Dispersion properties of slab waveguides with double negative material guiding layer and nonlinear substrate

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The dispersion properties of transverse electric nonlinear waves in a three-layer slab waveguide which consists of a double negative material (DNM) guiding layer sandwiched between an intensity-dependent refractive index substrate and semi-infinite linear dielectric cover are investigated. The dispersion properties for self-focusing and self-defocusing substrate nonlinearity are presented. The effects of the negative parameters of the DNM on the dispersion characteristics are investigated.

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1. INTRODUCTION

There has been considerable progress in the study of nonlinear electromagnetic waves. Much of this work has treated nonlinear waves guided by a single interface between nonlinear and linear media [1,2]. Nonlinear guided waves in three-layer structures have also received a great deal of attention [3]. They have been found to bear a strong potential for application in integrated optical devices for ultrafast signal processing [4]. One of the most interesting phenomena of nonlinear guided waves is that their propagation constants and field profiles can have power dependence. Moreover, a linear thin film bounded by nonlinear media is found to display peculiar properties, such as self-bending, optical bistability and generation of spatial solitary waves which may be applicable to optical switching and routing [5,6].

For most natural materials, electric permittivity (ε) and magnetic permeability (μ) are usually positive. In 1968, Veselago presented the electromagnetics of materials with simultaneously negative ε and μ [7]. Such materials were named double negative materials (DNMs). Many unusual features were reported for DNMs. These include reversal of Cherenkov radiation and Doppler effect [8], negative refractive index [9], and unusual nonlinearities [10]. Moreover, the wave vector **k**, the electric field vector **E**, and the magnetic field vector **H** form a left-handed system. DNMs have been studied extensively for a set of possible applications [11–16]. One of the main applications of DNMs is the so-called superlens, proposed by Pendry in 2000 [17].

When treating structures comprising nonlinear media, the Helmholtz equation has to be solved in each layer of the structure. Several forms of the characteristic equation have been obtained. The properties of any waveguide are determined by the guiding layer thickness, wavelength, and refractive indices of the layers. The propagation constant β can be determined from the characteristic equations for both TE and TM modes. The characteristic equation is a transcendental equation with no analytic solution known for it. Numerical techniques are usually employed to determine β. However, it is not trivial to evaluate β numerically either. To appreciate the complication involved, the modal index N was introduced such that $N = \beta/k$, where k is the free space wavenumber. The value of N is found to lie between n_f and n_s where n_f and n_s are the refractive indices of the guiding film and substrate, respectively. For most waveguide structures of practical interest, n_f and n_s are numerically very close to each other. Therefore, N differs from n_f or n_s very slightly, and hence an accurate determination of N is not an easy task. Moreover, the numerical results are valid for a specific set of waveguide parameters only. To avoid these difficulties, Kogelnik and Ramaswamy have shown that the properties of linear slab waveguides can be described by three generalized parameters [18]. These parameters are called normalized film thickness (V), asymmetry coefficient (a), and normalized guide index (b), which are given by

$$V = kt\sqrt{n_f^2 - n_s^2}, \tag{1}$$

$$a = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2}, \tag{2}$$

and

$$b = \frac{N^2 - n_s^2}{n_f^2 - n_s^2}, \tag{3}$$

where t is the guiding layer thickness and n_i is the refractive index of layer i.

The generalized parameters a and b are given in terms of the differences of index squares rather than the indices themselves. For a given waveguide structure operating at a specific wavelength, n_f, n_s, n_c, and t are known. The values of a and V may be calculated from these equations. The value of b can be determined numerically from the characteristic equation. There may be one or more solutions for b. Each solution of b corresponds to a guided mode. Once b is known, β and N can be evaluated.

In 1987, Chelkowski and Chrostowski extended this approach to a waveguide structure comprising a linear guiding layer and a nonlinear substrate [19]. In their paper, they defined a new power-dependent parameter (b_I) and were able to get a concise description of the dispersion properties at a given power where

$$b_I = \frac{\alpha E_o^2}{2(n_f^2 - n_s^2)}, \tag{4}$$

where E_o is the value of the field amplitude at the interface $y = 0$, and α is a constant depending on the parameters of the nonlinear substrate.

The scaling rules for a nonlinear guiding layer bounded by two linear media were investigated by Fontaine [20]. The scaling rules were then presented for waveguide structures comprising DNMs [21,22].

