

Decision algorithm of near-field microwave sounding

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Abstract: The resonant near-field microwave sounding is an actively developing area of researches the main aim of which is a diagnosis in medicine for different pathologies in human tissues. The microwave sounding is associated with visualization of heterogeneous dielectric mediums on the basis of scan data of tissue in the microwave range. With the help of microwave sounding, it is possible to determine dielectric capacitivity and conductivity of tissues of different physical nature. These electrodynamic characteristics depend on the physical features of environment, its structural, physical and chemical composition and may be used for diagnostic purposes in the medicine. In theoretical researches the decision algorithm of bidimensional reverse problems of change of dielectric capacitivity inhomogeneities in biological environment based on the scanned data of the diffraction field in the near zone of heterogeneities was offered. The algorithm is based on integral representation of the diffraction field using the "equivalent sources" and method of Lavrentiev regularization.

Keywords: Microwave sounding, reverse problem, regularization, dielectric capacitivity of tissues.

1. Introduction

In medical applications the definition of complex dielectric capacitivity of biological tissues is the basic information during the diagnosis for different pathologic process in human tissues, in particular, skin and oncological diseases [1-4].

The microwave sounding is for the pathologies diagnosis at early stages of its progression. This is due to the fact that during a disease the water content of tissues is changed and this in turn leads to changes in its dielectric capacitivity.

Besides that, during the microwave sounding there is a possibility to research hotbeds of disease with flat contrast for classical methods of supersonic and roentgen soundings. Some aspects of the new approach to the study of bidimensional reverse problems of the resonant near-field microwave sounding are discussed in this work. This approach is based on integral representations of the diffraction field using the "equivalent sources" [5] and method of Lavrentiev regularization [6].

The resonant near-field microwave sounding is an actively developing area of researches the main aim of which is diagnosis and visualization of heterogeneous

dielectric mediums on the basis of scan data of tissue in the microwave range [1,2]. With the help of microwave sounding, it is possible to determine dielectric capacitivity and conductivity of tissues of different physical nature. These electrodynamic characteristics depend on the physical features of environment, its structural, physical and chemical composition and may be used for diagnostic purposes [5].

2. Research method

The absence of effective solution of interpretation problem of the parameters of microwave sounding during its interaction with inhomogeneities of biological environment is the major obstacles on the way of technological development of the microwave sounding. It is necessary to take into account the whole range of such effects as dispersion, absorption, reflection, diffraction and etc. in the case where characteristic dimensions of inhomogeneities of biological environment are comparable with the wave-length of applied microwave sounding for wave advance process description in the studied biological environment.

This greatly complicates the construction of effective solutions of relevant reverse problems on restoration of the structure of inhomogeneities distribution. Despite the already developed methods for reverse problems solution, there is a need to develop new methods for the analysis of internal structure of biological environments, which more fully take into account the effects of the interaction of the microwave sounding with inhomogeneities in biological environments.

3. Research results

The two-dimensional area D is considered as electrodynamic model of the biological environment inhomogeneity. It is expected that the dielectric capacitivity of environment filling of area D is the function of two spatial variables x and y $\varepsilon(x, y)$. Outside the area D the dielectric capacitivity of environment is constant ε_0 , and magnetic permittivity aligns with magnetic permittivity of vacuum. Let the source of the microwave radiation is situated at the point $r_m = (x_m, y_m)$ which is outside the area D . This source

creates monochromatic cylindrical electromagnetic wave with electric-field intensity $\vec{E} = (0, 0, U^i)$:

$$U^i(x, y) = -\frac{iU_0}{4} H_0^{(1)}(k|r - r_m|), \quad (1)$$

where $r = (x, y)$, $k = \omega\sqrt{\varepsilon}/c$, ω – circular frequency,
 c – speed of light in vacuum,

$H_0^{(1)}(\dots)$ – Hankel function of first kind.

