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Dynamic Adverse Selection and the Supply Size

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Abstract

In this paper we examine the problem of dynamic adverse selection in a stylized market where the quality of goods is a seller's private information while the realized distribution of qualities is public information. We show that in equilibrium all goods can be traded if the size of the supply is publicly available to market participants. Moreover, we show that if exchanges can take place frequently enough, then agents roughly enjoy the entire potential surplus from exchanges. We illustrate these findings with a dynamic model of trade where buyers and sellers repeatedly interact over time. We also identify circumstances under which only full trade equilibria exist. Further, we give conditions for full trade to obtain when the realized distribution of qualities is not public information and when new goods enter the market at later stages.

JEL classification code: D82, L15.

Keywords: dynamic adverse selection; supply size; frequency of exchanges; asymmetric information.

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1 Introduction

Since the publication of the seminal work by Akerlof (1970), the problem of adverse selection has been widely investigated by economic theorists. One quite recent development in this regard is the exploration of the dynamics of exchanges under asymmetric information, and in particular of the phenomenon of dynamic adverse selection (see e.g., Hendel and Lizzeri, 1999; Janssen and Roy, 2002; Hendel et al., 2005; Moreno and Wooders, 2010).[1] Although several important aspects of dynamic adverse selection have been investigated, so far no attention has been given to the consequences of the public access to information about the supply size, e.g., the number of goods or services still on the market. In the present paper we explore this case, identifying the potential benefits accruing from the public availability of such a piece of information. We do this under the assumption of public knowledge of the realized distribution of qualities. In section 6 we show that this assumption can be relaxed and to what extent.

In a market where trade can take place sequentially, the public access to the supply size can solve dynamic adverse selection problems. The mechanism behind this result is actually very simple and can be explained with a short example. Suppose that the supply side is constituted by two sellers, one who wants to sell one unit of a high quality good and the other who wants to sell one unit of a low quality good. Qualities are private information of the sellers but it is public knowledge that there is one high quality good and one low quality good. The supply size tells agents how many goods – but not which qualities – are still circulating in the market. Upon arrival at the market, buyers are told that two goods are being supplied. On the first day of the market, some buyers start offering a low price, certain that for a low price they can only buy the low quality good. This choice is reasonable since offering a high price would lead to expected losses because of information asymmetries.

[1] A clear and formal account of dynamic adverse selection can be found in Bolton and Dewatripont (2005, Ch. 9) where the consequences of multi-stage contracting are analyzed.

Given the low price, the seller of the high quality good will not accept as for her accepting would mean a certain loss. What about the seller of the low quality good? She could opt to wait in the hope of higher prices in the following days. However, if she also rejects the offer, then the supply size does not change – two goods still circulate – and, hence, the following days buyers would face the same situation so that they would have no reasons to change their offer. Therefore, the seller of the low quality good finds it optimal to accept the low price offer, as the alternative would be indefinite waiting. As a result, the low quality good is sold and the supply size side shrinks by one unit. The second day of the market, upon disclosure of the fact that the supply size has diminished by one unit, buyers still on the market find it reasonable to offer a high price as they know that the day before there were one high quality good and one low quality good, and that the previous low price offer could be reasonably accepted only by the low quality seller. Finally, since there is no good reason to expect a higher offer in the future, the high quality seller accepts the high price offer and the market clears.

We note that the knowledge of the supply size allows to infer the exact quality of goods that remain unsold only if the realized initial distribution of qualities is common knowledge. This idea is reminiscent of the mechanism underlying the so-called "pac-man" conjecture for durable goods monopoly (Bagnoli et al., 1989) which has been opposed to the Coase conjecture, giving rise to a lively discussion on what exactly is the most reasonable prediction in durable goods markets with a monopolist (see von der Fehr and Kühn, 1995; Cason and Sharma, 2001). In short, the pac-man conjecture says that when consumers are not individually negligible, then the monopolist can make them pay their reservation prices (possibly discounted, depending on the discount factor and the distribution of reservation prices), i.e., the monopolist can discriminate prices and eat a bit of consumers' surplus in each trading stage. The mechanism underlying the pac-man conjecture is similar to the one presented here in that the monopolist can condition his price offers on the number of consumers still on the market. This allows him to induce consumers with high reservation prices to buy in the first periods at a higher price. Relevant similarities however end here since, differently from the model in Bagnoli et al. (1989), we focus on adverse selection in non-monopolistic markets, and hence we consider the case of many buyers and asymmetric information about qualities. In particular, in our model buyers know the initial distribution of qualities brought to the market by sellers, but cannot say which seller has what quality.

We remark that the knowledge of the *supply* size is relevant in our adverse selection model because sellers are the ones who are privately informed. If it is buyers to be privately informed (e.g., buyers' willingness to pay is private information while quality is observable, as in Bagnoli et al., 1989) then the relevant piece of information is not the supply size but the *demand* size. This is so because the piece of information that is relevant for mitigating the negative effects of asymmetric information is, in general, the size of the *informed side* of the market.

The fact that the public knowledge of the supply size is a potential solution to dynamic adverse selection problems is a theoretical finding which, we think, is interesting in itself, while its practical relevance depends much on the possibility that market participants have access to the necessary information. At least in some circumstances, the supply size can be quite a simple piece of information to retrieve, e.g., when the market is small and all participants can directly observe goods. However, not many real markets match this case, not even roughly. Large markets are often costly to monitor. Moreover, the anonymity of market participants makes it hard to keep track of the number of goods circulating at any given time. In fact, the supply size is not an individual information, but an intrinsically global one. This last consideration suggests an interesting possibility. Even if the supply size is not naturally available to market participants, it can be available to market authorities or external observers. Hence, such kind of information is a natural candidate for public disclosure by a dedicated authority, suggesting that our findings have perhaps a normative insight.

Another important issue is whether the public knowledge of the supply size is sufficient to let agents enjoy the whole potential surplus from exchanges, as would happen in the absence of asymmetric information on the quality of goods. This desirable outcome in general is not guaranteed because sequential trade may take a substantial amount of time and, hence, agents might be forced to wait long periods before enjoying their payoff which, reasonably, would be discounted accordingly. However, since the amount of time waited before exchanges take place has no special role in ensuring full trade, there is one straightforward way to increase total surplus: shortening the time between exchange opportunities. This entails that, if exchange opportunities are frequent enough, then agents roughly enjoy the whole potential surplus from exchanges.

Although these points are all conceptually simple, to prove them in a sufficiently general setup turns out to be quite a complicated task. In order to give readers the possibility to get the substance of our findings without forcing them to go through all technicalities and proofs, we begin the analysis (in section 3) by studying a simplified version of the model (whose general version is described in section 2). To keep things as simple as possible, we consider only two qualities (high and low) and assume that agents do not discount future payoffs. The structure of the game is as follows. The market exists for an infinite number of trading stages or until all goods are sold. Before the market begins the distribution of qualities brought to the market is announced. In each stage, firstly the number of goods still unsold is observed and, secondly, buyers make price offers at which they are willing to buy one good randomly chosen from any seller willing to sell; thirdly, sellers consider the list of price offers and decide whether to sell or not. We demonstrate that, in this setup, there exists a weak perfect Bayesian equilibrium leading to full trade in finite time.

The simplified model of section 3 shows with sufficient clarity why adverse selection can be solved by the possibility to make price offers conditional on the number of goods still on the market. However, the simple model is not rich enough to tackle other issues of interest which can arise when the number of qualities brought to the market is greater than two and future payoffs are discounted. In section 4 we study the general model presented in section 2 which can accommodate any number of qualities and the discounting of future payoffs. We show that also in this more general setup there exists a weak perfect Bayesian equilibrium leading to full trade. The proof of this result turns out to be substantially more complicated because, depending on qualities and discount factor, it can happen that goods of different qualities must be sold in equilibrium at the same price and in the same trading stage. Applying the model studied in section 4 we are also able (in section 5) to discuss issues related to welfare and, more specifically, to the losses due to the length of trading stages. In this regard we show that as the spell between exchanges tends to zero – or, equivalently, the discount factor tends to unity – full trade can still be obtained in finite time while total surplus tends to the entire potential surplus gainable from exchanges.

In section 6 we discuss extensions of the model that help to shed light on three important issues: uniqueness of full trade equilibria, the relevance of our findings when there is incomplete information on the realized distribution of qualities, and their relevance when new goods enter the market at later times. Firstly, we argue that intertemporal competition among buyers as well as intertemporal competition between one buyer and herself in the past can sustain equilibria where full trade does not emerge; in particular, we argue that this possibility arises when buyers can condition current price offers to past price offers, and we show that in three cases where intertemporal competition is at least partly impeded – i.e., buyers stay on the market just one trading stage, agents are extremely impatient, adverse selection is extreme – all weak perfect Bayesian equilibria lead to full trade. Secondly, we relax the assumption of public knowledge of the realized distribution of qualities by allowing each good to be randomly drawn from an interval of qualities, with intervals potentially overlapping; we prove that if trading stages take place frequently enough, then there exists a weak perfect Bayesian equilibrium that leads to full trade also under incomplete information on the distribution of qualities. Thirdly, we allow for the sequential arrival of sellers and buyers, and we prove that if agents can be distinguished on the basis of their wave of arrival, then there exists a weak perfect Bayesian equilibrium that leads to full trade.

In section 7 we discuss a variety of issues related to the nature and robustness of our results. We begin by assessing some important features of full trade equilibria. Then, we indicate a few reasonable generalizations of the model which would not substantially modify our results. Moreover, we identify which assumptions – and, therefore, what market characteristics – are crucial for the emergence of full trade and the realization of the entire potential surplus from trade.

Finally, we refer the reader to the last section of the paper for a detailed discussion of the relation between the present paper and the existing literature on adverse selection.

2 The model

We consider a market with n sellers and m buyers. We refer to the generic seller as seller i, with $i \in \{1, \ldots, n\}$, and to the generic buyer as buyer j, with $j \in \{1, \ldots, m\}$. We assume $m > n$ so that there is enough demand for all goods (for which $m \geq n$ is sufficient) and there is competition among buyers at every possible stage (which requires $m > n$). Each seller comes to the market with one good, and goods can differ by quality across sellers. The quality of a good is a private information of its seller, while the realized distribution of qualities that initially are on the market is public information. The market stays open from the first stage onwards, i.e., for periods $0, 1, 2, \ldots$, and buyers and sellers stay on the market until they complete a transaction.

In each time period the following things happen in order. The supply size – i.e., the number of goods still on the market – is disclosed to all market participants. Buyers simultaneously make price offers that are valid for the current period only and at which they are willing to buy one good. Price offers are public, i.e., observable by both buyers and sellers. After observing price offers, all sellers simultaneously decide whether to sell or not their goods (at the highest price offer). The allocation of goods is resolved as follows. The sellers who have accepted to sell are randomly paired with the buyers who have made the highest price offer: a transaction occurs for each random pair of one seller and one buyer. Clearly, some sellers (buyers) will not be able to sell (buy) if the number of sellers wishing to sell is larger (smaller) than the number of buyers making the highest offer. When a transaction occurs between a seller and a buyer, both the buyer and the seller exit the market and the good that has been exchanged is no longer traded. Figure 1 illustrates the timing of the game.