In a recent study, the nonlinearity in a DNM film bounded by two linear semi-infinite media with positive refractive index has been investigated [23]. It is found that novel surface and guided waves can be generated as a consequence of both the DNM and the nonlinearity. One of the most important aspects that have been observed is the possibility of controlling the total power flow through variation of the different parameters available in the nonlinear DNM layer.

In this work, we consider a three-layer nonlinear waveguide structure in which the guiding layer is assumed to be DNM. The study is limited to the case of a Kerr-type nonlinear substrate and to the TE modes. The dependence of the normalized guide index on the normalized film thickness is studied in detail for different mode orders and power-dependent parameters. The dispersion characteristics are investigated for both self-focusing and self-defocusing substrate nonlinearity. Moreover, the dispersion curves for different values of the negative parameters of the DNM are presented.

2. THEORY

The geometry of the proposed structure is illustrated in Fig. 1. A DNM of thickness t is assumed to be the guiding layer. It is characterized by an electric permittivity ε_f and a magnetic permeability μ_f. A linear dielectric material occupying the region $t < y$ is considered a cover layer. The substrate is assumed to be a Kerr-type nonlinear material which has a refractive index $n_s(I) = n_s + \eta I$, where I is the light intensity and η is the nonlinear refractive index.

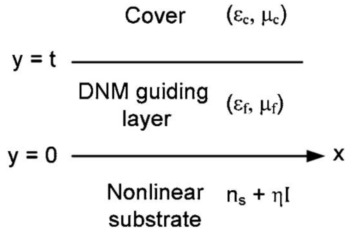

Fig. 1. Nonlinear slab waveguide configuration with a DNM is used as a guiding layer.

The wave equation of electromagnetic waves propagating along the x axis has the form

$$(\nabla^2 + k^2 n_i^2) E_z(y, x) = 0, \tag{5}$$

where n_i^2 stands for n_c^2 for $y > t$, n_f^2 for $0 < y < t$, and n_{sub}^2 for $y < 0$. For the substrate, $n_{\text{sub}}^2 = n_s^2 + \alpha|E|^2$, where $\alpha = \varepsilon_o c \eta n_s^2$ with c the speed of light in free space.

For s-polarized electromagnetic waves propagating along x axis, the x-dependence of the fields is given by $\exp(i\beta x)$ whereas the y-dependence is given in the cover layer ($y > t$) by

$$E_z(y) = \frac{E_o}{\cos\varphi} \cos(q_f t - \varphi) e^{-q_c(y-t)}, \tag{6}$$

$$H_x(y) = \frac{q_c}{i\omega\mu_o\mu_c} \frac{E_o}{\cos\varphi} \cos(q_f t - \varphi) e^{-q_c(y-t)}, \tag{7}$$

where $q_f = k\sqrt{n_f^2 - N^2}$, $q_c = k\sqrt{N^2 - n_c^2}$, and φ is an arbitrary constant.

In the DNM guiding layer ($0 < y < \mathrm{t}$),

$$E_z(y) = \frac{E_o}{\cos\varphi} \cos(q_f t - \varphi), \tag{8}$$

$$H_x(y) = \frac{q_f}{i\omega\mu_o\mu_f} \frac{E_o}{\cos\varphi} \sin(q_f t - \varphi). \tag{9}$$

In the nonlinear substrate ($y < 0$),

$$E_z(y) = A \frac{q_s}{\cosh[q_s k(y - y_s)]} \quad \text{for } \alpha > 0, \tag{10}$$

where $A = \sqrt{2/\alpha}$, $q_s = \sqrt{N^2 - n_s^2}$, and y_s is a constant related to the light intensity which can be calculated from the total guided wave power per unit distance along the wave front [1]. For $\alpha < 0$, $\sinh[q_s k(y - y_s)]$ replaces $\cosh[q_s k(y - y_s)]$ and $A = -\sqrt{-2/\alpha}$.