The diffraction field with intensity $\vec{E}^s = (0, 0, U^s)$ results from interaction of sounding wave (1) with inhomogeneity D . As it follows from [7], this field satisfies the integral equation:

$$U^s(x, y) = k^2 \int_D G(x, y, \bar{x}, \bar{y}) h(\bar{x}, \bar{y}) U(\bar{x}, \bar{y}) d\bar{x} d\bar{y}. \quad (2)$$

Here:

$$G(x, y, \bar{x}, \bar{y}) = -\frac{i}{4} H_0^{(1)}\left(k\sqrt{(x - \bar{x})^2 + (y - \bar{y})^2}\right), \quad (3)$$

$$U = U^i + U^s, \quad h(\bar{x}, \bar{y}) = \varepsilon(\bar{x}, \bar{y})\varepsilon_0^{-1} - 1.$$

The problem of determining the electromagnetic properties of inhomogeneity D is as follows. Let D^s is a general area without inhomogeneity D and (k_1, k_2) is an arbitrary interval of frequency parameter $k = \omega\sqrt{\varepsilon_0}/c$ of sounding wave (1). It is necessary to determine the dielectric capacitivity $\varepsilon(x, y)$ of inhomogeneity D using the diffraction field $U^s(x, y)$ known (measured) for frequency parameter $k \in (k_1, k_2)$ and points $(x, y) \in D^s$.

The construction for reverse problems solution is based on the equation (2). It is easily seen that this non linear integral equation relative to the dielectric capacitivity $\varepsilon(x, y)$. As the diffraction field non linear depends on function $\varepsilon(x, y)$, that's why in some cases is it possible to linearize the equation (2). (For example if h (see (2)) is sufficiently small for $(x, y) \in D$.) This is so called Born approximation [8]. In this case the equation (2) comes to linear integral equation of first kind relative to function $h(x, y)$:

$$U_s|_{D^s} = k^2 \int_D G(x, y, \bar{x}, \bar{y}) h(\bar{x}, \bar{y}) U^i(\bar{x}, \bar{y}) d\bar{x} d\bar{y}. \quad (4)$$

The regularization methods of ill-defined problem are used for the numerical solution of this equation.

In the general case when the integral operator defined in the right part (2) is non linear, the different iterative algorithms were offered based on the classical method of Newton – Kantorovich [9], which allows to linearize the connection between the desired dielectric capacitivity of inhomogeneity and its diffraction characteristics as a source of data. The effectiveness of these algorithms build the solution of the reverse problem is determined by the following factors:

- knowledge of a good initial approximation (there is no universal recommendation for its selection), available analytical or numerical ways for calculating of Fresnel non linear integral operator;
- performance characteristics of algorithms for constructing solutions of direct problems of diffraction (guaranteed calculation accuracy, speed, etc.).

Without going into details, we note that using these variants of linearization a sufficiently broad range of relevant, practically important reverse problems of remote sounding and nondestructive testing can be considered. However, for medical applications, when it is not enough information about the initial approximation (dielectric capacitivity of inhomogeneity of the biological environment) these algorithms, apparently, unsuitable.

The following results for new inversion algorithm does not require information of the first approximation.

Inversion algorithm. As previously noted, the base for construction of solution for considered reverse problem is the system of non linear integral equations:

$$U_s|_{D^s} = k^2 \int_D G(x, y, \bar{x}, \bar{y}) n(\bar{x}, \bar{y}) U(\bar{x}, \bar{y}) d\bar{x} d\bar{y}, \quad (5)$$

$$U = U^i + k^2 \int_D G(x, y, \bar{x}, \bar{y}) n(\bar{x}, \bar{y}) U(\bar{x}, \bar{y}) d\bar{x} d\bar{y}, \quad (x, y) \in D. \quad (6)$$

In this equations the unknown quantities are functions $U(x, y)$ and $n(x, y)$, and known quantities (basic data) is sounding wave U^i and diffraction field U^s in defined area D^s .

Specify the position of the reverse problem. Let in the area D^s set M points $r_m = (x_m, y_m)$, $m = 1, \dots, M$, and Q frequency parameters $k = \bar{k}_q, q = 1, \dots, Q$ in interval $\bar{k}_q \in (k_1, k_2)$. Let $U_m^{sq} = U^s(x_m, y_m)$ denote as value of the diffraction field obtained in the result of measurements with location of source of the sounding wave (1) at the point $r_m = (x_m, y_m)$ frequency parameters $k = \bar{k}_q$.

