BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LVII (LXI), Fasc. 5, 2011
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

USING LINEAR MATRIX INEQUALITY METHOD TO DESIGN MINIMUM-LENGHT FINITE IMPULSE RESPONSE LOOP FILTERS IN FIXED WiMAX PLL

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Received, April 5, 2011
Accepted for publication: September 19, 2011

Abstract. We designed Finite Impulse Response (FIR), digital filters by Semi-Definite Programming (SDP) using SeDuMi (self-dual minimization) toolbox software. The stability is assured using Linear Matrix Inequality (LMI) constraints. The minimum length FIR filter algorithm was used to proof that the order of the FIR filter, which was designed, is optimal for our design specifications. The proposed method gave better result with regard to all specifications of control signal, where we had the faster system and more stable than the other systems.

Key words: fixed WiMAX; FIR filter; LMI method; SDP algorithm; SeDuMi software.

1. Introduction

A PLL is an electronic control system that generates a signal that has a fixed relation to the phase of a “reference” signal (Hsieh & Huang, 1996). One of the specific tasks accomplished by PLLs include frequency synthesis. A frequency synthesizer is an electronic system used for generating a range of frequencies from a single fixed time base or oscillator to high frequencies (Abramovich, 2002). The frequency synthesizer is used in Fixed and Mobile

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WiMAX communication system. Fig. 1 shows the basic form of a PLL which consists of three fundamental functional blocks namely (Abramovich, 2002), a Phase Detector (PD), a Loop Filter (LF) and a voltage controlled oscillator (VCO), where F_{in} and F_{out} are input and output frequencies, respectively. There are many different methods to design the loop filters in PLL. One of the main methods used to design those filters is the LMI method (Chou *et al.*, 2006; Long *et al.*, 2007). Digital filters can be designed using a minimax method based on Semi-Definite Programming (SDP). FIR is an example of digital filters which has been designed using SDP as explained for example by Antoniou & Lu (2007). The stability of the filters was assured by using a single LMI constraint derived from the well-known Lyapunov theory (Boyd *et al.*, 1994). In this work, FIR is introduced as loop filter for PLL. Antoniou (2005), Al-Baroudi (1997) and Wu *et al.* (1999) designed the FIR filters using different algorithms.

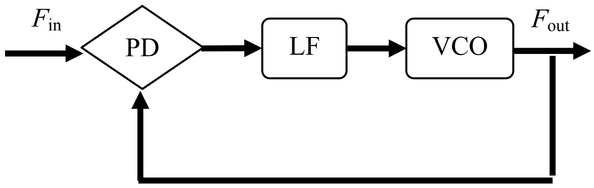

Fig. 1 – A basic PLL Block.

The primal-dual path following algorithm was used to solve the SDP problem by Antoniou and Lu (2007). In a previous work (Al-Quga, 2009), the optimal design of PLL Mobile WiMAX system was established, in which the digital filters (FIR) were designed using linear programming and convex programming (SDP). In this paper we introduced a new optimal design for FIR digital filters using LMI method. We will reformulate the filter design problem as an SDP problem and solve it by self-dual minimization algorithm. We will employ multi-objective control technique to deal with the various design objectives such as: Frequency Range used for Fixed WiMAX System (3.5…5.8 GHz), Stable System, Small Settling Time, Small Overshoot and Small Raising Time. Trade-off among the conflicting objectives will be made using SDP in conjunction with appropriate adjustment of certain design parameters. In our system the VCO noise is neglected (Kozak &Friedman, 2004). The structure of our paper is the following: section **2** introduced the design of optimal FIR digital filters using SDP method. Section **3** displays the results using the toolbox, SeDuMi and compares our results with others. Finally, the conclusion is given in section **4**.