Equation (10) can be rewritten as

$$E_z(y) = E_o[\cosh(q_s k y) - \sigma \sinh(q_s k y)]^{-1}, \tag{11}$$

where

$$\sigma = f(y_s)\left(1 - \frac{\alpha E_o^2}{2q_s^2}\right)^{1/2} = \begin{cases} \tanh(q_s k y_s) & \text{for } \alpha \geq 0 \\ \coth(q_s k y_s) & \text{for } \alpha \leq 0 \end{cases}, \tag{12}$$

where $f(y_s) = +1$ for $y_s \geq 0$ and -1 for $y_s \leq 0$.

The tangential component of the magnetic field in the substrate is given by

$$H_x(y) = \frac{q_s k E_o}{i\omega\mu_o\mu_s} \frac{[\sinh(q_s k y) - \sigma \cosh(q_s k y)]}{[\cosh(q_s k y) - \sigma \sinh(q_s k y)]^2}. \tag{13}$$

The continuity of E_z and H_x at $y = t$ and $y = 0$ leads to the dispersion relation

$$q_f t = \tan^{-1}\left(\frac{\mu_f}{\mu_c}\frac{q_c}{q_f}\right) + \tan^{-1}\left(\frac{\mu_f}{\mu_s}\frac{kq_s}{q_f}\sigma\right) + \pi m, \quad (14)$$

where $m = 0, 1, 2, \ldots$ is the mode order.

Equation (14) can be written in terms of the generalized parameters V, b, a, and b_I as

$$V\sqrt{1-b} - \tan^{-1}\left(\frac{\mu_f}{\mu_c}\sqrt{\frac{a+b}{1-b}}\right) \mp \tan^{-1}\left(\frac{\mu_f}{\mu_s}\sqrt{\frac{b-b_I}{1-b}}\right) - \pi m = 0. \quad (15)$$

The upper sign of the third term corresponds to $y_s > 0$ whereas the lower one corresponds to $y_s < 0$.

3. RESULTS AND DISCUSSION

The dispersion relation given by Eq. (15) has a reduced number of parameters compared to Eq. (14). To plot the universal dispersion curves, the function $V(b, a, b_I)$ has to be calculated. For a given mode order, Eq. (15) is solved numerically for V as a function of b for specific values of a and b_I. As $b_I \to 0$, the dispersion relation reduces to the one covering the linear waveguide structure [22]. As can be seen from Eq. (12), the limit $b_I \to 0$ is satisfied as $y_s \to \infty$. Consequently, the parameter b_I seems to be a more suitable power measure than y_s. We have to mention here that b_I also vanishes when $y_s \to -\infty$ that leads to the strongly nonlinear solution. Usually, the total energy flow in localized guided wave is taken as a nonlinear parameter defining the nonlinear regimes of the mode propagation. We will adopt b_I as a power parameter since we here concentrate our attention mainly on the new physics which DNM introduces into the considered structure.

In the calculations, we assume both self-focusing and self-defocusing nonlinear substrate. For self-focusing nonlinearity, η is taken to be positive and consequently α is positive. In this case, b_I must be positive and less than unity. However, b_I can be any negative number for self-defocusing nonlinearity in which both η and α are negative. Moreover, for $\alpha < 0$, the lower sign of the third term of Eq. (15), ($y_s < 0$), seems to be physically unacceptable because the field in the substrate becomes infinite at $y = y_s$.

For $a = 0$, Fig. 2 illustrates the dispersion curves of the slab waveguide with DNM guiding layer and self-focusing nonlinear substrate for $m = 0, 1$. The guiding layer is assumed to have $\mu_f = -6 + 0.01i$ whereas the cladding and substrate are assumed to be nonmagnetic materials with $\mu_c = \mu_s = 1$. The power parameter (b_I) is given the values 0, 0.2, 0.4, 0.6, and 0.8. Solid lines in the figure describe the cases for which the electric field maxima are contained in the DNM film. This case corresponds to $y_s > 0$. Many features should be clarified in the figure. First, it is well known that when a DNM guiding layer is surrounded by two linear dielectric media, the fundamental mode does not exist, whereas it exists in the proposed structure as the figure shows. It is clear that this is the influence of nonlinearity of the substrate. Second, the shape of the dispersion curves for the first mode is similar to that of the fundamental mode with a shift toward higher values of V. Third, the value of b does not exceed 1.0 in the solid lines. This means that these solutions correspond to guided modes. For surface modes, the value of b exceeds unity [24]. Fourth, for $b_I > 0$, there is no cutoff frequency at which $b = 0$. This behavior is in contradiction with that of the guided modes in linear waveguide structure [22]. Moreover, for a relatively high values of b_I (e.g., $b_I = 0.8$), the dispersion curves show a slight dependence of b on V. The solid curve corresponding to $b_I = 0.8$ is almost a straight line parallel to the V axis. In this case, the dispersion properties are crucially determined by the nonlinear parameters of the substrate. This property is similar to that mentioned in [23] by Boardman and Egan in 2009. They concluded that the thickness of the film plays a significant role, with the possibility of finding critical thicknesses at which the nonlinearity will have the most significant effect.

