Exact electronic Green functions in a Luttinger liquid with long-range interactions

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We compute the two-point (equal-time) electronic Green function in a Tomonaga-Luttinger system with long-range electron-electron interactions. We obtain an analytical expression for a ''super long-range'' potential of the form $V(x)=e^2d^{-\epsilon}/|x|^{1-\epsilon}$. As a consistency check of our computational technique we also consider the particular case of a Coulomb potential. Our result confirms the $\exp-C(\log x)^{3/2}$ long-distance behavior first obtained by Schulz.

Models of one-dimensional (1D) interacting electrons are relevant as testing grounds for new ideas and can help us to understand systems in higher dimensions. In particular they are useful to describe the behavior of strongly anisotropic physical systems in condensed matter, such as organic conductors,[1] charge transfer salts,[2] and quantum wires.[3] Probably the two most widely studied 1D systems are the Hubbard model[4] and the so-called ''g-ology'' model.[5] They are known to display the Luttinger liquid[6] behavior characterized by spin-charge separation and by nonuniversal (interaction dependent) power-law correlation functions. In these models one usually considers short-range electron-electron interactions. This picture works well for conductors in which the screening between adjacent chains reduces the range of interactions within one chain.[7] On the other hand, as the dimensionality of the system decreases, charge screening effects are expected to become less important and the long-range interaction between electrons seems to play a central role in determining the properties of the system. This assertion seems to be confirmed by experiments in GaAs quantum wires[3] and quasi-1D conductors.[8] From a theoretical point of view the effects of long-range interactions have been also recently discussed in connection to a variety of problems such as the Fermi edge singularity,[9] the insulator-metal transition[10] and the role of the lattice through umklapp scattering and size dependent effects.[11]

The effect of Coulomb forces on the single-particle Green function and on charge-density correlations in 1D systems have been previously investigated in a pioneering work by Schulz[12] using the conventional bosonization method.[13] The purpose of the present paper is to compute the electronic Green function in the presence of long-range interactions using an alternative path-integral bosonization technique previously developed in the context of quantum-field theories (QFT's).[14] We consider a nonlocal version of the Thirring model[15] described by the following (Euclidean) action

$$S=\int d^2x\,\bar{\Psi}i\not{\partial}\Psi-\int d^2x\,d^2y[V_{(0)}(x,y)J_0(x)J_0(y)+V_{(1)}(x,y)J_1(x)J_1(y)], \tag{1}$$

where $x=(\mathbf{x},\tau_x)$, and $J_0=\bar{\Psi}\gamma_0\Psi=\rho$, and $J_1=\bar{\Psi}\gamma_1\Psi=j$ are the charge-density and current-density operators, respectively. Let us also mention that, for simplicity, we shall take $v_F=1$ and $p_F=0$ throughout this article. The functions $V_{(\mu)}(x,y)$ are forward-scattering potentials. Setting $V_{(0)}=V_{(1)}=(g^2/2)\delta^2(x-y)$ one gets the covariant and local version of the Thirring model usually studied in the context of $(1+1)$ QFTs. On the other hand, the choice

$$V_{(0)}(x,y)=V(|\mathbf{x}-\mathbf{y}|)\,\delta(\tau_x-\tau_y),$$
$$V_{(1)}(x,y)=0, \tag{2}$$

yields the simplest version of the Tomonaga-Luttinger (TL) model with an instantaneous distance dependent potential and no current-current fluctuations. The main purpose of the present paper is to study this case for a long-range electron-electron potential, *different from the Coulomb interaction*, of the form

$$V(|\mathbf{x}-\mathbf{y}|)=\frac{e^2d^{-\epsilon}}{|\mathbf{x}-\mathbf{y}|^{1-\epsilon}}\,\delta(\tau_x-\tau_y), \tag{3}$$

where d is a length scale and ϵ is a small real number ($0<\epsilon\leqslant 1$). This interaction has been previously analyzed by using renormalization group techniques in order to explore the possibility of having Luttinger liquid behavior in more than one dimension[16] and in connection to the vacuum structure of the nonlocal Thirring model.[17]

