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Sensitivity of Left Handed Material Film-Superconductor Waveguide Sensors

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Abstract- The sensitivity of planar waveguide sensor containing dielectric, superconductor, and left-handed materials has been theoretically investigated. The proposed waveguide sensor in this study consists of a metamaterial film bounded by a dielectric cover and a superconductor substrate. The variation of the sensor sensitivity are found to be strongly depended on the metamaterial film, the film thickness and the temperature of the superconductor.

Key Words: Sensitivity, Left-handed materials, Superconductors, Waveguide sensors, Dispersion relation.

1. INTRODUCTION

Recently, there are a great and considerable attention of optical waveguide sensors which can effectively be used in the investigation of many physical phenomena [1, 2]. These sensors have attractive characterization such as resistance of the interference of electromagnetic radiation, capability of working very well with fiber networks and high sensitivity to the variation of physical parameters including pressure and temperature. The waveguide sensor consists of three layers: film, cladding (cover) and substrate. The sensing of such waveguides is accomplished by the evanescent tail in the cladding medium which causes an electromagnetic field into the substrate and cladding that senses an effective refractive index of the guided mode. This effective refractive index depends on the cladding thickness, dielectric permittivity and magnetic permeability. The principle of the waveguide operation depends on the variation in the refractive index of the cladding layer which leads to a variation in the effective refractive index of the guided layer which can be determined. The variation rate of the effective refractive index depending on the variation in temperature is defined as the temperature sensitivity. The optical waveguide sensors are therefore called temperature sensors. Optical waveguide sensors can be effectively used in many fields including liquids concentrations measurements, small tracing detection in chemicals, thickness measurements of metals, enzymes and biological microbes and bacteria, detection of drug vapor and the study of biological, physical and chemical processes [1,2].

The electric permittivity and magnetic permeability of a new fabricated materialwhich well known as left- handed material (LHM)are both simultaneously negative, therefore they also defined as a metamaterial (MTM) which refers to (beyond conventional material). LHM have not been found in nature. In 1968 Veslago [3] has introduced these materials for the first time. LHMs have many characteristics which are completely different from that of normal materials. Some of these are Negative Refraction, Reversed Cerenkov Radiation and Revered Doppler Effect. Left Handed Materials were first proposed for the possibility to be fabricated by Pendry [4,5]. LHM fabrication were first practically carried out by Smith in 2001 [6] using metallic materials in microstructure form for the microwave frequency range. This fabrication can be used in many interesting application such as LHM waveguides which allows the propagation of the electromagnetic radiation. Many interesting investigation were carried out after that including the study of electromagnetic radiation within LHM waveguide fabricated in multilayered form by Zhang et al.[7]. Negative shift for a Gaussain rays was shown by Kong et al.[8] using LHM structure which is not found using a normal material structure. The electromagnetic guided radiation were studied using a LHM structure by Cory et al.[9]. The linear guide radiations were investigated by Shadrivov et al. [10] in a LHM waveguide structure which showed the disappearance of the fundamental modes and the variation of the energy flux sign.

It is interesting to investigate the possibility of using superconductor materials in optoelectronics devices especially such kind of sensors where in some times we are in badly need to use sensors in very low temperature states. This leads us to use a superconductor material which is very good candidate to act as a sensor in such condition.

In superconductor materials, the density of conduction electrons equals (nn) normal electron density and (ns) superconducting electron density. Both the normal electron density and superconducting electron density depend on temperature. When temperature approaches zero, the superconducting electron density almost equals the conduction electron density. When temperature reaches the superconducting state phase transition i.e. the critical temperature Tc, the normal electron density equals the conduction electron density. Gracheva et al. [11] had investigated the characteristics of surface electromagnetic waves propagating through a superconductor-dielectric structure and found that these waves substantially depend on the frequency, the structure and the temperature. Hamada, Shabat et al. [12]had reported the propagation characteristics of TM surface waves in waveguide structure containing superconductors.