We now turn our attention to the dotted lines in the figure. The points of the dotted lines describe the waveguide dispersion when $y_s < 0$ and $\varphi < 0$. These points have been obtained by considering the positive sign of the third term of Eq. (15). In the fundamental mode, the dotted lines represent the cases for which the electric field maxima are contained in the nonlinear substrate. This can be interpreted in the language of geometrical optics. The incident ray penetrates

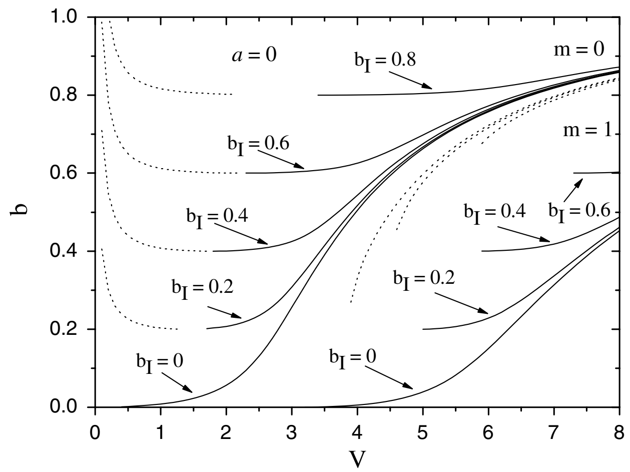

Fig. 2. Universal dispersion curves of the slab waveguide with DNM guiding layer and self-focusing nonlinear substrate for $a = 0$ and $m = 0, 1$. The dotted lines describe modes having maxima in the substrate ($y_s < 0$).

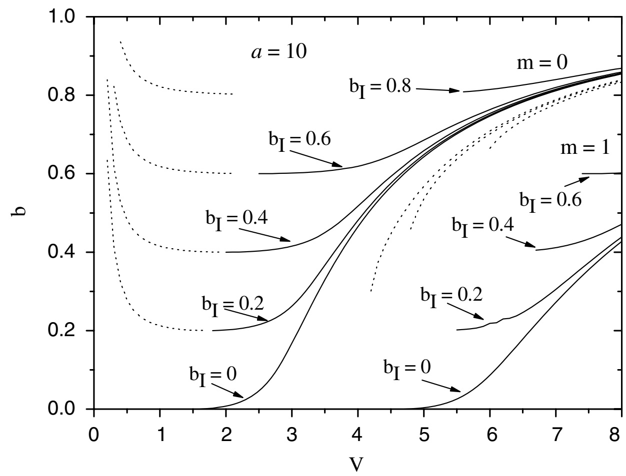

Fig. 3. Universal dispersion curves of the slab waveguide with DNM guiding layer and self-focusing nonlinear substrate for $a = 10$ and $m = 0, 1$. The dotted lines correspond to cases having $y_s < 0$.

the nonlinear substrate and turns back due to the changes in the refractive index induced by the intensity variation [19].

In Fig. 3, the dispersion curves are displayed for the slab waveguide with self-focusing nonlinear substrate for $a = 10$. As mentioned above, solid lines represent the cases for which the electric field maxima are contained in the guiding film ($y_s > 0$), whereas dotted lines describe the cases for which $y_s < 0$ and the electric field maxima are contained in the nonlinear substrate. The spatial distribution of the electric field is determined by the parameters V, a, and b_I provided that σ and φ are also dependent on these parameters. The guiding properties of nonlinear slab waveguide structures and the electric field distribution in space are completely determined by these parameters.

