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Forecasts of Female Breast Cancer Referrals Using Grey Prediction Model GM(1,1)

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Abstract

Breast Cancer forecasting is an important matter for governments, health sector investors, and other related companies. Although there are different forecasting models, choosing the suitable model is of great significance. This paper focuses on utilizing the performance of grey prediction model GM(1,1) to the monthly total number of women referrals for Breast Cancer in Gaza Strip\Palestine from January 2002 to December 2016. The results show that the GM(1,1) model exhibits good forecasting ability according to the MAPE criteria. Moreover, the forecasting results are compared with the results of exponential smoothing state space (ETS) and ARIMA models. The three techniques do similarly well in forecasting process. However, GM(1,1) outperforms the ETS and ARIMA techniques according to forecasting error accuracy measure MAPE.

Keywords: Female Breast Cancer Referrals, Grey Model, Exponential Smoothing Model, Box-Jenkins Model

1 Introduction

The problem of Breast Cancer is extremely spreading among women in all developed and developing countries. It is one of the most treatable types of cancer around the world; however, the survival rates of Breast Cancer in Gaza Strip are very low compared with the other countries. The blockade of Gaza Strip affects every stage of patients' diagnosis and treatment since they are interrupted when many drugs cannot be supplied to complete the treatment process. The World Health Organization (WHO) states that the approval rates for exit permits from Gaza Strip are decreased to 44 percent in October 2016, compared with 82 percent in 2014, and 93 percent in 2012. Therefore, forecasting Breast Cancer is of great importance for various sectors in Gaza Strip such as governments, health sector investors, and other related companies.

The Grey Model (GM) is a forecasting dynamic model and has been used in various applications especially health care. As a result, an extensive literature exists on the application of GM(1, 1) model in health care. To begin with, [4] used the Box-Jenkins modeling and GM(1,1) model approach to predict the hepatitis B incidence in Qian'an. Hepatitis B incidence was collected monthly and the models were performed to calculate hepatitis B incidence. Moreover, the study of [23] shows that the original predicted values of woman suicide are obtained by the GM(1,1) model, the Verhulst model and the GM(2,1) model. The results obtained from these various models show that the forecasting accuracy of the GM(1,1) is better than the other models. In addition, [6] used three models including the exponential smoothing model, the Grey model GM(1,1), and the modified Lotka-Volterra model (L.V.) to conduct forecasting analyses based on the data of foreign patients from 2001 to 2013 in six different places. The results showed that the L.V. model obtained the best forecasting performance with an 89.7% precision rate. Furthermore, [17] applied the forecasting Grey model GM(1,1) on data collected in the smart home project to improve health services for elderly people. The results of the study showed that the Grey Model is more efficient than the Box-Jenkins ARIMA model as it has more accurate forecasting values. Adding to the mentioned studies, [30] applied an improved Grey GM(1,1) model to predict blood glucose. The original data of blood glucose of type 2 diabetes is acquired by CGMS. Then the prediction model is established. The results show that the improved Grey GM (1,1) model has excellent performance in the prediction of blood glucose. Finally, [32] uses three Grey models; the traditional Grey GM(1,1) model, the Grey Periodic Extensional Combinatorial model (PECGM(1,1)), and the Modified Grey Model using Fourier Series (FGM (1,1)) to explore an effective human Echinococcossis forecasting model in Xinjiang. The results of their study demonstrate that the dynamic epidemic prediction model is capable of identifying the future tendency.

The section above contains a summary of forecasts of female breast cancer referrals using Gray Prediction Model GM(1,1). Moreover, an overview of previous literature is also added. The remainder of the paper is organized as follows. Section 2 discusses the methodology. Section 3 contains the application to real data. Section 4 consists of the conclusion of this paper.

2 Methodology

In this section, the grey information system and the most popular predicting model GM(1,1) are described.

2.1 Grey information system and GM(1,1) models

The complicated human society and nature contain various dynamic phenomena.

