## MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

## NATIONAL TECHNICAL UNIVERSITY «KHARKIV POLYTECHNICAL INSTITUTE»

**«Structural reliability of electrical objects.** Theory and examples of solving tasks»

# **METHODICAL INSTRUCTIONS** of course «Reliability and diagnostics»

by specialty 141 «Electric Power Engineering, Electrical Engineering and Electromechanics», knowledge field title 14 «Electrical engineering» for English speaking students

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### **INTRODUCTION**

Structural reliability of electrical objects is the ability of its electrical circuit to maintain consumer connections to power sources. Structural reliability is determined only by the structural connections of the elements of the system, that is, its scheme. Features of modes of operation of elements are not taken into account – their capacity is not limited. The calculation schemes take into account only those elements through which energy is transmitted to the consumer from the power source.

The main task of structural reliability is to develop and research methods to ensure the effectiveness of various objects (products, devices, systems) in the process of their design, manufacture and subsequent operation. Reliability theory establishes and studies the quantitative characteristics of reliability and investigates the relationship between reliability indicators and uptime.

Reliability and diagnostics of energy objects are based on mathematical disciplines such as probability theory, mathematical statistics, queuing theory, graph theory, and mathematical programming.

Structural reliability of energy objects is one of the most important topics of study in the study of specialty disciplines in the field of Power Engineering, Electrical Engineering and Electromechanics. Students in the specialty "Renewable Energy and High Voltage Engineering and Electrophysics" should have a clear understanding of the nature of structural redundancy issues, be able to evaluate the actual level of reliability through appropriate analysis and know the ways and means of ensuring trouble—free operation of power systems, subsystems and objects of renewable energy.

### 1. BASIC CONCEPTS OF THE COURSE

**Reliability** – the peculiarity of the object to perform the specified functions, keeping the set performance within the specified limits under the specified modes and conditions of operation for the required period of time or working cycles.

There are four characteristics to determining reliability, namely: the ability to perform specified functions, successful operation during operation, operating conditions, and operating time.

Reliability is a complex feature that, depending on the purpose of the electrical element and its operating conditions, may include trouble—free operation, durability, maintainability and safety.

**Failure** – the quality of the object to constantly maintain a state of performance for a certain time or a certain time.

**Durability** – the characteristic of an object to remain operable until it is destroyed or before the limit state occurs. The signs of the limit state are inserted by the normative–technical documents of the object.

**Boundary** – is a condition in which further operation of the facility is impossible due to insurmountable violations, safety or performance requirements, as well as care of parameters beyond the established limits. Longevity is quantified by resource or service life.

**Resource** – the operation of the object from the beginning of operation or its restoration after repair to the onset of the limit state.  $\gamma$  – Percentage Resource – or working cycles equal to or greater than  $\gamma$  % of the total batch.

**Service life** is the calendar duration of exploitation up to the limit state occurs. The necessary durability of electrical equipment is determined on the basis of long–term experience and reflected in the norms and standards for individual objects.

A resource can be measured by time or cycles. For products that operate continuously with uniform stale, the terms "service life" and "resource" are the same if the products are decommissioned no earlier than the limit state.

Along with the notion of limit state, the concept of rejection is used.

**Failure** – is an event that involves the disability of the object (or part of it). For electrical equipment, the failure criteria are non–performanc by the product of its basic functions, which, depending on its type and kind are: 1) fulfillment of the main purpose (for transformers – power supply to electric motors; for the electric motor – drive of the working machine; for the starter – remote control of the electric motor etc.); 2) providing electrical protection: mechanical, electrical and electromechanical interlocks; inspections of triggering of defenses and resetting of defenses; explosion protection; grounding of enclosures; 3) the possibility of maintenance and repair, which are specified in the regulatory and technical documens for the delivery of products or in the operating instructions.

In terms of performance requirements, electrical equipment is classified as renewable and non-renewable.

**Renewable** products, either those that can be restored after failure on site or in workshops, include: complete switchgear, complete transformer substations, circuit breakers, actuators, stationary test facilities, etc.

**Non-renewable** products include electric lamps, batteries, resistors, capacitors, high-voltage diodes, etc. items whose performance cannot be restored at all or is only possible at a specialized centralized repair facility.

## 2. DETERMINATION OF RELIABILITY INDICATORS BASED ON EXPERIMENTAL DATA

Reliability indicators are divided into single and comprehensive. Individual indicators quantitatively characterize one of the four properties of the product: failure—free, maintainability, durability or safety.

