# MULTIPLE CHIRAL BANDS BUILT ON THE SAME MANY-PARTICLE NUCLEON CONFIGURATION IN THE 100 MASS REGION* 

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Multi-particle-plus-triaxial-rotor (MPR) model calculations were performed for chiral partner bands associated with the multi-particle $\pi g_{9 / 2}^{-1} \otimes$ $\nu h_{11 / 2}^{2}$ nucleon configuration in the 100 mass region. Multiple chiral systems were found, but they may not necessarily form well-defined pairs of near-degenerate bands.

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## 1. Introduction

Chirality is a symmetry important in several fields of science. A structure is chiral if its image generated by reflection in a plane cannot be brought to coincide with itself by rotation. In the nuclear structure physics, the concept of chirality was first introduced twenty years ago by Frauendorf and Meng [1]. This symmetry is suggested to occur in nuclei where the angular momentum of the particle (an odd valence nucleon occupying a lower energy orbital in a high- $j$ shell), the hole (an odd valence nucleon occupying a higher-energy orbital in a high- $j$ shell) and the rotational core form an aplanar system, i.e. when the total angular momentum has significant projections along all three nuclear axes [1]. Hence, the three angular momenta

[^0]of the system can be arranged in a right- or in a left-handed systems. The right- and the left-handed systems will generate a pair of $\Delta I=1$ chiral bands which are degenerate, i.e. all properties of the two bands like excitation energies, $B(\mathrm{M} 1)$ and $B(\mathrm{E} 2)$ transition probabilities, quasiparticle alignments, moments of inertia, etc., should be the same [1].

Up to date, chiral candidates showing two-quasiparticle partner bands have been observed in several nuclei in $A \approx 80,100,130$ and 190 mass regions. Chiral partner bands associated with multi-quasiparticle configurations have been found in some odd-mass nuclei in $A \approx 100,130,190$ [2-6] and in an odd-odd nucleus in $A \approx 190$ [7] mass regions.

The existence of multiple chiral partner bands ( $\mathrm{M} \chi \mathrm{D}$ ) with different particle-hole nucleon configurations were proposed in a single nucleus [8], and experimentally confirmed in ${ }^{133} \mathrm{Ce}$ [9] and in ${ }^{78} \mathrm{Br}$ [10], where two pairs of chiral systems built on different nucleon configurations were found.

Contrary to $\mathrm{M} \chi \mathrm{D}$, where the multiple chiral systems differ from each other in their nucleon configurations and correspond to different triaxial deformations, previous calculations performed with a single shell particlerotor model found that more than one pair of chiral bands can exist in a single nucleus with the same particle-hole configuration $[1,11,12]$. The only multiplet of chiral bands built on the same nucleon configuration was discovered in ${ }^{103} \mathrm{Rh}$ [13] and possibly in ${ }^{194} \mathrm{Tl}$ [14]. In both nuclei (i.e. ${ }^{103} \mathrm{Rh}$ and ${ }^{194} \mathrm{Tl}$ ), the multiple chiral systems are associated with more than two quasiparticles and are formed at high excitation energy $(E>2 \mathrm{MeV})$, suggesting that perhaps chiral geometry can be formed easier for multiquasiparticle configurations involving high angular momenta.

The present work studies the formation and properties of multiple chiral bands built on the same many-particle nucleon configuration at high angular momenta in $A \approx 100,130,190$ nuclei. We have used the multi-particle-plus-triaxial-rotor (MPR) model [15-17] of Carlsson and Ragnarsson. The model does not include pairing, but it is expected that the pairing interaction does not affect significantly the single-particle properties of the bands, such as particle alignments which are investigated here. In addition, the calculations were performed for multi-particle configurations, where pairing is reduced due to a number of odd particles near the Fermi levels. Other results from this work were published previously in Refs. [18, 19].

In this paper, we present results on multiple chiral bands in $A \approx 100$ mass region. The energy spectra, the angular momentum components, the $K$-distributions, and the expectation values for the angles between the angular momenta of the odd proton $(p)$, the odd neutrons $(n)$, and the collective rotation $(R)$ were examined for the six lowest energy bands associated with the $\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}$ configuration.

