearly, the position of the maximum drift-wave amplitude for the real laboratory plasma moves further towards the plasma edge than in the theoretical plasma. The maximum wave amplitude is found at 2.39 for the theoretical case and 2.63 for the real laboratory plasma.

In the present work, the electric field is included by considering a sheared electric potential given by $\phi_0(r) = b_1 + b_2 r^2$, where b_1 and b_2 are constants [20]. This potential produces a sheared electric field $\vec{E}(r)$ and hence a non-sheared rotation frequency ω_E. If $\vec{E} = 0$ then, $\omega_E = 0$ and the eigenfunction $\phi_1(r)$ and the eigenvalue ω become as those obtained by [13].

The numerical results of the present work are found for the same mode number m = 6 as follows: $\omega_R = 4.9\times 10^5 s^{-1}$, $\omega_I = 1.1\times 10^4 s^{-1}$ which is slightly different from the measured value given above. This indicates that the inclusion of the $\vec{E}\times\vec{B}$ rotation into the theory is important and has an effect on the drift wave frequency. In Fig. 2 we have plotted the eigenfunction of our theory, the distribution of the radial wave shape from the solution with $\vec{E}\times\vec{B}$ rotation becomes more pronounced and the maximum amplitude is shifted further more to the edge of the plasma beam. The maximum wave amplitude is found at 2.9. This shift is attributed to the inclusion of both temperature gradient and $\vec{E}\times\vec{B}$ rotation as well as the density profile in the theory.

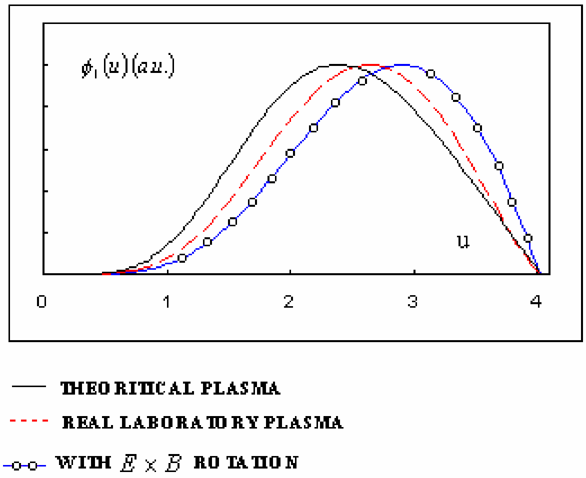

FIG. 2 THE EIGENFUNCTION $\phi_1(u)$ IN ARBITRARY UNITS (a.u.) FOR m = 6 MODE, FOR THE THREE CASES DESCRIBED ABOVE.

5. Conclusions

- We have presented a radial wave equation for drift waves in cylindrical geometry in which the gradients in electron temperature and the $\vec{E} \times \vec{B}$ rotation are included. Our calculations have been carried out by three procedures.
- In the first procedure, we only consider the gradients in electron density.
- The second procedure is concerned with both gradients in electron density and temperature.
- The third procedure is concerned with $\vec{E} \times \vec{B}$ rotation in addition to both gradients in electron density and temperature.
- We find that the drift wave frequency and the radial drift wave structure are influenced by the variation of the radial electron temperature and the $\vec{E} \times \vec{B}$ rotation. Therefore the inclusion of temperature variation and $\vec{E} \times \vec{B}$ rotation are very important to theory.

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