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Detection of significant climatic precession variability in early Pleistocene glacial cycles

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7 Abstract

Despite having a large influence on summer insolation, climatic precession is thought to account for little variance in early Pleistocene proxies of ice volume and deep-water temperature. Various mechanisms have been suggested to account for the dearth of precession variability, including meridional insolation gradients, interhemispheric cancellation of ice-volume changes, and antiphasing between the duration and intensity of summer insolation. We employ a method termed Empirical Nonlinear Orbital Fitting (ENOF) to estimate the amplitudes of obliquity and precession forcing in early Pleistocene proxies and their respective leads or lags relative to the timing of orbital variations. Analysis of a high-resolution North Atlantic benthic δ^{18} O record, comprising data from IODP sites U1308 and U1313, indicates a significantly larger precession contribution than previously recognized, with an average precession-to-obliquity amplitude ratio of 0.51 (0.30-0.76 95% confidence interval) in the rate-of-change of δ^{18} O between 3 and 1 Ma. Averaged when eccentricity exceeds 0.05, this ratio rises to an average of 1.18 (0.84-1.53). Additional support for precession's importance in the early Pleistocene comes from its estimated amplitude covarying with eccentricity, analyses of other benthic δ^{18} O records yielding similar orbital amplitude ratios, and use of an orbitally-independent timescale also showing significant precession. Precession in phase with Northern Hemisphere summer intensity steadily intensifies throughout the Pleistocene, in agreement with its more common identification during the late Pleistocene. A Northern Hemisphere ice sheet and energy balance model run over the early Pleistocene predicts orbital amplitudes consistent with observations when a cooling commensurate with North Atlantic sea surface temperatures is imposed. These results provide strong evidence that glaciation is influenced by climatic precession during the late Pliocene and early Pleistocene, and are consistent with hypotheses that glaciation is controlled by Northern Hemisphere summer insolation.

- ⁸ Keywords: Milankovitch, Pleistocene, precession, glacial cycle, spectral analysis, orbital
- 9 forcing.

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¹⁰ 1. Introduction

Adhémar (1842), Croll (1864), and Murphy (1869) each argued in turn that ice ages are initiated by 11 changes in the duration and intensity of the seasons. Adhémar (1842) suggested that long winters 12 favor the growth of ice sheets, while Croll (1864) proposed that cold winters are the most favorable, 13 and Murphy (1869) that cool summers, unable to melt the previous winter's snowfall, are responsible. 14 All three yet agreed that "only one hemisphere, the northern or the southern, has a glacial climate 15 at the same time" (Murphy, 1869), citing the important influence of climatic precession on the 16 duration and intensity of seasons. Climatic precession, whose influence is anti-phased between the 17 hemispheres, arises from the precession of Earth's rotational axis, at a period of 25.7 ky, which when 18 combined with apsidal precession gives an overall period that ranges between 18 and 24 ky. Its effect 19 on climate is modulated by the orbital eccentricity. 20

Milankovitch (1941) rejected the view that hemispheres were alternately glaciated, arguing instead 21 that a Northern Hemisphere glacial advance has global consequences by changing the planetary 22 albedo. Using more accurate calculations of variations in obliquity – which produce in-phase changes 23 in insolation across both hemispheres – than had been available to his predecessors, Milankovitch 24 predicted the timing of ice ages from variations in the summer caloric half-year, or the insolation 25 averaged over the half of the year that maximizes the resulting value, on which precession and 26 obliquity have nearly equal influence. A number of studies have confirmed Milankovitch's hypothesis 27 in the late Pleistocene; for example, sea-level highstands demonstrate variability consistent with 28 Milankovitch's hypothesized precession phase (e.g. Broecker et al., 1968), and other climate proxies 29 feature obliquity and precession amplitudes consistent with Milankovitch's hypothesized forcing 30 (Hays et al., 1976; Imbrie et al., 1992). But whether Milankovitch's theory, or indeed any theory 31 calling upon precession as causing fluctuations in ice volume, also holds for the early Pleistocene and 32 late Pliocene is much less clear, because δ^{18} O measured in benthic foraminifera has been interpreted 33 to vary almost exclusively at the obliquity period of 41 ky during this earlier epoch, with apparently 34

³⁵ negligible variance at precession periods (Ruddiman et al., 1986).

Several explanations have been proposed for the 41-ky ice ages. Raymo and Nisancioglu (2003) 36 proposed that the meridional insolation gradient, which varies primarily at the obliquity period, 37 could drive heat and moisture fluxes that would control the ice sheet mass balance, but this would 38 require that the ice sheet be equally sensitive to lower-latitude and local insolation. Loutre et al. 39 (2004) asked whether the mean annual insolation, which features no precession variance, could 40 drive long-term climate variability, and also proposed a role for the meridional insolation gradient 41 in driving poleward moisture transport. Huybers (2006), modifying the caloric half-year model of 42 Milankovitch (1941), argued that the influence of precession-induced changes in insolation intensity 43 were canceled by an opposing change in summer duration. The degree of precession cancellation 44 would depend on several factors, including the average global temperature and the meridional extent 45 and thickness of the ice sheet (Huybers and Tziperman, 2008). 46

Contrasting with explanations excluding precession from the forcing, Raymo et al. (2006), extending 47 the model of Murphy (1869), argued that local summer intensity does control ablation, but its anti-48 phased response between the hemispheres results in a cancellation of the precession signal in δ^{18} O 49 records. This would require that the Southern Hemisphere contribution to changes in δ^{18} O nearly 50 balance and be well-mixed with that of the Northern Hemisphere despite the Northern ice sheets 51 lying at comparatively low latitude (Alley, 1991), with an ablation margin exposed to insolation 52 more strongly influenced by precession. The foregoing mechanisms are not mutually exclusive, and 53 Tabor et al. (2015) suggested that a combination of factors including eccentricity modulation of 54 the precession amplitude and obliquity's longer period relative to precession increase the obliquity 55 response relative to precession. 56

⁵⁷ Evidence for the presence of precession in early Pleistocene glacial cycles is not entirely absent. An ⁵⁸ analysis of globally-distributed benthic δ^{18} O records suggested that the amplitude of climate vari-⁵⁹ ability at precession frequencies follows the long-term amplitude modulations of climatic precession

throughout the Plio-Pleistocene (Lisiecki and Raymo, 2007), and precession has long been identified 60 in early Pleistocene planktonic δ^{18} O records for use in the construction of age models (Shackleton 61 et al., 1990). A further important finding is that not all early Pleistocene glacial cycles are symmetric 62 (Ashkenazy and Tziperman, 2004; Lisiecki and Raymo, 2007), with rates of deglaciation appearing 63 to exceed those of glaciation in some cycles. This asymmetry suggests a nonlinear response to or-64 bital forcing and is qualitatively consistent with late Pleistocene climate variability, albeit of smaller 65 magnitude (Huybers, 2007). Because precession does not change the net annual insolation, only re-66 distributing it across seasons (Rubincam, 1994), its appearance in climate proxies must result from a 67 nonlinearity in the climate response (Rubincam, 2004) or how it is recorded (Huybers and Wunsch, 68 2003). Precession might therefore be expected to accompany apparently nonlinear variability during 69 the early Pleistocene, just as it does similar asymmetric variability in the late Pleistocene. These 70 factors motivate our revisiting the substantiality of precession forcing in early Pleistocene glacial 71 cycles. 72

73 2. Data

⁷⁴ We evaluate a composite, high-resolution benthic δ^{18} O record from the North Atlantic ocean span-⁷⁵ ning 3 Ma to the present, referred to as L2H19 (Figure 3a). L2H19 comprises data from IODP Site ⁷⁶ U1313 between 3 Ma and 2.42 Ma and from Site U1308 thereafter. The combined record features an ⁷⁷ average temporal resolution of 0.3 ky from 1.62 Ma to present, 1.3 ky from 2.42 to 1.62 Ma, and 2.1 ⁷⁸ ky from 3.0 to 2.42 Ma. L2H19 complements analysis of an average over many records, or a stack, ⁷⁹ because of its high resolution and because small differences in the age models of individual records ⁸⁰ may smooth δ^{18} O variability when records are averaged (Huybers and Wunsch, 2004).