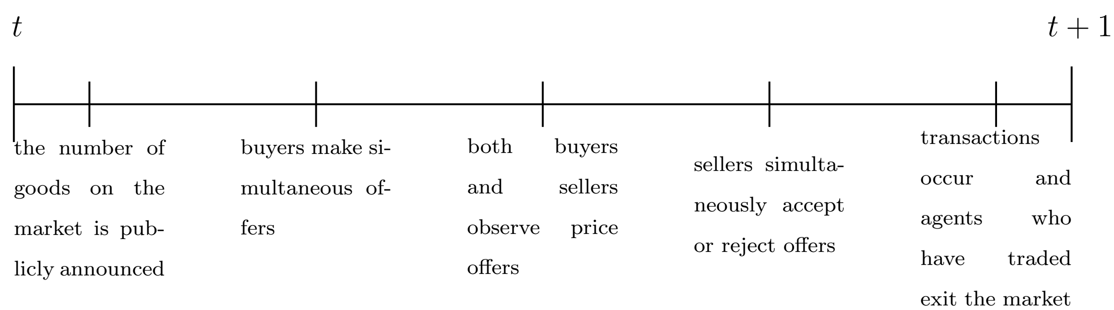

Figure 1: Timing of the stage game.

Payoffs are assigned as follows. Let $0 < \delta \leq 1$ be a discount factor common to all players, which can be interpreted as a measure of either progressive good deterioration or agents' impatience or a combination of both. All buyers are identical so they have the same payoff structure. We denote with b_i the reservation price of good i for buyers and with s_i the reservation price of good i for seller i. A possible interpretation of b_i and s_i is that they are the present values of the stream of services granted to the owner of good i.[2] Alternatively, one can think that agents have appropriate options. We note that the payoff of buyers if they buy nothing is 0. To simplify notation, we also normalize at 0 the payoff of sellers in

[2] We note that this interpretation does not work for $\delta = 1$ if goods provide an infinite stream of services.

the case they do not sell the good – i.e., we systematically subtract s_i from the payoff of seller i. Therefore, considering present values at time t', a buyer who obtains good i for price p at time $t > t'$ evaluates the transaction $\delta^{t-t'}(b_i - p)$, while seller i evaluates the same transaction $\delta^{t-t'}(p - s_i)$. Without loss of generality, we assume that $i' > i$ implies $s_{i'} \geq s_i$ and $b_{i'} \geq b_i$, meaning that goods are ordered by increasing quality. Moreover, we impose that all goods brought to the market can potentially generate a surplus for both trading parties, i.e., $b_i > s_i$ for all i.

We introduce some further notation to simplify the exposition. We denote with G the set of all goods, with $G(t) \subseteq G$ the set of goods still unsold at time t, with $g^t = ||G(t)||$ its cardinality, and with $S(t) \subseteq G$ the set of goods sold at time t. We refer to g^t equivalently as the supply size or as the number of unsold goods at time t (we simply use g when we do not refer to a specific time period). A generic price offer is denoted with $p \in \mathbf{R}_+$, a generic vector of price offers at time t (one for each buyer on the market) is denoted with $\mathbf{p} \in \mathbf{R}_+^{m-n+g(t)}$, and the maximum price offer in $\mathbf{p}$ is indicated with $p_{\max}(\mathbf{p})$.

The informational structure is as follows. At each stage t buyers and sellers are informed about g^t. Moreover, they remember all previous price offers. In addition, sellers also observe current offers before taking a decision about selling. Therefore, at each stage a buyer j knows $(\mathbf{g}^t, \mathbf{P}^{t-1})$: the sequence $\mathbf{g}^t = (g^0, g^1, \ldots, g^t)$ of observed numbers of unsold goods from time 0 to time t, and the sequence $\mathbf{P}^{t-1} = (\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^{t-1})$ of buyers' price offers from time 0 to time $t-1$. At each stage a seller i knows $(\mathbf{g}^t, \mathbf{P}^t)$: the sequence $\mathbf{g}^t = (g^0, g^1, \ldots, g^t)$ of observed numbers of unsold goods from time 0 to time t, and the sequence $\mathbf{P}^t = (\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^t)$ of all price offers including the list of offers at time t.

A strategy for buyer j is a function π_j that assigns a price offer $p \in \mathbf{R}_+$ to each of her information sets. A strategy for seller i is a function ρ_i that assigns a response of either 1 or 0 to each of her information sets, where 1 means willingness to sell and 0 means refusal to sell. A strategy profile is $(\boldsymbol{\pi}, \boldsymbol{\rho})$, where $\boldsymbol{\pi}$ is the m-dimensional vector of buyers' offer functions and $\boldsymbol{\rho}$ is the n-dimensional vector of sellers's response functions. We say that a strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ leads to *full trade* when sooner or later every good is sold, that is, when there exists some $\hat{t}$ such that $\bigcup_{t \leq \hat{t}} S(t) = G$.

Finally, throughout the paper we focus on the weak perfect Bayesian equilibrium (wPBE hereafter) as a solution concept. In brief, a strategy profile is wPBE if for every player, for every information she may obtain in the game, there exists a belief on the previous play of the game such that, in the presumption that subsequent play by other players unfolds as prescribed by the strategy profile, there does not exist a strictly profitable deviation from the behavior prescribed for the remaining part of the game. Beliefs are unrestricted everywhere but along the equilibrium path, where instead they are derived by means of Bayes rule (see e.g., Mas-Colell et al., 1995).

3 Full trade with two sellers and no discounting

In this section we illustrate how the information about the supply size can be exploited to obtain an equilibrium strategy profile that leads to full trade. The basic idea is that price offers are conditioned to the current number of unsold goods in such a way that: first, buyers can buy goods of the lowest possible quality without incurring in expected losses, second, only goods of lowest quality can be sold without incurring in losses, and, third, sellers of lowest quality cannot do better by refusing to sell. With such price offers, sellers of lowest quality find it optimal to sell, so that the goods of the lowest quality exit the market and the size of the informed side of the market shrinks accordingly. Iterating such price offers in the subsequent stages leads to trade goods of increasing quality at increasing prices.

We note that while for $\delta < 1$ the waiting time can be used as a screening device to obtain full trade (see Janssen and Roy, 2002), for $\delta = 1$, the public knowledge of the supply size is crucial for the obtainment of full trade. The reason is that conditional price offers can dissuade sellers from refusing to sell in the hope of higher future offers since such higher offers will never arrive: if current offers are rejected then the number of goods still on the market stays put, implying that price offers will be the same in the next period.

Given the illustrative aim of the current section, we assume $\delta = 1$ and only two sellers, i.e., $n = 2$, so that we may simply speak of a low quality good L and a high quality good H (we also use L and H as identifying subscripts), with $s_L < s_H$ and $b_L < b_H$. We will sometimes refer to the model with $\delta = 1$ and $n = 2$ as the *simplified* model.

We are now ready to state our result for this setup.

PROPOSITION **1.** *In the model with $n = 2$ and $\delta = 1$, there exists a wPBE that leads to full trade in 2 trading stages.*

Proof. We now show that the following profile of strategies is a wPBE of the above game:

buyers: at every time period t, if on the market then offer b_L if $g = 2$, and b_H if $g = 1$;

seller L: at every time period t, if on the market then accept any offer larger or equal than b_L if $g = 2$, and accept any offer larger or equal than b_H if $g = 1$;

seller H: if on the market then accept any offer larger or equal than b_H.

We consider decisions at a generic period and we call p_{max} the maximum price offer in that period. We begin the check from seller L. Consider first the case $g = 2$. If $p_{max} < b_L$, then seller L finds it strictly profitable to wait the next period, when she will receive an offer equal to b_L. If $b_L \leq p_{max} < b_H$, then seller L finds it optimal to accept (which is strictly profitable if $p_{max} > b_L$), since in case of refusal she will receive a price offer equal to b_L in the next period and at every future period if she goes on rejecting offers (because seller H always rejects offers strictly lower than b_H). If $p_{max} \geq b_H$, then seller L finds it optimal to accept (which is strictly profitable if $p_{max} > b_H$), since in case of refusal she will receive a price offer equal to b_H in the next period (because seller H accepts the current maximum price offer) and in every future period if she goes on rejecting offers. Now consider the case

$g = 1$. The price offers in the next period – and in every future period if seller L goes on rejecting offers – will be equal to b_H. Therefore, if $p_{max} < b_H$ then seller L finds it strictly profitable to wait, while if $p_{max} > b_H$ then seller L finds it strictly profitable to accept, and she is indifferent between accepting and waiting in case $p_{max} = b_H$.

We now consider seller H. If $p_{max} < b_L$, then seller L finds it strictly profitable to wait for one period (if $g = 1$) or two periods (if $g = 2$): if good L is still on the market then this time no sale takes place, the next period good L will be sold at price b_L, and in the following period seller H will receive an offer equal to b_H; if instead good L has already been sold then this time no sale takes place, and the next period seller H will receive an offer equal to b_H. If $b_L \leq p_{max} < b_H$, then seller H finds it strictly profitable to delay selling at least until the next period: if good L is still on the market then it is sold at this time, otherwise no sale takes place; in any case, the next period seller H will receive an offer equal to b_H. If $p_{max} \geq b_H$, then seller H finds it optimal to accept (which is strictly profitable if $p_{max} > b_H$), since in case of refusal she will receive a price offer equal to b_H in the next period (because the number of goods still on the market will be 1 since seller L, if still on the market, accepts the current maximum price offer) and in every future period if she goes on rejecting offers.

Finally we consider a generic buyer at a generic information set in period t. If $g = 2$, then following the equilibrium strategy profile grants a payoff equal to 0, and no positive payoff can ever be obtained since seller L and seller H do not accept offers lower than b_L and b_H, respectively. If $g = 1$, then, if the information set is on the equilibrium path, then the only admissible belief requires that the good still on the market is H while, if the information set is not on the equilibrium path, then we choose the belief that the good still on the market is H. In any case, there is no strictly ninient deviation, since following the equilibrium strategy profile yields a null payoff, and no positive payoff can ever be obtained since seller H does not accept offers lower than b_H. □

We end this section with two remarks. The first remark is on efficiency. In the simplified model every wPBE that leads to full trade is efficient, but this result crucially hinges on $\delta = 1$. In fact, when exchanges at later stages are worth less than exchanges in the current stage, equilibria where transactions occur sequentially do not allow to obtain the whole surplus. In section 5 we come back to this issue showing that, if the time between any two subsequent rounds of market exchanges can be made arbitrarily small, then efficiency is recovered in the limit (Proposition 3).

The second remark is on the difficulty of obtaining full trade when $\delta = 1$ and the realized distribution of qualities is public knowledge, but the supply size is not observed. Consider the setup of the simplified model with the only difference that the number of goods still on the market is not publicly observable. The obtainment of full trade encounters a substantial difficulty: a low current price offer combined with a high *unconditional* price offer in a later period – which is necessary to convince seller H to sell – induces seller L to reject the current low price offer, since seller L knows that her current rejection will not prevent buyers from making the high offer in the future. This suggests that, even if the realized distribution of qualities is public knowledge, when $\delta = 1$ in order to obtain full trade additional public information is required.[3]

4 Full trade with any number of sellers and discounting

In this section we remove the simplifying assumptions applied in section 3 and prove the same existence result for a more general model. Our objective is to show that it is possible to assign the selling of each good in G to some time period – possibly having more goods sold at the same time – in such a way that the specified order is sustainable as wPBE, for a given value of the discount factor. In order to do this we rely on an algorithm which can be applied to any $G' \subseteq G$ yielding – when G' is non-empty – a partition of G' and a function

[3] In a setup with re-sales, an alternative to the public knowledge of the supply size is the public knowledge of the goods' vintage (Hendel et al., 2005).