These phenomena are divided into three different types. To begin with, “the white system” is when people know the origin and development of the phenomena. On the other hand, “the black system” is when people know nothing about the phenomena. Finally, “the grey system” is when people know part of the phenomena and not everything. This type of phenomena causes feelings of uncertainty and suspicion which urges modern scientific research to find a solution.
Grey System Theory is a new theoretical system that is based on the establishment of “less information uncertainty”. It is introduced by Deng Julong in the period from late 1970s to early 1980s. The birth of the Grey System Theory was in 1982 when Deng Julong published an article entitled “The Control of Grey System” in the UK publication [7]. Grey System Theory has been developed very quickly and was applied broadly in various fields. Moreover, it has reflected powerful science vitality and academic status as it contains system analysis, information processing, modeling, forecasting, planning control and decision-making ([7], [9], and [26]).

The main point of grey information system theory is to use data to extract the actual laws in system. This idea is known as grey sequence generation [19]. The grey model (GM) as a substantial part in the grey system is a forecasting dynamic model and has been successfully used in many applications like science and technology, agriculture, health care, industry and other fields [27] and [20]. The most widely used grey forecasting model is GM (1, 1), which is the general notation of “Grey Model (First Order, One Variable)”. This model is a time series prediction model with time varying coefficient. Moreover, it is not required to utilize all data from the original time series to build the GM(1,1), at least four data points needed. The process of modeling of the GM(1,1) can be summarized in the following steps [21] and [22].

Step 1. Denote the original non negative time series data by

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)),$$

where the superscript (0) represents the original raw data sequence and $x^{(0)}(k)$ represents the time series data at time k, and $k = 1, 2, \cdots, n,$ for $n \geq 4.$

Step 2. Using a first-order accumulated generation operator (AGO) to transform the original sequence $X^{(0)}$ into the one-time accumulated generating sequence (1-AGO), which is an increasing sequence denoted by $X^{(1)}$ and is written as follows:

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)),$$

where $x^{(1)}(1) = x^{(0)}(1)$ and $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),$ as $k = 1, 2, \cdots, n.$ For example, the following $X^{(0)}$ represent the raw data sequence with 6 samples:

$$X^{(0)} = (3, 6, 2, 4, 5, 8)$$

implementing the AGO, then the 1-AGO sequence $X^{(1)}$ will be

$$X^{(1)} = (3, 9, 11, 16, 20, 28)$$

It is clear that $X^{(1)}$ have a distinguished discipline and shows a clear growth trend rather than the original sequence $X^{(0)}$. (Figure 1) displays the trend of both sequences.

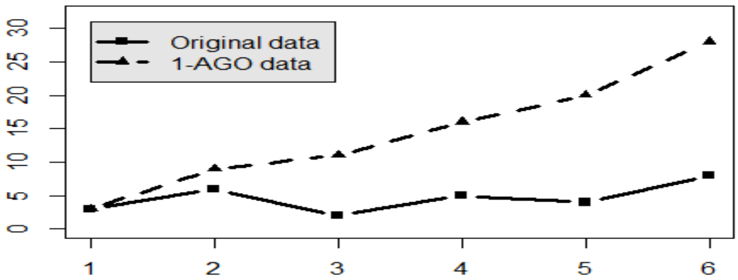

Figure 1: The trend of original and accumulative generating numbers

Step 3. From the (1-AGO) sequence $X^{(1)}$, the GM(1,1) model can be characterized by the first order differential equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \qquad (1)$$

which is known as whitenization (or image) equation of the following grey differential equation [21].

$$x^{(0)}(k) + a.z^{(1)}(k) = b, \; k = 2, 3, \cdots, n. \qquad (2)$$

where the parameters a and b of the GM(1,1) model are the developmental coefficient and the grey control quantity coefficient respectively. Moreover, $z^{(1)}(k)$ is referred to as the generated sequence of the consecutive neighbors of $X^{(1)}$ and is expressed as;

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), \; k = 2, 3, \cdots, n. \qquad (3)$$

Step 4. In order to obtain a solution of Equation (2), the parameters a and b can be evaluated using the first order linear differential equation (1), the following matrix form will be obtained:

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}.$$

This matrix form can be written as

$$\mathbf{Y} = \mathbf{B}\mathbf{v} \tag{4}$$

where,

$$\mathbf{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix} \tag{5}$$

where $\mathbf{B}$ and $\mathbf{Y}$ are the accumulated $n \times 2$ matrix and a constant $n \times 1$ vector respectively. According to Equation (3), $z^{(1)}(k)$ is the k^{th} background value. Therefore, the matrix $\mathbf{B}$ above can also be written as:

$$\mathbf{B} = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ -\frac{1}{2}(x^{(1)}(3) + x^{(1)}(4)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix},$$

It is worth noting that in Equation (4), the vector $\mathbf{Y}$ and the matrix $\mathbf{B}$ can be derived from the original data, but the vector of coefficients $\mathbf{v}$ has to be estimated. In order to provide a solution for Equation (4), the well-known least squares solution (LS) is usually employed to obtain LS approximation. As a result, Equation (4) can be written as follows:

$$\mathbf{Y} = \mathbf{B}\hat{\mathbf{v}} + \varepsilon \tag{6}$$

where ε is an error term, the vector $\mathbf{v}$ can be estimated using the following matrix deviation formula.

$$\hat{\mathbf{v}} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (\mathbf{B}^T \mathbf{B})^{-1} (\mathbf{B}^T \mathbf{Y}). \tag{7}$$

2.1.1Forecasting

Step 5. After obtaining the parameters $\hat{a}$ and $\hat{b}$, substituting in Equation (2) to get the solution $X^{(1)}$ at time k as follows:

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{\hat{a}}{\hat{b}} \right) . e^{-\hat{a}(k-1)} + \frac{\hat{b}}{\hat{a}}, \ \ k = 1, 2, \dots, n \tag{8}$$

where $\hat{x}^{(1)}(k)$ is the predicted value of $x^{(1)}(k)$ at time k. Therefore, Equation (8) is called the forecasting equation for the GM(1,1) model.

Step 6. The forecasted values of the original raw data $\hat{X}^{(0)}$ can be computed based on the (1-IAGO), one time inverse accumulated generating operation. The predicted values can be expressed as follows:

$$\hat{X}^{(0)} = \left(\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(n) \right)$$

where:

$$\begin{aligned} \hat{x}^{(0)}(1) &= x^{(0)}(1) \\ \hat{x}^{(0)}(k) &= \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \ \ k = 2, 3, \dots, n \end{aligned} \tag{9}$$

Moreover, Equation (9) can be expressed as follows:

$$\hat{x}^{(0)}(k) = \left(x^{(0)}(1) - \frac{\hat{a}}{\hat{b}} \right) . e^{-\hat{a}(k-1)} . (1 - e^{-\hat{a}}), \ \ k = 2, 3, \dots, n \tag{10}$$

2.1.2Model Accuracy Examination

In the literature, there are many distinct execution measures which can be used in order to get the accuracy of the forecasting models. In this paper, three accuracy measures have been used to test the accuracy of GM(1,1) model, they are the absolute relative error (ARE), mean absolute percentage error (MAPE) and posterior difference test (C value). These accuracy measures are defined in Table 1.

Table 1: Accuracy Measures

Acronyms	Definition	Formula
ARE	Absolute relative error	$\frac{\left\|x^{(0)}(k)-\hat{x}^{(0)}(k)\right\|}{x^{(0)}(k)}$
MAPE	Mean absolute percentage error	$\frac{1}{n}\sum_{k=1}^{n}\frac{\left\|x^{(0)}(k)-\hat{x}^{(0)}(k)\right\|}{x^{(0)}(k)}*100\%$
C value	C value	$C=\frac{S_e}{S_x}$

where $x^{(0)}(k)$ is the original value at time k, $\hat{x}^{(0)}(k)$ is its forecasting value, S_e and S_x represent the standard deviation of the residuals and standard deviation of the original sequence $X^{(0)}$, and n is the number of points in $X^{(0)}$.

A prediction can be classified in distinct Levels relying on the grades of MAPE [18] obtained in the study as shown in Table 2.