Extensive theoretical and experimental works of the optical waveguide sensors have been carried out [13-15]. Various waveguide structures containing different materials such as metamaterials, nonlinear dielectric media, and metals have been investigated theoretically. The sensitivity enhancement in optical waveguide sensor containing left-handed materials has been reported as in ref. [16]. Other waveguide sensors consisting of nonlinear media have also investigated to find out the effect of the nonlinearities on such structures. The plasmon characteristics and the sensitivity has recently been investigated in graphene waveguide sensor where it was found that the sensitivity can be controlled by changing the frequency and thickness [16].

This study aimed to theoretically investigate the sensitivity of planar waveguide structure sensor containing superconductor and left handed materials.

2. ANALYSIS OF TRANSVERSE ELECTRIC GUIDED MODES

In our proposed multilayers waveguide structure, as shown in Fig. 1, a LHM core (region 2) of thickness d is sandwiched between a dielectric material of water (region 1) characterized by refractive index n=1.33 which does not depend on frequency and a superconductor material (region 3) of permittivity ε_3 that depends on frequency.

The superconductor permittivity is a function of temperature which can be written as [11]:

$$\varepsilon_2 = \varepsilon_p - \frac{\omega_0^2}{\omega^2}\left(1 + \frac{iv\theta^4}{\omega - iv}\right) \tag{1}$$

where ε_p is the lattice contribution to the total permittivity of the superconductor, ω_0 is the plasma frequency, $\theta = T/T_c$ is the normalized temperature and T_c is the critical temperature, i.e. temperature of the phase transition to the superconducting state, and ν is the frequency of collisions of superconducting electrons. The coordinate is displayed in Fig. 1.

In the case of a transvers electric (TE) mode, where the electric field is polarized in the direction of the y-axis. The electric field E_y could be written in the form:

$$E_y(x,z,t) = E_x \exp(-i(wt - \beta z)),$$

where $\beta = k_0 N$ is the propagation constant in the direction of the z-axis, and N is the effective refractive index.

The electric field for the three layers could be written:

$$E_y(x) = \begin{cases} A_1 \exp(-\alpha_1(x-d)), & x > d \\ A_2 \cos(k_2 x + \varphi), & 0 \le x \le d \\ A_3 \exp(\alpha_3 x), & x > 0 \end{cases} \tag{2}$$

where $\alpha_1 = (\beta^2 - k_0^2 n_1^2)^{1/2}, \alpha_3 = (\beta^2 - k_0^2 n_3^2)^{1/2}$ are the evanescent rates in region 1 and region 3. $k_2 = (k_0^2 n_1^2 - \beta^2)^{1/2}$ is the transverse wave number in region 2, which could either be real or imaginary. n_1, n_2, and n_3 are the refractive index in the three regions respectively.

The TE modes nonzero electric field could be obtained as follows:

$$H_x(x) = -\frac{\beta}{\omega\mu} E_y(x) \text{ and } H_z(x) = \frac{i}{\omega\mu}\frac{\partial E_y(x)}{\partial x} \tag{3}$$

The magnetic field nonzero tangential component could be estimated by using Equations (2) and (3) as follows

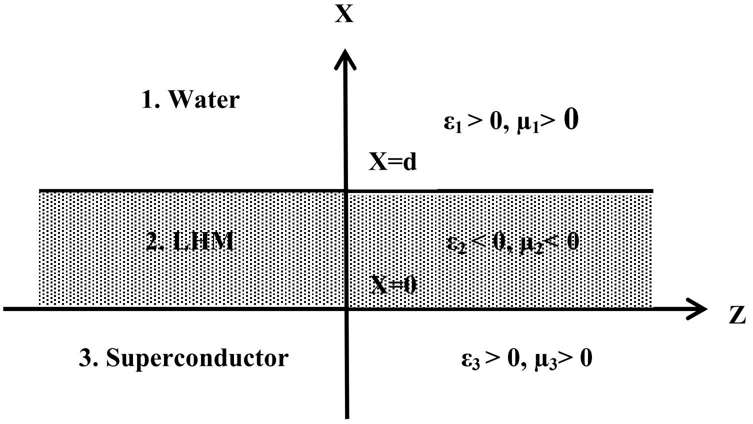

Fig. 1: A LHM waveguide structure Geometry. Region 2 is a LHM film, region 1 is a water dielectric cover and region3 is a superconductor substrate.