One of the most important concepts of probability theory is the concept of "accidental size". Random is called the value resulting from the study can take on certain values, and it is unknown in advance which ones.

Under time of analysis of reliability of power equipment by methods of mathematical statistics we have to deal with discrete and continuous random ones values.

Discrete can only get separated from each other values, and continuous ones - continuously fill a certain gap in the number axis. Discrete random variables, in particular, include the number of products, failures, the number of equipment failures over the period considered etc.

Continuous random variables include failure times or between two consecutive failures, unit recovery time, etc.

For a quantitative comparison of events (such as equipment failures) by the degree of their likelihood requires each of them to be associated with a number that the more likely the event. The number called the event probability is a numerical measure of the objective possibility of this event.

The concept of "probability events "is related to the studied concept of" event frequency ". Unit of measurementprobabilities are determined based on two opposite types of events: that is, one that will inevitably occur as a result of the study and impossible, which cannot occur during the study.

For the former, the probability is equal units, for the second - zero. Therefore, the probability of any event is in the interval from 0 ... till 1.

Complex indicators characterize the product's ability to perform its functions in the process of operation, taking into account that periodically maintenance will be carried out and, if necessary, repair of the product.

Complex indicators provide estimates of the type: how credibly it is that the product will be operational at any given time; what will be the efficiency of use of the product in a given period of operation, taking into account the fact that it may occur

and eliminate failures. In other words, the complex indicators depend on the reliability and maintainability of the product.

Indicators of failure. The most commonly used are the following single failure rates: the probability of failure–free operation P(t), failure rate  $\lambda(t)$ , average time to first failure  $T_I$ , average failure time T. The latter is applicable to renewable products and the other to non–renewable products. In the particular case, if the reliability of the recoverable product before the first failure is evaluated, the indicators P(t),  $\lambda(t)$ ,  $T_I$  can also be applied.

The probability of failure—free operation is the probability that within the given time t objects failure will not occur. This parameter is related to the probability of failure of the relation

$$P(t) = 1 - Q(t), \tag{2.1}$$

where: P(t) – the probability of failure–free of the object over time t; Q(t) – the probability of failure of the object over time t.

The probability of failure Q(t) refers to the auxiliary indicators of failure.

Function P (t) is monotonically declining and the function Q (t) – monotonically increasing, and  $0 \le P(t) \le 1$ , P(0) = 1,  $P(\infty) = 0$ .

Typical function graph P(t) is shown on the picture 2.1. Value P(t) determines the proportion of workable products at time  $t_i$ .

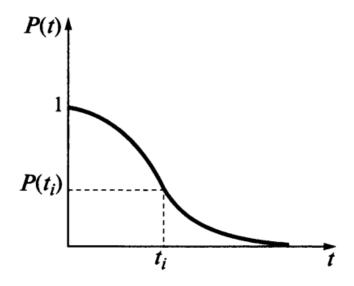


Figure 2.1 – Change in the probability of failure of the product over time

For the experimental determination of reliability indicators, a sufficiently large batch of the same type of products is tested and the failure times are recorded. The test program may change depending on the goal.

For P(t) the following statistical estimate is used:

$$P(t) = \frac{N(t)}{N_0} = 1 - \frac{n(t)}{N_0},\tag{2.2}$$

where  $N_0$  – number of products put to the test;

N(t) – number of products remaining in service at time t (failed in the process of testing products are not being repaired);

n(t) – the number of products that failed within the time interval (0, t).

The failure rate is the conditional probability of failure of a product at a certain point in time, provided that there were no failures up to that point:

$$\lambda(t) = \frac{dQ(t)}{dt} \frac{1}{(1-Q(t))} = -\frac{1}{p(t)} \frac{dP(t)}{dt},$$
(2.3)

For reliable systems in which  $P \rightarrow I$ , the failure rate is approximately equal to the density of the distribution of failure time. Statistically, failure rate is defined as the proportion of products that fail per unit time after time t:

$$\lambda(t) = \frac{n(t + \Delta t)}{N(t)\Delta t},\tag{2.4}$$

where: n(t) and  $n(t + \Delta t)$  – the number of items that have failed according to time t and  $(t + \Delta t)$ ;

N(t) – the number of good products at a time t.

If 
$$t = 0$$
, then N  $(t) = N_0$ .