Before considering the ''super long-range'' potential described above, we shall sketch our functional approach to bosonization in a general case. As shown in Ref. 15 the partition function of the model defined by Eq. (1) can be solved, in the path-integral framework, by means of a chiral change of fermionic variables. This procedure allows to obtain a bosonized effective action in terms of scalar fields that are naturally identified with the collective modes (plasmons) of the system. Since this method has been described many times in the literature, here we shall skip the details (the interested reader is referred to Refs. 14 and 15 and references therein). Let us only say that using a Hubbard-Stratonovich-like identity, which amounts to introducing an auxiliary vector field A_μ, the partition function of the general model given by Eq. (1) can be written in terms of a functional fermionic determinant as

$$Z=\int \mathcal{D}A_\mu\,\det(i\not{\partial}+g\not{A})e^{-S[A]}, \tag{4}$$

with

$$S[A]=\frac{1}{2}\int d^2x\, d^2y\, V^{-1}_{(\mu)}(x,y)A_\mu(x)A_\mu(y), \tag{5}$$

where $V^{-1}_{(\mu)}$ is such that

$$\int d^2z\, V^{-1}_{(\mu)}(z,x)V_{(\mu)}(y,z)=\delta^2(x-y). \tag{6}$$

Decomposing $A_\mu(x)$ in longitudinal and transverse pieces

$$A_\mu(x)=\epsilon_{\mu\nu}\partial_\nu\phi(x)+\partial_\mu\eta(x), \tag{7}$$

where ϕ and η are boson fields (to be associated to the normal modes of the system) and applying, as anticipated, functional bosonization techniques to express the fermionic determinant in terms of ϕ and η, one finally obtains

$$Z=N\int \mathcal{D}\phi\,\mathcal{D}\eta\, e^{-S_{\rm bos}}, \tag{8}$$

where N is a normalization constant and $S_{\rm bos}$, already written in momentum space, reads

$$S_{\rm bos}=\frac{1}{(2\pi)^2}\int d^2p[\hat{\phi}(p)\hat{\phi}(-p)A(p)+\hat{\eta}(p)\hat{\eta}(-p)B(p) + \hat{\phi}(p)\hat{\eta}(-p)C(p)] \tag{9}$$

with

$$A(p)=\frac{1}{\pi}p^2+\frac{1}{2}[\hat{V}^{-1}_{(0)}(p)p_1^2+\hat{V}^{-1}_{(1)}(p)p_0^2], \tag{10}$$

$$B(p)=\tfrac{1}{2}[\hat{V}^{-1}_{(0)}(p)p_0^2+\hat{V}^{-1}_{(1)}(p)p_1^2], \tag{11}$$

$$C(p)=[\hat{V}^{-1}_{(0)}(p)-\hat{V}^{-1}_{(1)}(p)]p_0p_1, \tag{12}$$

and

$$\Delta=C^2(p)-4A(p)B(p). \tag{13}$$

Let us now focus our attention on the one-particle fermionic propagator

$$\langle\Psi(x)\overline{\Psi}(y)\rangle=\begin{pmatrix}0 & G_R(x,y)\\ G_L(x,y) & 0\end{pmatrix}. \tag{14}$$

The above depicted bosonization scheme can be easily employed to evaluate this expression. First of all it is straightforward to verify that the nonzero components of the Green function factorize as

$$G_{R,L}(x,y)=G^{(0)}_{R,L}(x,y)B_{R,L}(x,y), \tag{15}$$

where $G^{(0)}_{R,L}(z)=1/2\pi(\tau_z^2+z^2)(\tau_z\pm iz)$ describe free right and left propagation, and

$$B_{R,L}(x,y)=\frac{\int D\hat{\Phi}\,D\hat{\eta}\,e^{-[S_{\rm bos}+S_{R,L}(x,y)]}}{\int D\hat{\Phi}\,D\hat{\eta}\,e^{-S_{\rm bos}}}. \tag{16}$$