$$E_y(x) = \begin{cases} \dfrac{\alpha_1 A_1}{i\omega\mu_1} \exp(-\alpha_1(x-d)), & x > d \\ \dfrac{k_1 A_2}{i\omega\mu_2} \sin(k_2 x + \varphi), & 0 \le x \le d \\ \dfrac{\alpha_3 A_3}{i\omega\mu_3} \exp(\alpha_3 x), & x > 0 \end{cases} \tag{4}$$

If the boundary conditions have been matched at x=0 and x=d, the dispersion relation is calculated as follows:

$$k_2 d = m\pi + \tan^{-1}\left(\frac{\mu_2\alpha_1}{\mu_1 k_2}\right) + \tan^{-1}\left(\frac{\mu_2\alpha_3}{\mu_3 k_2}\right) \tag{5}$$

The evanescent field sensor sensitivity could be written as the change of the effective refractive index versus the change of the cover refractive index n_1 as follows [2],

$$S = \frac{\partial N}{\partial n_1} \tag{6}$$

Differentiating the dispersion relation (equation 5) with respect to N to get the sensitivity:

$$S = \frac{n_1\mu_2^2}{N\mu_1}\left\{\eta_{12}(1+\eta_{12}^2)\left[k_2 d + \frac{\mu_2^2 + \mu_1^2\eta_{12}^2}{\mu_1^2\eta_{12}(1+\eta_{12}^2)} + \frac{\mu_2^2 + \mu_3^2\eta_{32}^2}{\mu_3^2\eta_{32}(1+\eta_{32}^2)}\right]\right\}^{-1} \tag{7}$$

where $\eta_{12} = \frac{\mu_2\alpha_1}{\mu_1 k_2}$, and $\eta_{32} = \frac{\mu_2\alpha_3}{\mu_3 k_2}$.

3. NUMERICAL RESULTS AND DISCUSSIONS

Computation of the sensitivity of the proposed waveguide structure were performed by using a Maple Software starting by solving the dispersion equation. The roots of the dispersion relation were fed to the sensitivity equation to find out the sensitivity.

In our study, the propagation characteristics in the proposed structure are firstly investigated and displaced as shown in Fig. 2 where the effective index (N) is plotted against the normalized frequency ($K_o d$) at reduced temperature of θ =0.6 for different TE modes m=1, 2, 3, 4, 5. It can be noticed from the Fig. that the TE_0 guided mode does not exist for any normalized frequency of the LHM waveguide. This is in contrast with the conventional dielectric waveguide where

TE_0 guided mode can be found in the case of frequency beyond a cutoff values. It is convenient to mention here that the existence of fundamental mode is observed by Dong et al. [17] using a four-layer slab waveguide containing nihility core.

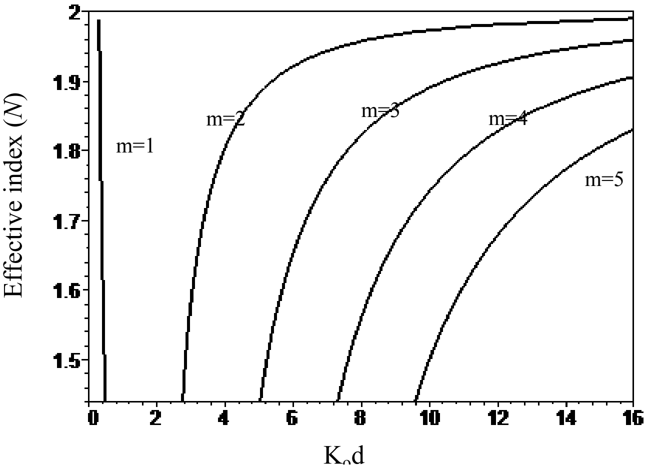

Fig. 2: The effective index (*N*) versus the normalized frequency (K_od) and θ=0.6 .