Table 2: The values of the MAPE

MAPE	$\leq 10\%$	$(10\%-20\%)$	$(20\%-50\%)$	$>50\%$
Forecasting ability	High	Good	reasonable	weak

Moreover, the prediction accuracy level depends on the obtained value of the ratio of the posterior difference (C value). It is categorized as explained in Table 3.

Table 3: The grades of the C value

C value	<0.35	[0.35,0.5)	[0.5,0.65)	≥ 0.65
prediction accuracy	OK	qualified	barely qualified	unqualified

2.2 Comparison

In this section, the GM(1,1) model is compared with several well-known models, the Box-Jenkins model namely, Autoregressive Integrated Moving Average (ARIMA) models and the Exponential Smoothing (ETS) models.

2.3 .1 Exponential Smoothing State Space Model

Exponential smoothing techniques are another well-known type of forecasting models. They are considered simple but very useful in adjusting time series forecasting. The early works of [3], [11] and [31] have initially introduced these methods. The purpose behind forecasting using exponential smoothing is to appoint exponentially decreased weights to observations as they go back in age, meaning recent observations have a larger weight than the old ones.

Pegels [24] was the first to classify exponential smoothing techniques and propose a taxonomy of the trend component and seasonal component. Pegels' taxonomy was later extended by [10], who added damped trend to the classification. This extension is then modified by [13], before the final extension is proposed by [28] who extended the classification to include damped multiplicative trends [15].

2.3.2 ARIMA Model

A non-stationary time series $\{X_t\}$ is said to follow a non-stationary autoregressive integrated moving average (ARIMA) denoted by $\text{ARIMA}(p,d,q)$ if it is expressed as:

$$\Phi_p(\mathbf{B})\nabla^d Y_t = \mu + \Theta_q(\mathbf{B})\varepsilon_t \tag{11}$$

where ε_t are identically and independently distributed as $N\left(0, \sigma^2\right)$, $t = 1, 2, \cdots, N$ and N is the number of observations, d is the order of non-seasonal differences and ∇ is the non-seasonal differencing operator, $\nabla = 1 - \mathbf{B}$. μ is the mean of a series assuming that after differencing it is stationary. As mentioned above, $\mathbf{B}$ is the backshift operator is used to simplify the representation of lag values by $\mathbf{B}X_t = X_{t-1}$. In addition, $\Phi_p(\mathbf{B})$ and $\Theta_q(\mathbf{B})$ are the autoregressive polynomial of $\mathbf{B}$ for order p as well as the moving average polynomial of $\mathbf{B}$ for order q respectively where:

$$\Phi_p(\mathbf{B}) = 1 - \phi_1\mathbf{B} - \phi_2\mathbf{B}^2 - \cdots - \phi_p\mathbf{B}^p \tag{12}$$

and

$$\Theta_q(\mathbf{B}) = 1 - \theta_1\mathbf{B} - \theta_2\mathbf{B}^2 - \cdots - \theta_q\mathbf{B}^q \tag{13}$$

More details can be found in [16], [5], [25], [1], and [2].

Parameters Estimation

The estimation of parameters for ARIMA model is a nonlinear problem that requires some special processes such as the maximum likelihood method or nonlinear least-squares estimation. At this stage of model building, the estimated parameter values should minimize the sum of squared residuals. For this purpose, many software packages are applicable for fitting ARIMA models. In this current study, the forecast package in R software will be used. To choose the best ARIMA model based on observation data, we use the corrected Akaike Information Criterion (AICc) [16].

3. Application to Real Data

We use a data set about the total number of women referred for breast cancer in Gaza strip which is registered by the Palestine Health Information Center (PHIC) in the Ministry of Health (MOH) from January 2000 to December 2016. Table 4 presents some descriptive statistics for this data set.

Table 4. Descriptive Statistics for Monthly Total Referrals 2000:1-2016:12

Statistics	Obs.	Min.	1st Qu	Mean	Std.Dev.	3rd Qu.	Max.
Total Referral	204	6	17	42.74	33.89	57	145

We use the first 192 observations from January 2000 to December 2015 as training sample to create the fitted models and the remaining 12 observations from January 2016 to December 2016, which are employed as test objects to compare their performance.