Figure 4 shows a comparison between the dispersion properties of the proposed waveguide structure with self-focusing substrate for different values of the asymmetry coefficient a. For a given value of b_I, the dispersion curves are similar in shape to each other for different values of a with a slight shift toward higher values of V as the asymmetry coefficient increases. As b_I increases, this shift is reduced gradually. For $b_I = 0.4$, this shift is less than that associated to $b_I = 0.2$. This is an important property which indicates that, as the nonlinearity of the substrate increases, it becomes the dominant parameter of the structure and the effect of other parameters is reduced.

The dispersion curves for self-defocusing nonlinear substrate are plotted in Fig. 5 for $a = 0$ and $a = 10$ for both the fundamental and first modes. In this case, the power parameter b_I can be any negative number. The figure shows the cases for which $b_I = 0$, -0.4, and -1. Generally, the shape of the dispersion curves for $m = 0$ and $m = 1$ is almost the same. The normalized cutoff frequency increases slightly with increasing absolute value of b_I. In analogy to Fig. 2, the value of b doesn't exceed unity for all values of b_I which means that we have guided modes. A high degree of similarity is seen for the curves corresponding to $a = 0$ and those corresponding to $a = 10$. The figure reveals that increasing a for a specific value of b_I shifts the dispersion curve toward higher values of V.

It is significant to compare between the dispersion properties in self-focusing and self-defocusing nonlinearities. Let us consider, for example, the curve with the parameters $a = 10$ and $b_I = 0.4$ in Fig. 4 and the corresponding one with $a = 10$ and $b_I = -0.4$ in Fig. 5. It is clear that both curves coincide at large values of the normalized thickness due to the high confinement of the wave in the guiding film, and there is no effect of the substrate nonlinearity. At low values of the normalized thickness, the two curves split with a major difference between them. In the case of self-defocusing nonlinearity, there is a cutoff thickness at which $b = 0$ and the modal index is equal to that of the substrate, which means that all the power of the mode propagates in the substrate. The self-focusing nonlinearity does not have a cutoff thickness.

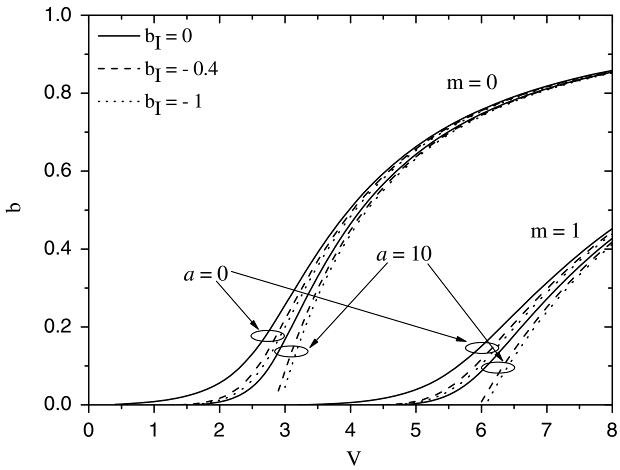

Fig. 5. Universal dispersion curves of the slab waveguide with DNM guiding layer and self-defocusing nonlinear substrate for $a = 0$, 10 and $m = 0$, 1.

We now investigate the effect of the DNM on the dispersion properties. In the following we assume the asymmetry coefficient to be zero ($a = 0$) and the fundamental mode ($m = 0$). We first investigate the difference between a positive index material (PIM) and a DNM guiding layers in Figs. 6 and 7 for self-focusing and self-defocusing substrates, respectively. The dispersion curves for PIM and DNM guiding layers are similar in shape with a significant difference. As can be seen from the two figures, the use of DNM as a guiding layer shifts

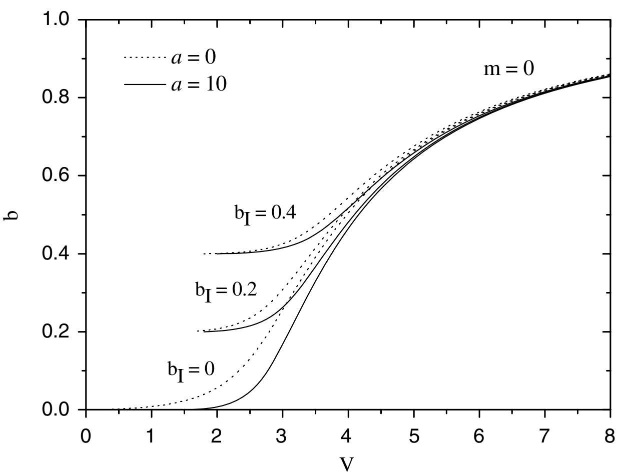

Fig. 4. Universal dispersion curves of the slab waveguide with DNM guiding layer and self-focusing nonlinear substrate for $a = 0$ and $a = 10$ for the fundamental mode.