The choice to patch U1313 between 3 Ma and 2.42 Ma was made for purposes of having a complete and undistorted record. Although U1308 extends to beyond 3 Ma, it contains a hiatus between 2.6 and 2.65 Ma that skips Marine Isotope Stages (MIS) G1 and G2, and low sedimentation rates or a short hiatus distorts MIS 97 and 98 (Hodell and Channell, 2016). U1313 provides a more complete record in the late Pliocene (Bolton et al., 2010), and its benthic δ^{18} O values closely align with those of U1308 where they overlap except with minor amplitude mismatches from 2.7 to 2.9 Ma. Both records are drilled at similar water depths, with U1313 at 3426m and U1308 at 3882m, and the shipboard spliced composite sections were checked for completeness. U1313 and U1308 are spliced together by taking mean values across an overlap of 10 ky at 2.42 Ma (Figure A1).

⁹⁰ Five other benthic δ^{18} O records that extend into the early Pleistocene are also evaluated, and their ⁹¹ characteristics are listed with those of L2H19 in Table 1. Two records, one from DSDP Site 607 ⁹² and the second from ODP Sites 980 and 981, are from the North Atlantic. The three other records, ⁹³ from ODP Sites 677, 846, and 849, are from the equatorial Eastern Pacific.

To better ensure that our results neither diminish nor over-represent orbital variability, two separate
chronologies are considered, based on orbitally-tuned and depth-derived records (Lisiecki and Raymo,
2005; Huybers, 2007). Details of depth-age relationships are given in Appendix A.

⁹⁷ 3. Indeterminate precession estimates from spectral analysis

98 3.1. Method

⁹⁹ Following the convention of past studies (Hays et al., 1976; Imbrie and Imbrie, 1980), we first attempt ¹⁰⁰ to quantify orbital forcing of early Pleistocene glacial cycles using spectral analysis, evaluating the ¹⁰¹ rate-of-change of δ^{18} O, i.e., $d\delta^{18}$ O/dt. Analysis of the rate of change, rather than the magnitude, ¹⁰² follows from major ice sheets having long response times (Weertman, 1964) and is supported by co-¹⁰³ herence between $d\delta^{18}$ O/dt and summer-integrated energy in the obliquity band (Roe, 2006; Huybers, ¹⁰⁴ 2006). Analyzing $d\delta^{18}$ O/dt also permits for better separating sensitivity to orbital forcing from the

Age model				Orbitally-t	uned, 3-1 Ma	Depth-Derived, 2-1 Ma		Original Reference(s)
Record	$\sigma,$ ‰, 2-1 Ma	Lat., $^{\circ}$	Lon., $^{\circ}$	Age, Ma	Δt , ky	Age, Ma	Δt , ky	
L2H19	0.37	$50N^*$	24^*W	3.00	0.97	2.0	0.48	Channell et al. (2016); Hodell and Channell (2016);
								Obrochta et al. (2014) ; Bolton et al. (2010)
LR04	0.28	-	-	3.00	2.35	-	-	Lisiecki and Raymo (2007)
H07	0.25	-	-	-	-	2.00	1.00	Huybers (2007)
DSDP 607	0.35	41N	33W	3.00	3.89	2.00	3.55	Ruddiman et al. (1989)
ODP 677	0.31	1N	84W	2.60	1.74	1.95	1.63	Shackleton et al. (1990)
ODP 846	0.29	3S	91W	3.00	2.36	1.77	2.71	Mix et al. (1995a)
ODP 849	0.29	0	111W	3.00	3.20	2.00	3.49	Mix et al. (1995b)
ODP 980/981	0.34	55N	17W	3.00	2.08	1.95	2.85	Oppo et al. (1998); McManus et al. (1999);
								Mc Intyre et al. (1999) ; Flower et al. (2000) ;
								Raymo et al. (2004)

Table 1: Properties of marine sediment cores. Listed are the standard deviation of δ^{18} O between 2 and 1 Ma, location, estimated age of the oldest sample, and mean sampling interval. Age and sampling interval are given as available for both orbitally-tuned and depth-derived age models. The coordinates of L2H19 are listed as those of IODP Site U1308, but note that between 3 and 2.42 Ma, data are from Site U1313.

influence of the relative periods of orbital variations, with previous studies noting that obliquity's
longer period amplifies it relative to precession when integrating a model that relates orbital forcing
to the rate-of-change of ice volume (Huybers and Tziperman, 2008; Tabor et al., 2015).

 $d\delta^{18}O/dt$ is constructed by taking the first-difference of $\delta^{18}O$ then dividing by the time between 108 samples, after linearly interpolating to an even spacing of 0.5 ky. Although some samples in L2H19 109 are taken at smaller intervals than this, repeating the analysis using a spacing of 0.25 ky leads to 110 only minor differences in estimates. Power spectral density (PSD) is computed using Thomson's 111 multi-taper method with three tapers (Percival and Walden, 1993). A noise floor is estimated by 112 fitting a power law to the PSD (see Figure 1 and Appendix B). The variance attributed to obliquity 113 and precession is the net integral between the power spectrum and the noise floor in their respective 114 frequency bands, defined to be $1/41 \pm 1/125 \text{ ky}^{-1}$ and $1/21 \pm 1/125 \text{ ky}^{-1}$, including negative values 115 where the noise floor exceeds the power spectrum. The integrals are taken as a fraction of the total 116 area under the power spectrum, then multiplied by the time series' variance. The square root of this 117 result is reported in amplitude units of 0.001 % ky⁻¹, or meg⁻¹ky⁻¹. 118

¹¹⁹ Nearly all Pleistocene records feature a red background continuum, with δ^{18} O typically following ¹²⁰ a spectral power law ranging from 0.8 to 2 that breaks down at or above 1/100 ky⁻¹ (Wunsch, ¹²¹ 2003). A process following a power law of 2 may be produced by a random walk in which its value ¹²² at the next time-step is the sum of its current value and a random number drawn from a Gaussian ¹²³ distribution. Differencing such a record therefore whitens its spectrum (Bracewell, 1986), consistent ¹²⁴ with the approximately flat noise floor in $d\delta^{18}$ O/dt (Figure 1).

The 95% confidence interval for the power spectrum is estimated from the approximate χ^2 -distribution of the spectral estimator (Percival and Walden, 1993), with further details given in Appendix B. To obtain approximate upper 95% confidence bounds for orbital amplitudes, the procedure for calculating orbital amplitudes is applied to the upper confidence bound of the power spectrum, and vice-versa for the lower bound. For the power spectrum area used as the denominator, power is taken as the central estimate, except in the orbital band where it is taken as the upper or lower 95% confidence bound of the power spectrum. Where a ratio of precession to obliquity amplitude is reported, the upper and lower 95% intervals are approximated by dividing the upper and lower precession estimate by the central obliquity estimate.

¹³⁴ 3.2. Spectral estimates of orbital forcing amplitudes

We divide records into segments representing 3-2 Ma and 2-1 Ma to allow direct comparison with depth-derived records, which end at or near 2 Ma. Spectral amplitude estimates do not rule out the possibility of significant early Pleistocene precession forcing, but substantial uncertainties preclude a confident interpretation of results.

Averaged from 3-2 Ma, the estimated precession amplitude in the orbitally-tuned L2H19 is 6.8 (0.0-16.8 95% confidence interval) $meg^{-1}ky^{-1}$, compared with an obliquity amplitude of 30.7 (22.9-42.1) $meg^{-1}ky^{-1}$, giving a precession-to-obliquity amplitude ratio of 0.22 (0.00 - 0.55). Over 2 to 1 Ma, the precession amplitude averages a larger 24.0 (13.4-38.6) $meg^{-1}ky^{-1}$ and, unlike in the previous million years, is statistically significant. A much smaller increase in the obliquity amplitude of 4.4 $meg^{-1}ky^{-1}$ is observed between the two time periods, such that the precession-to-obliquity amplitude ratio increases to 0.68 (0.38-1.10) in the later interval.