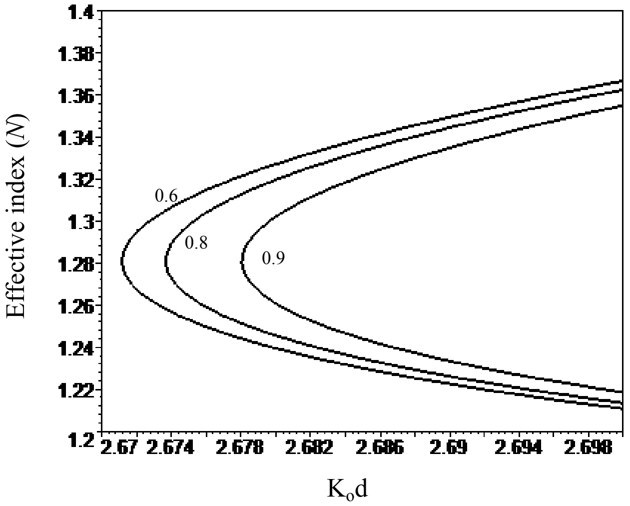

Fig. 3: The effective index (*N*) versus the normalized frequency (K_od) for different values of θ (0.6,0.8,0.9) and m=2.

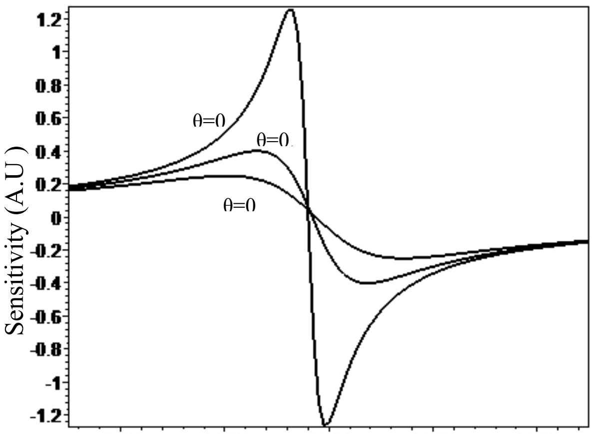

Fig. 4: The Sensitivity (S) versus *N* for different values of the normalized temperature (θ).

By increasing the normalized frequency K_od, the waveguide of LHM get more high-order guided modes. At very small value of the normalized frequency all the guided modes do not exist in the waveguide. It is obvious from Fig. 2 that in the LHM waveguide each guided mode has a certain cutoff frequency and if at a specific frequency guide mode appears, there is no other guided mode of other-order can also appear at that frequency. This is an excellent feature of the LHM waveguide which can be used as bandpass filters with no need of the use of a resonant structures. The cutoff frequency, as is apparent from the Fig., is getting of a higher value with a higher guided mode.

The presented data in Fig. 3, clearly show the effective index (N) is plotted against the normalized frequency Kodfor different values of reduced temperature θ of the superconductor (0.6, 0.8, 0.9).As can be seen from the Fig. a portion of a negative slope in the dispersion relation curve is formed and this does not exist in the conventional waveguide. For the normalized frequency there are two solutions for the guided mode. One solution is related to the LHM (the portion of the guided mode inside the metamaterial layer) and a backward wave is formed where the net total power flow is antiparallel to the direction of modal phase flow. The other solution is related to region 1 and region 2 (the portion of the guided mode outside the metamaterial region) and a forward wave exists where the power flow is parallel to the modal phase flow.

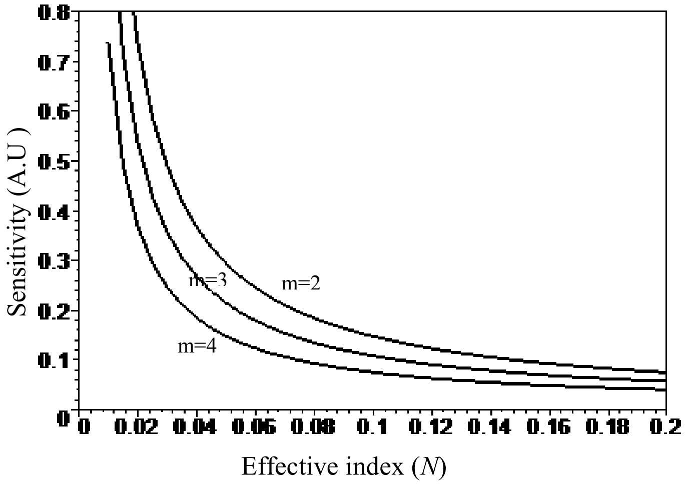

Fig. 5: The Sensitivity (S) versus *N*for different values of the modes order (m) and θ=0.8.