3.1 Breast Cancer Total Referral by GM(1,1)

Let us use the original data sequence denoted by $X^{(0)}$, from January 2000 to December 2015 to build the forecasting GM(1,1) model to predict the data of the 12 months from January 2016 to December 2016.

Following the process of modeling of the GM(1,1) shown above, the grey prediction model GM(1,1) is constructed as follows:

$$\hat{\mathbf{v}} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -0.0145 \\ 6.6113 \end{bmatrix}$$

According to Equation 10, the equation of forecasting is,

$$\begin{aligned} \hat{x}^{(0)}(k) &= \left(9 + \frac{0.0145}{6.6113}\right) e^{0.0145(k-1)}.(1 - e^{0.0145}), \\ &= -0.1315.e^{0.0145(k-1)}, \ k = 2,3,\ldots,192 \end{aligned} \tag{14}$$

Then, to test the accuracy of the GM(1,1) formula (14), a programming code was conducted based on R. The result are:

C value= 0.2462968,

C value <0.35, GM(1,1) The prediction accuracy level is: OK.

3.2 Breast Cancer Total Referral by ETS State Space Model

When fitting an exponential smoothing model with a state space approach, the ets() function in forecast package [12] and [14] was used. It is utilized to choose the suitable model automatically on the basis of maximum likelihood method (MLE) and then to calculate the point forecasts for the total number of women referred for breast cancer in Gaza strip for the 12 months from January 2016 to December 2016.

When applying the function, the results showed that the best performing model was ETS(M, A, N), that is a model with multiplicative error, additive trend and no seasonality. The exponential smoothing state space ETS(M, A, N) model is:

$$
\begin{aligned}
y_t &= \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}19.2346\\ 0.3576\end{bmatrix}(1+\varepsilon_t), \\
x_t &= \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}19.2346\\ 0.3576\end{bmatrix}+\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}19.2346\\ 0.3576\end{bmatrix}\begin{bmatrix}0.2616\\ 0.0000\end{bmatrix}\varepsilon_t
\end{aligned}
\tag{15}
$$

where the state vector $x_t = (\ell_t, b_t)^T$, includes the level and growth components respectively. Moreover, [15] illustrated the structures and notations of exponential smoothing state space models.

3.3 Breast Cancer Total Referral by ARIMA Model

Based on the methodology summarized above, to calculate the point forecasts for the total number of women referred for breast cancer in Gaza strip for the 12 months from January 2016 to December 2016, the results showed that the best ARIMA model was $\text{ARIMA}(1,1,1)$:

$$(1-1.2353\mathbf{B}+0.2353\mathbf{B}^2)\,Y_t = 0.5344+(1+0.8180\mathbf{B})\,\varepsilon_t \tag{16}$$

That is the final equation of the best ARIMA model is

$$Y_t = 0.5344+1.2353Y_{t-1}-0.2353Y_{t-2}+\varepsilon_t+0.8180\,\varepsilon_{t-1} \tag{17}$$

3.4 Results

The comparison of the predictive accuracy for the above three forecasting models is presented in Table 5. The mean absolute percentage error (MAPE) is utilized, which measures the predicting accuracy of model using a statistical method as defined in Table 1.

Table 5 explains that the showing of the grey prediction model GM(1,1) is better than ETS(M,A,N) and ARIMA(1,1,1) based on the mean absolute percentage error (MAPE). The MAPE value of training sample data (2000-2015) for GM(1,1), ETS(M,A,N), and ARIMA(1,1,1) is 19.525%, 32.083%, and 32.480% respectively. Similarly, the MAPE of testing sample data for GM(1,1), ETS(M,A,N), and ARIMA(1,1,1) is 9.880%, 13.700%, and 13.717% respectively. The above results show that the Grey prediction model, GM(1,1) has higher accuracy of forecasting than other forecasting models. Moreover, according to the grade values of the MAPE criteria in Table 2, the MAPE value of GM(1,1) model is categorized in good category. This demonstrate that the GM(1,1) is able to execute prediction very well.