2. Design of PLL Filter

The Loop Filter, VCO (very stable crystal oscillator with a divider by N-programmable divider in the feedback loop) and PD together comprise a frequency synthesizer. We considered a real design problem of a frequency synthesizer loop filter with the general specifications as shown in Table 1 (Al-Batta, 2009). The design process is divided into several stages. We first present the overall blocks of frequency synthesizer, then select the integer value of *N* according to reference frequency and resolution.

Table 1
Design Specifications

Parameter	Specification
Frequency range, [GHz]	3.5 – 5.8
Resolution, [kHz]	240
Overshoot	Less than 2%
Settling time	Less than 5 μs

2.1. Fractional-*N* PLL Block Diagram

The fractional-*N* PLL block diagram is showed in Fig. 2 (Al-Batta, 2009). We use the Multi-Modulus Fractional PLL with the following properties:

a) Fractional value between *N* and $2N-1$ (64…127).
b) Sigma Delta Modulator (programmable resolution).
c) Large reference (20 MHz) for good tradeoff with settling time.
d) Reduced *N* impact on phase noise by 45 dB over integer *N*.

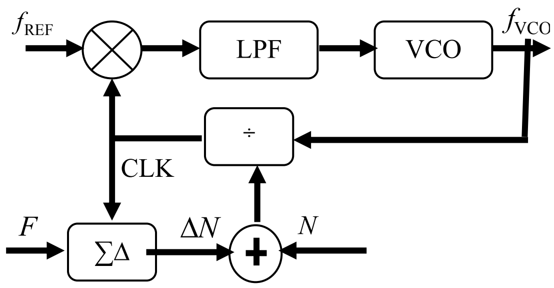

Fig. 2 – Basic fractional-*N* PLL block diagram.

As a result, for $N = 76…125$, it can be produced frequencies situated in the range 1,533 MHz…2,533 MHz which can be up converted to range 3,500 MHz…5,800 MHz (Al-Batta, 2009).

2.2. FIR Low Pass Filter Design by LMI

In this section we designed FIR filter using a minimax method based on SDP. In fact, the SDP approach can be used to design FIR filters with arbitrary amplitude and phase responses including certain types of filters that cannot be designed with other methods such as low-delay FIR filters with approximately constant passband group delay (Antoniou & Lu, 2007). Consider an FIR filter of order N, characterized by the general transfer function

$$H(z) = \sum_{n=0}^{N} b_n z^{-n}. \tag{1}$$

The frequency response of such a filter can be expressed as

$$H(\omega) = \sum_{n=0}^{N} b_n \mathrm{e}^{-\mathrm{j}n\omega} = b^T c(\omega) - \mathrm{j} b^T s(\omega). \tag{2}$$

Let $H_d(\omega)$ be the desired frequency response and assume a normalized sampling frequency of 2π. In a minimax design we need to find a coefficient vector, b, that solves the optimization problem

$$\text{Minimize } \delta, \tag{3}$$

$$\text{subject to } W^2(\omega)\left|H(\omega) - H_d(\omega)\right|^2 \le \delta \text{ for } \omega \in \Omega. \tag{4}$$

We can write

$$W^2(\omega)\left|H(\omega) - H_d(\omega)\right|^2 = W^2(\omega)\Big\{ \left[b^T c(\omega) - H_r(\omega)\right]^2 + \\ + \left[b^T c(\omega) + H_r(\omega)\right]^2 \Big\} = \alpha_1^2(\omega) + \alpha_2^2(\omega). \tag{5}$$

Using eq. (5), the constraint in eq. (4) becomes

$$\delta - \alpha_1^2(\omega) - \alpha_2^2(\omega) \ge 0 \text{ for } \omega \in \Omega. \tag{6}$$

It can be shown that the inequality (6) holds if and only if

$$D(\omega) = \begin{bmatrix} \delta & \alpha_1(\omega) & \alpha_2(\omega) \\ \alpha_1(\omega) & 1 & 0 \\ \alpha_2(\omega) & 0 & 1 \end{bmatrix} \ge 0, \text{ for } \omega \in \Omega. \tag{7}$$