Spectral estimates in other records follow a similar pattern to L2H19 (Table 2). No orbitally-tuned records give a significant precession amplitude estimate between 3 and 2 Ma. Between 2 and 1 Ma, the presence of precession is more ambiguous, with four out of six tuned records – DSDP 607, ODP 846, ODP 849, and the LR04 stack – giving significant precession amplitude estimates. When evaluating records on the depth-derived chronology between 2 and 1 Ma, the importance of precession is even less clear, with only two records, L2H19 and the LR04 stack, giving significant spectral estimates of precession.



Figure 1: Power spectra of $d\delta^{18}O/dt$. Columns are organized by the longitude associated with each record from west to east, and rows according (bottom). The 95% confidence interval is indicated as estimated for individual spectral estimates (red) and for summing energy across the orbital band (blue; see Appendix B). Bands over which obliquity and precession power are summed (gray shading) are at $1/41 \pm 1/125$ ky⁻¹ and $1/21 \pm 1/125$ ky⁻¹. All twenty-one obliquity-band spectral power estimates are significant at the 95% confidence level (indicated by black stars), but only seven indicate correspondingly significant climatic precession. In the last column, the upper two panels refer to the LR04 stack, and the bottom panel refers to the H07 to spectra using orbitally-tuned timescales for 3-2 Ma (top), orbitally-tuned timescales for 2-1 Ma (middle), and depth-derived timescales for 2-1 Ma stack.

Further ambiguity arises from precession's upper bound being poorly constrained. While only 7 of the 21 time-series evaluated give significant spectral estimates of precession, the upper 95% confidence limit on the precession estimate is greater than obliquity's lower 95% limit in four tuned records between 3 and 2 Ma, every tuned record between 2 and 1 Ma, and two records on the depth-derived timescale between 2 and 1 Ma, such that it is not always possible to rule out precession having a larger amplitude than obliquity.

It could be that Fourier-based estimation is sub-optimal for determining orbital amplitudes on ac-159 count of several basic assumptions being violated. The precession signal does not appear stationary, 160 as is implicitly assumed, with an apparently growing amplitude over the Pleistocene. Furthermore, 161 major frequency modulations associated with precession, such as those induced by variations in 162 eccentricity's amplitude (Burns, 1976), lend ambiguity to the energy band within which the preces-163 sion signal is contained. It is also clear from Table 2 that L2H19's orbital amplitude estimates are 164 substantially larger than for other records over 2-1 Ma, and possible reasons for orbital amplitude 165 differences between records are addressed in sections 4.2 and 7.1. 166

¹⁶⁷ None of these factors necessarily explain why some records give statistically significant precession ¹⁶⁸ where others on the same chronology or across the same time interval do not. On one hand, we are ¹⁶⁹ concerned that assessing multiple records can lead to false positives and, on the other, that the high ¹⁷⁰ degree of variability in spectral estimates implies that the presence of significant precession cannot ¹⁷¹ be ruled out. These considerations prompt our exploring a complimentary method for assessing ¹⁷² whether precession variability is present in the early Pleistocene.

4. Empirical Nonlinear Orbital Fitting (ENOF)

¹⁷⁴ In developing a method that is complimentary to spectral analysis, we seek to utilize precession's ¹⁷⁵ known frequency and amplitude modulations and to allow for non-stationarity in the amplitude of

													recession	(9.7-27.0)	ı	(4.1-10.5)	(2.3-13.4)	(1.3-12.6)	(5.0-14.7)	(2.5-13.0)	(0.4 - 16.8)	0		~	
										2-1 Ma	ENOF	Obliquity P ₁	2.7 (22.2-43.2) 17.6	ı	1.0 (19.8-28.3) 6.8	3.4 (20.8-36.6) 6.4	1.8 (16.4-33.4) 5.7	2.9 (17.6-28.9) 9.2	3.0 (16.7-29.7) 6.9	8.2 (9.1-28.7) 7.2	mate 95% confidence		tere are equivalent to		
							Depth-derived,	ectrum	Precession	23.2 (11.6-38.1) 32		8.2 (2.5 - 13.9) 24	7.7 (0.0 - 19.7) 28	$0.0 (0.0 - 15.8) 2_4$	10.0 (0.0 - 21.8) 22	$0.0\ (0.0 - 12.4)$ 25	9.6 (0.0 - 25.6) 1	values give approxi)	estimates reported h					
												Power Sp	Obliquity	32.9 (21.6 - 49.3)	ı	26.5 (21.9 - 31.4)	30.0 (21.1 - 41.4)	26.7 (17.7 - 40.0)	27.0 (18.8 - 39.3)	24.5 (17.3 - 33.5)	19.8 (6.0 - 34.0)	stimates. Bracketed		nce. Power spectral	
		OF	Precession	8.6 (3.9-14.7)	7.0(3.4-11.6)	8.3 (3.5-14.1)	3.8(0.0-12.3)	7.8(2.9-14.5)	5.6(0.7-12.0)	10.4 (5.3-17.0)		OF	Precession	22.4 (14.0-31.7)	15.8 (10.2 - 21.9)		19.6 $(13.8-26.1)$	$16.3 \ (8.9\text{-}24.9)$	$13.6 \ (7.6\text{-}20.5)$	13.7 (8.6-19.5)	10.5(2.0-20.9)	sctral and ENOF es		statistical significar	
1 0 0 11	ed, 3-2 Ma	EN	Obliquity	31.0 (24.1-38.1)	23.5 (18.5-28.6)	24.0 (18.4-30.1)	21.2 (12.0-31.3)	21.5 (14.9-28.6)	16.4 (9.4-23.6)	25.0 (17.9-32.4)	ed, 2-1 Ma	EN	Obliquity	34.5(24.2-45.1)	25.4 (19.2 - 31.9)	1	28.8 (22.1-36.0)	$25.5 \ (16.6-34.8)$	22.6 (16.6-29.5)	23.9 (17.9-30.3)	17.7 (8.4 - 28.5)	ributions from spe	•	old font indicates s	
	Urbitally-tun	ectrum	Precession	6.8(0.0-16.8)	$2.7\ (0.0\text{-}10.8)$	$6.5\ (0.0-16.1)$	0.0(0.0-22.2)	6.9(0.0-17.1)	5.8(0.0-16.2)	9.3(0.0-19.7)	Orbitally-tun	ectrum	Precession	24.0 (13.4 - 38.6)	15.8 (9.4 - 24.2)	1	20.8 (13.3 - 30.9)	14.2(0.0 - 25.8)	12.6 (2.3 - 22.3)	13.8 (6.6 - 22.1)	12.2 (0.0 - 26.3)	nd precession conti	4	neg^{-1} ky ⁻¹ , and be	
		Power Sp	Obliquity	30.7 (22.9 - 42.1)	24.7 (19.1 - 31.8)	$27.2 \ (19.9 - 36.4)$	23.3 (12.0 - 40.3)	23.7 (16.8 - 33.6)	14.6 (6.4 - 23.9)	$24.9 \ (17.0 - 35.4)$		Power Sp	Obliquity	35.1 (24.1 - 52.0)	26.9 (20.2 - 36.2)		28.7 (20.5 - 39.8)	27.5 (19.4 - 39.8)	25.4 (18.0 - 36.0)	24.7 (17.9 - 33.4)	19.4 (9.7 - 32.1)	udes of obliquity a	1	tes are in units of 1	
	Age model	Method	Forcing term	L2H19	LR04	DSDP 607	ODP 677	ODP 846	ODP 849	$ODP \ 980/1$	Age model	Method	Forcing term	L2H19	LR04	70H	DSDP 607	ODP 677	ODP 846	ODP 849	$ODP \ 980/1$	Table 2: Amplit	•	intervals, estima	

Table 2: Amplitudes of obliquity and precession contributions from spectral and ENOF estimates. Bracketed values give approximate 95% confidence
intervals, estimates are in units of $meg^{-1}ky^{-1}$, and bold font indicates statistical significance. Power spectral estimates reported here are equivalent to
those shown in Figure 1. 95% confidence intervals are narrower for ENOF than for spectral analysis, such that ENOF estimates differ from spectral
stimates in indicating significant precession contributions.

the precession response over time. The method we propose below is similar to spectral analysis, which can be represented as a least-squares fit of sinusoids to a time-series (e.g. Bracewell, 1986), but modified to be a least-squares fit of the orbital elements themselves that is sequentially fit using a sliding window.