Moreover, Fig. 4 illustrates the sensitivity versus the effective index for (N) various values of reduced temperature of the superconductor. It shows that at smaller values of the reduced temperature, the sensitivity is getting higher values. So that the sensitivity can be tuned and controlled by the temperature of the superconductor. In addition Fig. 5 displays the sensitivity versus the effective index (N) for various values of modes (m=2,3,4). It can be noticed that the higher modes gets lower sensitivity.

Fig. 6 shows the sensitivity versus the effective wave index for various values of the film thickness. The sensitivity is decreasing slightly rapidly and has bigger values for thicker LHM film.

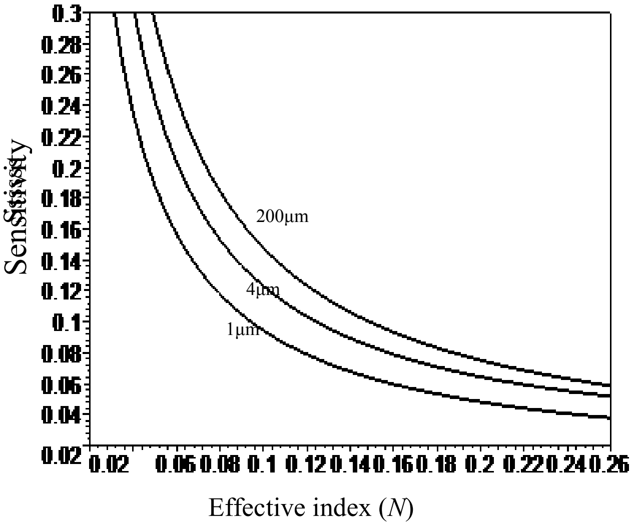

Fig. 6: The Sensitivity (S) versus *N* for different values of *the thickness (d) at m=2 and θ=0.8.*

Fig. 7 displays the sensitivity versus the normalized temperature (θ) for various of normalized thickness of the LHM film. It shows that the proposed structure is highly sensitive to the properties of the film thickness. The sensitivity is approaching the maximum as the normalized temperature is greater than 0.9 and then is decreasing rapidly. After the normalized temperature is approaching unity, the superconductor becomes as like the dielectric medium and the proposed sensor structure behaves as conventional sensors studied before [18,19].

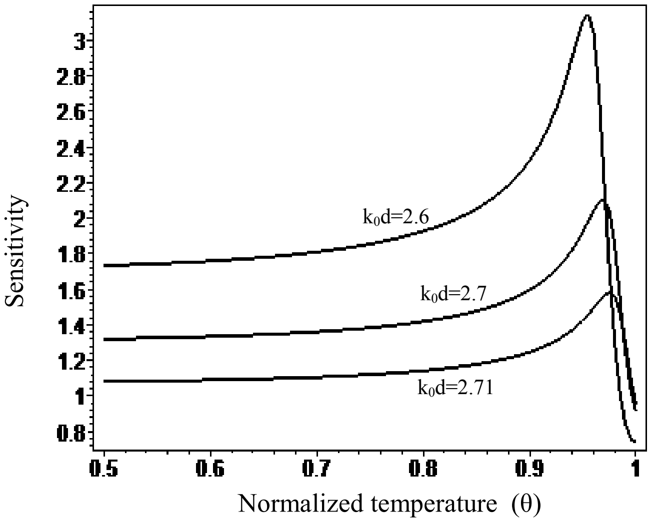

Fig. 7: The Sensitivity (S) versus θ for different values of K_0d and m=2.

4. CONCLUSION

To the best of knowledge to the authors, it is the first time, the proposed waveguide structure containing both superconductor and left- handed materialsas a model of sensor have been investigated. The high values of the sensitivity could be used by tuning some physical parameters such as the film thickness, the superconductor temperature, and the values of the permittivity and permeability of left handed materials. The proposed waveguide structure could be used in different environment where such sensors are in badly need of such temperature related to the superconductors.

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