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Electron Transport in a Quantum Wire: Effect of a High-Frequency Electromagnetic Field

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الملخص

دراسة الانتقال الالكتروني في سلك كمي تحت تأثير مجال كهرومغناطيسي عالي التردد

في هذه الورقة سوف نتعرض لدراسة خواص الانتقال الالكتروني في سلك كمي تحت تأثير مجال كهرومغناطيسي عالي التردد على جزء محدد من السلك بتطبيق تقريب الكتلة الفاعلة للإلكترون الحر، وسوف نقوم باشتقاق مصفوفة التشتت من خلال تطبيق شروط الاستمرار ، وقد لوحظ علـى الرغم من أن الالكترونات تعاني فقط من التصادمات العرضية مع الفوتونات إلا أنـه وجـد عمليـة انعكاس لحركة الالكترونات في الاتجاه المضاد ، كذلك عندما يكون تردد المجال المطبق فـي حالـة رنين مع أول مستويين عرضيين للطاقة فإنه يحدث انتقال داخلي لحركة الإلكترون وبالتالي ازديـاد حالة الانتقال على شكل خطوة، وذلك كدالة في طاقة الإلكترون الساقطة ، علاوة على ذلـك سـيتم دراسة تأثير طاقة التفاعل المتبادل.

ABSTRACT

In this paper, we investigate the electron transport properties in a semi-conductor quantum wire, where a finite-rang high-frequency electromagnetic field in the ballistic limit is imposed. Within the effective mass free-electron approximation, the scattering matrix for the system has been formulated by means of a time dependent mode matching method. Some interesting properties of the electron transmission for the system have been shown. It is found that, although the electrons in a nanowire only suffer from lateral collisions with photons, the reflection of electrons also takes place. And when the frequency of the electromagnetic field is resonant with the two lateral energy levels, the field induced inter subband transition dominates the process, and there is a step-arising on the transmission as a function of the incident electron energy. Moreover, the transmission dependence on the mode coupling is also discussed.

Keywords: Quantum transport; Nanostructures; Electron waveguides; Time-dependent field

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INTRODUCTION:

The discovery of conductance quantization has motivated a great deal of research interests in the quantum transport phenomena in semi-conductor quantum wires [1]. In the ballistic regime and at low temperature quantum coherent effects will dominate the electron transport properties of a mesoscopic system. One of the most important features is that, when the lateral size of a quantum wire varies, the conductance shows an histogram structure and each step has an height of ($2e^2/h$) or integer of it [2].

The electron transport properties of the quantum wire formed on a two-dimensional electron gas (2DEG) can be affected by many factors. The presence of disorders in a quantum wire generally leads to a suppression of the conductance plateaus below integer values [3], and the coupling among wire and leads has also been accounted for [4]. However, there has been growing interest in the time-dependent transport for quantum wire systems in recent years, such as presence of a time-modulated potential [5] and quantum pumping [6]. Further, when a quantum wire is illuminated under an external electromagnetic (EM) field, due to the inelastic scattering of electrons by photons many new features have been observed experimentally [7] and predicted theoretically [8]. The technique of applying an external field is of particular interest, since no additional current and voltage probes have to be attached to the sample which may disturb the system properties. It is therefore of great interest in basic physics aspect to study the time-dependent transport properties of quantum structures on semi-conductor 2DEG systems, which will have to operate at very high frequencies, require detailed knowledge of their frequency and time-dependent transport behavior.

The values of the lateral energy level separation and the Fermi energy are of the order of 1 ~ 100 meV for typical semi-conductor quantum wires. This corresponds to frequencies of the range of 0.25 ~ 25 THz, which is available in experiments with the development of the ultra fast laser technology [9]. When the Fermi level is below the lowest lateral level of the neck part of a quantum wire, electrons can not go through without the assistance of an external EM field. However, under the field illumination, electrons in the wire can absorb energy of photons and go through this geometric barrier (the neck) [10]. Therefore, in the region of barrier the electron transmission is determined by the combined effect of the external EM and the wire lateral shape variation.

In the present paper, we consider electron transport in a straight quantum wire illuminated by a transversely polarized THz EM field. Within the effective mass free-electron approximation, the scattering matrix for the system has been formulated through a time-dependent mode matching method [11]. Using two numerical examples we demonstrate some interesting electron transmission properties for this system.

The paper is organized as follows. In Sec. II we present the problem for a straight quantum wire in terms of a single-electron time-dependent Schrödinger equation, and calculate the electron transmission probability through the system in the framework of Landauer-Büttiker formalism [12]. In Sec. III, we illustrate the dependence of the electron transmission on the incident energy, and field parameters respectively. Finally, Sec. IV gives a conclusion of the paper.