Average time up to failure of  $T_1$  is the mathematical expectation of time to failure of a non-renewable product. for finding statistical evaluation of the  $T_1$  test leads to failure of all products in the party, with:

$$T_1 = \frac{1}{N_0} \sum_{i=0}^{N_0} \tau_i, \tag{2.5}$$

where  $\tau_i$  – time to failure of i–th product;

 $N_0$  – number of products put to the test.

Average working cycle up to failure time T is the mathematical expectation of failure object. The definition of T is as follows: a batch of products is put to work, the resulting failures are eliminated and again include the products in the work. Statistical estimate of average failure rate:

$$T = t/r(t), (2.6)$$

where: t – total product time;

r(t) – the total number of failures over time t.

Maintainability indicators. Maintenance – the property of the product, which is in adaptability to the maintenance and restoration of working condition through maintenance and repair.

Quantitatively system recovery is evaluated by the following indicators: probability of recovery  $P_{e}(\tau)$ , average recovery time  $T_{e}$  and the intensity of recovering  $\mu(\tau)$ , which mathematically correspond to the considered reliability indicators: failure probability Q(t), average failure time T and failure rates  $\lambda(t)$ .

The probability of recovery is the probability that the product will be restored after failure within a specified time under certain repair conditions. By analogy with the probability of failure, this figure can be imagined as the probability that the random recovery time of the product will not exceed the specified.

Statistical estimate of recovery probability:

$$P_{s}(t) = \frac{n_{\mathsf{B}}(\tau)}{N_{\mathsf{R}}},\tag{2.7}$$

where:  $n_{\theta}(\tau)$  – the number of products recovered over time  $\tau$ ;  $N_{\theta}$  – the number of items to be recovered.

Quantitative function Pe (t) is usually determined by other measures of recovery: average recovery time and recovery rate.

The most obvious indicator of recovery is the average recovery time, which means the mathematical expectation of a random variable – the recovery time of a product after failure.

Recovery time of a failing product includes the time for looking for a defective item, the time it was replaced or repaired, and the time to check for serviceability after repair.

The average recovery time is determined by both the product properties (adaptability to repair) and other factors (qualification of the service personnel, its technical equipment).

Statistical estimate of average recovery time  $T_e$ :

$$T_{s} = \frac{1}{m} \sum_{i=1}^{m} \tau_{i}, \qquad (2.8)$$

where:  $\tau_i$  – time spent restoring the product when i–failure; m – total number of updates.

In the presence of several sets of identical equipment, it is necessary to sum up the intervals of recovery time for all instances and divide this amount by the total number of failures.

# 3. STRUCTURAL RELIABILITY AND TYPICAL ELEMENT CONNECTION SCHEMES

All technical objects consist of elements. The elements can physically be interconnected in many different ways.

For the visual representation of the compounds of the elements are used different types of schemes: Structural, functional, principle, etc. Each has its own purpose and allows you to analyze how a particular product works. To analyze the level of reliability of the calculation of its indicators, special schemes, called structural reliability schemes, are used.

Structural reliability diagram is a visual graphical representation of the conditions under which the element, object, system, device, etc. works or does not work.

To draw up a structural diagram of reliability, analyze the process of functioning of the object, study the functional relationships between the elements, types of failures and causes of their occurrence. Such research requires high engineering and mathematical erudition. The degree of fragmentation of an object into elements depends on the specific task of the calculations. The same connection in the schematic diagram may have a completely different connection in the structural diagram of the reliability. The main failures of electric objects are the failures of the type "breakage" and "short circuit".

Let the object consist of two diodes VD1 and VD2, physically connected in parallel. If the short circuit type fails, the circuit will fail when either of the two diodes fails. Therefore, the structural diagram of the reliability for this case is depicted as a sequential connection of elements. Otherwise, in the case of failure type "breakage" the parallel circuit of diodes will fail only in case of failure of two diodes. Therefore, the structural diagram of the reliability will be a parallel connection of elements, figure 3.1.

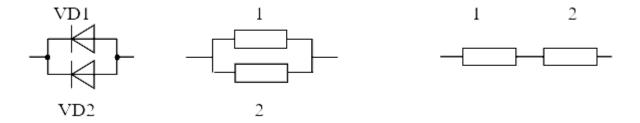


Figure 3.1 – Structural diagram of the reliability of two diodes VD1and VD2: parallel combination – "break", serial – "short circuit"

Structural reliability is the resultant reliability of the system at a given structure and known values of the reliability of all its elements (elements).

Suppose that the system consists of n different elements. Consider the reliability of a system that has different compounds of elements under the following assumptions:

- the elements are independent, that is, the resource of the individual is independent of each other, or the failure of one element does not change the reliability of the other elements;
- -state of system elements (good faulty) uniquely determine the state of the whole system;
  - after failures, the items are not restored.