180 4.1. Method

¹⁸¹ Previous studies have represented the orbital influence on glacial mass balance using a flexible ¹⁸² index consisting of a weighted sum of obliquity and climatic precession (e.g. Imbrie and Imbrie, ¹⁸³ 1980), where selecting appropriate weights and climatic precession phase allows for reproducing ¹⁸⁴ most parameterizations of insolation forcing. Equating this index to the rate-of-change of δ^{18} O has ¹⁸⁵ been shown to produce a good fit in the late Pleistocene (Imbrie and Imbrie, 1980; Roe, 2006) for ¹⁸⁶ the same reasons given in section 3.1. Following these previous studies, we posit a functional form ¹⁸⁷ for the orbital forcing of ice sheet mass balance as,

$$\frac{\mathrm{d}V}{\mathrm{d}t} = A_o(t)\varepsilon(t - \Delta t_o) + A_p(t)e\sin(\varpi(t - \Delta t_p)) + \eta(t).$$
(1)

The rate-of-change of ice volume, dV/dt, is represented as a combination of obliquity, ε , and climatic 188 precession forcing. Climatic precession has an amplitude controlled by orbital eccentricity, e, and 189 phase equal to the longitude of perihelion, ϖ , taken relative to the fixed vernal equinox. A_p and 190 A_o are the amplitudes respectively associated with obliquity and precession, Δt_o and Δt_p are the 191 respective time offsets, and all four parameters are permitted to vary over time. Time offsets arise 192 from age model error, the seasonal sensitivity of δ^{18} O, and lags in the response to orbital forcing. 193 Noise and processes not otherwise accounted for are represented by η . The values used for e, ϖ , and 194 ε are from Berger and Loutre (1991). 195

Taking ϖ relative to the vernal equinox gives a model in which $\Delta t_p = 0$ implies that maximum rates of melting and ocean warming occur when perihelion aligns with Northern Hemisphere summer solstice. Initial estimates for A_p , A_o , Δt_p and Δt_o are all set to zero, consistent with the rateof-change of δ^{18} O being in phase with peak Northern Hemisphere summer intensity, but Δt_p and Δt_o could allow for precession variability of essentially any phase because they are allowed to vary within ± 10 ky. No parameter is needed for a direct offset in the precession angle ϖ , because the time offset Δt_p captures virtually all of the structure that such a parameter might introduce. Note that a positive value of Δt_p or Δt_o indicates that observations lag orbital variations, and a negative value indicates a lead.

Values of A_p , A_o , Δt_p and Δt_o are estimated by a nonlinear least-squares fit of equation 1 to $d\delta^{18}O/dt$ 205 over a box-car window of length τ ky, centered on a time t. The rate-of-change is constructed by 206 a first-difference of δ^{18} O after linear interpolation to even spacing of 0.5 ky. As with spectral 207 analysis, using a finer interpolation scheme for L2H19 only produces small changes in estimates. 208 Fitting is undertaken using a Matlab optimization routine which uses a trust-region algorithm for 209 parameter estimation (see Appendix C). Before evaluation, $d\delta^{18}O/dt$ is smoothed with a 6 ky filter 210 to minimize the influence of noise in parameter estimation. Note that smoothing a record resolved at 211 frequencies well above that of climatic precession is expected to minimally influence orbital energy, 212 whereas coarsely sampling a sediment core instead aliases unresolved high-frequency energy to lower 213 frequencies (Pisias and Mix, 1988). Each window is shifted relative to the previous by k ky, and 214 the fit repeated, until the end of the time-series is reached. Each t is therefore associated with τ/k 215 estimates for a parameter, which are averaged to obtain a single time-continuous estimate. Unless 216 otherwise noted, we use $\tau = 50$ ky and k = 5 ky. Choosing k to be substantially less than τ permits 217 for sufficient continuity to obtain a smooth estimate of the time-evolution of orbital amplitudes. To 218 obtain time-averaged amplitudes for each of precession and obliquity, the square root of the variance 219 of the first and second terms in equation 1, respectively, is taken. 220

Like in spectral analysis, the presence of background noise causes ENOF to overestimate orbital variance. The bias is a function of the signal-to-noise ratio (SNR), and is estimated from a Monte ²²³ Carlo approach in which ENOF is run against repeated realizations of synthetic time-series contain²²⁴ ing both orbital variance and noise. The bias correction, analogous to subtracting the noise floor in
²²⁵ spectral analysis, is made by linearly interpolating the noise fraction of the total amplitude against
²²⁶ the mean bias for that fraction, as estimated from synthetic tests (see Appendix D).

²²⁷ 95% parameter confidence intervals for A_p and A_o are approximated under an assumption of asymp-²²⁸ totic normality of the estimator, with further information given in Appendix C. Similarly to spectral ²²⁹ analysis, when reporting a ratio of precession to obliquity amplitudes, the accompanying confidence ²³⁰ interval comprises the lower and upper 95% confidence bounds for precession divided by the central ²³¹ obliquity estimate. Because estimates of Δt_p and Δt_o are strongly influenced by age model error ²³² and bound-constrained to avoid overfitting, confidence intervals are not expected to be informative ²³³ for these parameters and are therefore not estimated.

²³⁴ 4.2. Time-averaged orbital forcing amplitudes and time offsets

²³⁵ Unlike with spectral analysis, ENOF estimates of the precession amplitude in L2H19 are significant ²³⁶ throughout the full 3-1 Ma period. Between 3 and 2 Ma, ENOF gives a precession amplitude of 8.6 ²³⁷ (3.9-14.7) meg⁻¹ky⁻¹ and an obliquity amplitude of 31.0 (24.1-38.1) meg⁻¹ky⁻¹, which together ²³⁸ yield a precession-to-obliquity ratio of 0.28 (0.13-0.47). The precession estimate rises to 22.4 (14.0-²³⁹ 31.7) between 2 and 1 Ma, whereas the obliquity amplitude increases by only 3.5 meg⁻¹ky⁻¹, such ²⁴⁰ that their ratio rises to 0.65 (0.41-0.92) between 2 and 1 Ma (Table 2).

The apparent detection of significant precession in the 3-2 Ma interval is supported by ENOF estimates in other records, with all but one orbitally-tuned record giving significant precession estimates over this time period (Table 2). Confidence in the significance of precession in the early Pleistocene is bolstered by a finding that ENOF precession estimates are significant in every record on both the orbitally-tuned and depth-derived chronologies between 2 and 1 Ma. Note, however, that central estimates of the precession-to-obliquity amplitude ratio average 50% lower in depth-derived records



Figure 2: Precession amplitude relative to obliquity in early Pleistocene benchic $d\delta^{18}O/dt$, as estimated by ENOF (red) and spectral analysis (blue), in (a) orbitally-tuned records for 3-2 Ma, (b) orbitally-tuned records for 2-1 Ma, and (c) depth-derived records for 2-1 Ma. Circles represent best estimates, and bars represent the 95% confidence range. Estimates from ice sheet model results described in section 6 are also included. The label for the composite records, "LR04 (a,b) / H07 (c)", refers to the LR04 stack in panels (a) and (b), and the H07 stack in panel (c).

relative to orbitally-tuned records (Figure 2b,c). Differences in precession estimates between age
models is further discussed in section 7.1.