These values of the diffraction field are used as basic data for solution of the reverse problem.

The first step of the algorithm construction consists of finding of "equivalent sources" with equation (5). Derive the function of "equivalent sources" by formula:

$$W(x, y) = n(x, y)U(x, y). \quad (7)$$

Here $U(x, y) = U^i(x, y) + U^s(x, y)$ – full field on the area of inhomogeneity, $n(x, y) = \varepsilon(x, y)\varepsilon_0^{-1} - 1$. It is obvious that $W(x, y)$ – finite function, equal to zero outside the area of inhomogeneity.

Assume that the area of inhomogeneity D is in rectangle $[a, b] \times [c, d]$. Split this rectangle to squares of side h and centers at points $\bar{r}_n = (\bar{x}_n, \bar{y}_n), n = 1, 2, \dots, N$. Let h is so small, that function $W(x, y)$ in rectangle with centers at points $\bar{r}_n = (x_n, y_n)$ takes constant value equal to $W(x, y) = W(\bar{x}_n, \bar{y}_n) = W_n$. The integral in (5) can be approximately represented as follows

$$\int_D G(x_m, y_m, \bar{x}, \bar{y}) W(\bar{x}, \bar{y}) d\bar{x} d\bar{y} \cong \sum_{n=1}^N G_{mn} W_n. \quad (8)$$

We can show that if the function $W(x, y)$ is continuously differentiable then the formula error (8) has order h^2 . The values G_{mn} are calculated in an explicit form

$$G_{mn} = \frac{ik\bar{a}\pi}{2} \begin{cases} H_0^{(1)}(k|r_m - \bar{r}_n|) J_1(k\bar{a}), r_m \neq \bar{r}_n, \\ H_1^{(1)}(k\bar{a}) + \frac{2i}{k\bar{a}\pi}, r_m = \bar{r}_n, \end{cases} \quad (9)$$

where $\bar{a} = \frac{h}{\sqrt{\pi}}$, $H_0^{(1)}(\dots), H_1^{(1)}(\dots)$ – a Hankel cylindrical function of first kind. Substitute (8) and (5) we have

$$U_m^{sq} = \sum_{n=1}^N G_{mn}^q W_n, \quad m = 1, \dots, M, \quad q = 1, \dots, Q. \quad (10)$$

The index q denotes, that values G_{mn} are calculated with the frequency parameter $k = \bar{k}_q$.

From the equation (10) we can denote the function value of "equivalent source" at points (\bar{x}_n, \bar{y}_n) in the area of inhomogeneity D . Lets use method of Lavrentiev regularization [6].

Assume (10) in matrix form:

$$U^{sq} = G_q W. \quad (11)$$

Here

$$U^{sq} = (U_1^{sq}, \dots, U_M^{sq})^T, W = (W_1, \dots, W_N)^T, G_q = \left\| G_{mn}^q \right\|_{\substack{m=1, \dots, M \\ n=1, \dots, N}}$$

, index T denotes an operation of transposition.

In general case matrix G_q is rectangle with $M < N$. That's why equation system (11) is no defined (the number of equation is smaller than dimensionality). The method of regularization [6] is applied for its solution. Derive the new unknown column-vector by formula:

$$W = G_q^* V, \quad (12)$$

where $V = (V_1, \dots, V_M)^T, G_q^*$ – is conjugate matrix G_q .

Substitute (12) in (11) we will have:

$$U^{sq} = G_q G_q^* V. \quad (13)$$

It is easy to see $G_q G_q^*$ – square matrix size $M \times M$. In accordance with (5), together with equation (13), we view the equation:

$$U^{sq} = G_q G_q^* V + \alpha V, \quad (14)$$

where $\alpha > 0$.

For every $\alpha > 0$, equation (14) has unique solution:

$$V_\alpha = (\alpha I + G_q G_q^*)^{-1} U^{sq}. \quad (15)$$

Besides that as shown in (5), with $\alpha \rightarrow 0$ such that $\delta/\alpha \rightarrow 0$:

$$\|V - V_\alpha\|^2 = \sum_{q=1}^Q |V_q - V_{\alpha q}|^2 \rightarrow 0.$$

Value δ is determined by measurement errors of the diffraction field U_m^{sq} , and V – normal solution of equation (13).