$H(k, G')$ that assigns each element of the partition of G' to a distinct time period k, so that $H(0, G')$ is the subset of goods assigned to period 0 (current period), $H(1, G')$ is the subset of goods assigned to period 1 (next period), and so on and so forth. Intuitively, k indicates how many periods in the future goods in $H(k, G')$ will be sold. In this reagard it is also useful to define the function $\tau(i, G')$ indicating how many periods in the future good i will be sold, i.e., $\tau(i, G') = k$ such that good $i \in H(k, G')$. The algorithm used for the construction of the partition of G' and function $H(k, G')$ is described in detail in the Appendix. What matters here is that the algorithm allows us to establish some properties of function $H(k, G')$ (which are stated in Lemma 1 below) that turn out to be crucial for proving the existence of a wPBE that leads to full trade (see Proposition 2 below).

We also introduce some new notation. For any $G' \subseteq G$ subset of goods, we denote with $\beta(G')$ the average value for buyers of goods in G', i.e., $\beta(G') = \frac{\sum_{\text{good } i \in G'} b_i}{||G'||}$, with $\beta(G') = 0$ if $G' = \emptyset$. Furthermore, we call $G^+(g)$ the set of g goods with highest index in G, i.e., $G^+(g) = \{n - g + 1, \ldots, n\}$, with $G^+(0) = \emptyset$.

Lemma **1.** *There exists a function $H(k, G')$ satisfying the following properties:*

> ***Property 1.*** *For every good i and every k, if good i belongs to $H(0, G')$, then $\beta(H(0, G')) - s_i \geq \delta^k(\beta(H(0, G''(k))) - s_i)$, with $G''(1) = (G' \setminus H(0, G')) \cup \{good\ \ i\}$ and $G''(k) = (G''(k-1) \setminus H(0, G''(k-1))) \cup \{good\ \ i\}$;*
>
> ***Property 2.*** *For every good i and every $k < \tau(i, G')$, the following inequality holds: $\beta(H(k, G')) - s_i < \delta^{\tau(i,G')-k} [\beta(H(\tau(i, G'), G')) - s_i]$.*

Proof. See the Appendix. □

Property 1 concerns the non-optimality of deferring the time of selling, while Property 2 concerns the non-optimality of anticipating the time of selling. The meaning of these properties is better seen by considering the case where buyers, in a given time period, offer a price equal to the discounted average valuation of the goods associated with that time period, i.e., buyers offer $\delta^k \beta(H(k, G'))$ in k periods from now. In such a situation, Property 1 says that if seller i is among those who are selected to sell in the current time period, then she does not find it strictly profitable to reject current offers and sell in any future period. Property 2, instead, says that if seller i is selected to sell in k periods from now, then she does not find it strictly profitable to accept an offer made in previous time periods.

The following proposition is a generalization of Proposition 1. In the proof we propose a strategy profile which leads to full trade and we check that it is indeed a wPBE. Figure 2 illustrates what happens over time when buyers and sellers follow such equilibrium strategy profile.

PROPOSITION **2.** *In the model with $n \geq 2$ and $0 < \delta \leq 1$, there exists a wPBE that leads to full trade in at most n trading stages.*

Proof. Preliminarily, we let i^* be equal to i if good $i \in G^+(g)$ and i^* be equal to the minimum index in $G^+(g)$ if good $i \notin G^+(g)$.[4] We consider $(\boldsymbol{\pi}, \boldsymbol{\rho})$ as the strategy profile proposed for equilibrium, where:

> **buyers:** for every buyer j, for every information $(\mathbf{g}^t, \mathbf{P}^{t-1})$, if buyer j is on the market then she makes the price offer $\pi_j(\mathbf{g}^t, \mathbf{P}^{t-1}) = \beta(H(0, G^+(g^t)))$;
>
> **sellers:** for every seller i, for every information $(\mathbf{g}^t, \mathbf{P}^t)$, if seller i is on the market then she accepts to sell, i.e., $\rho_i(\mathbf{g}^t, \mathbf{P}^t) = 1$, if and only if $p_{\max}(\boldsymbol{p}^t) \geq \delta[\beta(H(\tau(i^*, G^+(g^t)), G^+(g^t)))]$.

The strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ leads to sell some goods at every period until all goods are sold, which happens in at most n trading stages.

Check for buyers. Consider a generic buyer j and a generic information $(\mathbf{g}^t, \mathbf{P}^{t-1})$. If such information is along the equilibrium path, then Bayes rule forces buyer j to believe that the goods still on the market are $G^+(g)$. If instead the information is off-equilibrium, then

[4] We suppress the arguments of i^*, which is evidently a function of both i and $G^+(g)$, for notational simplicity.

we choose a belief for buyer j such that she thinks that the goods still on the market are $G^+(g)$. By the definition of the proposed strategy profile and by construction of function β, if buyer j follows the equilibrium strategy from now on, then she gains a null expected payoff. We have now to check that no deviation consisting in choosing a different action at this information set and/or at following information sets allows buyer j to gain a positive expected payoff.

We first consider deviations at $(\mathbf{g}^t, \mathbf{P}^{t-1})$. The strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ requires other buyers (that exist since $m - n + g^t \geq 2$ for every t due to $m > n$) to offer $\beta(H(0, G^+(g)))$. Buyer j will buy nothing if she offers less, and she will realize a negative expected payoff if she offers more, because of sellers' proposed strategies and the construction of functions β and H.

We now turn our attention to future deviations. Consider a future information set $(\mathbf{g}^{t'}, \mathbf{P}^{t'-1})$ that can be reached from buyer j's belief at $(\mathbf{g}^t, \mathbf{P}^{t-1})$ through some sequence of actions by buyer j when other players follow their proposed strategies. Bayes rule implies – because a seller of lower quality following the proposed strategies surely accepts when a seller of higher quality accepts – that buyer j believes that the goods still on the market at $(\mathbf{g}^{t'}, \mathbf{P}^{t'-1})$ are $G^+(g^{t'})$. This allows us to conclude – similarly to what done when considering deviations at $(\mathbf{g}^t, \mathbf{P}^{t-1})$ – that deviations lead either to null or to negative expected payoffs.

Check for sellers. Consider a generic seller i and a generic information $(\mathbf{g}^t, \mathbf{P}^t)$. We treat beliefs similarly to what done for buyers. If information $(\mathbf{g}^t, \mathbf{P}^t)$ is along the equilibrium path, then Bayes rule forces seller i to believe that the goods still on the market are $G^+(g^t)$. If instead the information is off-equilibrium, then we choose a belief for seller i such that she thinks that the goods still on the market are either $G^+(g^t)$ if good $i \in G^+(g^t)$, or $G^+(g^t-1) \cup \{\text{good } i\}$ if good $i \notin G^+(g^t)$; this latter case is due to the fact that seller i knows that her own good is still on the market.

We first consider deviations that anticipate the time of selling with respect to that in the proposed strategy profile. Suppose that $\delta\beta(H(h-1, G^+(g^t))) \leq p_{\max}(\boldsymbol{p}^t) < \delta\beta(H(h, G^+(g^t)))$, with $h \leq \tau(i, G^+(g^t))$ and where we set $\beta(H(-1, G)) = 0$ for notational consistency. If other sellers follow the proposed strategies, then sellers of goods in $H(0, G^+(g^t)) \cup \ldots \cup H(h-1, G^+(g^t))$ accept the current offer. If seller i rejects, then at next time buyers will offer $\beta(H(0, G^+(g^t) \setminus \bigcup_{r=0}^{h} H(r, G^+(g^t)))$, that is equal to $\beta(H(h, G^+(g^t)))$. Applying an analogous reasoning, we may extend this result to future time periods, obtaining that if seller i rejects for $\tau(i, G^+(g^t)) - h + 1$ time periods, then she will receive the following list of price offers in present value: $p_{\max}(\boldsymbol{p}^t)$, $\delta\beta(H(h, G^+(g^t))), \delta^2\beta(H(h+1, G^+(g^t))), \ldots, \delta^{\tau(i, G^+(g^t))-h+1}\beta(H(k, G^+(g^t)))$. From $p_{\max}(\boldsymbol{p}^t) < \delta\beta(H(h, G^+(g^t)))$ and property 2 we get that any decision to accept when the proposed strategy prescribes to reject is not strictly preferred.

We now consider deviations that delay the acceptance time with respect to that in the proposed strategy profile. We first assume $i^* = i$. We consider any future $(\mathbf{g}^{t'}, \mathbf{P}^{t'})$ that can be reached from seller i's belief at $(\mathbf{g}^t, \mathbf{P}^t)$ through some sequence of actions by seller i (more precisely, rejections) when other players follow their proposed strategies. Bayes rule implies that seller i believes that the goods still on the market at $(\mathbf{g}^{t'}, \mathbf{P}^{t'})$ are $G^+(g^{t'}-1) \cup \{\text{good } i\} = G^+(g^{t'})$ (the last inequality because $i^* = i$). Property 1 allows us to establish that rejecting a maximal price offer at the current or at any future information set when the proposed profile prescribes to accept cannot be strictly preferred for seller i. Finally, we note that when seller i is different from seller i^*, and in particular when $i < i^*$, to postpone selling is more costly so that the same conclusion holds a fortiori. $\square$

5 Welfare analysis and the frequency of trading stages

Janssen and Roy (2002) show that in the presence of a discount factor, i.e., $\delta < 1$, full trade can be obtained without giving any additional public information. Indeed, when agents discount future payoffs the waiting time before exchanges take place can be used as a screening device: lower quality sellers face a larger cost of waiting than high quality

time period	0	1	...	k	...	$\tau(n,G)$
sold goods	$H(0,G)$	$H(1,G)$	...	$H(k,G)$	...	$H(\tau(n,G),G)$
price	$\beta(H(0,G))$	$\beta(H(1,G))$	...	$\beta(H(k,G))$	...	$H(\beta(\tau(n,G),G))$
buyers' payoff	0	0	...	0	...	0
sellers' payoff	$\beta(H(0,G))-s_i$, for good $i \in H(0,G)$	$\delta(\beta(H(1,G))-s_i)$, for good $i \in H(1,G)$	...	$\delta^k(\beta(H(k,G))-s_i)$, for good $i \in H(k,G)$	...	$\delta^{\tau(n,G)}(\beta(H(\tau(n,G),G))-s_i)$, for good $i \in H(\tau(n,G),G)$

Figure 2: Timing of the wPBE in Proposition 2. In time 0 the group of goods of least quality $H(0,G)$ is sold at price $\beta(H(0,G))$ generating a null expected payoff for buyers and a payoff of $\beta(H(0,G)) - s_i$ for sellers whose good is in $H(0,G)$. In time $k = 1,\ldots,\tau(n,G)$ the goods of the k-th quality group $H(k,G)$ are sold at price $\beta(H(k,G))$ generating a null expected payoff for buyers and a payoff of $\delta^k(\beta(H(0,G)) - s_i)$ for sellers whose good is in $H(k,G)$.

sellers, so that there always exists a delay of exchanges long enough to convince low quality sellers not to wait. This, however, imposes a non-negligible cost since part of the surplus must be burned to sort qualities over time. This cost is also non-reducible in a sensible way: reducing impatience or increasing the frequency of exchange opportunities – i.e., increasing the discount factor – implies that high quality sellers have to sell in more distant periods in order to convince low quality sellers not to wait. This is confirmed by Janssen and Roy (2002) who finds that, when the discount factor tends to 1, the length of time required for all exchanges to occur tends to infinity. Despite full trade, efficiency is lost and cannot be recovered even in the limiting case.