In this equation one has defined

$$S_{R,L}(x,y)=-\frac{1}{(\pi)^2}\int d^2p[\pm\hat{\phi}(p)+i\hat{\eta}(p)] \times(e^{-ip\cdot x}-e^{-ip\cdot y}). \tag{17}$$

At this point we notice that the evaluation of $B_{R,L}(x,y)$ can be carried out by means of a convenient shift of the fields $\hat{\phi}(p)$ and $\hat{\eta}(p)$. This standard procedure gives

$$B_{R,L}(x,y)=\exp[I_{R,L}(x,y)], \tag{18}$$

where $I_{R,L}$ is a functional of the potentials given by

$$I_{R,L}(x,y)=-\frac{1}{\pi^2}\int d^2p\,\sin^2\left[\frac{1}{2}p(x-y)\right]\times\frac{(2/\pi)\hat{V}_{(0)}(p)\hat{V}_{(1)}(p)p^2+[\hat{V}_{(0)}(p)-\hat{V}_{(1)}(p)](p_0\mp ip_1)^2}{(2/\pi)p^2[\hat{V}_{(1)}(p)p_0^2+\hat{V}_{(0)}(p)p_1^2]+p^4}. \tag{19}$$

This result gives a very general expression for the two-point electronic correlator as functional of density-density $[\hat{V}_{(0)}(p)]$ and current-current $[\hat{V}_{(1)}(p)]$ interaction potentials. It can be used as an alternative route to analyze the effect of long-range interactions on a Luttinger system. Of course, if one considers the local and covariant case $\hat{V}_{(0)}(p)=\hat{V}_{(1)}(p)=g^2/2$ one easily obtains the well-known Thirring behavior:

$$B^{\rm Thirring}_{R,L}(x-y)\propto|x-y|^{-(1/2)(g^2/\pi)^2/(1+g^2/\pi)}. \tag{20}$$

Let us now proceed with our main task and specialize formula (19) for the TL model defined by Eq. (2). Having $\hat{V}_{(1)}(p)=0$ and $\hat{V}_{(0)}(p)=\hat{V}_{(0)}(p_1)$ greatly simplifies the integrand in Eq. (19) which allows us to perform the integral in p_0. Moreover, since we are concerned with the equal-time correlator, we take $\tau_x=\tau_y$. In this case there is no distinction between I_R and I_L and one gets

$$I_{R,L}(|\mathbf{x}-\mathbf{y}|,\tau_x=\tau_y) =I(|\mathbf{x}-\mathbf{y}|)=-2\int_0^\infty\frac{d\mathbf{p}}{\mathbf{p}}\left[\frac{\frac{1}{\pi}\hat{V}(\mathbf{p})+1}{2\sqrt{\frac{2}{\pi}\hat{V}(\mathbf{p})+1}}-\frac{1}{2}\right] \times[1-\cos(|\mathbf{x}-\mathbf{y}|\cdot\mathbf{p})], \tag{21}$$

where we have used $p_1=\mathbf{p}$. It is convenient to check the validity of this equation by considering a nontrivial potential such as the Coulomb interaction

$$V(|\mathbf{x}-\mathbf{y}|)=\frac{e^2}{\sqrt{|x-y|^2+d^2}} \tag{22}$$

with Fourier transform given by

$$\hat{V}(\mathbf{p})=2e^2K_0(d\mathbf{p}), \tag{23}$$

where d is a very small length scale. This computation is interesting for two reasons. First, one should stress that Eq. (19) has been checked only for local (short-range) potentials: the covariant Thirring model [see Eq. (20)] and the short range TL model.[15] On the other hand, the electronic correlator for a Coulomb potential has been computed by Schulz[12] using standard (operational) bosonization[13] and thus our computation will give an independent confirmation by means of a different approach. To achieve this goal we consider a large distance approximation of Eq. (19) that consists of inserting $|\mathbf{x}-\mathbf{y}|^{-1}$ in the lower limit of Eq. (21) and disregarding the cosine term, which is valid for $1/d \gg 1/|\mathbf{x}-\mathbf{y}|$, we obtain