## 3.1 Sequential connection of elements

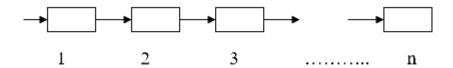


Figure 3.2– Structural diagram of the suential connection of elements

With a serial connection, the failure of any element causes a system failure. In this case, the probability of failure of the system is defined as the product of the values of the probability of failure of individual elements, namely:

$$P_{\Sigma} = p_1 \cdot p_2 \cdot p_3 \cdots p_n = \prod_{i=1}^n p_i$$
(3.1)

If the probability of failure–free operation of the elements is subordinate to exponential distribution:

$$P_i = e^{-\lambda_i t}, \tag{3.2}$$

where  $\lambda_i$  – failure rate of the i–th element,

t – operating time, then the reliability of the system (TRW – time reliability work, uptime) is:

$$P_{\Sigma} = \prod_{i=1}^{n} e^{-\lambda_{i}t} = e^{-\lambda_{\Sigma}t}$$
, (3.3)

where 
$$e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \cdot e^{-\lambda_3 t} \cdots e^{-\lambda_n t} = e^{-(\lambda_1 + \lambda_2 + \cdots + \lambda_n)t} = e^{-\lambda_{\Sigma} t}$$
,
$$\lambda_{\Sigma} = \lambda_1 + \lambda_2 + \cdots + \lambda_n = \sum_{i=1}^n \lambda_i$$
 (3.4)

The corresponding average of uptime defines the average time to failure or:

$$T = \sum_{i=1}^{n} \left(\frac{1}{T_i}\right)^{-1} . {(3.5)}$$

If the reliability is the same, then  $P_Z = p^n = e^{-nxt} = e^{-\lambda_{\Sigma}t}$ .

The probability of failure is determined  $Q_i = 1 - P_i = 1 - \prod_{i=1}^{n} p_i$ .

## 3.2 Parallel connection of elements

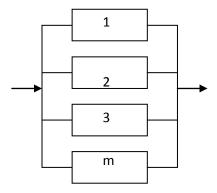


Figure 3.3– Structural diagram of the parallel connection of elements

The elements are connected in parallel if the system failure occurs only when all the elements fail.

If the probabilities of failure of individual elements are equal  $p_1$ ,  $p_2$ ,  $p_3$ , ...... $p_i$ , then the failure probabilities will be equal  $q_1 = 1 - p_1$ ;  $q_2 = 1 - p_2$ ; ...... $q_m = 1 - p_m$ , and the resulting unreliability of the scheme, that is, the probability of failure of the whole system is equal:

$$q_{\Sigma} = q_1 \cdot q_2 \cdot \dots \cdot q_m = \prod_{j=1}^{j=m} q_j = \prod_{j=1}^{j=m} (1 - p_j)$$
(3.6)

The reliability of this scheme, consisting of parallel connected blocks:

$$p_{\Sigma} = 1 - q_{\Sigma} = 1 - \prod_{j=1}^{j=m} (1 - p_{j})$$
(3.7)

For the case where the blocks have the same unreliability q1 = q2 = .... qm = q, we have:

$$q_{\Sigma} = q_n^m = (1 - p)^m \tag{3.8}$$

$$p_{\Sigma} = 1 - (1 - p)^m \tag{3.9}$$

If p is subject to an exponential distribution, that is:  $p = e^{-\lambda t}$ ,

than 
$$p_{\Sigma} = 1 - (1 - e^{-\lambda t})^m$$
.

We define the average time (mathematical expectation) as:

$$Tmidl = \int_{0}^{\infty} p(t)dt = \int_{0}^{\infty} \left[ 1 - (1 - e^{-\lambda t})^{m} dt \right] = \left\{ y = 1 - e^{-\lambda t}; dy = -\lambda e^{-\lambda t} dt; dt = -\frac{dy}{\lambda e^{-\lambda t}} = \frac{dy}{\lambda (1 - 1 + e^{-\lambda t})} = \frac{dy}{\lambda (1 - 1 + e^{-\lambda t})} \right\} = \frac{dy}{\lambda (1 - 1 + e^{-\lambda t})} = \frac{dy}{\lambda (1 -$$

$$\int_{0}^{\infty} (1 - y^{m}) \frac{dy}{\lambda (1 - y)} = \frac{1}{\lambda} \left\{ \int_{0}^{1} y^{m-1} dy + \int_{0}^{1} y^{m-2} dy + \dots \int_{0}^{1} dy \right\} = \frac{1}{\lambda} + \frac{1}{2\lambda} + \frac{1}{3\lambda} + \dots + \frac{1}{m\lambda}$$
 (3.10)

If n=2,  $T=\frac{1}{\lambda}+\frac{1}{2\lambda}=\frac{3}{2}\cdot\frac{1}{\lambda}=\frac{3}{2}T_0$ , where  $T_0$  average running time of one element.