Instead, in our model the discount factor is only an element of realism and does not play any essential role in allowing a full trade outcome. Proposition 2, where we can have $\delta = 1$, substantiates this claim. It also implies that efficiency is not impossible to achieve. This is evident for the case where $\delta = 1$, since trading at later stages does not reduce surplus. What is most important, however, is that we are able to get arbitrarily close to efficiency also for the case in which agents effectively discount future payoffs. In particular, in the following we argue that if the spell between any two subsequent stages of trading is made smaller and smaller, then efficiency is recovered in the limit.

If δ is the discount factor for one period of time and trading stages are Δt time distant from each other, then the discount factor between the current stage and the next stage is $\delta^{\Delta t}$. We define the *frequency of trading stages* as $1/\Delta t$. We measure the welfare of an outcome with the sum, over all agents (buyers and sellers), of individual payoffs. We refer to the maximum welfare that is attainable with an outcome supported by a wPBE as the *maximum attainable welfare*.[5] The entire potential surplus from exchange is equal to $\sum_{i=1}^{n}(b_i - s_i)$.

PROPOSITION **3.** *As the frequency of trading stages increases, the maximum attainable welfare tends to the entire potential surplus from exchanges.*

Proof. We note that in the wPBE that we have used in the proof of Proposition 2 all transactions occur within n trading stages. Since the surplus extracted from the exchange of good i is at least $\delta^n(b_i - s_i)$, it clearly converges to $b_i - s_i$ as the frequency of trading stages increases. □

Proposition 3 suggests that, in our setup, the disclosure of information about the supply size may be usefully combined with an increase in the frequency of trading stages.

6 Extensions

In this section we discuss extensions of the model that help to shed light, in turn, on: the uniqueness of full trade equilibria, the relevance of our findings under incomplete information on the realized distribution of qualities, and their relevance when we allow for new goods entering the market at later times.

[5] Such maximum attainable welfare is well defined, since there is a finite number of outcomes with a welfare larger or equal than the welfare attained by the wPBE shown to exist in Proposition 2.

Intertemporal competition and uniqueness of full trade equilibria In Propositions 1 and 2 we have shown that a wPBE exists where every good is traded. However, we have been silent on whether other wPBEs exist that do not lead to full trade. In the following we argue that such non-full trade equilibria might actually exist, but their existence rests on a form intertemporal competition among buyers that requires price offers to be observed by competing buyers.

Two types of intertemporal competition can arise. The first type is not tremendously relevant as it is effective in preventing full trade only for $\delta = 1$. The following sketched example illustrates. Suppose that there exists a wPBE such that some goods remain unsold. Call i the minimum quality of unsold goods. Consider a time period where, according to the equilibrium strategies, no good is sold and will ever be sold. Consider then a buyer who is still on the market in such time period and who may offer a price p strictly comprised between s_i and b_i, in the hope to induce seller i to sell her good. Note that seller i would indeed profit from selling. However, such unexpectedly high offer is observable by other buyers, who may adopt a strategy that reacts to high offers by retaliating in the next period with a price offer equal to b_i. Such a retaliation induces seller i to reject the initial unexpectedly high offer. Note that this retaliation strategy does not lead to any loss for the buyers adopting it, and effectively makes the deviating buyer indifferent between deviating and not deviating (since in any case she makes a payoff equal to zero).[6]

The second type of intertemporal competition is more relevant as it exploits the information about the supply size and does not disappear for $\delta < 1$. As such, it can be considered a byproduct of the public knowledge of g. The following sketched example illustrates. Suppose that in a wPBE there are two goods on the market that are never sold, call them L and

[6] If $\delta < 1$, then a buyer could offer a price p strictly comprised between b_i and δb_i, in the attempt to gain from the purchase of unsold goods. This price offer grants to seller i a positive payoff in case of exchange, and it cannot be outperformed by any future retaliating price offer by other buyers, unless they offer more than b_i which would imply incurring in a loss.

H. Consider a buyer who deviates from her equilibrium strategy in the hope to gain some surplus from the purchase of unsold goods, and suppose that she makes a price offer strictly comprised between b_L and δb_L. Other buyers may react by offering $(b_L + b_H)/2$ in the next period, which in the case $(b_L + b_H)/2 > s_H$ may convince both seller L and seller H to reject the current offer and accept theirs. This kind of intertemporal competition could be contrasted by the initial buyer by considering a different behavior: making a price offer strictly comprised between $(b_L + b_H)/2$ and $\delta(b_L + b_H)/2$, which could result in the purchase of both goods. Indeed, other buyers cannot make more appealing offers in the next period without incurring in a loss, if they want to buy both goods. However, they may take advantage of the situation and try to buy only good H, by means of a reaction strategy that exploits the information conveyed by g. More precisely, they may offer b_H in the next period conditional on the fact that g decreases by one. Given these strategies, seller L obtains a positive payoff from accepting the initial deviation and cannot hope to earn more by rejecting it, since in such a case other buyers would not make a higher offer in the following period; instead, seller H finds it optimal to wait until such a higher offer, provided that the discount factor is close enough to 1. Note that the described reaction strategy is not costly for the buyers adopting it. The only one who loses is the buyer who attempts the initial deviation, that would be strictly profitable (in expectation) in the case both seller L and seller H accept it, but it is not in the case only seller L accepts to sell. In conclusion, this type of intertemporal competition turns out to be effective in preventing the emergence of full trade even in the presence of a time discount, although not for any value of the discount factor.

However, intertemporal competition can be really effective in preventing full trade only if buyers stay on the market for more than one trading stage. Indeed, if at each stage new buyers arrive and stay for just one stage (as in, e.g., Daley and Green, 2012), then intertemporal competition of either type is precluded when it is conditional on price offers, since no buyer stays long enough to retaliate the behavior of other buyers. It remains possible, however, to condition offers on the supply size, but this turns out not to be sufficient to impair full trade. The following variant of our setup and the related proposition make this point precise.

Assume that, at the end of every stage $t \geq 0$, all buyers currently on the market exit, substituted by $m_{t+1} > n$ new buyers entering the market at time $t+1$. Buyers entering the market at $t+1$ know the setup of the game, but only observe $g(t+1)$. Crucially, buyers entering the market at $t+1$ do not observe any price offer made at $t' \leq t$. Call this setup the model with *one-period buyers.*

PROPOSITION **4.** *In the model with one-period buyers, every wPBE leads to full trade in at most n trading stages.*

Proof. By contradiction, suppose that there exists a strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ that is wPBE and does not lead to full trade. Hence, $(\boldsymbol{\pi}, \boldsymbol{\rho})$ must induce an equilibrium play for which there exists $\bar{t}$ such that $S(\bar{t}) = S(t) \neq \emptyset$ for all $t > \bar{t}$. We next show that, if this is the case, then buyers have a strictly profitable deviation.

Note that all sellers whose goods are in $S(\bar{t})$ never sell along the equilibrium path. This requires that along the equilibrium path no offer is ever strictly greater than s_{i^-} where good i^- is of the least quality in $S(\bar{t})$, i.e., no offer can be strictly greater than what the seller of the good of least quality still on the market expects to obtain if she refuses to sell. Otherwise, at least the seller of good i^- would find it optimal to accept some offer at some time.

Consider a buyer offering a price p at time $t \geq \bar{t}$ such that $s_{i^-} < p < b_{i^-}$. If accepted, p grants to the buyer a payoff of $\delta^t(b_{i^-} - p)$, which is strictly greater than her equilibrium payoff, that is zero. Let us now consider the decision that the seller of good i^- faces. If she rejects p at t, then for every $t' > t$ the new buyers entering the market will not recognize that a deviation from the equilibrium play has occurred, unless $g(t') \neq g(t) = g(\bar{t})$. And sellers, even if they do recognize that a path of out-of-equilibrium play has been activated, will not change their behavior because of sequential rationality: sellers must eventually reject any offer that is lower than their own evaluation. Therefore, $g(t') = g(t) = g(\bar{t})$ for every $t' > t$, and the sequence of offers for $t' > t$ which is induced by $(\boldsymbol{\pi}, \boldsymbol{\rho})$ after the deviating offer p at t will be the same as along the equilibrium path. This implies that no offer at $t' > t$ will be larger than s_{i^-}, as shown in the previous paragraph. Hence, we can conclude that the seller of good i^- must accept offer p at t, which grants her a payoff of $\delta^t(p - s_{i^-})$ that is strictly greater than her null payoff in equilibrium.

□

We conclude by observing that even when competing buyers stay on the market for more than one period, there are important situations where the type of intertemporal competition which can hinder full trade is not likely to emerge. A first case is what we may refer to as *extreme adverse selection*, i.e., it does not exist $U \subseteq G$ such that $||U|| > 1$ and $\beta(U) \geq s_i$ for every good $i \in U$. In words, extreme adverse selection is a situation where goods of different qualities cannot be sold in the same period because buyers' offers should be so high to convince sellers to accept that they would entail a negative expected payoff for the offering buyer. In the example sketched above relative to the second type of intertemporal competition, extreme adverse selection implies that $(b_L + b_H)/2 < s_H$. Under extreme adverse selection, only the first type of intertemporal competition can be at work, so that the presence of a non-negligible discounting of future payoffs is enough to ensure that every wPBE leads to full trade. The following proposition formalizes the result.

Proposition **5.** *In the model with extreme adverse selection, if $\delta < 1$ then every wPBE leads to full trade in at most n trading stages.*

Proof. By contradiction, suppose that there exists a strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ which does not lead to full trade. Hence, under $(\boldsymbol{\pi}, \boldsymbol{\rho})$, there must exist $\bar{t}$ such that $S(\bar{t}) = S(t) \neq \emptyset$ for all $t > \bar{t}$. We next show that buyers have a strictly profitable deviation.

Preliminarily, note that along the equilibrium path all sellers whose goods are in $S(\bar{t})$ never sell and buyers never make offers strictly greater than s_{i^-}, where good i^- is of the least quality in $S(\bar{t})$ (see proof of Proposition 4).