$D(\omega)$ is positive defined for the frequencies of interest (Antoniou & Lu, 2007). If we write

$$x = \begin{bmatrix} \delta \\ b \end{bmatrix} = \begin{bmatrix} \delta \\ b_0 \\ b_1 \\ M \\ b_N \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 0 \\ M \\ 0 \end{bmatrix}, \tag{8}$$

then matrix $D(\omega)$ is *affine* with respect to x. If $S = \{\omega_i : 1 \le i \le M\} \subset \Omega.$ is a set of frequencies which is sufficiently dense on Ω, then a discretized version of eq. (5) is given by

$$F(x) \ge 0, \tag{9}$$

where

$$F(x) = \operatorname{diag}\{D(\omega_1), D(\omega_2), \Lambda, D(\omega_M)\} \tag{10}$$

and the minimization problem in eq. (3) can be converted into the optimization problem

$$\text{Minimize } c^T x \tag{11}$$

$$\text{subject to } F(x) \ge 0, \tag{12}$$

where

$$c = \begin{bmatrix} 1 \\ 0 \\ M \\ 0 \end{bmatrix}. \tag{13}$$

The problem in eq. (11) is an SDP one. We have shown that FIR design with LMI constraints which minimizes the value of the squared weighted error, δ, between the designed FIR low pass filter and the desired low pass loop filter can be cast as SDP feasibility problems. In fact, many extensions of the problem can be handled by simply adding a cost function and/or LMI constraints to our SDP formulation. The other problem is finding the optimal order of FIR filter which gives the desired specifications.

2.3. Minimum-Length FIR Design

The length of an FIR filter is a quasi-convex function of its coefficients (Wu *et al.*, 1996). Hence, the problem of finding the minimum-length FIR filter given magnitude upper and lower bounds is as following:

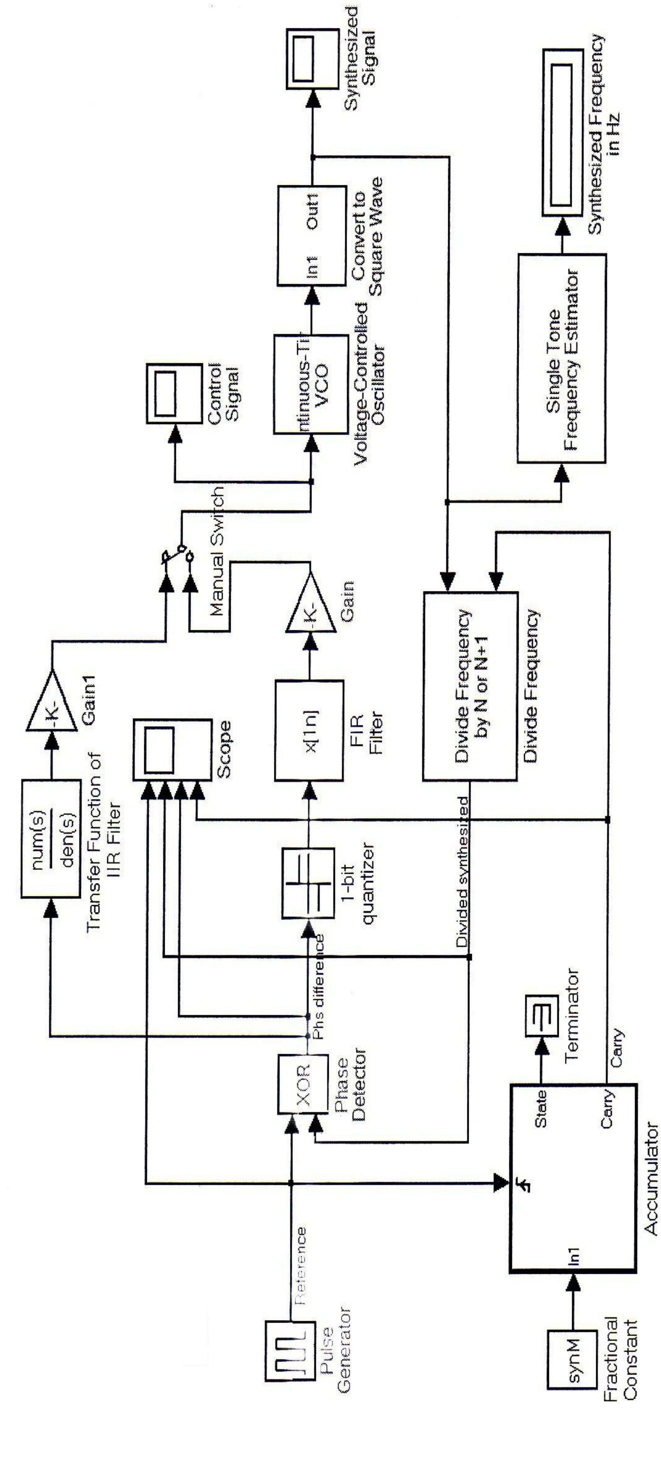

Fig. 3 – PLL frequency synthesizer simulation model.

$$\text{Minimize } G, \tag{14}$$

$$\text{subject to } L(\omega_i) \le |H_N(\omega_i)| \le U(\omega_i),\ (i = 1, \Lambda, M), \tag{15}$$

where $G = N + 1$ is the length of an FIR filter, $L(\omega)$ – the lower magnitude bound and $U(\omega)$ – the upper magnitude bound. The problem (14) is quasi-convex and can be solved using bisection on N. We will use the Theorem 1 established by Antoniou & Lu, (2007), to solve (14). The Theorem 1 proof was given by Al-Batta (2009). Thus, we can formulate the SDP feasibility problem as

$$\text{Find } r \in R^n \text{ and } P = P^T \in R^{(G-1)\times(G-1)}, \tag{16}$$

$$\text{subject to } L^2(\omega_i) \le R(\omega_i) \le U^2(\omega_i),\ \omega_i \in \Omega, \tag{17}$$

$$\begin{bmatrix} P - A^T PA & C^T - A^{TPB} \\ C - B^T PA & D + D^T - B^T PB \end{bmatrix} \ge 0. \tag{18}$$

The SDP feasibility problem (16) can be cast as an ordinary SDP and solved efficiently. Each iteration of the bisection in (14) involves solving an SDP feasibility problem in (16).

3. Analysis Comparison

The design process is divided into two steps presented as follows: the first step is to design FIR digital low pass filter using SDP algorithm. The second step is to simulate the designed filters, discuss the results, and compare them with others. To simulate the designed filters we used MATLAB Simulink and SeDuMi to construct simulation module shown in Fig. 3.

3.1. FIR Low Pass Filter

To design FIR we implemented the SDP algorithm with parameters given in Table 2. We begin the design process by using SeDuMi toolbox to solve the SDP problem in eq. (11). Using MATLAB code, the FIR filter magnitude response is shown in Fig. 4. Note that FIR phase was not taken into consideration. Fig. 4 shows the designed FIR filter specifications as in Table 2. When we simulate the obtained FIR filter with Fixed WiMAX simulation block diagram shown in Fig. 3, the simulation works properly and the correct output frequency has been achieved. The control signal using this FIR filter is shown

Table 2
Comparison in Frequency and Control Signal Responses of FIR Digital Filters Design

Parameter	FIR filter design (SDP by CVX)	FIR filter design (SDP by SeDuMi)
Frequency response of FIR digital filters design		
Filter order	19	19
Passband frequency, [rad/s]	0.006π	0.006π
Stopband frequency, [rad/s]	0.2π	0.2π
Maximum pass band ripple, [dB]	0.4	0.3167
Stop band attenuation, [dB]	44.5	44.2459
Control signal response of FIR digital filters design		
Settling time, [μs]	0.4998	≈ 0.22
Rise time, [μs]	≈ 0.25	0.125
Maximum overshoot, [%]	zero	Zero