Parallel connection usually occurs when all the elements perform the same function. One element is sufficient for its implementation, others play the role of reserve. This type of reservation is a loaded reserve. In this case, the elements are usually the same and have an equal CBD. For them, the expression (3.9) holds. There is also substitution reservation and sliding reservation. It should be noted that if the reliability of the elements is subject to exponential law, then the resulting reliability will no longer obey this law or:

 $p_1 = e^{-\lambda_1 t}$ ;  $p_2 = e^{-\lambda_2 t}$ ;..... $p_m = e^{-\lambda_m t}$ 

$$p_{\Sigma} = 1 - \left(1 - e^{\lambda_1 t}\right) \left(1 - e^{\lambda_2 t}\right) ... \left(1 - e^{\lambda_m t}\right).$$
(3.11)

## 3.3 Parallel-sequential connection of elements

There are two common cases of connecting elements.

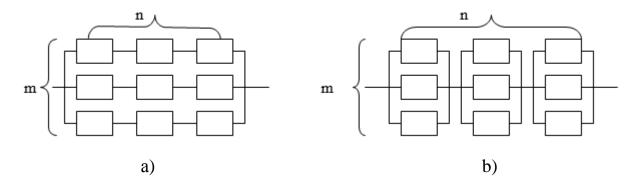


Figure 3.4 – Mixed element schematics:

- a) there are "m" parallel chains of "n" sequentially included elements;
- b) "n" groups are connected in series with "m" in parallel connected identical elements.

In scheme a) there are m parallel chains of n sequentially included elements. To simplify the calculations, we assume that the systems (a, b) are made of the same elements of the CBD. The BSR of each chain (serial connection of elements) will be equal to:

$$p_{\Sigma}^{1} = p^{n}. \tag{3.12}$$

Probability of refusals

$$q^{1} = 1 - p_{\Sigma}^{1} = 1 - p^{n}. \tag{3.13}$$

For m chains in parallel, the reliability of each failure is equal  $q_1$ , the probability of failure is equal:

$$q_{\Sigma} = (q^1)^m = (1 - p^n)^m$$
 (3.14)

Reliability of the whole system:

$$p_{\Sigma} = 1 - q_{\Sigma} = 1 - (q^{1})^{m} = 1 - (1 - p^{n})^{m}. \tag{3.15}$$

Consider  $m \to \infty$ , obsessed  $p_{\Sigma} \to 1$ , that is, parallel connection of chains from identical blocks increases the reliability of the circuit.

If  $n \to \infty$ , then  $p_{\Sigma} \to 0$ ;

If  $n \to \infty$  and  $m \to \infty$ , then  $p_{\Sigma} \to 0$ .

In scheme b) is connected in series "n" groups of "m" in parallel connected identical elements. Group reliability (parallel connection):

$$p_{\Sigma}^{1} = 1 - (1 - p)^{m}, \tag{3.16}$$

Reliability of the whole system:

$$p_{\Sigma} = (p_{\Sigma}^{1})^{n} = [1 - (1 - p)^{m}]^{n}. \tag{3.17}$$

If  $m \to \infty$  value  $p_{\Sigma} \to 1$ , if  $n \to \infty$ , then  $p_{\Sigma} \to 0$ . Finally at  $m \to \infty$  and  $n \to \infty$ , then  $p_{\Sigma} \to 1$ .

In the general case, when a series connection of n groups having different numbers of m parallel connected elements with different reliability, the unreliability of any i–th group of mi elements is defined as:

$$q_{i} = \prod_{j=1}^{j=m_{i}} q_{j} = \prod_{j=1}^{j=m_{i}} (1 - p_{j}),$$
(3.18)

and the value of reliability (the probability of failure–free operation)

$$p_{i} = 1 - q_{i} = 1 - \prod_{j=1}^{j=m_{i}} (1 - p_{j})$$
(3.19)

The resulting circuit reliability:

$$p_{\Sigma} = \prod_{i=1}^{i=n} p_i = \prod_{i=1}^{i=n} \left[ 1 - \prod_{j=1}^{j=m_i} (1 - p_j) \right].$$
 (3.20)

### 4. EXAMPLES OF TASKS SOLVING

## **Task 4.1**

Calculation of structural reliability of non-renewable objects.