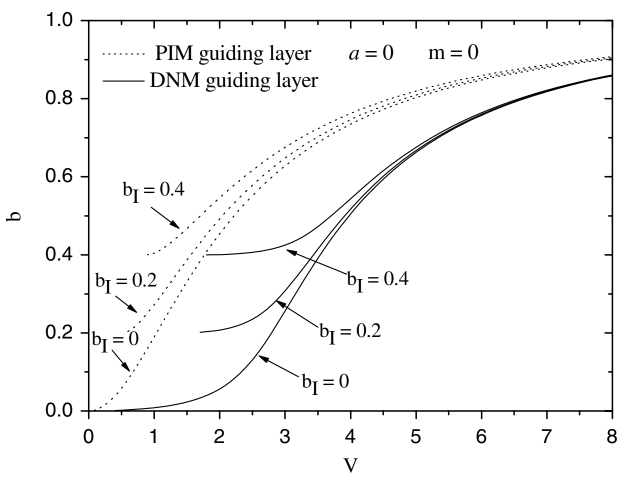

Fig. 6. Universal dispersion curves of the slab waveguide with DNM and PIM guiding layer and self-focusing nonlinear substrate for $a = 0$ for the fundamental mode.

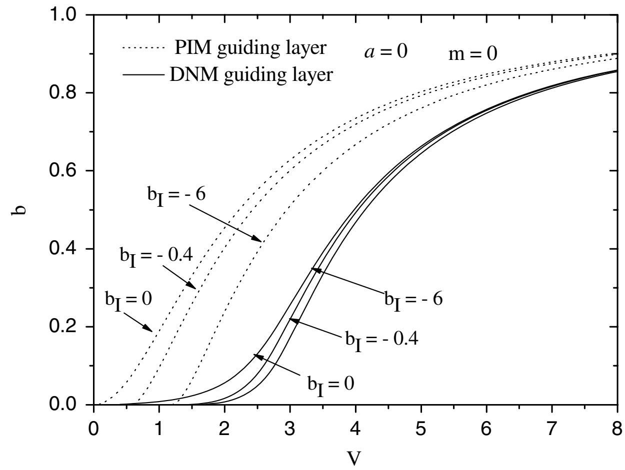

Fig. 7. Universal dispersion curves of the slab waveguide with DNM and PIM guiding layer and self-defocusing nonlinear substrate for $a = 0$ for the fundamental mode.

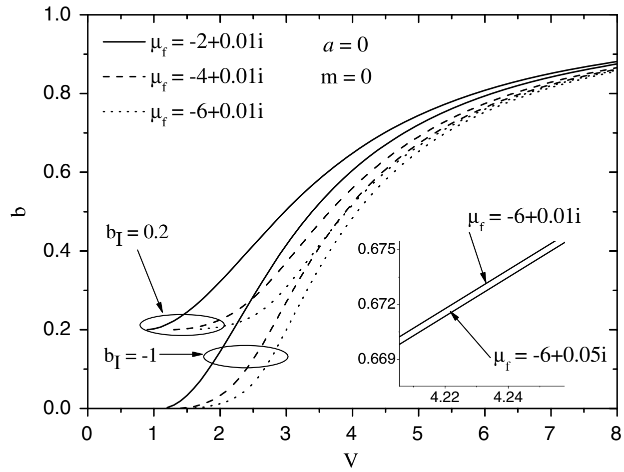

Fig. 8. Universal dispersion curves of the slab waveguide with DNM guiding layer and nonlinear substrate (self-focusing and self-defocusing) for the fundamental mode for $a = 0$ and different values of the DNM permeability.