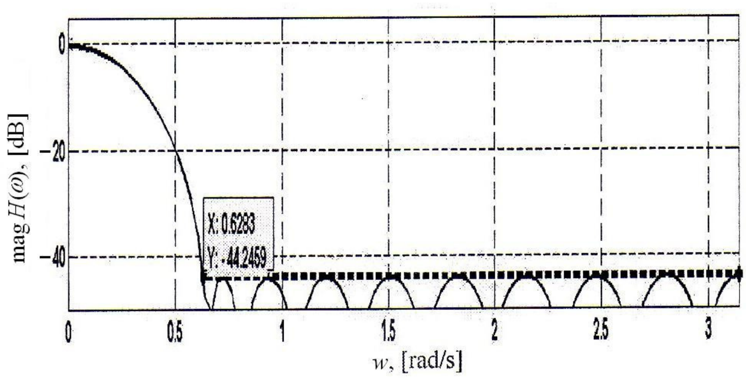

Fig. 4 – FIR filter magnitude response.

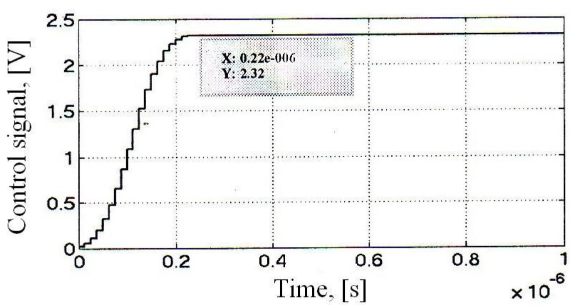

Fig. 5 – The control signal of VCO input using designed FIR filter.

in Fig. 5 which explores the results from Table 2. We have shown that FIR design with minimizing the value of the squared weighted error, δ, between the designed FIR low pass filter and the desired low pass loop filter (ideal case). In fact, the other problems of finding the minimum-length FIR filters give the same results. As a result, we conclude that designing FIR digital filter by using SeDuMi toolbox to solve the SDP problem produced a very fast stable system compared with used method by Al-Quqa (2009). Table 2 summarizes these differences.

4. Conclusions

A novel loop filter design method for PLL loop filter is proposed taking into consideration various design objectives. The SDP based method for the design of the digital FIR low pass loop filter was used under LMI constraint. This constraint minimizes the value of the squared weighted error, δ, between the designed FIR low pass filter and the desired low pass loop filter (ideal case). The minimum length FIR filter algorithm was used to proof that the order of the FIR filter which was designed is optimal for our design specifications. MATLAB was used to run SDP formulation of the design problem's equation and solved it using SeDuMi (Self-Dual Minimization) toolbox. Then, the resultant FIR filter was used in designing fractional-*N* synthesizer as a loop filter to derive the desired fixed WiMAX frequency. The proposed method gave better result with regard to all specifications of control signal, where we had the faster system and more stable than the system with FIR filter which was designed using SDP method and simulated using toolbox software (CVX).

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UTILIZAREA METODEI LMI ÎN PROIECTAREA FILTRELOR DE BANDĂ PLL ÎN SISTEME WIMAX CU RĂSPUNS FINIT LA IMPULS DE LUNGIME MINIMĂ

(Rezumat)

S-au proiectat filtre digitale cu răspuns finit la impuls (FIR) prin programare semi-definită (SDP) utilizând pachetul de programe SeDuMi (minimizare auto-duală). Stabilitatea este asigurată de constrângerile LMI. Pentru a demonstra că ordinul filtrului FIR proiectat este optim, în condițiile de proiectare date, s-a utilizat algoritmul de minimizare a lungimii filtrului FIR. Metoda propusă a dat rezultate mai bune în ceea ce priveşte toate specificațiile semnalului de control unde sistemul a fost mai rapid şi a avut o utilitate mai bună decât celelalte sisteme.