The system is a series connection of elements of the structural scheme of reliability. The known failure rates of each element are known.

Determine the failure rate of the system, the probability of failure–free operation and the probability of system failure at time  $t=100000\,\mathrm{hours}$ .

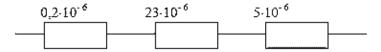


Figure 4.1 – Series connection of elements

Decision.

## The first method is:

To determine the failure rate we use the formula 3.4:

$$\lambda_{\rm C} = \sum_{i=1}^{n} \lambda_i = 0.2 \cdot 10^{-6} + 23 \cdot 10^{-6} + 5 \cdot 10^{-6} = 28.2 \cdot 10^{-6} \text{ hour}^{-1}.$$

The probability of failure–free operation under the exponential law of distribution of reliability indices:

$$P_{\rm C}(100000) = e^{-28.2 \cdot 10^{-6} \cdot 10^5} = e^{-2.82} = 0.0596$$

The probability of failure

$$Q_{\rm C}(100000) = 1 - P(100000) = 1 - 0.0596 = 0.9404$$

## The second method is:

We determine the probability of failure—free operation of each element

$$P_1(100000) = e^{-0.2 \cdot 10^{-6} \cdot 10^5} = e^{-0.02} = 0.9802$$

$$P_2(100000) = e^{-23 \cdot 10^{-6} \cdot 10^5} = e^{-2.3} = 0.1003$$

$$P_3(100000) = e^{-5 \cdot 10^{-6} \cdot 10^5} = e^{-0.5} = 0.6065$$

Using the formula of the theory of failure-free operation for the series connection of elements, we determine the probability of failure-free operation of the system

$$P_C(100000) = P_1(100000) \cdot P_2(100000) \cdot P_3(100000)$$
  
= 0,9802 \cdot 0,1003 \cdot 0,6065 = 0,0596

The conclusions we draw from the solution of the problem:

- 1. The more elements that make up a system with a series of elements, the higher the failure rate and therefore the lower the reliability of the system.
- 2. The total probability of failure of the system below the probability of failure of the most reliable element.

## **Task 4.2**

The bulbs are connected through an ammeter, the reliability of each respectively:

$$P_1 = 0.9$$
;  $P_2 = 0.85$ ;  $P_3 = 0.95$ .

Find the probability that there will be no current in the electrical circuit.

Decision.

$$Q = 1 - P_1 \cdot P_2 \cdot P_3 = 1 - 0.9 \cdot 0.85 \cdot 0.95 = 0.273.$$

## **Task 4.3**

The system is a parallel connection of elements in the structural diagram of reliability. The known failure rates of each element are known.

Determine the probability of failure–free operation and the probability of failure system time point t = 100000 hours.

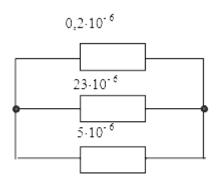


Figure 4.2 – Parallel connection of elements

Decision.

We determine the probabilities of failure-free operation of each element

$$P_1(100000) = e^{-0.2 \cdot 10^{-6} \cdot 10^5} = e^{-0.02} = 0.9802$$
  
 $P_2(100000) = e^{-23 \cdot 10^{-6} \cdot 10^5} = e^{-2.3} = 0.1003$   
 $P_3(100000) = e^{-5 \cdot 10^{-6} \cdot 10^5} = e^{-0.5} = 0.6065$ 

Determine the failure probabilities of the elements

$$Q_1(100000) = 1 - 0.9802 = 0.0198$$
  
 $Q_2(100000) = 1 - 0.1003 = 0.8997$   
 $Q_3(100000) = 1 - 0.6065 = 0.3935$ 

Determine the probability of failure of the system with the parallel connection of elements in the structural scheme of reliability:

$$Q_C(100000) = Q_1(100000) \cdot Q_2(100000) \cdot Q_3(100000)$$
  
= 0.0198 \cdot 0.8997 \cdot 0.3935 = 0.007

The probable probability of system failure

$$P_{\rm C}(100000) = 1 - Q_{\rm C}(100000) = 1 - 0.007 = 0.993$$

The failure rate of the system with parallel connected elements:

$$lnP_{C}(100000) = ln e^{-\lambda_{c} \cdot 10^{5}}$$

$$ln0,993 = -\lambda_{c} \cdot 10^{5}$$

$$\lambda_{c} = \frac{ln0,993}{-10^{5}} = \frac{0,007025}{10^{5}} = 0,07025 \cdot 10^{-6}$$

The conclusions we draw from the solution of the problem:

- 1. The more elements that make up a system with a parallel connection of elements, the lower the failure rate and, therefore, the higher the reliability of the system.
- 2. The total probability of failure of the system above the probability of failure of the most reliable element.