So, the function of "equivalent source" with (15) and (12) may be represented as follows:

$$W = G_q^* (\alpha I + G_q G_q^*)^{-1} U^{sq}. \quad (16)$$

The parameter value α (regularization parameter) must be consistent with measurement errors δ of the diffraction field. We can do it on basis of principle of the residual ([10]:

$$\|U^{sq} - G_q G_q^* V_\alpha\| = \delta. \quad (17)$$

The value α is determined from the equation (17) for this error δ .

The next step is to calculate the full field in the area of inhomogeneity D . We will use the equation (6) and formula (16) for the function of "equivalent source". From (6) we have:

$$U_n^q = U_n^{iq} + \sum_{p=1}^N G_{np}^q W_p^q, \quad (18)$$

where $U_n^q = U(\bar{x}_n, \bar{y}_n)$, $U_n^{iq} = U^i(\bar{x}_n, \bar{y}_n)$ - values of the full field and the sounding wave at the points $\bar{r}_n = (\bar{x}_n, \bar{y}_n)$, $W_n^q = W(\bar{x}_n, \bar{y}_n)$. The index q determines that all values are calculated for frequency parameter $k = \bar{k}_q$, $q = 1, \dots, Q$.

So, the formulae (16) and (18) allow to calculate the function of "equivalent source" and the full field in the area of inhomogeneity using basic data (diffraction field and sounding wave). Let's see the reconstruction algorithm of the dielectric capacitivity $\varepsilon(x, y)$ in the area of inhomogeneity

Let $\varepsilon_n = \varepsilon(\bar{x}_n, \bar{y}_n)$, $n = 1, \dots, N$ values of unknown dielectric capacitivity are at the point $\bar{r}_n = (\bar{x}_n, \bar{y}_n)$. So the function of "equivalent source" and the full field are known at this points, derive the quadrature functional:

$$\Phi(\varepsilon_n) = \sum_{m=1}^M \sum_{q=1}^Q \left| W_{nm}^q - (\varepsilon_n \varepsilon_0^{-1} - 1) U_{nm}^q \right|^2. \quad (19)$$

Here W_{nm}^q и U_{nm}^q - values of the function of "equivalent source" and the full field calculated at the point $\bar{r}_n = (\bar{x}_n, \bar{y}_n)$ with location of the source of sounding wave at the point $\bar{r}_m = (\bar{x}_m, \bar{y}_m)$ and frequency parameter $k = \bar{k}_q$.

Determine values ε_n , which give minimum to functional (19). For this is enough to find the solution of equation:

$$d\Phi/d\varepsilon_n = 0 \quad (20)$$

From (20) with (19), after transformations we have:

$$\varepsilon_n = \varepsilon_0 + \frac{\sum_{m=1}^M \sum_{q=1}^Q (U_{nm}^q)^* W_{nm}^q}{\sum_{m=1}^M \sum_{q=1}^Q |U_{nm}^q|^2}, \quad (21)$$

where * is complex conjugation operation.

So, formula (21) give the solution of reverse problem of the reconstruction of dielectric capacitivity according to the data of the microwave sounding.

4. Conclusion

The analysis of problems in medicine shows that the microwave sounding is necessary for early diagnosis of hotbeds of diseases in human tissues (for example, oncological diseases). Due to its delicacy the microwave sounding can surpass the classic sounding methods of hotbeds of tissue diseases using supersonic and roentgen. This is due to the fact that water content is changed in hotbeds of tissue diseases and human skin.

Finally we can make 2 main conclusions from our research:

1. It is necessary to use the obtained results in the performance of bidimensional reverse problems with heterogeneous dielectric mediums for development of the method of microwave sounding of hotbeds of diseases in human tissues.
2. It is necessary to use the developed algorithm for the reconstruction of dielectric capacitivity of tissues on the measured values of the diffraction field near inhomogeneities.

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