Consider a buyer offering a price p at time $t \geq \bar{t}$ such that $p > s_{i^-}$ and $\delta b_{i^-} < p < b_{i^-}$, which is possible since $\delta < 1$. If accepted, p grants to the buyer a payoff of $\delta^t(b_{i^-} - p)$, which is strictly greater than her equilibrium payoff, that is zero. Consider now the decision of the seller of good i^- at t. If she accepts, her payoff is $\delta^t(p - s_{i^-})$ which is strictly greater than her null payoff in equilibrium. The seller of good i^- can get at least $\delta^t(p - s_{i^-})$ only if she accepts an offer $p' \geq p/\delta^{t'-t}$ made at $t' > t$, . However, any such offer would entail a negative expected payoff for the buyer making it, because of $\delta < 1$ and extreme adverse selection, so it cannot occur in a wPBE. Hence, seller of good i^- must accept offer p at t. □

A second case where intertemporal competition does not hinder full trade is what we may call *extreme impatience*, i.e., there do not exist $U \subseteq G$ and good $i \in U$ such that $\delta \geq b_i/\beta(U)$. In words, extreme impatience is a situation where the time discount is so low that an offer which is near enough to the buyers' valuation for the single good of least quality still on the market cannot be beaten by higher offers in future periods. So, under extreme impatience intertemporal competition is impossible and hence non-full trade equilibria cannot exist. The following proposition formalizes the result.

PROPOSITION **6.** *In the model with extreme impatience, every wPBE leads to full trade in at most n trading stages.*

Proof. The proof goes analogously to the proof of Proposition 5. Here we just highlight the differences.

The price offer p at $t \geq \bar{t}$ made by the deviating buyer is such that that $p > s_{i^-}$ and $\delta\beta(\bar{U}) < p < b_{i^-}$, where $\bar{U} \subseteq S(\bar{t})$ maximizes $\beta(U)$ over all $U \subseteq S(\bar{t})$ such that $i^- \in U$. Note that $\bar{U}$ exists since $S(\bar{t})$ is finite. By extreme impatience and the maximality of $\beta(\bar{U})$, it follows that any offer $p' \geq p/\delta^{t'-t}$ made at $t' > t$ is such that $p' > \beta(U)$ for any $U \subseteq S(\bar{t})$ such that $i^- \in U$. This implies that p', if accepted at least by seller i^-, entails a negative expected payoff for the buyer offering it. □

Our last remark is about welfare. Under extreme impatience total surplus is bounded away from the entire potential. The reason is that a strong enough discounting is needed – i.e., the discount factor is bounded away from unity – so that the frequency of trading stages cannot be made arbitrarily high. We note that this is not necessarily the case under extreme adverse selection and that is never the case for the model with one-period buyers.

The working of the mechanism in practice: Complete versus incomplete information The possibility to implement full trade equilibria in real markets by means of the mechanism described in this paper depends on the possibility that real markets satisfy – or are made satisfy – the assumptions of the model. Fortunately enough, we can think of providing controlled market environments (for instance, online platforms) where most of our assumptions, if not all, can be satisfactorily approximated.

The one assumption which is probably hardest to satisfy in real markets, even if controlled, is that of complete information, i.e., the public knowledge of the realized distribution of qualities. In many real markets there is incomplete information about who are the market participants and what are their goods or preferences. In the spirit of Harsanyi, this can be always thought of as a case where agents who actually participate in the market are drawn from a known ex-ante distribution of agent types. Unfortunately, for a full trade equilibrium to be sustained by price offers conditional on the supply size, each seller must believe that from some point onward refusing to sell would not be followed by an increase in price offers, and hence each seller must believe that her acceptance plays a crucial role at some stage of the mechanism. Moreover, all buyers must expect not to make losses. This suggests that incomplete information could make this type of full trade equilibria unsustainable for two reasons: first, uncertainty can make it optimal to wait even for the seller of lowest realized quality, in the expectation that even lower quality sellers are in the market; second, buyers' lack of information about the realized distribution of qualities can be such that any offer capable of convincing the seller of lowest realized quality (that may be fairly high) to sell makes them incur in expected losses. We remark, however, that incomplete information does not necessarily imply that full trade equilibria cannot be sustained, although it may make sustainability less likely. With the following example we show that for some incompleteness in information full trade equilibria can still be obtained with the proposed mechanism.

Consider a stylized job market where Ann and Bob are two job candidates (who act as sellers), and they receive offers from a number (larger than two) of employers (who act as buyers). Following our model, we should assume that the skills of the two candidates are common knowledge, allowing uncertainty about the identity of the two candidates (in others words, agents do not know who is who). Instead, we assume that each job candidate's skill is randomly drawn from a uniform distribution over $[0, 1]$. Ann's skill is denoted by x_a, and Bob's skill is denoted by x_b. We now introduce some limitation to the extent of incomplete information. In particular, we assume that prior to entering the market, job candidates have been engaged in a joint work generating an observable output which discloses to both job candidates and employers the sum of candidates' skills, $\bar{x} = x_a + x_b$. Seller 1 is assumed to be the job candidate with the lower skill, while the other is seller 2.[7] In case they both have the same skill, i.e., $x_1 = x_2 = 0.75$, then we assume that Ann acts as seller 1. Finally, we assume that $\delta = 1$,[8] and that sellers' valuation coincides with their skill, while buyers' valuation is equal to sellers' valuation multiplied by a, with $a > 1$.

Suppose for the sake of concreteness that $\bar{x} = 1.5$. We consider a strategy profile where each buyer offers $a0.625$ if both candidates are on the market, and offers $a0.875$ when only one candidate is left on the market. Moreover, the candidate who has the lower skill behaves as follows independently of her realized skill: she accepts any offer larger or equal than $a0.625$ if two candidates are on the market, and she accepts any offer larger or equal than $a0.875$ if she is the only candidate in the market. Finally, the candidate who has the higher

[7] We remark that both sellers, knowing their own skill and the sum of the skills, can infer the skill of the other candidate and hence their rank.

[8] Actually, this assumption is not needed. We make it to provide a simpler exposition of the example.

skill behaves as follows independently of her realized skill: she accepts any offer larger or equal than $a0.875$ whatever number of candidates are in the market. We now briefly argue that the described strategy profile is wPBE if $a \geq 0.75/0.625 = 1.2$. Substantially, we have to ensure that the reservation wage s_1 of the candidate of lower skill is lower or equal than the offer by buyers when two candidates are in the market, i.e., $0.75 \leq a0.625$. Similarly, we have to ensure that the reservation wage s_2 of the candidate with higher skill is lower or equal than the offer by buyers when one candidate is left, even in the worst case – that occurs when $x_2 = 1$, i.e., $1 \leq a0.875$. We observe that $0.75 \leq a0.625$ implies $1 \leq a0.875$. What remains to check follows the proof of Proposition 1. We conclude this example by noting that in order to obtain full trade in a standard setup where there is only one stage of interaction, we would require the expected value for employers of a random draw to be larger or equal than the reservation wage of the candidate with highest skill, i.e., $1 \leq a0.5$ which means $a \geq 2$.

In the following, we proceed to identify and generalize the idea driving the result in the above example. Essentially, the public knowledge of the sum of skills for the two job candidates makes it possible to say that the lower type and the higher type have skills in different ranges: the lower type has an actual skill that is drawn from $[0.5, 0.75]$, the higher type has an actual skill that is drawn from $[0.75, 1]$. The actual value for the seller is known only to herself, while buyers has to make offers on the basis of their expectations, but as long as for both the lower type and the higher type we have that the expected value for buyers is larger than the highest value for sellers, then full trade can be achieved by conditioning the private offers on the supply size.

Let us first adjust the formal setup to the case of incomplete information. Suppose that each good i has a value for the seller which is drawn from $[\underline{s}_i, \overline{s}_i]$. No special assumption is made on the ex-ante distribution of quality over each quality interval and on the correlations across intervals. The expected value for a buyer is denoted with $\mathbb{E}b_i$. Without loss of generality, we order goods on the basis of buyers' expectations, i.e., $\mathbb{E}b_i \leq \mathbb{E}b_{i+1}$ for $i = 1, \ldots, n-1$. We observe that for every seller i, a strategy is now dependent of her realized valuation, i.e., a strategy π_i assigns either 1 or 0 to each combination of information set $(\mathbf{g}^t, \mathbf{P}^t)$ and type s_i. The following assumption is crucial, and can be intended as a reinforcement of $s_i < b_i$ for every good i. Here we assume that $\overline{s}_i < \mathbb{E}b_i$ for every good i. This ensures that the expected value of good i for a buyer is larger than the maximum value that seller i can assign to such a good; hence all sellers would prefer selling the good at such a price instead of keeping it. We observe that the assumption under consideration is compatible with proper cases of adverse selection, i.e., when a price equal to the expected value for a buyer of a generic good in the market is not enough to convince high quality sellers. Clearly, if goods' qualities are drawn from very similar ranges, then the assumption made turns out to be very close to rule out proper adverse selection. Instead, if ranges of goods' qualities are rather distinct one from the other, then the assumption is more easily satisfied. This is obtained by exploiting constraints that may arise in a specific setup (like the knowledge of the sum of the candidates' skills) and that introduce limitations in the incompleteness of information. At the extreme, constraints may yield different pointwise ranges of qualities, and in such a case we come back to the model of section 2.

There are some additional issues that must be fixed in order to obtain full trade in an incomplete information setup. So far we have not discussed how goods are ordered in terms of sellers' valuations. Even if we think that sellers' ex-ante expected valuations follow the same order as buyers' ones, however we are not sure that realized qualities for sellers respect such order. This creates problems when we apply the algorithm behind Lemma 1, since for a good with a lower index but a higher realized quality it is less costly to delay selling. Moreover, even if realized qualities are ordered in the same way of ex-ante expected qualities, however the precise way in which goods are bundled according to the algorithm used for Lemma 1 is dependent of the whole vector of realized qualities; this creates serious problems to buyers to figure out the proper way of bundling, and hence to make correct offers. However, if distinct sellers find it optimal to accept a plan where each one sells in a distinct trading stage, and this is true whatever type they are, then the above mentioned difficulties are overcome and full trade can be reached. Such a fine sorting of sold goods over time can arise in equilibrium if the discount factor is sufficiently close to unity and ties between buyers' expected valuations are ruled out. As discussed in section 5 with regard to welfare, in our model the mechanism that allows for full trade exploits the dynamic setup in the sense of the sequentiality of trading stages, while the passing of real time is not crucial. This creates the room for the following result:

PROPOSITION **7.** *In the model with incomplete information on the distribution of qualities, if* $\overline{s}_i < \mathbb{E}b_i$ *for* $i = 1, \ldots, n$, $\mathbb{E}b_i < \mathbb{E}b_{i+1}$ *for* $i = 1, \ldots, n-1$, *and the frequency of trading stages is high enough, then there exists a wPBE that leads to full trade in at most* n *trading stages.*

Proof. We propose a strategy profile that, for any vector of sellers' types $(s_1, \ldots, s_n)$, tells agents to behave as in the profile used in the proof of Proposition 2, with the difference that no bundling is allowed, so that one distinct good is sold at each trading stage. This is as if we replace function H with another function, $\widehat{H}$, that we prove to satisfy the properties stated in Lemma 1, for any vector of realized qualities.

Preliminarly, let $\widehat{H}(k, G')$ be a function that assigns to time period k the singleton containing the good with the $(k+1)$-th lowest index in G'. Note that $\widehat{H}$ assigns distinct goods to distinct trading stages, with rising quality over time.

We show that function $\widehat{H}$ satisfies both Property 1 and Property 2 of Lemma 1. It is trivial to show that Property 1 holds; in fact, for any vector of realized qualities, $G' = G''(k)$ for any k, since goods are sold one per period, and if the seller of the good assigned to the current trading stage refuses to sell, then no good is sold and in the next trading stage the same set of goods stays on the market.