As is the case with spectral analysis, L2H19 features significantly larger orbital amplitudes than other records (see Table 2). More generally, orbital amplitudes are systematically higher in North Atlantic records than in Pacific records, and this may partly reflect greater variance from deep-water temperature in North Atlantic benthic δ^{18} O records (Duplessy et al., 1980; Waelbroeck et al., 2002). There does not appear to be a substantial difference in the relative sensitivity to precession versus obliquity between basins, with the ratio between the two averaging 0.64 for North Atlantic and 0.60 for Eastern Pacific orbitally-tuned records between 2 and 1 Ma.

The estimated time offsets, Δt_p and Δt_o , serve to quantify the timing of orbitally-induced variations in δ^{18} O. Several factors make it unwise to place much weight on individual time offset estimates, however, including (i) that age uncertainties are up to half the period of precession (Huybers, 2007), (ii) that there remains significant uncertainty regarding what controls the precession phase in global climate records (e.g. Kawamura et al., 2007), (iii) that orbitally-tuned records have previously been aligned to a Northern Hemisphere ice-volume curve, and (iv) that the ENOF model is centered about a Northern Hemisphere summer forcing. Despite these factors, agreement in Δt_p and Δt_o across several records and age models would provide confidence that the estimated precession variability represents a consistent set of physical processes.

Mean Δt_p is no more than ± 1 ky in records on both age models and over both 3-2 Ma and 2-1 Ma. 265 In orbitally-tuned records, Δt_p averages -0.2 ky between 3 and 2 Ma, and 0.1 ky between 2 and 1 266 Ma, and in depth-derived records, Δt_p averages 0.5 ky between 2 and 1 Ma. Similarly, mean Δt_o 267 does not exceed 3 ky in any record on either age model, averaging 0.5 ky over 3-2 Ma and 1.6 ky 268 over 2-1 Ma in orbitally-tuned records, and 2.4 ky over 2-1 Ma in depth-derived records. Given age 269 uncertainties of ± 6 ky for tuned records (Lisiecki and Raymo, 2005) and ± 10 ky for depth-derived 270 records (Huybers, 2007), Δt_p and Δt_o in $d\delta^{18}O/dt$ may be interpreted as being indistinguishable 271 from in-phase with the intensity of Northern Hemisphere summer. 272

The algorithm may occasionally fail to adequately search the parameter space, returning a Δt_p value equal to its initial estimate, though this may also occur if the initial estimate is optimal. Excluding values of Δt_p that are equal to the initial estimate before averaging does not lead in qualitatively different results, with all records giving a mean Δt_p of less than 2 ky and a mean Δt_o less than 3.5 ky.

²⁷⁸ 5. Detailed analysis of orbital forcing in L2H19

An analysis of the temporal variability and trends in the estimated precession component of benthic δ^{18} O would further inform the physical origin of the signal and whether it can confidently be attributed to precession forcing. Of the records we have evaluated, L2H19 has the highest sampling resolution by a wide margin (Table 1) and features high sedimentation rates through the Pleistocene (Channell et al., 2016; Hodell and Channell, 2016). In light of these exceptional properties, we are
motivated to more closely evaluate Pleistocene precession variability using L2H19.

²⁸⁵ 5.1. Further evidence for precession forcing in early Pleistocene benthic $\delta^{18}O$

A common method for testing whether precession-band variance in a δ^{18} O record is of physical 286 origin is to pass the record through a narrow band-pass filter admitting only precession frequencies, 287 typically $1/18 \text{ ky}^{-1}$ to $1/24 \text{ ky}^{-1}$, and measure the correlation of the filtered signal's amplitude with 288 that of eccentricity (e.g. Shackleton et al., 1990). Filtering can produce eccentricity-like amplitude 289 modulations in orbitally-tuned records even when no relationship with eccentricity exists (Huybers 290 and Aharonson, 2010), though methods have been proposed to overcome this problem. Zeeden et al. 291 (2015) proposed that filtering records using a wider band, e.g. 1/8 to 1/35 ky⁻¹, then calculating the 292 instantaneous amplitude of the filtered time-series using the Hilbert transform and smoothing the 293 resulting curve, permits for avoiding spurious detections of eccentricity modulation. ENOF offers a 294 simpler alternative. Undertaken fully in the time domain, ENOF does not require filtering a record 295 to identify amplitude variability over time, thereby avoiding the primary concern raised by Huybers 296 and Aharonson (2010). 297

If ENOF is repeated with eccentricity excluded from the climatic precession term of equation 1, A_p is responsible for capturing all amplitude modulations of precession. The correlation of A_p with eccentricity then provides an independent test for the presence of precession. Because ENOF can only resolve variations on timescales equal to or longer than the length of its fitting window, eccentricity is smoothed by a window of the same length prior to computing the correlation.

The correlation of A_p with eccentricity in the orbitally-tuned L2H19 between 3 and 1 Ma is 0.46. The significance of this correlation is evaluated against a null hypothesis, $H_{0,ecc}$, that A_p varies independently of eccentricity. A probability distribution associated with $H_{0,ecc}$ is formed by repeating the eccentricity-independent fit 10^4 times on phase-randomized versions of the input data. ³⁰⁷ Phase randomization preserves the Fourier amplitudes and power spectrum of the time-series, but ³⁰⁸ destroys meaningful amplitude modulations (see Appendix E for details). The 99% significance level ³⁰⁹ for $H_{0,ecc}$ is found to be 0.38, allowing for a conclusion that the eccentricity modulation in L2H19 ³¹⁰ is highly significant. Variations in the window width or window shift (*h* or *k*, respectively, in sec-³¹¹ tion 4) only induce small variations in the correlation between eccentricity and A_p (Appendix C.2). ³¹² Repeating the test with a 100-ky ENOF fitting window gives an equally significant result.

³¹³ 5.2. Trends in forcing amplitudes

Previous work has generally described the Pleistocene as involving a distinct transition between 314 a "41-kyr world", featuring apparently obliquity-dominated glacial cycles, and the later "100-kyr 315 world", with more strongly expressed precession (e.g. Elderfield et al., 2012). Some studies have 316 alternately described a more gradual progression in glacial-cycle characteristics (Huybers, 2007; 317 Lisiecki and Raymo, 2007). An ENOF fit using a 100-ky window over the past 3 Ma indicates that 318 precession's contribution to L2H19's $d\delta^{18}O/dt$ rose over the Pleistocene, with no abrupt transition 319 (Figure 3c). Precession's amplitude increased at a rate of $(16.21 \pm 0.42) \times 10^{-3} \text{ meg}^{-1} \text{ky}^{-2}$ between 320 3 Ma and the present, roughly five times faster than the growth rate in obliquity amplitude, $(2.82 \pm$ 321 $(0.29) \times 10^{-3} \text{ meg}^{-1} \text{ky}^{-2}$ (Figure 3). These results accord with an earlier finding that the spectral 322 power of precession-band frequencies in δ^{18} O increases in amplitude over the Pleistocene (Lisiecki 323 and Raymo, 2007). Note that eccentricity itself has slightly larger amplitude on average in the late 324 Pleistocene, but recomputing the precession amplitude trend using a linearly de-trended eccentricity 325 negligibly changes the result. 326

The residual between the ENOF fit and $d\delta^{18}O/dt$ gradually increases between 3 and 1 Ma, a trend that can largely be explained by an increase in the variance of $d\delta^{18}O/dt$ over the same interval. A linear regression between the moving variance of $d\delta^{18}O/dt$ and the moving variance of the ENOF residual using a box-car window of 250 ky yields an R^2 of 0.91. Additional amplitude variability



Figure 3: Trends in orbital forcing in $d\delta^{18}O/dt$ owing to precession and obliquity over the Pleistocene and late Pliocene, using data from L2H19, a high-resolution benthic $\delta^{18}O$ record. (a) L2H19, with select Marine Isotope Stages labeled, and its geomagnetic polarity reversal stratigraphy (Channell et al., 2008, 2016). (b) The rate-of-change of $\delta^{18}O$, smoothed with a 6ky 2nd-order Butterworth filter (gray), and ENOF fit to the time-series (blue) (c) Envelope of the precession contribution from 3 to 0 Ma (red line) and orbital eccentricity (black line). (d) same as (c), but for obliquity, and where the black line represents the envelope of obliquity. Envelopes are calculated as the magnitude of the Hilbert transform of the ENOF-estimated precession and obliquity contributions, and for the calculated values of obliquity. Shaded areas represent the 95% confidence intervals, and black trend-lines indicate a linear least-squares fit. The ENOF fit uses a 100 ky window, but results are similar when using a 50 ky window (see Appendix C). The *y*-axis is reversed in panels (a) and (b).

is superimposed upon precession's amplitude trend due to its modulation by eccentricity, as can
be recognized in the time domain. For example, the peak at 1.75 Ma in Figure 3a apparently
corresponds to a strong precession peak during an eccentricity maximum.