Formulation:

We consider the electron transmission through a straight 2D quantum wire, which is depicted in Fig. 1 schematically. The quantum wire is smoothly connected the two electron reservoirs (leads) at each end. The x-axis longitudinally is along the wire, and the y-axis describes the transverse direction. The range $0 \le x \le l$ in the quantum wire is illuminated under transversely polarized THz EM field.

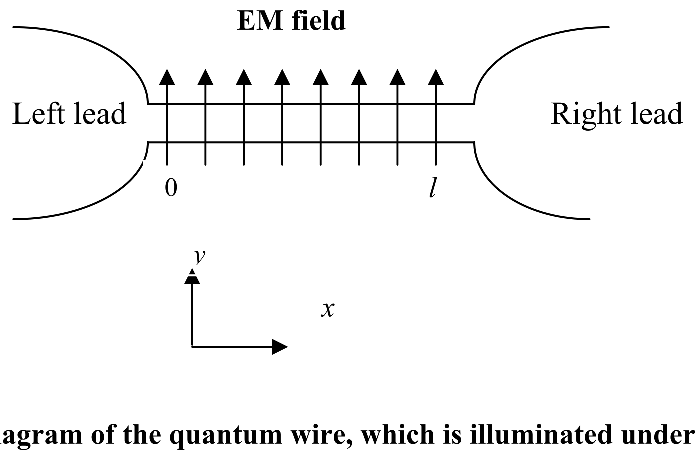


FIG. 1. Schematic diagram of the quantum wire, which is illuminated under a transversely polarized EM field in the range $0 \le x \le l$. The quantum wire is connecting to two leads (reservoirs).

The field vector potential can be described as

$$\vec{A}(t) = \frac{\varepsilon}{\omega}\cos(\omega t)\hat{y} \tag{1}$$

with angular frequency ω and amplitude ε ($\hat{y}$ is the unit vector in the polarized direction).

Within an effective mass approximation, the single particle time-dependent Schrödinger equation in the field illuminated region is

$$\left[-\frac{\partial^2}{\partial x^2} + (-i\frac{\partial}{\partial y} + eA)^2 + V(y)\right]\Psi(x,y,t) = i\frac{\partial}{\partial t}\Psi(x,y,t), \tag{2}$$

where we have adopted the unit of $\hbar = 2m^* = 1$. In the Hamiltonian, $V(y)$ presents a transverse confining potential in the form of a hard-wall which confines electrons to the wire and to the reservoirs. We attempt to find the electron wave function in the expansion form

$$\Psi(x,y,t) = \exp(ikx)\sum_n a_n(t)\Phi_n(y) \tag{3}$$

where k is the longitudinal momentum and $a_n(t)$ presents the time-dependent amplitudes. $\Phi_n(y)$ are the eigenfunctions of the transverse motion without an EM field, which is dependent on $V(y)$. We only consider the transition between the two lowest transverse energy levels with the Fermi level between them. Using the method in Ref. [11] to solve the time-dependent Schrödinger equation (2), and then considering the scattering of the two interfaces between the field illuminated region and the clean region separately [13].

When an electron with total incident energy E emits from the left reservoir to the left interface at $x = 0$, transmission and reflection will take place simultaneously. Because the electron has certain probability of absorbing photon after penetrating the interface, the transition from the lower mode to the upper mode happens. So there are two energy components of E and $E + \hbar\omega$ in the reflected wave in the region of $x < 0$.

$$\begin{aligned}\Psi(x,y,t) = [&\exp\{i(k_1x - Et)\} + c_1\exp\{-i(k_1x + Et)\}]\Phi_1(y) + \\ &c_2\exp\{-i(k_2x + (E+\omega))t\}\Phi_2(y)\},\end{aligned} \tag{4}$$

where $\Phi_n(y)$ (n =1,2) are transverse eigenfunctions with eigenvalues ε_n in the clean region (without illumination), c_1, c_2 are the reflection coefficients of the two modes respectively, and

$$k_1 = (E - \varepsilon_1)^{1/2}, \quad k_2 = (E - \varepsilon_2 + \omega)^{1/2} \tag{5}$$

are their associated wave-vectors. Consequently, we can obtain the electronic wave function in the region of $x > 0$ as

$$\begin{aligned}\Psi(x,y,t) &= [c_+ \exp\{i(k_+x - Et)\} + c_- \exp\{i(k_-x - Et)\}]\Phi_1(y) \\ &+ [G_+c_+ \exp\{i(k_+x - (E+\omega))t\} + G_-c_- \exp\{i(k_-x - (E+\omega))t\}]\Phi_2(y),\end{aligned} \tag{6}$$

where c_+, c_- are the transmission coefficients of the two field-split modes, and the constants

$$G_\pm = \frac{\mp\gamma \pm (\gamma^2 + \xi^2)^{1/2}}{\xi} \tag{7}$$

where $\gamma = \omega - (\varepsilon_2 - \varepsilon_1)$ is the detuning, and ξ is the two-mode coupling constant. The two electron wave-vector in the illuminated region are

$$q_\pm = \sqrt{E - \varepsilon_1 + \frac{\gamma}{2} \mp (\gamma^2 + \xi^2)^{1/2} / 2} \tag{8}$$