We turn to Property 2. Since $\mathbb{E}b_1 < \mathbb{E}b_2 < \ldots < \mathbb{E}b_n$, there exists $\overline{\delta} \in (0,1)$ such that $\mathbb{E}b_1 < \delta^{\Delta t}\mathbb{E}b_2 < \ldots \delta^{(n-1)\Delta t}\mathbb{E}b_n$ for $\delta^{\Delta t} \in (\overline{\delta}, 1]$. Therefore, if the frequency of trading stages $1/\Delta t$ is high enough, then $\delta^{\Delta t} \in (\overline{\delta}, 1]$. From $\mathbb{E}b_1 < \delta^{\Delta t}\mathbb{E}b_2 < \ldots \delta^{(n-1)\Delta t}\mathbb{E}b_n$ we obtain that, for every $i, i' \in G'$ such that goods i and i' are sold respectively in period k and k' according to $\widehat{H}$, with $k > k'$, $\mathbb{E}b_{i'} - s_i < \delta^{k-k'}(\mathbb{E}b_i - s_i)$, i.e., there is no convenience for seller i to anticipate the time of selling. We remark that this holds whatever realized valuation $s_i \in [\underline{s}_i, \overline{s}_i]$.

Consider now the following strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$, where $i^* = i$ if $i \geq n+1-g^t$ and $i^* = n+1-g^t$ if $i < n+1-g^t$:

buyers: for every buyer j, for every information $(\mathbf{g}^t, \mathbf{P}^{t-1})$, if buyer j is on the market then she makes the price offer $\pi_j(\mathbf{g}^t, \mathbf{P}^{t-1}) = \mathbb{E}b_{n+1-g^t}$;

sellers: for every seller i who is on the market, for every information $(\mathbf{g}^t, \mathbf{P}^t)$ and every type s_i, if seller i is on the market then she accepts to sell, i.e., $\rho_i(\mathbf{g}^t, \mathbf{P}^t; s_i) = 1$, if and only if $p_{\max}(\boldsymbol{p}^t) \geq \delta\mathbb{E}b_{i^*}$.

We observe that, for any vector of realized qualities $(s_1, \ldots, s_n)$, the above strategy profile corresponds to that proposed in the proof of Proposition 2, if we replace function $H(k, G')$ with $\widehat{H}(k, G')$.

What remains to do are the checks that no deviation is strictly profitable for buyers and sellers. These checks follow almost identically the checks done in the proof of Proposition 2, to which we refer.

□

We close our analysis of this extension that accommodates incomplete information with a remark on welfare. The fact that the emergence of full trade relies, among other things, on a sufficiently high frequency of trading stages allows us to easily combine Proposition 7 with Proposition 3. In particular, an increase in the frequency of trading stages turns out to be desirable for two reasons under incomplete information; one is that it allows full trade as an equilibrium outcome, the other is that it leads to a welfare that is arbitrarily close to the entire potential surplus from exchanges.

Non-stationary environment: Arrival of new sellers and buyers So far we have assumed that all buyers and sellers are on the market since the first trading stage. In particular, we have assumed that neither new sellers nor new buyers enter the market at later times. We observe that allowing the arrival of new buyers does not alter our results in any substantial way, since we have already imposed that buyers outnumber goods in all trading stages – by assuming that $m > n$ in the first stage – and that each buyer buys just one good, so that additional buyers do not play any significant role. Instead, the arrival of new sellers with new goods can be a threat to full trade, since it makes the supply size less informative about the actual distribution of qualities on the market. This is due to the fact that when new goods enter the market both the supply size and the distribution of qualities change. So, the issue here is how to let agents correctly infer the current distribution of qualities when the initial distribution of qualities does not change only because lower quality goods are sold but also because new goods enter the market. One possibility is to have different markets for goods arrived at different dates. This can be obtained in the case sellers are not anonymous with respect to the date of their arrival to the market. In the following we briefly explore this case, arguing that the substance of Proposition 2 still holds.

Consider a situation where buyers and sellers arrive to the market in distinct waves over time. A generic wave $w \in W$, with either $W = \{1, \dots, \bar{w}\}$ or $W = \mathbb{N}$, is made of n^w sellers and m^w buyers, who enter the market at time $t(w)$. We assume $m^w > n^w$ for all w. While the number of buyers and sellers may differ across waves, we assume that the number of sellers is bounded above by $\bar{n}$. A seller coming in wave w is denoted with (i, w), and her reservation price is $s_{(i,w)}$. A buyer coming in wave w is denoted with (j, w), and buyers' reservation price for good (i, w) is $b_{(i,w)}$. As in the baseline model, buyers and sellers in each wave are ordered with natural numbers, on the basis of the quality of their goods. More precisely, for each (i, w) and each (j, w), we have that $i \in \{1, \dots, n^w\}$ and $j \in \{1, \dots, m^w\}$, and for each (i, w) and (i', w) with $i' > i$, we have that $s_{(i',w)} \geq s_{(i,w)}$ and $b_{(i',w)} \geq b_{(i,w)}$. Moreover, each good brought to the market can potentially generate a surplus for both trading parties, i.e., $b_{(i,w)} > s_{(i,w)}$ for each (i, w).

We denote with $G(w)$ the set of goods in wave w and with $G(w, t)$ the set of goods in wave w that are on the market at time t. The supply size of wave w at time $t \geq t(w)$ is $g^t(w) = ||G(t, w)||$, and we denote with $g^t = (g^t(w))_{w:\, t(w) \leq t}$ the list of supply sizes at t. We observe that some $g^t(w) \in g^t$ might be zero, if all goods of such wave are already been sold within time t.

The distribution of qualities for each wave is assumed to be public knowledge. The quality of the good possessed by a generic seller (i, w) remains her private knowledge, but it is publicly known that such seller belongs to wave w. Moreover, at each stage t buyers and sellers are informed about g^t. We maintain that $\mathbf{g}^t = (g^0, g^1, \dots, g^t)$. To simplify the setup, we also suppose that upon arrival to the market past prices and supply sizes are known.

At time t, a generic buyer (j, w'), who has arrived to the market – i.e., with $t(w') \geq t$ – and who is still on the market, chooses a wave w whose goods have already entered the market and some of them are still unsold – i.e., with $t(w) \geq t$ and $g^t(w) > 0$ – and makes a price offer $p \in \mathbf{R}_+$ to the sellers of that single wave. We indicate with $(p, w)^t_{(j,w')}$ the price-wave offer of buyer (j, w') made at time t, and with $\mathbf{p}^t(w)$ the set of price-wave offers made at stage t by any buyer on the market to the sellers of wave w; let $p_{\max}(\mathbf{p}^t(w))$ be the maximum price offer in $\mathbf{p}^t(w)$ which, if non-existent because $\mathbf{p}^t(w) = \emptyset$, we conventionally set equal to 0;[9] further, let $\mathbf{P}^t(w) = (\mathbf{p}^0(w), \mathbf{p}^1(w), \dots, \mathbf{p}^t(w))$ and $\mathbf{P}^t = (\mathbf{P}^t(w))_{w:\, t(w) \leq t}$, so that $(\mathbf{g}^t, \mathbf{P}^{t-1})$ and $(\mathbf{g}^t, \mathbf{P}^t)$ still indicate information sets for, respectively, buyers and sellers on the market at t.

[9] A seller who accepts a non-existent offer – that we have modeled as null price offer – can be interpreted as dismissing her own good; clearly, since all valuations are assumed to be positive, this case will never occur.

A strategy for buyer (j, w) is a function $\pi_{(j,w)}$ that assigns a price-wave offer to each of her information sets. A strategy for seller (i, w) is a function $\rho_{(i,w)}$ that assigns a response of either 1 or 0 to each of her information sets, where 1 means willingness to sell and 0 means refusal to sell. A strategy profile is $(\boldsymbol{\pi}, \boldsymbol{\rho})$, where $\boldsymbol{\pi}$ is the collection of buyers' offer functions and $\boldsymbol{\rho}$ is the collection of sellers's response functions.

In this setup the following result holds:

PROPOSITION **8.** *In the model where buyers and sellers enter the market in waves, if agents are not anonymous with respect to waves then there exists a wPBE where each good is sold in at most $\bar{n}$ trading stages.*

Proof. Preliminarily, denote with $G^+(w, g)$ the set of g goods with highest quality in wave w; for every (i, w), let i^* be equal to i if good $i \in G^+(w, g)$ and i^* be equal to the minimum index in $G^+(w, g)$ if good $i \notin G^+(w, g)$.Consider the following profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$:

buyers: for every (j, w), for every information $(\mathbf{g}^t, \mathbf{P}^{t-1})$, if buyer (j, w) is on the market then she offers $\pi_{(j,w)}(\mathbf{g}^t, \mathbf{P}^{t-1}) = (\beta(H(0, G^+(w, g^t(w)))), w)$;

sellers: for every (i, w), for every information $(\mathbf{g}^t, \mathbf{P}^t)$, if seller (i, w) is on the market then she accepts to sell, i.e., $\rho_{(i,w)}(\mathbf{g}^t, \mathbf{P}^t) = 1$, if and only if
$p_{\max}(\mathbf{p}^t(w)) \geq \delta[\beta(H(\tau(i^*, G^+(w, g^t(w))), G^+(w, g^t(w))))]$.

For each wave w, the strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ leads to sell some goods to some buyers of wave w in each trading stage, starting from the first stage after the arrival of wave w and until all goods arrived with wave w are sold. Since $n^w \leq \bar{n}$ for all w, every good is sold in at most $\bar{n}$ trading stages from its arrival to the market.

Check for buyers. Consider a generic buyer (j, w) and a generic information $(\mathbf{g}^t, \mathbf{P}^{t-1})$. If such information is along the equilibrium path, then Bayes rule forces buyer (j, w) to believe that the goods still on the market are $(G^+(w', g))_{w' \colon t(w') \leq t}$. If instead the information is off-equilibrium, then we choose a belief for buyer (j, w) such that she thinks that the goods still on the market are $(G^+(w', g))_{w' \colon t(w') \leq t}$.

For fixed choice of w, the fact that the generic buyer (j, w) cannot gain by changing her price offer follows from the check for buyers in the proof of Proposition 2.

Consider deviations at $(\mathbf{g}^t, \mathbf{P}^{t-1})$ which involve making a price offer to wave $\hat{w} \neq w$. The strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ requires buyers of wave $\hat{w}$ (that exist since $m^{\hat{w}} - n^{\hat{w}} + g^t(\hat{w}) \geq 2$ for every t due to $m^{\hat{w}} > n^{\hat{w}}$) to offer $(\beta(H(0, G^+(\hat{w}, g))), \hat{w})$. Buyer (j, w) will buy nothing if she offers less, and she will realize a negative expected payoff if she offers more, because of sellers' proposed strategies and the construction of functions β and H.

Consider deviations at a future information set $(\mathbf{g}^{t'}, \mathbf{P}^{t'-1})$ that can be reached from buyer (j, w)'s belief at $(\mathbf{g}^t, \mathbf{P}^{t-1})$ through some sequence of actions by buyer (j, w) when other players follow their proposed strategies, and that involve making a price offer to wave $\hat{w} \neq w$. Bayes rule implies that buyer (j, w) believes that the goods of wave $\hat{w}$ still on the market at $(\mathbf{g}^{t'}, \mathbf{P}^{t'-1})$ are $G^+(\hat{w}, g^{t'}(\hat{w}))$. Again, this leads either to null or to negative expected payoffs for (j, w).