³³⁴ 6. Predictions of an ice sheet and energy-balance model

To examine a possible physical explanation for the observed orbital ratios in benthic δ^{18} O records and 335 the gradual increase in precession amplitude relative to obliquity, we revisit the coupled Northern 336 Hemisphere ice sheet and energy-balance model of Huybers and Tziperman (2008), hereafter the 337 EBM. The EBM represents a parabolic ice sheet in a two-dimensional transect from the equator to 338 the North Pole, and is forced with diurnally averaged daily insolation across all latitudes, permitting 339 it to capture the full seasonal temperature cycle. The model computes heat fluxes across a two-340 layer atmosphere and a subsurface layer, with an ice sheet that freely deforms in accordance with a 341 shallow-ice approximation and sits above a deformable sediment layer. 342

A number of important factors are not included. There is no ocean, and the model does not simulate geochemical interactions such as those that may drive a CO₂ feedback (Broecker, 1982), nor the possible influence of sea ice (Gildor and Tziperman, 2000) or orbitally-forced fluctuations in the volume of the Antarctic ice sheet (Raymo et al., 2006). Though these limitations are important to note, the model has been shown to produce ice-volume variability that was previously interpreted to contain little precession (Huybers and Tziperman, 2008), allowing us to evaluate the EBM output in the same context as the observations.

³⁵⁰ 6.1. Orbital forcing in a two-million year model run with a cooling atmosphere

We analyze a single run of the EBM starting at 3.1 Ma, the first 100 ky of which is excluded from analysis as an equilibration period, using the parameters listed in Appendix F (Figure 4). A cooling ³⁵³ of the background climate is imposed for consistency with an observed long-term cooling trend. To ³⁵⁴ impose this cooling, the height of the atmospheric radiation emission level is linearly lowered from ³⁵⁵ 7.20 km to 7.06 km between 3.1 and 1 Ma, which cools the Northern Hemisphere surface temperature ³⁵⁶ by 2.2°C. Cooling averaged between 45°N and 70°N is 2.7°C, consistent with an estimated North ³⁵⁷ Atlantic sea-surface-temperature cooling of 2.8°C over 3 to 1 Ma (Lawrence et al., 2010).

Orbital forcing amplitudes are estimated from differenced ice volume, and only ENOF results are 358 reported because spectral analysis produces very similar central estimates, giving precession-to-359 obliquity amplitude ratios within 0.01 of those estimated from ENOF. Averaged over 3-2 Ma, ENOF 360 estimates a precession-to-obliquity amplitude ratio of 0.55 (0.49-0.61 95% confidence interval) in 361 simulated ice volume. This ratio exceeds that estimated in the observations, which average 0.32. 362 The ratio grows to to 0.71 (0.68-0.75) between 2 and 1 Ma, a shift caused almost entirely by an 363 increase in the precession contribution. During this later interval, the partitioning of orbital energy 364 in the EBM is consistent with orbitally-tuned records, in which the ratio averages 0.62 (see Figure 365 4 for a comparison of L2H19 against simulated ice volume). 366

The upward trend in the model ice volume's precession amplitude can be understood as an ice sheet's 367 response to gradually cooling atmospheric temperatures. Cooling reduces melting both because of 368 a shorter melt season and less intense melting therein, leading to growth of the ice sheet and a 369 southward shift of the ablation margin. Both a shorter melt season and a more southerly melting 370 line will lead to greater precession variability (see Huybers and Tziperman (2008) and sensitivity 371 tests in Appendix F.1), where the former heightens the sensitivity to summer intensity, and the 372 latter exposes the melting line to long-term insolation variations that are more strongly influenced 373 by precession. 374

Following past approaches (Raymo and Nisancioglu, 2003; Raymo et al., 2006), we have treated $\delta^{18}O$ as being indicative of ice volume, though it is also sensitive to the local deep-water temperature. Previous efforts to deconvolve benchic $\delta^{18}O$ into its respective components have used an inverse



Figure 4: Simulated variations in ice volume between 3 and 1 Ma. (a) Model ice volume (black) and L2H19 on an orbitally-tuned age scale (red) for 3-2 Ma. Variations in obliquity (black) and the climatic precession parameter ($e\sin(\varpi)$, light gray) are below. (b) Similar to (a) but for 2-1 Ma and with L2H19 also shown on a depth-derived age scale (blue). δ^{18} O values are de-trended over each 1 Ma segment. Select Marine Isotope Stages are labeled for reference.

model of an ice sheet couple to a simplified representation of deep-water temperature (e.g. Bintanja 378 and van de Wal, 2008) or empirical isolation of the temperature component from independent mea-379 surements of calcite Mg/Ca ratios (e.g. Elderfield et al., 2012). There remain large uncertainties in 380 the relative amplitudes of ice volume and deep-water temperature in benthic δ^{18} O, however (Bin-381 tanja et al., 2005), making it difficult to produce a pure ice-volume curve from which to directly 382 estimate orbital forcing. The phase of precession in any such deconvolution is also uncertain because 383 it is sensitive to factors including the lag in the response to orbital forcing, amplitude of Southern 384 Hemisphere contributions, and seasonal sensitivity of the deep-water temperature component, each 385 of which is uncertain. 386

³⁸⁷ 7. Discussion and conclusions

³⁸⁸ 7.1. Differences in precession estimates across records and methods

The detection and interpretation of orbital variability in climate proxies would be straightforward if orbital variations were unmodulated sinusoids, records were sampled at high resolution, sedimentation rates were stable across space and time, and noise and nonlinearity were small. Clearly, orbitally-induced changes in radiative forcing and the subsequent proxy recording of the response are not so simple. It is therefore unsurprising that various proxy records have engendered different interpretations when evaluated with different methods, on different chronologies, or compared against different physical models.

Spectral analysis detects significant precession in just 7 out of the 21 samples across records, evaluated on two different 1 Ma intervals, and with two different age models (Table 2). In contrast, ENOF identifies a significant precession contribution in 20 out of 21 samples. The higher rates of detection with ENOF corresponds with the algorithm producing 95% confidence intervals that are, on average, 30% narrower than for spectral estimates. Higher detection rates are only physically

meaningful, however, if they are not borne of an increased probability of false detection. Thus, a 401 series of tests on synthetic time series are undertaken to explore differences between ENOF and 402 spectral analysis. These tests are described in Appendix D and briefly summarized here. Synthetic 403 signals are formulated by adding background noise similar to that found in the observations (Figure 404 1) with obliquity and climatic precession signals. The relative skill of ENOF versus spectral analysis 405 depends on the amplitude of the orbital signal relative to noise, and in the case of a high signal-406 to-noise ratio, there is negligible difference between the two methods' skill (Figures D1 and D2), as 407 found in analysis of the EBM's simulated ice volume in section 6. 408