## 2. The MPR calculations in $A \approx 100$

In the 100 mass region, the MPR calculations were performed for chiral bands associated with the three-particle $\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}$ configuration. The quadrupole deformation was set to $\varepsilon_{2}=0.25$, while values of the triaxiality parameter $\gamma=20^{\circ}$ and $30^{\circ}$ were considered, because these nuclear deformation parameters are typical for the chiral bands in the 100 mass region. For instance, the chiral bands in ${ }^{103,105} \mathrm{Rh}[13,20]$ were associated with $\varepsilon_{2} \approx 0.23-0.30$ and $\gamma \approx 15^{\circ}-22^{\circ}$. The ${ }^{109} \mathrm{Sb}$ nucleus was chosen since the Fermi level for the valence odd proton was situated at the highest energy $\pi g_{9 / 2}$ orbitals. The $\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}$ configuration was described in the MPR calculations as nine protons placed among the 10 orbitals of the $\pi g_{9 / 2}$ shell and two neutrons placed among the 12 orbitals of the $\nu h_{11 / 2}$ shell. Standard parameters for the Nilsson potential [21] and irrotational moments of inertia for the core were used. When calculating electromagnetic transition probabilities, standard attenuation of the spin $g$-factor $g_{\mathrm{s}}=0.7 g_{\mathrm{s}, \text { free }}$ for the odd particle(s) was used (attenuation of $60 \%$ or $70 \%$ is considered standard, see, for instance, Refs. $[13,20,22])$. The $g$-factor of the core was taken as $g_{\mathrm{R}}=Z / A$, which is typical for the nuclei in this mass region [13, 20, 22].

## 3. Results and discussion

The calculations yielded several bands associated with the configuration of interest in $A \approx 100$ mass region (see Fig. 1). To test for a possible chiral symmetry, the orientations of the total angular momenta for the calculated six lowest energy bands were examined. It was found that the expectation values for the angles between the angular momenta of the odd proton $(p)$,


Fig. 1. Calculated excitation energies $E$ for the six lowest energy bands for the $\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}$ chiral partner bands at $\gamma=20^{\circ}, 30^{\circ}$ and $\varepsilon_{2}=0.25$. The calculated bands are labelled Bands $1,2,3,4,5$ and 6 according to their excitation energy at low spin.
the odd neutrons ( $n$ ) and the collective rotation are large, suggesting that the vectors form 3-dimensional chiral geometry. Furthermore, the calculated total angular momentum has major contributions from the proton angular momentum on the long axis, from the neutron angular momentum on the short axis, and from the angular momentum of the core on the intermediate axis. Since large angular momenta are involved in this $\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}$ configuration, the Coriolis effects are also considerably stronger. It is well-known that at high spins the Coriolis effects tend to change the orientation of the angular momenta of the particles and holes, increasing the probability for planar contributions, and making the formation of chiral geometry less probable. One can better evaluate the magnitude of the possible non-chiral (i.e. planar) contributions to the wave functions by looking at the distributions of the projections of the total angular momentum along the three nuclear axes, shown in Figs. 2, 3 and 4. Each distribution peaks at the most likely projection of the total angular momentum. Figures 2, 3 and 4 show that the optimal conditions for forming a 3 -dimensional system occur at spins $I=16.5 \hbar$ for Bands 1 and $2, I=15.5 \hbar$ for Bands 3 and $4, I=14.5 \hbar$ for Bands 5 and 6 , where all three distributions have maxima at non-zero projections. The next step was to examine the near-degeneracy of the yrast and yrare chiral systems.


Fig. 2. Calculated probability distributions for the projections of the total angular momenta on the short (s), intermediate (i) and long (l) nuclear axes for Bands 1 and $2\left(\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}\right.$, nature $)$ at $\gamma=30^{\circ}$ and $\varepsilon_{2}=0.25$.


Fig. 3. Calculated probability distributions for the projections of the total angular momenta on the short (s), intermediate (i) and long (l) nuclear axes for Bands 3 and $4\left(\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}\right.$, nature $)$ at $\gamma=30^{\circ}$ and $\varepsilon_{2}=0.25$.

$\mathrm{A} \sim 100, \gamma=30^{0}, \varepsilon_{2}=0.25$



Fig. 4. Calculated probability distributions for the projections of the total angular momenta on the short (s), intermediate (i) and long (l) nuclear axes for Bands 5 and $6\left(\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}\right.$, nature $)$ at $\gamma=30^{\circ}$ and $\varepsilon_{2}=0.25$.