the dispersion curves toward higher values of V for the same value of b_I. The curves of a PIM core saturate at higher values of b than that of a DNM core for both self-focusing and self-defocusing substrates. This means that, for a given value of V and b_I, the value of the normalized guide index is reduced when using a DNM guiding layer compared to that of a PIM guiding layer. The reduction of the normalized guide index leads to the decrease of the modal index of the guided mode and hence the less guidance of the wave in the core layer. In this case, the evanescent field in the surrounding media is enhanced. This property serves in slab waveguide applications requiring the enhancement of the evanescent field, such as optical waveguide sensing [11–14]. The principle of operation of optical waveguide sensors is based on the evanescent tail of the modal field in the cover medium. The guided wave extends as an evanescent field into the cover and substrate media and senses a modal index of the guided mode. The modal index of the propagating mode depends on the structure parameters, e.g., the guiding layer thickness and refractive indices of the media constituting the waveguide. As a result, any change in the refractive index of the cover results in a change in the modal index of the guided mode. The basic sensing principle of planar waveguide sensors is to measure the changes in the modal index due to changes in the index of the cover medium.

The dispersion curves are shown for $a = 0$ and $m = 0$ for different values of the DNM guiding layer permeability in Fig. 8 for self-focusing and self-defocusing substrates. As the two figures show, for constant V and b_I, b is enhanced with decreasing the absolute value of the real part of μ_f. Lossless DNM does not exist in practical designs. It represents an ideal case. In all present designs, DNM with an absorption coefficient was obtained. DNMs with minimal absorption coefficient have shown an interesting property. It was verified that DNMs with a small absorption coefficient can focus light onto an area smaller than a square wavelength in near fields [25]. This super-resolution is attributed to an important feature of DNMs, which is amplification of evanescent waves. Therefore, for the sake of reality, we assume the DNM permeability to have the form $\mu_f = \mu_r + \mu_i\ i$, where μ_r and μ_i are the real and imaginary parts of μ_f. It is significant to study the effect of the imaginary part of μ_f on the dispersion curves. The inset of Fig. 8 shows the dispersion curves of the proposed structure for two different values of the imaginary part of the DNM permeability. As the inset reveals, when the imaginary part of μ_f increases, the normalized guide index is a little bit reduced. Therefore, a DNM guiding layer with minimal absorption coefficient is recommended to attain high guidance of the wave in the core layer.

It is worth mentioning that the transition from TE to TM polarization is simply by replacing μ_i of the ith layer by ε_i in the dispersion relation for many waveguide structures. In nonlinear waveguide configurations, the case is completely different. In TM polarization mode, the nonlinear medium exhibits the biaxial nature of the refractive index because TM polarizations involve two nonzero electric-field components. In this situation, in addition to the parameters V, a, b, and b_I, more normalized parameters are needed to treat TM polarization [26]. Therefore, the treatment of the TM case has to be done in a separate work.

Finally, it is significant to compare the present work with our previous paper, cited as [21], in which a nonlinear guiding layer is surrounded from one or both sides with a DNM. In the previous work, we found that the range of the normalized film thickness in which there are propagating modes is dependent on the nonlinearity constant of the guiding layer. In the current work, the value of the normalized thickness at which the wave guidance begins is determined by the power parameter of the nonlinear substrate. In the previous work, when both the cladding and substrate are DNMs, the cutoff thickness is found to have a significant dependence on the nonlinearity constant. In the current work, the cutoff thickness of the self-defocusing structure is found to be dependent on the power parameter, mode order, and asymmetry coefficient.

4. CONCLUSION

In conclusion, we have investigated the optical characteristics of nonlinear waveguide structures comprising DNMs. Three independent parameters have been used to determine the dispersion properties. These parameters are the normalized film thickness V, asymmetry coefficient a, and mode power measure b_I. For self-focusing and self-defocusing nonlinear configuration, the fundamental mode is found to exist in contrary to the waveguide structure comprising DNM guiding layer surrounded by two linear dielectric media. For the first mode, the dispersion curves are similar to that of the fundamental mode with a shift toward higher values of V. No cutoff frequency is found for self-focusing nonlinear substrate for $b_I > 0$. The use of DNM as a guiding layer in the proposed structure is found to be constructive for applications requiring the enhancement of the evanescent field in the surrounding layers, such as optical sensing. The concept of controlling the dispersion properties by the nonlinear parameters of the substrate and not by the guiding layer thickness at some values of b_I is a very attractive feature. This means that the proposed structure can have strong design implications in optoelectronics.

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