Signal-to-noise ratios are estimated for observations by dividing the total energy above the noise 409 floor in both orbital bands by the sum of all other energy in the power spectrum, giving an average 410 of 0.20 and ranging from 0.03 to 0.35 across the 21 different samples. The one exception is the 411 H07 stack for which the signal-to-noise ratio is 0.92, possibly because noise is suppressed through 412 averaging multiple records. Note that because obliquity has a larger amplitude than precession, the 413 signal-to-noise ratio for each orbital component differs, but we do not account for this distinction. 414 For a signal-to-noise ratio of 0.075, our synthetic results indicate that 95% of ENOF estimates will 415 fall within a range that is 36% narrower for ENOF than for spectral estimates, and for a signal-to-416 noise ratio of 0.25 the range is 26% narrower. These empirical coverage intervals are thus consistent 417 with the 30% narrower confidence intervals determined for ENOF relative to spectral analysis (Table 418 2) and with ENOF having greater statistical power for identifying climatic precession. 419

In contrast to its better identifying orbital variations in the presence of noise, ENOF is relatively more sensitive to age model errors than our spectral analysis approach (Figure D3). Greater sensitivity can be understood in that ENOF fits only to a specific time-series of orbital forcing, whereas spectral estimates are insensitive to phase shifts, and our approach of summing orbital energy across a frequency band permits for recovering energy dispersed from the exact frequencies associated with orbital variations. This accords with ENOF giving lower central estimates of precession amplitudes than spectral analysis in all but two depth-derived records (Table 2).

Estimates of the amplitude of obliquity variability are less sensitive to age model errors than pre-427 cession on account of its longer period leading to smaller phase differences for a given perturbation 428 in time (Figure D3). This difference in sensitivity makes it useful to consider the ratio of precession 429 to obliquity variability in records, but we find a surprisingly large difference in these ratios when 430 transitioning from orbitally-tuned to depth-derived age models. Estimates of precession-to-obliquity 431 amplitude ratios in depth-derived records average 50% lower than for orbitally-tuned records if using 432 ENOF and 58% lower if using spectral analysis. Age errors relative to orbital variations having a 433 standard deviation between 2 and 5 ky are only expected to decrease the precession amplitude by an 434 average of <1% to 28% (Figure D3); age errors having a standard deviation of 10 ky is required for 435 a > 50% reduction in precession amplitude, but then a large decrease in the amplitude of obliquity 436 is also anticipated. We speculate that orbital tuning accounts for a portion of the otherwise larger-437 than-expected decrease in orbital energy. The first step of orbital tuning for records included in the 438 LR04 stack involves mapping δ^{18} O variations onto a target curve representing June 21 insolation 439 at 65°N (Lisiecki and Raymo, 2005; Imbrie and Imbrie, 1980), which primarily contains climatic 440 precession variability. Tuning δ^{18} O records to a precession target using a dynamic time warping 441 algorithm similar to that involved in the LR04 alignment procedure has been shown to artificially 442 increase estimates of the precession amplitude (Proistosescu et al., 2012). The combination of depth-443 derived ages being untuned and their containing age errors may account for estimates of precession 444 amplitude being much smaller when using depth-derived as opposed to orbitally-tuned ages. 445

A final consideration for differences in orbital amplitudes relates to variable sampling resolution across records. Coarsely sampling marine sediment records aliases high-frequency variability into lower-frequency bands (Pisias and Mix, 1988). Moreover, sampling intervals above the Nyquist frequency may not suffice for fully resolving orbital variability because of uneven sampling, time uncertainty, and the amplitude and frequency modulation present in obliquity and climatic precession.

Between 2 and 1 Ma, the sampling resolution in depth-derived records vary by more than an order 451 of magnitude, with L2H19 averaging 60 data points over a 21 ky precession cycle, and DSDP 607 452 averaging just 5.5 points. If L2H19 were sampled at a resolution equal to that of the DSDP 607 453 record, a 16% lower precession amplitude estimate would be expected from either ENOF or spectral 454 analysis, compared to an essentially unchanged estimate of the obliquity amplitude (Figure D5). 455 Although possibly only a coincidence, the DSDP 607 estimate of the amplitude of precession using 456 the orbitally-tuned age model is, in fact, 15% lower than for L2H19, where both records are from 457 the North Atlantic and obtained at similar water depths. 458

⁴⁵⁹ 7.2. Interpretation of orbital amplitudes and trends

Despite some evidence for the presence of precession in early Pleistocene glacial cycles (e.g. Lisiecki 460 and Raymo, 2007), a long-standing view has persisted that precession variability in early Pleistocene 461 benthic δ^{18} O is mostly negligible in amplitude (Ruddiman et al., 1986), motivating explanations for 462 obliquity-paced glacial cycles (Raymo and Nisancioglu, 2003; Loutre et al., 2004; Raymo et al., 2006; 463 Huybers, 2006; Tabor et al., 2015). Our finding that early Pleistocene δ^{18} O records contain signif-464 icant precession variability nearly in phase with Northern Hemisphere summer intensity suggests 465 that proposed mechanisms to continuously suppress precession may be less relevant to the early 466 Pleistocene than previously believed. 467

One possibility is that, in accordance with classical Milankovitch theory, glaciation is controlled by Northern Hemisphere summer insolation (Milankovitch, 1941; Huybers, 2006). In such a model, the observed gradual increase in precession amplitude (Figure 2) is explained on the basis of a southward extension of the ice sheet ablation margin induced by global cooling, making variations in ice volume more sensitive to precession. An alternative interpretation, consistent with the proposal of Murphy (1869), is that the early Pleistocene represents a transitional period in which the net precession contribution to global ice volume shifts from Southern to Northern Hemisphere dominance. Such a ⁴⁷⁵ model also holds global cooling responsible for the increase in precession amplitude, but instead im-⁴⁷⁶ plies that anti-phased precession-forced ice volume fluctuations between hemispheres, which would ⁴⁷⁷ cause cancelation of the δ^{18} O precession signal (Raymo et al., 2006), became increasingly imbal-⁴⁷⁸ anced from Northern Hemisphere ice sheets growing in size, and possibly, Antarctica becoming more ⁴⁷⁹ stable.

Neither model excludes the possible importance of other factors. For example, erosion of regolith by 480 an ice sheet (Clark and Pollard, 1998) could contribute to less frequent full deglaciations, keeping 481 the ablation zone relatively far south and exposed to precession-dominant insolation. Past efforts to 482 model the ice sheet response to real or idealized orbital forcing have yielded ice volume fluctuations 483 ranging from precession-dominant (e.g. Nisancioglu, 2004) to obliquity-dominant (e.g. Berger et al., 484 1999). The variety in these results as well as the possible importance of mechanisms not included 485 in the EBM suggests a need for further empirical analyses and simulation to better determine the 486 characteristics of early-Pleistocene glacial variability and its causes. 487

Two further important questions have not been explored here. First, the origin of glacial cycle 488 asymmetry, which has been identified throughout the full Pleistocene (Ashkenazy and Tziperman, 489 2004; Lisiecki and Raymo, 2007), remains uncertain, and further exploration of the connections 490 between precession forcing, asymmetry, and age model accuracy would be useful. Second, it has 491 been suggested that both the phase and amplitude of Pleistocene 100-ky variability are sensitive 492 to precession (e.g. Imbrie et al., 1993). Lisiecki (2010) proposed that strong precession forcing 493 disrupts 100-ky variability in δ^{18} O over the past 5 Ma, but raised the question of "how precession" 494 modulation could suppress 100-kyr glacial cycles during the early Pleistocene 41-kyr world when the 495 23-kyr power of δ^{18} O is negligible". Caballero-Gill et al. (2019) suggested that 100-ky glaciations also 496 occurred during the Pliocene, and proposed that a nonlinear response to precession is responsible. 497 Our finding of significant early-Pleistocene precession variability suggests that further investigation 498 into the links between orbital eccentricity, precession forcing, and quasi-100-ky climate variability is 499

500 needed.

A revised description of the early Pleistocene as featuring significant and gradually increasing precession amplitude in phase with Northern Hemisphere summer intensity recasts the time period as supporting, rather than contradicting, the long tradition of models that invoke changes in summer insolation as controlling global ice volume fluctuations (Murphy, 1869; Milankovitch, 1941; Hays et al., 1976; Imbrie and Imbrie, 1980; Raymo et al., 2006). Whether the early Pleistocene accords more readily with the model of Milankovitch or that of Murphy remains to be seen.