Check for sellers. Consider a generic seller (i, w) and a generic information $(\mathbf{g}^t, \mathbf{P}^t)$. We treat beliefs similarly to what done for buyers. If information $(\mathbf{g}^t, \mathbf{P}^t)$ is along the equilibrium path, then Bayes rule forces seller (i, w) to believe that the goods still on the market are $(G^+(w', g^t(w')))_{w' \colon t(w') \leq t}$. If instead the information is off-equilibrium, then we choose beliefs for seller (i, w) such that for any wave w' that has already reached the market (i.e., with $t(w') \leq t$), for $w' \neq w$ seller (i, w) believes that $G^+(w', g^t(w'))$ is the set of goods of wave w' still on the market, while for wave w seller (i, w) believes that either $G^+(w, g^t(w))$ if good $i \in G^+(w, g^t(w))$ or $G^+(w, g^t(w) - 1) \cup \{\text{good } i\}$ if good $i \notin G^+(w, g^t(w))$.

We observe that since sellers can only accept price offers that are associated to their wave of arrival, sellers have no additional potentially profitable deviations with respect to those checked in the proof of Proposition 2. □

While the profile considered in the proof of Proposition 8 is such that goods of wave w are sold only to buyers of wave w, we remark that other equilibria are possible, where buyers might choose to make offers to a wave that is different from their own. The profile chosen allows us to repeat many of the arguments used in the proof of Proposition 2.

We briefly comment on timing and welfare. Let waves of new agents arrive every ΔT units of time, and let trading stages take place every Δt units of time. Note that between one trading stage and the subsequent trading stage there can arrive either none, one, or more than one wave, depending on the length of ΔT with respect to Δt. We stress that while ΔT might well be exogenously fixed, Δt could be purposedly shortened in order increase the frequency of trades and, hence, increase total surplus.

Lastly, let us stress that if we let agents know from the beginning the number of goods and the distribution of qualities of the goods as well as the time of entry, then a result similar to that of Proposition 8 can be obtained under full anonymity, i.e., a good's wave is not publicly known so that price offers cannot be conditioned on waves. The reason of this is that buyers would be able to anticipate the correct distribution of qualities from the knowledge of the supply size and of the timing of wave entries. However, to prove this result the model would require further substantial adjustments, which we leave for future research.

7 Discussion

In this section we discuss several aspects and implications of our model. For greater clarity, we organize the discussion in topic-specific paragraphs.

Two features of full trade equilibria The wPBE considered in the proof of Proposition 2 has two important features shared with any other wPBE that leads to full trade in the model presented in section 4. The first feature is the so-called *skimming property*: lower expected quality goods are sold earlier and at a lower price than higher quality goods. The skimming property is a quite common feature in dynamic models of trade under asymmetric information.[10] This feature is inherently related to the strategic situation that we consider: if higher qualities were to be sold first then buyers would have to make high price offers when low quality goods are still on the market, and such high offers would induce the low quality sellers to accept, implying a negative expected payoff for buyers. We note that the skimming property emerges also in all extensions explored in section 6, but some clarifications are needed. In the case of arrival of new sellers, the skimming property holds only between one wave and the next one. In the case of incomplete information on the realized distribution of qualities it can occur that a good of higher realized quality is sold earlier than a good of lower realized quality – as it has a lower expected quality – but in this case the higher quality good is sold at a lower price, in accordance with the skimming property.

The second feature which characterizes full trade equilibria is that all surplus goes to sellers. This feature depends crucially on the fact that buyers always outnumber sellers so that there is competition among buyers whatever the number of goods still on market. We stress that different assumptions about the relative number of buyers and sellers can lead to equilibria where buyers obtain part of the surplus. In the following discussion, we briefly sketch the case of a monopsonistic buyer where her surplus is not zero.

Some assumptions that are not easily relaxable A crucial assumption for the working of our model is that when a good is sold to a buyer, such good is no longer traded in that market. The reason why this assumption is crucial lies in the fact that the size of the informed side of the market only allows to separate lowest qualities from the rest of the goods, implying that if the lowest quality goods do not leave the market once sold then higher quality goods will never be sold. Of course, this does not mean that re-sales cannot take place but requires,

[10] The property is usually stated in models of durable goods monopoly, see for instance Fudenberg et al. (1985) and Gul et al. (1986), but it can also be found in a literature closer to ours, see for instance Janssen and Roy (2002).

as for instance in Hendel et al. (2005), that buyers can at least distinguish goods already sold once from goods never sold before in that market. This extra information plays a role similar to that of non-anonymity with respect to waves in the extension where new sellers arrive sequentially to the market.

Another relevant assumption is the perfect observability of the supply size. Indeed, as indicated by the analysis in Levine and Pesendorfer (1995), if the piece of information on which price offers are conditioned upon is not perfectly observable, then deviations from the full trade equilibrium can become strictly profitable as imperfect observability can hide opportunistic behavior.

Other assumptions that can be relaxed There are many other details of our model which have a non-negligible role, but they can be considered as less important for the gist of our results. For instance, a different bargaining structure might be assumed (we might consider sellers making price offers or sellers and buyers alternating in making offers), price offers might not be a public information (we might use a matching model and only consider bilateral interactions), the relative number of buyers and sellers might be different or buyers might be willing to buy more than one good (for instance, we might consider a monopsonistic buyer, as discussed below). We might also opt for a Walrasian framework instead of considering strategic price setting. Each of these variants should be considered with care, and some technical issues are likely to arise. However, the mechanism that exploits information on the supply size to enforce full trade is still applicable to such variants. For instance, consider the variant where there is a monopsonistic buyer who never exits the market, that makes a take-it-or-leave-it offer to sellers in every time period, and who is willing to buy whatever number of goods, provided the price is low enough. In this variant of our model we can find a set of strategy profiles that are wPBE and lead to full trade, with the only important difference with respect to what shown in this paper that the entire surplus would be earned by the monopsonistic buyer. The different distribution of surplus is not surprising as it clearly depends on the absence of competition among buyers.

Another assumption that might be dispensed with is the homogeneity of buyers' in their valuations of goods. Under heterogeneous valuations the competitive pressure on the buyers' side might decrease, possibly allowing some buyers to obtain a positive surplus. Moreover, as shown by Roy (2012), some issues regarding the optimal sorting of buyers over time can arise. However, full trade equilibria would not be ruled out unless we allow for some buyer type having a reservation price for some quality that is lower than seller's reservation price. We might also let the discount factor vary across agents. This would have the intuitive consequence of creating differences in agents' valuation that evolve over time. We note that the procedure associated with Lemma 1 would work in a similar way and with similar outcomes for what concerns the grouping of sellers over time.

Durables versus non-durables Finally, we want to stress that the goods brought to the market in our model are best interpreted as durable goods, at least when $\delta < 1$. This implicitly points to a potentially interesting issue: how to extend our results to a model where agents discount future payoffs and goods have the standard features of non-durable goods – such as exhaustion upon consumption. Indeed, if the goods brought to the market are non-durable, then some of the arrangements which can reduce the loss due to adverse selection, such as leasing and secondary markets, become ineffective (see Waldman, 2003, for a comprehensive discussion of the functioning of real markets for durable goods). Moreover, delays in exchanges are intrinsically useless as screening devices since the seller's cost of delaying a sale is the same for high quality and low quality goods, because goods do not provide a stream of services over time. On the contrary, our mechanism based on the public knowledge of the size of the informed side of the market does not seem to suffer from this problem, and it may prove to be effective in obtaining both full trade and efficiency, provided that trading stages are made frequent enough. We are aware, however, that some further reasons for the emergence of non-full trade equilibria could arise because of the features of non-durable goods, such as the possibility that a seller consumes her own good at an initial stage of the game in the belief that no acceptable offer will me made in the future.

8 Related literature

Dynamic adverse selection has recently been considered with an emphasis on the efficiency-enhancing role of multiple-stage contracting. In particular, it has been shown that if traders discount future payoffs then the delay of exchanges can fruitfully work as a screening device, potentially allowing market transactions that would never be made in the traditional static framework of Akerlof (1970). Janssen and Roy (2002) show that delaying exchanges of high quality goods can effectively screen qualities and lead to full trade in a Walrasian dynamic setup (see also Janssen and Karamychev, 2002; Janssen and Roy, 2004, for an analysis of exchange cycles). The screening mechanism is based on the fact that sellers with higher-quality goods have greater incentive to wait for higher prices.

Our model is similar to that in Janssen and Roy (2002) under many respects, but there are important differences. One is about the informational structure: in our model both the realized distribution of qualities and the supply size are public information. Another important difference is the mechanism that allows the sorting of qualities over time: the possibility of conditioning price offers on the supply size. The relevance of the latter difference can be appreciated by considering the welfare loss due to the delay of exchanges as the discounting becomes arbitrarily small. In Janssen and Roy (2002) the welfare loss does not vanish since, as the discounting diminishes, the delay required for efficient sorting becomes consequently larger. Instead, as we show in section 5, in our model the welfare loss tends to disappear when the discount factor tends to unity.

A related body of literature has focused on the role of contract types (e.g., leasing rather than selling) and the interaction between new and used good markets. Hendel and Lizzeri (1999) investigate both the role and the existence of markets for used durables under dynamic adverse selection. They find that the used-goods market never shuts down and, more importantly, that distortions due to information asymmetries are smaller than when re-sale is not allowed. Hendel and Lizzeri (2002) study the role of leasing contracts in durable goods markets. They show that, under dynamic adverse selection, leasing contracts for durables can improve welfare, although they remain imperfect tools. Johnson and Waldman (2003) find similar results, also showing that buybacks can further reduce inefficiencies due to asymmetric information.[11] Of particular interest here is the contribution by Hendel et al. (2005) who show that, if consumers observe the number of times a durable good has changed hand, then the combination of multiple secondary markets and endogenous assignment of new goods can completely eliminate the inefficiencies caused by asymmetric information. A common element between our paper and Hendel et al. (2005) is that both provide a solution to dynamic adverse selection problems that does not exploit the delay in exchanges as a screening device but the availability of extra information.

In recent years there has been an upsurge of interest in the relation between the overall informational structure and the likelihood of market failures due to adverse selection. Indeed, there are various aspects of the informational structure that can turn out to be relevant – e.g., the number and the quality of information, the distribution of information among agents – and it is not always straightforward to understand whether they mitigate or exacerbate adverse selection problems. Some basic facts, however, are now established. Kessler (2001) considers a lemons market where the seller can be uninformed with some probability and shows that welfare is non-monotonic in the amount of information on qualities. Levin (2001) shows that greater information asymmetries can reduce the gains from trade, although better information on the uninformed side unambiguously improves trade when demand is downward sloping. Creane (2008) proves that, in a pooling equilibrium where a monopolist sells a product of unknown quality to a group of consumers, welfare

[11]See also Johnson and Waldman (2010) for an extension of the model including moral hazard problems concerning the maintenance choices of owners of non-used-goods.

can be locally decreasing in the fraction of informed consumers. Sarath (1996) considers the issue of information disclosure to market participants and shows that entrusting the choice of (unverifiable) public information quality to traders who benefit from such information leads to inefficiencies, while delegating the choice of information quality to an independent agent who cannot share trade profits results in efficient implementation.[12]

Each of these contribution sheds some light on the role of informational structures in adverse selection problems, but none of them considers the case of dynamic adverse selection. As shown by Hörner and Vieille (2009) a dynamic environment can rise new and specific informational issues. They compare the effects of public versus private price offers in a dynamic adverse selection model with information and payoff structures as in Akerlof (1970). Differently from our model, they consider a situation where there is a unique seller who bargains sequentially with potential buyers until agreement is reached, if ever. Interestingly, Hörner and Vieille (2009) find that trade always eventually occurs when offers are private – i.e., buyers cannot observe past offers made by other buyers – while bargaining often ends at an impasse when offers are public. This happens because, with public offers, buyers can compete intertemporally deterring each other from making offers that lead to trade (see section 6 on this).