Table 5. Prediction values and performance evaluation of actual values, GM(1,1), ETS(M,A,N), and ARIMA(1,1,1) for part of years of study (2012:1-2016:12).

Year		Actual value	GM(1,1)		ETS(M,A,N)		ARIMA(1,1,1)	
			value	ARE	value	ARE	value	ARE
2012	Jan	54	54.241	0.004	51.917	0.039	51.498	0.046
	Feb	47	55.035	0.171	52.82	0.124	54.010	0.149
	Mar	53	55.84	0.054	51.655	0.025	51.495	0.028
	Apr	57	56.657	0.006	52.364	0.081	53.590	0.060
	May	82	57.487	0.299	53.935	0.342	55.560	0.322
	Jun	58	58.328	0.006	61.634	0.063	66.665	0.149
	Jul	37	59.182	0.600	61.041	0.65	59.848	0.618
	Aug	43	60.048	0.396	55.110	0.282	51.156	0.190
	Sep	47	60.927	0.296	52.300	0.113	51.492	0.096
	Oct	25	61.818	1.473	51.271	1.051	52.025	1.081
	Nov	47	62.723	0.335	44.757	0.048	42.337	0.099
	Dec	62	63.641	0.026	45.701	0.263	48.772	0.213
2013	Jan	59	64.573	0.094	50.322	0.147	55.118	0.066
	Feb	77	65.518	0.149	52.950	0.312	55.527	0.279
	Mar	79	66.477	0.159	59.598	0.246	64.080	0.189
	Apr	83	67.45	0.187	65.031	0.216	67.675	0.185
	May	56	68.437	0.222	70.089	0.252	71.814	0.282
	Jun	69	69.438	0.006	66.761	0.032	62.991	0.087
	Jul	68	70.455	0.036	67.704	0.004	67.552	0.007
	Aug	62	71.486	0.153	68.139	0.099	67.807	0.094
	Sep	74	72.532	0.020	66.891	0.096	65.747	0.112
	Oct	67	73.594	0.098	69.108	0.031	70.482	0.052
	Nov	90	74.671	0.170	68.914	0.234	68.609	0.238
	Dec	64	75.764	0.184	74.788	0.169	78.324	0.224
2014	Jan	78	76.873	0.014	72.323	0.073	70.007	0.102
	Feb	85	77.998	0.082	74.166	0.127	75.165	0.116
	Mar	100	79.139	0.209	77.357	0.226	79.011	0.210
	Apr	95	80.298	0.155	83.638	0.120	86.770	0.087
	May	75	81.473	0.086	86.968	0.160	87.500	0.167
	Jun	88	82.665	0.061	84.195	0.043	80.927	0.080
	Jul	34	83.875	1.467	85.548	1.516	85.682	1.520
	Aug	54	85.103	0.576	72.422	0.341	63.976	0.185
	Sep	90	86.348	0.041	67.96	0.245	67.276	0.252
	Oct	56	87.612	0.565	74.083	0.323	80.292	0.434

Table 5. (Continued): Prediction values and performance evaluation of actual values, GM(1,1), ETS(M,A,N), and ARIMA(1,1,1) for part of years of study (2012:1-2016:12).