In order to do that, the chiral bands had to be grouped in chiral pairs. It turned out that this is not easy. In order to group the bands in pairs, we tried to determine whether some of them have larger similarity based on the orientation of their angular momenta. In particular, we compared the angular momentum projections of the odd proton, odd neutrons and collective rotation. The projection of the rotational angular momentum along the intermediate axis was found to be similar for the six lowest energy bands (see Fig. 5 (a) and (d)). The $\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}$ configuration considered here involves a contribution of one proton hole with an angular momentum oriented along the long nuclear axis. It was found that the average angular momentum projection of this proton remains similar for all six bands (see Fig. 5 (c) and (f)), while the average angular momenta projection of the neutron particles along the short nuclear axis may differ (see Fig. 5 (b) and (e)).


Fig. 5. (a) and (b) Calculated angular momenta of the core $j_{R}$ along the nuclear intermediate $y$-axis, (b) and (e) neutrons $j_{n}$ along the nuclear short $x$-axis, and (c) and (f) proton $j_{p}$ along the nuclear long $z$-axis for the six lowest energy bands with $\pi g_{9 / 2}^{-1} \otimes \nu h_{11 / 2}^{2}$ configuration, and at $\gamma=20^{\circ}, 30^{\circ}$ and $\varepsilon_{2}=0.25$.

One may examine the energies of the bands shown in Fig. 1 (a) and attempt to group the bands into chiral pairs. We have already analysed the $B(\mathrm{E} 2)$ transition probabilities, which allowed us to identify a few band crossings, as for instance the crossing of Bands 1 and 2, see Fig. 1 (a). Based on the excitation energies Bands 1 and 2 would be assigned as chiral partners forming the yrast chiral system, while Bands $3,4,5$ and 6 which are lying at higher energies would form two yrare chiral systems although the grouping into the yrare chiral pairs might not be obvious.

However, the plot of the projections of the angular momenta of the odd neutrons along the nuclear short axis, Fig. 5 (b), raises some questions. One notices that Bands 1 and 2, the assumed partners in the yrast chiral pair, show the largest difference in the neutron angular momenta projections among all six bands. These two bands exhibit a difference in the angular momenta projections on the short axis of about $2 \hbar$ for $I>16.5$. This casts doubts whether the obvious grouping of Bands 1 and 2 into a chiral pair is correct, and whether the excited chiral systems might not exhibit better chiral symmetry because they might have a closer similarity in their intrinsic structure.

Another example in the 100 mass region is shown in Fig. 5 (e), where the six bands are calculated for $\gamma=20^{\circ}$. In this case, Bands 1 and 2 are again close in energy and would naturally be interpreted as partners forming the yrast chiral system. However, examining the projections of the neutron angular momenta on the short axis suggests a different grouping of the bands; for $I>17.5$ the pairs with similar angular momenta projections are Bands 2 and 3, Bands 5 and 6, and Bands 1 and 4. This is very different from the simple coupling based on their excitation energies. It should be stressed that these calculations show that bands that are near-degenerate in excitation energy may not necessarily have nearly identical intrinsic nature, i.e. the projections of the angular momenta of the odd nucleons may not have the same behaviour as a function of spin.

In Fig. 5 (e), one notices that the projection of the angular momentum of the neutrons on the short axis for Band 1 decreases rather fast at higher spins, while for Band 2 it remains nearly constant. This probably indicates that the nature of these two bands is not very similar, and that their good energy near-degeneracy does not necessarily mean close similarity in their intrinsic nature. Furthermore, this fast loss of neutron alignment along the short axis for Band 1 suggests that the chiral geometry might not be optimal. On the other hand, Band 1 is the lowest energy band; it will be easily observed experimentally and most likely will be assigned into a chiral system. This indicates that it is desirable to observe as many members of the chiral multiplet as possible, and to examine them carefully in order to group them into pairs and discuss whether they form good chiral symmetry systems.

In summary, it was found that the two lowest energy bands (Bands 1 and 2) in some chiral nuclei in the 100 mass region show somewhat different behaviour of the angular momentum projections of the odd neutrons. These bands are often the only bands observed experimentally. Furthermore our calculations show that in order to search for best chiral symmetry, one needs to study not only the two lowest energy bands, but also as many excited bands as possible. It is quite possible that the excited bands will couple into pairs with more similar geometry of the intrinsic angular momenta as a function of spin, and show closer intrinsic structure than the two lowest energy bands. It is also concluded that to couple chiral bands into pairs with similar nature, one needs to consider the projections of the angular momenta along the nuclear axes.

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