The boundary conditions, the continuity of the wave function and the conservation of current density, should be imposed at the interface of $x = 0$. Continuously connecting the two above wave-functions and their differentials gives the following four equations for the coefficients $c_{1,2}$, and $c_{+,-}$

$$\begin{aligned} 1 + c_1 &= c_+ + c_-, \\ c_2 &= c_+G_+ + c_-G_-, \\ k_1 - c_1k_1 &= c_+q_+ + c_-q_-, \\ -c_2k_2 &= c_+G_+q_+ + c_-G_-q_- \end{aligned} \tag{9}$$

With the solution of these algebraic equations in Eq. 9 both the transmission and reflection matrix for the left interface can be expressed respectively as

$$r = [(k_1 + q_+)(k_1 + q_-)]^{-1}\begin{bmatrix} k_1^2 - q_+q_- & 0 \\ k_1(q_- - q_+) & 0 \end{bmatrix}, \tag{10}$$

$$t' = k_1\begin{bmatrix} (k_1 + q_+)^{-1} & 0 \\ (k_1 + q_-)^{-1} & 0 \end{bmatrix} \tag{11}$$

Now, we consider the electron transmission through the whole wire. This needs to derive the total scattering matrix, which can be expressed in the transmission matrix and reflection matrix on each interface. The total transmission matrix is just the anti-diagonal sub-matrix of the total scattering matrix in the symmetry system case.

Because of the similarity of the two interfaces one has not to match the wave functions at the right interface $x = l$, but we need to know the transmission and reflection matrix of electron emitting from right to left for the left interface. In this case the electronic wave-function in the illuminated region ($0< x< l$) is

$$\begin{aligned}\Psi(x,y,t) = {} & [c_+^e \exp(-ik_+x) + c_+^e \exp(-ik_-x) + c_+^r \exp(ik_+x) + \\ & c_-^r \exp(ik_-x)] \exp(-iEt)\Phi(y) \\ & + [G_+c_+^e \exp(-ik_+x) + G_-c_+^e \exp(-ik_-x) + G_+c_+^r \exp(ik_+x) + \\ & G_-c_-^r \exp(ik_-x)] \exp(-i(E+\omega)t)\Phi(y)\end{aligned} \qquad (12)$$

where $c_\pm^e$ are the coefficients of the electron emitting from right to left, $c_\pm^r$ are the associated reflection coefficients. Correspondingly, the transmitted electron wave-function in the region of $x<0$ is

$$\Psi(x,y,t) = c_1^t \exp(-i(k_1x + E\,t))\Phi(y) + c_2^t \exp(-i(k_2x + (E+\omega)t)\Phi(y) \quad (13)$$

where c_1^t and c_2^t are the transmission coefficients. These two wave functions also satisfy the continuous condition at $x = 0$, from which we obtain the transmission and reflection matrix from right to left for this interface, r' and t, respectively. Consequently, the total transmission matrix through the two interfaces (the whole system) is [11]

$$t_{tot} = t(1 - X\,r'X\,r')^{-1} X\,t', \qquad (14)$$

where X is the transfer matrix between the two interfaces of the system and is given by:

$$X = \begin{bmatrix} \exp(iq_+l) & 0 \\ 0 & \exp(iq_-l) \end{bmatrix} \qquad (15)$$

Therefore, according to Landauer-Büttiker's formulation the total electron transmission probability through the whole system is:

$$T = Tr[t_{tot}^* \, t_{tot}] = |Q_1 + Q_2|^2 + |Q_1 - Q_2|^2 \qquad (16)$$

with Q_1 and Q_2 are given by:

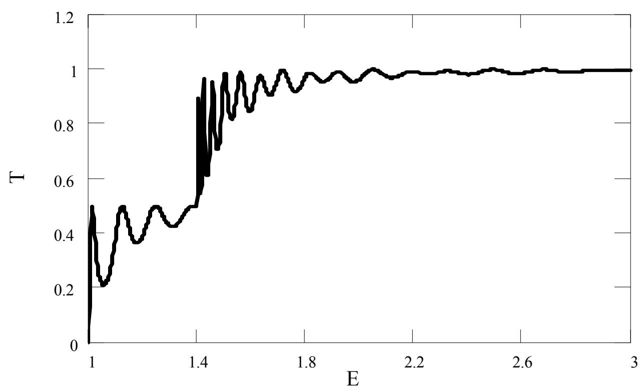
$$Q_1 = \frac{2k_1q_+}{\exp(-iq_+l)(k_1+q_+)^2 - \exp(iq_+l)(k_1-q_+)^2} \qquad (17)$$

and

$$Q_2 = \frac{2k_1q_-}{\exp(-iq_-l)(k_1+q_-)^2 - \exp(iq_-l)(k_1-q_-)^2} \qquad (18)$$