	Nov	84	88.895	0.058	69.711	0.170	68.278	0.187
	Dec	98	90.196	0.080	73.806	0.247	78.138	0.203
2015	Jan	99	91.516	0.076	80.492	0.187	85.456	0.137
	Feb	127	92.855	0.269	85.691	0.325	88.565	0.303
	Mar	119	94.214	0.208	96.854	0.186	102.559	0.138
	Apr	94	95.593	0.017	103.005	0.096	104.077	0.107
	May	90	96.992	0.078	101.007	0.122	96.769	0.075
	Jun	112	98.412	0.121	98.485	0.121	95.004	0.152
	Jul	121	99.852	0.175	102.378	0.154	103.683	0.143
	Aug	112	101.314	0.095	107.607	0.039	109.362	0.024
	Sep	103	102.796	0.002	109.114	0.059	108.133	0.050
	Oct	110	104.301	0.052	107.872	0.019	105.489	0.041
	Nov	104	105.828	0.018	108.786	0.046	108.366	0.042
	Dec	113	107.376	0.050	107.892	0.045	106.568	0.057
MAPE(%) 2000-2015				19.525		32.083		32.480
2016	Jan	97	108.948	0.123	109.586	0.130	110.265	0.137
	Feb	83	110.543	0.332	106.651	0.285	104.494	0.259
	Mar	140	112.160	0.199	100.822	0.280	97.696	0.302
	Apr	119	113.802	0.044	111.428	0.064	119.218	0.002
	May	126	115.468	0.084	113.766	0.097	114.645	0.090
	Jun	102	117.158	0.149	117.324	0.150	118.768	0.164
	Jul	122	118.872	0.026	113.673	0.068	110.477	0.094
	Aug	141	120.612	0.145	116.209	0.176	117.689	0.165
	Sep	118	122.377	0.037	123.051	0.043	126.812	0.075
	Oct	145	124.169	0.144	122.088	0.158	120.204	0.171
	Nov	112	125.986	0.125	128.439	0.147	131.480	0.174
	Dec	119	127.830	0.074	124.496	0.046	120.577	0.013
		97	108.948	0.123	109.586	0.130	110.265	0.137
MAPE(%) (2016)				9.880		13.700		13.717

The appropriate curves are displayed in Figure 2. The forecasting results clearly point out that the values predicted by these models are very close to the original values of the total number of women referred for breast cancer in Gaza strip from 2011 to 2016.

As we can see from Table 5 and Figure 2, the Grey prediction model, GM(1,1) displays a preferable performance in trend forecasting.

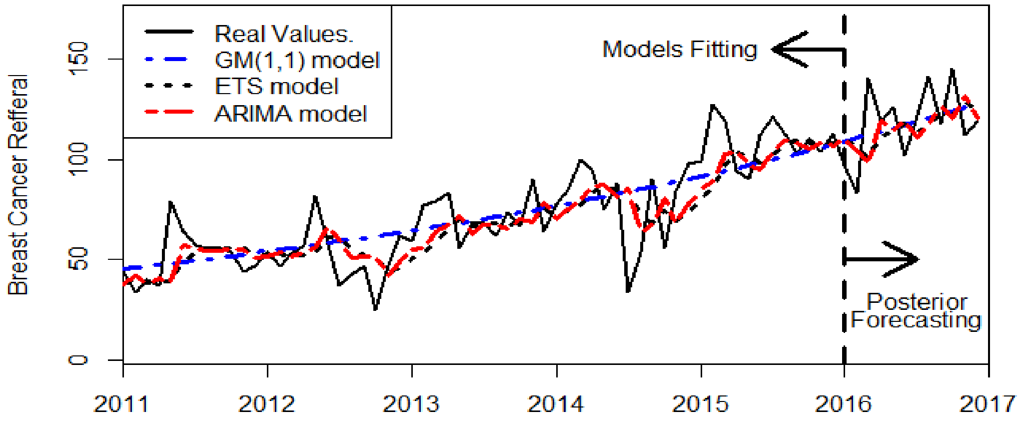

Figure 2: The Comparison between the real values and the forecasted results by the three propose models.

4 Conclusion

The main objective of this paper is to forecast the monthly total number of women referrals for Breast Cancer in Gaza/Palestine for the 2002:1-2016:12 period. Three models are used as an attempt to select the most appropriate fit. They are a prediction grey model GM(1,1), an exponential smoothing state space (ETS))model, and a Box-Jenkins ARIMA model. The results show that the GM(1,1) model exhibits good forecasting ability according to the MAPE criteria.

The analysis recognizes that the GM(1,1) model has good prediction results than the exponential smoothing state space (ETS) and ARIMA models based on the MAPE. However, we can note that although prediction grey model GM(1,1) is not widely applied in the forecasting of Breast Cancer. In addition, the empirical results assert the importance of the GM(1,1) application.

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