Further evidence of the importance of the observability of price offers is provided by Moreno and Wooders (2010) who show that in a dynamic model of decentralized trade all goods entering the market are sold at some stage, notwithstanding asymmetric information on goods quality (see also Blouin, 2003, for a full trade result under decentralized trade). Decentralization of exchanges is modeled with a random matching mechanism where uninformed buyers make a take-it-or-leave-it offer to the informed seller they are matched with. If the seller accepts, then exchange takes place and both agents exit the market. If the seller

[12] Along this line of research there are also contributions which consider screening strategies. For instance, Lewis and Sappington (1994) show that, if the seller can make discriminating offers to the buyer, then the effects of a more informed buyer are ambiguous on the seller's welfare (see also Ottaviani and Prat, 2001).

rejects the offer, then both agents remain in the market and are randomly matched again. Such a matching mechanism entails private price offers, preventing all forms of intertemporal competition (see again section 6 on this). Also in this case, full trade emerges thanks to the cost associated with the delay of exchanges that works as a screening device.[13] Moreno and Wooders (2012) further investigate decentralized trade, showing that it can perform better than centralized trade if the trade horizon is finite and the cost associated with the delay of exchanges is low enough. They also explore the role of taxes and subsidies. Camargo and Lester (2012) study the impact of policies aimed at mitigating the lemon problem in decentralized asset markets, showing that insuring buyers from the risk of buying a lemon can have ambiguos effects.

Decentralized trade in the form of random matching is also considered by Kultti et al. (2012) who study a dynamic adverse selection model where matching between sellers and buyers randomly generates a competitive situation that varies across different matchings. Interestingly, Kultti et al. (2012) find that a stationary equilibrium with equally frequent sales of all qualities is feasible for a narrower range of quality distributions than in Akerlof (1970). This happens when trading frictions – in the sense of Moreno and Wooders (2010) – are large enough because frictions increase buyers' ability to extract positive surplus from low quality goods when they are matched in a favorable competitive situation. In our model this effect is absent since trade is not decentralized and buyers have no chance to find themselves in a favorable competitive situation.

Another paper that deals with the role of public information in markets plagued by dynamic adverse selection is Daley and Green (2012), which investigates the market for financial assets under asymmetric information with the public disclosure, at eSee Boukouras

[13] Moreno and Wooders (2010) refer to "frictions" as both the discounting of future gains and the possible delay in matching with a trading partner. In line with Janssen and Roy (2002), as frictions go to zero payoffs tend to the ones obtained in the single-period competitive equilibrium because, although traders become more patient, delay increases even more.

and Koufopoulos, 2013a, for the design of a mechanism that deals with the case where the realized distribution of types is not a public information; in particular, a two-stage mechanism is suggested that can, as the number of agents grows, approximate the ïňĄrst best.ach trading stage, of an information that affects the future value of the traded asset. Daley and Green (2012) show that, depending on beliefs, in equilibrium there can be immediate trade, no trade at all, or partial trade. There are various differences between our model and this one – e.g., private price offers – but the most important ones regard again which pieces of information are public knowledge.

One recent contribution which is very much related to our paper is that by Boukouras and Koufopoulos (2013b), which focuses on the design of a static direct revelation mechanism in an economy plagued by adverse selection where the realized distribution of types is public information. Importantly, their mechanism obtains both full trade and full surplus. To compare this paper to ours, we can interpret our analysis as focusing on more decentralized decision-making where the public authority has limited possibility of intervention (only providing the information about the supply size and, at most, increasing the frequency of trading stages). Moreover, as shown by our Proposition 7 of Section 6, the information on the supply size can turn out to be enough even if we allow for some degree of incomplete information. See Boukouras and Koufopoulos (2013a) for the design of a mechanism that deals with the case where the realized distribution of types is not a public information; in particular, a two-stage mechanism is suggested that can, as the number of agents grows, approximate the first best.

Finally, our paper is also partly related to the literature on sequential bargaining with one-sided incomplete information (see, among others, Fudenberg et al., 1985; Evans, 1989; Vincent, 1989; Deneckere and Liang, 2006). This literature studies a buyer and a seller who bargain over a unit of an indivisible good, with agents' valuations being a private information. The basic findings of this stream of literature are that the existence of a dynamic dimension together with some form of discounting of future payoffs can lead to intertemporal price discrimination and to the use of waiting times as screening devices (Gul and Sonnenschein, 1988). Thus, efficiency is in general not attained in these models. In particular, as discussed for Janssen and Roy (2002) and others, the welfare loss due to the discounting of future payoffs can persist even if the frequency of bargaining stages is increased since more stages are then needed to effectively screen qualities. In our model too there is price discrimination in full trade equilibria with prices (and qualities sold) increasing over time – a result often referred to as the skimming property. However, since delays are not used as screening devices, by making bargaining stages more frequent the efficient outcome can be reached in the limit.

Appendix

Proof of Lemma 1 We use the following algorithm to build up a function $H(k, G')$ that satisfies the properties in Lemma 1.

Algorithm. The algorithm applies to any $G' \subseteq G$.

Step 0. Define a function τ which assigns to every good i in G' a natural number – interpreted as time – as follows: $\tau(i, G') = ||\{\text{good } \ell \in G' : \ \ell \leq i\}||$. Initially this function simply assigns each good to a time period according to its index, but then, as the algorithm unfolds, some goods will be moved to earlier periods and grouped together with other goods. If $G' \neq \emptyset$, let $H(k, G') = \{\text{good } i \in G' : \ \tau(i, G') = k\}$ be the function assigning to each time period a set of goods to be sold, as given by function τ (of whom $H(k, G')$ might be thought of as a sort of inverse). If $G' = \emptyset$, let $H(k, G') = \emptyset$. Note that function $H(k, G')$ is defined only for values of k which are no lower than 0 and not greater than the maximum value of function τ over G'. Note also that function $\beta(H)$, as defined in section 4 for a non-empty set of goods, can be intepreted as the price offer whose expected value for buyers is zero if all and only the sellers in H accept the offer.

Preliminary check. If $G' = \emptyset$, exit the algorithm.

Step 1. Set k equal to the maximum value of function τ over G', and indicate with i the good with the minimum index in $H(k, G')$.

Step 2. Consider the following inequality, where good $i \in H(k, G')$:

$$\beta(H(k-1, G') \cup \{\text{good } i\}) - s_i \geq \delta \left[\beta(H(k, G')) - s_i\right] .$$

Seller i is for the moment assigned to period k, i.e., $\tau(i, G') = k$. The left-hand side represents the value for seller i of accepting the price offer made in $k-1$ periods from now, while the right-hand side represents the value for seller i of accepting the price offer made in k periods from now.

Update and go back to step 1. If the inequality is satisfied, change the image of i according to τ from k to $k-1$, while leaving the image of other elements of G' unchanged, and then go back to step 1.

Update and go back to step 2. If the inequality is not satisfied and $k \geq 2$, then set k equal to $k-1$ and, subsequently, indicate with i the good with the minimum index in $H(k, G')$, and then go back to step 2.

Exit algorithm condition. If the inequality is not satisfied and $k = 1$, then exit the algorithm.

It is easy to check that a function τ is uniquely selected when exiting the algorithm. As a consequence, a function H is selected as well.

We are now ready to check the properties stated in Lemma 1.

Check of property 1. We show that the inequality holds for $k = 1$, then the result can be extended to any k by iteration. The check trivially holds with equality if $||H(0, G')|| = 1$, so we limit our attention to the case $||H(0, G')|| \geq 2$. We first consider good i having the maximum index in $H(0, G')$, and we apply the algorithm to $G''(1)$. Clearly, good i belongs to $H(0, G''(1))$. We consider the following assignment of goods to time periods: the goods in $G' \setminus G''(1)$ are associated to the first $||G' \setminus G''(1)||$ periods, one per time period, so that good 1 is associated to time period 0, good 2 is associated to time period 1, ..., good

$||G' \setminus G''(1)||$ is associated to time period $||G' \setminus G''(1)|| - 1$, and then the subsets of goods found by means of the algorithm applied to G' are associated to the following time periods, so that $H(0, G''(1))$ is associated to time period $||G' \setminus G''(1)||$, $H(1, G''(1))$ is associated to time period $||G' \setminus G''(1)|| + 1, \ldots$. We note that this particular assignment of goods to time periods is an intermediate state in the algorithm applied to G' (for this it is crucial that good i has the maximum index in $H(0, G')$). Therefore, we know that the algorithm moves good i to a previous time period, and $\beta(\{\text{good } i-1\} - s_i \cup \{\text{good } i\}) \geq \delta(\beta(H(0, G''(1))) - s_i)$. The algorithm will in general entail iterations of steps 1 and 2. Note that at every iteration of step 2, the discounted values of β computed on the subsets of goods associated with k and $k-1$ do not decrease (while the discounted values of β computed on all other subsets of goods are unaffected). So, focussing our attention on good i, we have a chain of inequalities of the form: $H(0, G') - s_i \geq \ldots \geq \delta(\beta(H(0, G''(1))) - s_i)$. Finally, when good i does not have minimum index in $H(0, G')$, the result holds a fortiori because the reservation value s_i is lower or equal (and hence waiting is more costly), and the average value $\beta(H(0, G''(1)))$ is easily seen to be lower or equal as well.

Check of property 2. When the algorithm terminates we have that $\beta(H(\tau(i, G') - 1) \cup \{\text{good } i\}) - s_i < \delta\left[\beta(H(\tau(i, G'))) - s_i\right]$ holds. Hence, $\beta(H(\tau(i, G') - 1)) - s_i < \delta\left[\beta(H(\tau(i, G'))) - s_i\right]$. Note that the latter inequality holds for any seller other than seller i too, and in particular it holds for any seller with an index lower than i that is associated to earlier time periods. So, if we consider good i' belonging to $H(\tau(i, G') - 1)$, it must be that $\beta(H(\tau(i, G') - 2)) - s_{i'} < \delta\left[\beta(H(\tau(i, G') - 1)) - s_{i'}\right]$. Note also that this same inequality holds a fortiori if we replace s_i with $s_{i'}$, implying that $\beta(H(\tau(i, G') - 2)) - s_i < \delta\left[\beta(H(\tau(i, G') - 1)) - s_i\right] < \delta^2\left[\beta(H(\tau(i, G'))) - s_i\right]$. Proceeding along this line of argument, the property is established. □

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