$$G_{R,L}(\mathbf{x}-\mathbf{y})=\pm iC(\Lambda)\,\mathrm{sign}(\mathbf{x}-\mathbf{y})\exp\left[-\alpha\left(\log\frac{|\mathbf{x}-\mathbf{y}|}{\beta d}\right)^{3/2}\right], \tag{24}$$

where α and β are two constants that depend on e^2 and $C(\Lambda)$ depends on an ultraviolet cutoff Λ. This is our first nontrivial contribution. We have obtained, as anticipated, the behavior previously found by Schulz.[12]

Now we undertake the computation of $G_{R,L}(\mathbf{x}-\mathbf{y})$ for the so-called ''super long-range'' potential given by Eq. (3), whose Fourier transform is

$$\hat{V}(\mathbf{p})=2e^2\cos\left(\frac{\pi}{2}\epsilon\right)\Gamma(\epsilon)(d|\mathbf{p}|)^{-\epsilon}. \tag{25}$$

Inserting this expression in Eq. (21) one gets

$$I(\xi)=-\int_\lambda^\infty\frac{d|\mathbf{q}|}{|\mathbf{q}|}\left[\frac{|\mathbf{q}|^{-\epsilon}+2}{2\sqrt{|\mathbf{q}|^{-\epsilon}+1}}-1\right][1-\cos(\xi|\mathbf{q}|)], \tag{26}$$

where it has been natural to define the new variable

$$\xi=\left[\frac{4}{\pi}e^2\cos\left(\frac{\pi}{2}\epsilon\right)\Gamma(\epsilon)\right]^{1/\epsilon}\frac{\mathbf{x}-\mathbf{y}}{d}. \tag{27}$$

Note that we have also introduced a small quantity λ to be set equal to zero at the end of our computation. Although $I(\xi)$ is infrared convergent, it is convenient to split it into two pieces: $I(\xi)=I_1(\xi)-I_1(\xi=0)$ with

$$I_1(\xi)=\frac{1}{2}\int_\lambda^\infty\frac{d|\mathbf{q}|}{|\mathbf{q}|}\left[\frac{|\mathbf{q}|^{-\epsilon}+2}{\sqrt{|\mathbf{q}|^{-\epsilon}+1}}-2\right]\cos(\xi|\mathbf{q}|). \tag{28}$$

The first ξ-independent piece can be very easily computed yielding

$$I_1(\xi=0)=-\frac{1}{\epsilon}\lambda^{-\epsilon/2}-\log\lambda-\frac{1}{\epsilon}(\log 4-1)+O(\lambda^{\epsilon/2}). \tag{29}$$

The evaluation of $I_1(\xi)$ is more involved, as expected. It can be achieved by splitting the integration region into two subintervals $(\lambda, 1)$ and $(1, \infty)$, then performing series expansions of the corresponding square root factors, and finally integrating term by term. Putting all this together and taking the limit $\lambda\to 0$ one finally obtains