RESULTS AND DISCUSSION:

In this section, we compute the transmission probability from Eq. (16). The physical parameters are chosen to be a high-mobility GaAs-$Al_xGa_{1-x}As$ heterostructure [14] with electron effective mass $m^* = 0.067\,m_e$ (where m_e is the free electron mass). We choose the hard-wall transverse confining potential and the unit of energy $E^* = \varepsilon_1 = \hbar^2 k_F^2 / 2m^* = 9\,\text{meV}$, which corresponds to the unit of time $t^* = \hbar / E_F = 7.32 \times 10^{-14}$ s. correspondingly, the field frequency unit $\omega^* = 1/t^* = 13.66$ THz and a length unit $a^* = 1/k_F = 79.6\,\text{Å}$. In the following we systematically present some



numerical examples, while restrict our attention to the energy range ($\varepsilon_1, \varepsilon_2$).

FIG. 2. Transmission probability *T* versus the incident energy *E* (in units of ε_1) in the resonant case (γ =0, $\omega = \varepsilon_2 - \varepsilon_1$) with ε = 0.8, ω = 1 and *l* =39.

Fig. 2 represents the calculated transmission probability versus the total energy *E* in the resonant case (γ = 0, $\omega = \varepsilon_2 - \varepsilon_1$) with ε = 0.8 and ω = 1. We clearly see that a step raising of transmission occurs at energy $E = \varepsilon_1 + \xi/2 = 1.4$. This interesting phenomenon can be explained by the field-induced inter-sub-band transition [11]. When an electron penetrate through the interface, the transverse levels of the electron in the field illuminated region are dressed and one electron mode is split into the two time-dependent modes with the longitudinal momentum q_+ and q_-, respectively. When $E < \varepsilon_1 + \xi/2$, q_+ is imaginary and its corresponding mode is an evanescent mode which contributes nothing to the transmission so that the total transmission probability is suppressed to an half value.

Further, when $E > \varepsilon_1 + \xi/2$ the both modes become propagating and all contribute to the transmission.

We also note that there is some resonance oscillations on the transmission spectrum in Fig. 2. These oscillations are physically from the interference of the forward and backward going electron waves induced by the two interfaces of the illuminated region along the transport direction.

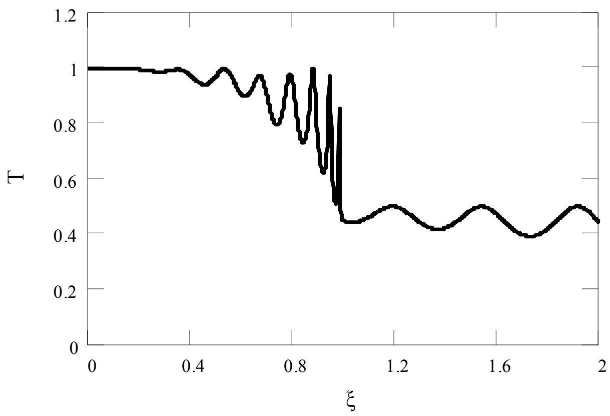


FIG. 3. Transmission probability *T* versus the mode coupling ξ with *E* = 1.5ε_1, the system parameters are the same as in Fig. 2.

Next, we consider the calculated *T* as function of a mode coupling ξ for resonant frequency case and energy of *E* = 1.5ε_1. From Fig. 3, we see that the transmission probability *T* have a step dropping at point ξ = 1. However, it shows apparently coherence pattern. When ξ > 1 the mode corresponding to q_+ becomes an evanescent one so that total transmission probability is suppressed to around 0.5. However, when ξ < 1 the both two modes are propagating and the total transmission is the coherence superposition of these two modes.

In general for a fixed electron incident energy, a different combinations of the field parameters (i.e. $\xi = \varepsilon/\omega$) results in a different transmission dependence.

CONCLUSION:

We have theoretically investigated the electron transport properties for a straight semi-conductor quantum wire partially illuminated by an external THz EM field in the ballistic limit. Within the effective free-electron approximation, a single-particle time-dependent Schrödinger equation was established and the scattering matrix for the system was formulated. The numerical examples predicate that a step arising on the transmission probability versus the electron incident energy and mode coupling occurs in the case of the field frequency resonant with the lateral energy spacing of the two lowest levels. The physical origin is mainly the coherent field-induced inter-sub-band transition.

Therefore, from the results of this work we conclude that parameters E, ω, and ε can control the characteristics of electron transmission in a quantum wire. These effects may be useful for understanding basic physics of nano-structures and for micro-electronic devices.

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