#### **Task 4.4**

The high-voltage substation is powered by three independent power supplies:

1) wind farms,

- 2) solar power plant
- 3) thermal power plant.

With reliability, respectively:

$$P_1 = 0.85$$
;  $P_2 = 0.8$ ;  $P_3 = 0.9$ .

Determine the probability of uninterrupted operation of the high-voltage substation.

Decision:

$$P = 1 - P_1 \cdot P_2 \cdot P_3 = 1 - (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) = 1 - (1 - 0.85) \cdot (1 - 0.8) \cdot (1 - 0.9) = 0.997.$$

## **Task 4.5**

In the wiring diagram there are m=2 parallel circuits with n=2 sequentially included elements. In this case, each element has the probability of trouble–free operation Pi=0,9.

Find the reliability of such a scheme.

Decision:

$$P_{\Sigma} = 1 - (1 - p^n)^m = 1 - (1 - 0.9^2)^2 = 1 - (1 - 0.81)^2 = 1 - 0.19^2 = 1 - 0.0361 = 0.9639$$

### **Task 4.6**

In the wiring diagram there are m=3 parallel circuits of n=3 sequentially included elements. In this case, each element has the probability of trouble–free operation Pi=0,9.

Find the probability of trouble–free operation of the scheme.

Decision:

$$P_{\Sigma} = 1 - (1 - 0.729)^3 = 1 - 0.271^3 = 1 - 0.0199 = 0.98009$$

## **Task 4.7**

In the wiring diagram, n = 2 groups are connected in series with m = 2 identical elements connected in parallel. In this case, each element has the probability of trouble–free operation  $\underline{P}_i = 0.9$ . Find the reliability of such a scheme.

Decision:

$$P_{\Sigma} = \{1 - (1 - p^m)\}^n = \{1 - (1 - 0.9^2)\}^2 = (1 - 0.01)^2 = 0.999^2 = 0.9801$$

### **Task 4.8**

In the wiring diagram, n = 3 groups are connected in series with m = 3 identical elements in parallel. In this case, each element has the probability of trouble–free operation Pi = 0.9. Find the probability of trouble–free operation of the scheme.

Decision:

$$P_{\Sigma} = \{1 - (1 - 0.9)^3\}^3 = (1 - 0.001)^3 = 0.999^3 = 0.99700$$

## **Task 4.9**

A structural diagram of the reliability of the mixed connection of system elements (parallel-serial). The probabilities of failure-free operation of the elements included in the system are known. To determine the probability of failure of the system:

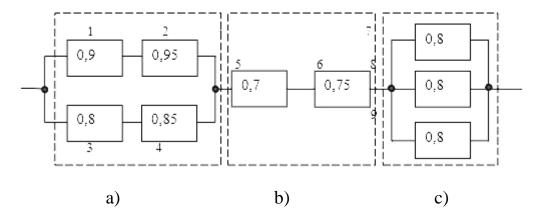


Figure 4.3 – Parallel–serial connection of elements

Decision:

It is necessary to reduce the complex structure of a mixed connection to a scheme that will contain only a serial connection of elements.

For this:

1. Determine the probability of failure–free operation of the serial connection of elements 1 and 2:

$$P_{12} = P_1 P_2 = 0.9 \cdot 0.95 = 0.855$$

2. Determine the probability of failure—free sequential connection of elements 3 and 4:

$$P_{34} = P_3 P_4 = 0.8 \cdot 0.85 = 0.68$$

3. The part of scheme a) is a parallel connection of elements 1, 2 and 3.4. Hence the probability of trouble–free operation of the scheme

$$P_{\alpha} = 1 - (1 - P_{12})(1 - P_{34}) = 1 - (1 - 0.855)(1 - 0.68) = 1 - 0.145 \cdot 0.32$$
  
