On the flexibility of Home-Away pattern sets

Roel Lambers (Speaker) * Dries Goossens † Frits C.R. Spieksma ‡

1 Introduction

We consider the scheduling of a sports league in a round robin format (see De Werra (1988)), i.e., there is a set of teams $\mathcal{T}$ and every team $t \in \mathcal{T}$ has to play every other team of the league exactly k times. If k is equal to 1, the format is called a Single Round Robin (SRR), if k is equal to 2, the format is called a Double Round Robin (DRR) - for instance, most major soccer leagues play according to a Double Round Robin format. In such a format, matches are played in rounds, periods in which a team $t \in \mathcal{T}$ can play at most 1 match. If the league consists of an even number of teams, say $|\mathcal{T}| = 2n$, a compact schedule for SRR needs $2n - 1$ rounds in which every team plays exactly 1 match every round.

We assume, as in practice, that in such a compact schedule, from the viewpoint of a team, each match is played either at home, or away. This gives rise to a Home-Away pattern (HAP) for each team. Deciding upon which matches are played in which round is of great practical importance, and a popular way to do so is a first-break-than-schedule approach (see Rasmussen and Trick (2008)). In such an approach, first a HAP-set is chosen for each team, resulting in a set of patterns denoted by $\mathcal{H}$. Next, the individual matches are assigned to rounds in a way that is compatible with $\mathcal{H}$. The goal of this contribution is to investigate to what extent the choice of $\mathcal{H}$ leaves room for the assignment of individual matches to rounds. Indeed, we will show that not all HAP-sets $\mathcal{H}$ are created equal and some HAP-sets allow more flexibility than others.

2 Measures for the flexibility of a HAP-set

A *schedule* for a compact SRR specifies for each match (t, t'), with $t, t' \in \mathcal{T}$, in which of the $2n-1$ rounds it is played. A schedule S is *compatible* with HAP-set $\mathcal{H}$ if every team is scheduled home and away according to $\mathcal{H}$. We say two schedules

*r.lambers@tue.nl. Department of Mathematics and Computer Science, Eindhoven University of Technology, Eindhoven, Nederland

†dries.goossens@ugent.be. Department of Business Informatics and Operations Management, Ghent University, Gent, Belgium

‡f.c.r.spieksma@tue.nl. Department of Mathematics and Computer Science, Eindhoven University of Technology, Eindhoven, Nederland

	1	2	3	4	5	6	...
t_1	H	A	H	A	H	A	...
t_2	H	A	A	H	A	H	...
t_3	H	A	H	A	A	H	...

Table 1: A sample of a CPS HAP-set

S_1, S_2 are *match-distinct* if for every match (t, t'), with $t, t' \in T$, the round in which it is planned in S_1, differs from the round it is planned in S_2.

We now introduce three measures, $\text{width}(\mathcal{H}), \text{spread}(\mathcal{H}), \text{AFP}(\mathcal{H})$ representing the flexibility of HAP-set $\mathcal{H}$.

Definition 1 The *width* of a HAP-set, denoted by $\text{width}(\mathcal{H})$, is equal to the maximum size of a set of pair-wise match-distinct schedules compatible with $\mathcal{H}$. The width of any infeasible HAP-set is 0, since there are no schedules compatible with $\mathcal{H}$. The width of any feasible HAP-set is at least 1.

We call the spread of a match (t, t') the number of distinct rounds in which the match can be scheduled in any feasible schedule compatible with $\mathcal{H}$.

Definition 2 The *spread* of a HAP-set $\mathcal{H}$, denoted by $\text{spread}(\mathcal{H})$, is given by summing the spread of a match over all matches. A higher spread points to more flexibility.

Definition 3 The *absolute fixed part*, denoted as $\text{AFP}(\mathcal{H})$ is the number of matches that have a fixed round for any schedule compatible with $\mathcal{H}$. This is the same as the number of matches with spread equal to 1. A larger AFP points to diminished flexibility.

3 The Canonical Pattern Set

In a HAP, if in two consecutive rounds $r, r+1$ a team needs to play either both matches at home or both matches away, the HAP has a break in round r. A HAP with only one break is called *single break*. Breaks are often seen as unwanted. There are feasible HAP-sets for a SRR in which every team has at most 1 break, so called single break HAP-sets (see De Werra (1988) and Briskorn (2008)).

The *Canonical Pattern Set (CPS)* is such a single break HAP-set, specifically one in which breaks only occur in even rounds (see Table 1 for an illustration of the CPS). To generate a schedule compatible with the CPS, one can use the *Circle Method*, which is a very popular way to schedule leagues worldwide (see Goossens and Spieksma (2012)).

4 Results

Let $CPS(2n)$ denote the canonical pattern set for $2n$ teams. Then:

Theorem 1. *For each $n \geq 2$: width$(CPS(2n)) = 1$.*
Moreover, all feasible single break HAP-sets $\mathcal{H}$ have width$(\mathcal{H}) = 1$.

Theorem 2. *For each $n \geq 2$:*

$$spread(CPS(2n)) = \frac{n}{6}\left(10n^2 - 9n + 11\right) - \lceil \frac{n}{2} \rceil.$$

Theorem 3. *For each $n \geq 2$: AFP$(CPS(2n)) = n$.*

As convenient as it may be to use, the CPS has some significant drawbacks in terms of flexibility, especially when leagues grow bigger, even when only comparing to other single break HAP-set. The number of matches that are tied down to one specific round grows for the CPS, while in other HAP-sets, this is not necessarily the case. Also, there are HAP-sets with higher spread when n gets bigger. All this is shown in Table 2 below, where AFP$(*)$, spread$(*)$ denotes the best score that can be achieved on the measures by a single break HAP-set that is not CPS, given the league size.

2n	spread(CPS)	spread(*)	AFP(CPS)	AFP(*)
4	**10**	-	**2**	-
6	**35**	-	**3**	-
8	**88**	76	**4**	**4**
10	**177**	**177**	5	**4**
12	314	**332**	6	**4**
14	507	**557**	7	**4**
16	768	**864**	8	**4**

Table 2: Boldfaced denotes the better score in the respective measure.

References

[1] D. de Werra (1988), *Some models of graphs for scheduling sports competitions*, Discrete Applied Mathematics **21**, 47-65.

[2] D.R.Goossens, F.C.R. Spieksma (2012), *Soccer schedules in Europe: an overview*, Journal of Scheduling **15**, 641-651

[3] D. Briskorn (2008), *Sports Leagues Scheduling: Models, Combinatorial Properties, and optimization*, New York, Springer.

[4] R. Rasmussen, M. Trick (2008), *Round Robin Scheduling: A survery*, European Journal of Operational Research **188**, 617-636