$$\begin{aligned}I(\xi)=&\frac{1}{4}\sum_{n=0}^{\infty}(2a_n+a_{n+1})\frac{1}{\mu_n}[\Phi(\mu_n,\mu_n+1;i|\xi|)\\&+\Phi(\mu_n,\mu_n+1;-i|\xi|)]+\frac{1}{4}\sum_{n=0}^{\infty}(a_n\\&+2a_{n+1})|\xi|^{\nu_n}[e^{i\pi/2\nu_n}\Gamma(-\nu_n,i|\xi|)+e^{-i\pi/2\nu_n}\Gamma\\&\times(-\nu_n,-i|\xi|)]-\frac{1}{4}|\xi|^{\epsilon/2}\left[e^{-i\pi/4\epsilon}\Gamma\left(-\frac{\epsilon}{2},-i\left|\xi\right|\right)\right.\\&\left.+e^{i\pi/4\epsilon}\Gamma\left(-\frac{\epsilon}{2},i\left|\xi\right|\right)\right]-Ci(|\xi|)+\frac{1}{2}|\xi|^{\epsilon/2}\Gamma\\&\times\left(-\frac{\epsilon}{2}\right)\cos\left(\frac{\pi}{2}\epsilon\right)+\log|\xi|+\left(C+\frac{1}{\epsilon}-\frac{1}{\epsilon}\log 4\right),\end{aligned} \tag{30}$$

where $\Phi(a,b,z)$ is Kummer's confluent hypergeometric function, $\Gamma(\alpha,x)$ is the incomplete Gamma function, and $Ci(z)$ is the cosine integral function. We have also defined the coefficients $\mu_n=(n+\frac{1}{2})\epsilon$, $\nu_n=(n+1)\epsilon$, and $a_n=(-1)^n\Gamma(n+1/2)/\Gamma(1/2)\Gamma(n+1)$. Formula (30) is our main result. Indeed, by recalling that

$$G_{R,L}(\xi)=G^0_{R,L}(\xi)\exp I(\xi), \tag{31}$$

one sees that Eq. (30) gives a complicated but analytical and exact result for the fermionic propagator of the TL model in the presence of the long-range interaction given by Eq. (3).

Of course, it is now interesting to study the long-distance behavior of our solution ($|\xi|\gg 1$). A careful evaluation of the dominant contributions to Eq. (30) for this case yields

$$G_{R,L}(\xi)\propto\pm i\ \ \mathrm{sign}(\xi)\exp\left[\frac{1}{2}\Gamma\left(-\frac{\epsilon}{2}\right)\cos\left(\frac{\pi}{4}\epsilon\right)\left|\xi\right|^{\epsilon/2}\right]. \tag{32}$$

Note that, exactly as it happens with Schulz's solution, the above function decays faster that any power law. Moreover, the larger the range of the potential (larger ϵ) the faster is the Green function decay. Let us also mention that the definition of the long-distance regime is different in both cases [see Eq. (27)], since the quantity ϵ determines the range of the potential. This does not prevent us from comparing both long-distance decays for definite values of ϵ. A simple numerical comparison allows us to verify that the results are increasingly similar when distances increase.

It is also interesting to notice that the long-distance regime can be obtained following a much simpler route, analogous to the one employed to get this regime for the Coulomb case [see the paragraph preceding Eq. (24)]. However, we must mention that the condition that allows to drop the oscillating integrals leads, in the present case, to a result that is

only valid for small ϵ. Under this condition, replacing Eq. (3) in Eq. (21) and disregarding the cosine term gives

$$G_{R,L}(\xi) \propto \pm i \ \ \mathrm{sign}(\xi) \exp\left[-\frac{1}{\epsilon} \left| \xi \right|^{\epsilon/2} \right], \tag{33}$$

which coincides with Eq. (32) for $\epsilon \ll 1$. We want to stress here that this agreement is an important consistency check of our exact result since it has been derived in a very straightforward way, *without using Eq. (30)*.

In summary, we have shown how to compute, through path-integral methods, the equal-time fermionic one-particle Green function in a simple version of the TL model. In particular we have obtained a rather involved but exact analytical expression for this propagator in the case of a ''super long-range'' potential [see Eq. (3)]. This result is indeed of academic interest, since most of the explicit results found in the literature actually correspond to long-distance regimes (see, for instance Ref. 12). In passing, and as a consistency check of our computation, we have given a path-integral derivation of the Green function corresponding to the Coulomb case, a well-known result previously found by Schulz.

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