= 1 - 0.0464 = 0.953

4. The structure of the plot of the scheme b) is a series connection of elements 5 and 6, then:

$$P_b = P_5 P_6 = 0.7 \cdot 0.75 = 0.525$$

The structure of the section of the scheme c) is a parallel connection of elements 7, 8 and 9. Given that the elements have the same reliability, then:

$$P_c = 1 - (1 - P_7)(1 - P_8)(1 - P_9) = 1 - (1 - 0.8)^3 = 1 - 0.2^3 = 1 - 0.008 = 0.992 \ .$$

We determine the probability of failure of the system, that is, the connection of serial elements:

$$P_{\Sigma} = P_a \cdot P_b \cdot P_c = 0.953 \cdot 0.525 \cdot 0.992 = 0.497.$$

# 5. METHODS OF CALCULATING THE RELIABILITY OF SYSTEMS WITH STRUCTURAL REDUNDANCY WITHOUT RECOVERING

Complex systems are split into serial–parallel systems; systems having the elements "triangle" and "star"; cross–link systems.

Consider the basic method of determining reliability indicators for the above—mentioned structures of complex systems, provided that the failure of the failed elements is not possible. Collapsing method. Consistently parallel structures of complex systems are very common.

For such structures, the coagulation method is an effective method. This method is based on the sequential transformation of the structure of the system and reducing it to the main combination of elements. Consider this method as an example of the scheme shown in the figure 5.1

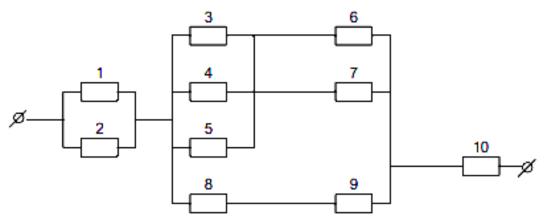


Figure 5.1 – Schematic of a complex system

The collapse method consists of several steps.

The first step looks at all the parallel connections that are replaced by equivalent elements with the corresponding reliability metrics.

In Scheme 5.1, the following parallel elements are: 1 and 2; 3, 4 and 5; 6 and 7. After the first stage, the converted scheme is as follows (Figure 5.2):

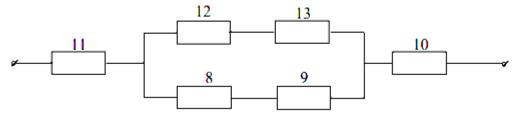


Figure 5.2 – Equivalent circuit after the first conversion stage

The reliability characteristics of the circuit elements are:

$$p_{11} = 1 - (1 - p_1)(1 - p_2);$$

$$p_{12} = 1 - (1 - p_3)(1 - p_4)(1 - p_5);$$

$$p_{13} = 1 - (1 - p_6)(1 - p_7).$$

Consider all consecutive elements that are replaced by equivalent ones. In the example, these are the elements: 8 and 9; 12 and 13. The scheme looks like 5.3:

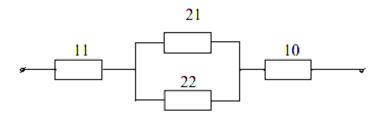


Figure 5.3 – Equivalent circuit after the second conversion step

Characteristics of element reliability after the second stage:

$$p_{21} = [1 - (1 - p_3)(1 - p_4)(1 - p_5)][1 - (1 - p_6)(1 - p_7)];$$
  
$$p_{22} = p_8 p_9$$

In the third stage, all parallel connections are replaced, which are replaced by equivalent ones. These are elements 21 and 22. The diagram will look like 5.4:

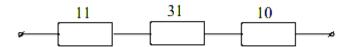


Figure 5.4 – Equivalent circuit after the third stage of conversion

We define the reliability parameters:

$$p_{31} = 1 - (1 - p_{21})(1 - p_{22}) =$$

$$= 1 - \{1 - [1 - (1 - p_3)(1 - p_4)(1 - p_5)[1 - (1 - p_6)(1 - p_7)]\}(1 - p_8 p_9).$$

As a result of all the transformations, we get a consistent connection of the elements. In the fourth stage for the sequential structure of the system we determine the probability of its uptime:

$$P_{\Sigma} = p_{11} p_{31} p_{10}$$

The clotting method is an effective method of determining the reliability of series—parallel structures of complex systems without recovery.

The number of elements has little effect on the complexity of calculations. The disadvantage of this method is to determine the limitation of parallel—serial schemes.

However, for most reliability and diagnostics tasks, this method and methods for determining the probability of failure–free operation in parallel, sequential, and parallel–sequential circuits are informative and allow the determination of reliability with high probability.

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Самостійне електронне видання