507 8. Acknowledgements

PRL and PJH were supported by National Science Foundation award 1338832. DAH was supported
by Natural Environmental Research Council grant NE/R000204/1 and Engineering and Physical
Sciences Research Council grant EP/S030417/1. DAH thanks D. Schrag and the Harvard University
Center for the Environment for supporting a sabbatical during AY 2018-2019. This research used
samples and/or data provided by the International Ocean Discovery Program (IODP).

Appendices

⁵¹⁴ Appendix A. Depth-age relationships

Orbitally-tuned chronologies are developed by matching to LR04. The LR04 age model is constructed 515 by alignment of the LR04 stack with the ice-volume model of Imbrie and Imbrie (1980), where ice-516 model parameters are adjusted over time to give an increasingly asymmetric target and longer 517 response time owing to larger ice sheets. Tuning is achieved first by maximizing the correlation 518 between the stack and ice model, then further adjusting the timescale to be in phase with the ice 519 model's obliquity component. The re-tuning to obliquity allows the precession phase to vary between 520 glacial cycles. The depth-derived approach uses the orbitally-independent age model of Huybers 521 (2007), referred to as H07. H07 is constructed by graphic correlation of 14 benthic and planktic 522 records, and alignment of synchronous geomagnetic and isotopic events across records, between 523 which age is linearly interpolated with depth after correcting for downcore compaction. 524

L2H19's orbitally-tuned chronology is based on alignment to the LR04 stack by identification of isotopic events occurring both in LR04 and L2H19, with an assumption of linear sedimentation rates between them (Bolton et al., 2010; Channell et al., 2016; Hodell and Channell, 2016). The age model is supplemented by correlation of sediment lightness to ODP Site 609 between the present and 76 ka (Obrochta et al., 2014), and to the NGRIP record between 76 and 110.5 ka (Hodell and Channell, 2016).

To convert L2H19 to a depth-derived timescale, it is aligned with the benthic δ^{18} O record of Site 607 on the H07 age model. Site 607 is chosen because it is closest geographically to the records comprising L2H19, being drilled at the same location as U1313 in the North Atlantic, and is therefore likely to have the most similar glacial cycles. Note, however, that the H07 age model for Site 607 is based on a global calibration with other records in the H07 stack. Isotopic event ages used in calibrating H07



Figure A1: Pleistocene δ^{18} O records and age models for L2H19. (a) The orbitally-tuned L2H19 benthic δ^{18} O record comprising data from IODP Sites U1313 (red) and U1308 (black). (b) L2H19 (black) after alignment to a depth-derived timescale for DSDP 607 (red), where black markers are age control point used for calibration. The orbitally-tuned L2H19 is also shown (gray) for comparison. Records in panel (b) are de-trended to enable direct comparison, and select Marine Isotope Stages are labeled in both panels.

are adopted as age control points (ACPs) for L2H19, with a total of 48 events identified between 536 2 and 1 Ma. The ACPs of L2H19 and Site 607 were aligned by assigning to L2H19's ACPs the 537 ages of Site 607's ACPs, then linearly interpolating age with depth between these points. Note 538 that the depth-derived timescale ends at 2 Ma, because too few independent $\delta^{18}O$ records exist 539 beyond 2 Ma to adequately constrain age-depth relationships without orbital assumptions. The 540 five other individual records included in this study are also evaluated on both orbitally-tuned and 541 depth-derived age scales, following the age models of LR04 and H07, respectively. They are used 542 here as published in Lisiecki and Raymo (2005) and Huybers (2007). 543

⁵⁴⁴ Appendix B. Spectral estimates

All power spectra are computed with Thomson's multi-taper method (Percival and Walden, 1993)
using three tapers. The noise floor for the power spectrum is computed by a least-squares fit of a
power-law,

$$y = A f^q, \tag{B.1}$$

to the power spectrum, where the parameters A and q are chosen to minimize the sum of squared 548 residuals between y and the power spectrum. Frequencies below $1/150 \text{ ky}^{-1}$, between $1/41 \pm 1/125$, 549 between $1/21 \pm 1/125$, and above $1/5 \text{ ky}^{-1}$ are excluded from the fit. If the net variance in a 550 band is negative with respect to the estimated noise floor, the amplitude is assigned a value of zero. 551 The noise floor is approximately white in the rate-of-change of $\delta^{18}O$ (Figure 1). Across all records 552 on both the orbitally-tuned and depth-derived age models and both time intervals evaluated (3-2 553 Ma and 2-1 Ma), A averages 0.02 ± 0.02 and q averages -0.01 ± 0.2 , where the range represents one 554 standard deviation. 555

The 95% confidence interval for power spectral density is estimated from the approximate χ^2 distribu-556 tion of the spectral estimator (Percival and Walden, 1993). The interval is given by $\left[\frac{\nu S(f)}{Q_{\nu}(p)}, \frac{\nu S(f)}{Q_{\nu}(1-p)}\right]$, 557 where S is the estimate at frequency f, ν is the equivalent degrees of freedom, and $Q_{\nu}(p)$ and 558 $Q_{\nu}(1-p)$ are the p = 0.025 and 1 – p = 0.975 percentage points on the χ^2_{ν} distribution with ν 559 degrees of freedom. For an estimate at a given frequency, ν is approximately equal to 2K where K 560 is the number of tapers used in the multi-taper analysis. The overall orbital amplitude estimate has 561 more degrees of freedom, however, because it is obtained by summing energy across several spectral 562 estimates in an orbital band. There are B_f/B_W independent spectral estimates in an orbital band, 563 where $B_f = 2/125 \text{ ky}^{-1}$ is the width of an orbital band and B_W is the spectral bandwidth, which 564 represents the frequency range across which spectral estimates decorrelate. An orbital amplitude 565 estimate therefore has approximately $\frac{2KB_f}{B_W}$ degrees of freedom. 566

Several aspects of the confidence interval for spectral estimates are uncertain. A simple power-law 567 representation of the noise floor (equation B.1) is assumed, and its coefficients are influenced by the 568 choice of frequency range over which it is fit, here 1/150 to 1/5 ky⁻¹. The variance of the spectral 569 estimator is not perfectly χ^2 -distributed when deterministic variance is partially responsible for the 570 spectral peak, as the χ^2 approximation is most accurate for a random process (Percival and Walden, 571 1993). To check the applicability of the approximate confidence intervals for the purposes of this 572 study, we compute the fraction of synthetic tests, described in Appendix D, for which the true 573 precession amplitude lies within the confidence interval. For tests with SNR=0 (pure noise), 0.075, 574 0.25, 0.5, 1, and 2, the estimated confidence interval contains the true amplitude in 93.8%, 97.3%, 575 98.5%, 99.7%, 99.99% and 100% of tests after bias correction, implying the coverage is approximately 576 correct for noisy signals and slightly conservative for less noisy signals. The average approximate SNR 577 of 0.20 in observations implies that the coverage interval is expected to be generally appropriate and 578 in some cases slightly conservative. Note that for SNR<1, the confidence interval is less conservative 579 than for ENOF (see Appendix D.1), suggesting that the relative uncertainty of spectral analysis is 580 not being overstated in our comparison of methods. 581

An alternative method for estimating uncertainties is to evaluate the significance of orbital-band 582 spectral energy relative to the expected distribution of the null hypothesis of no orbital energy. In 583 this approach, an approximate χ^2 confidence interval is constructed for the position of the noise 584 floor, and an orbital amplitude estimate is considered significant if the integrated energy between 585 the power spectral density and the 95% level for the noise floor in an orbital band exceeds zero. 586 We elect not to use this approach because it does not provide an estimate of the uncertainty range 587 associated with the orbital amplitude estimate, only providing a measure of the estimate's statistical 588 significance. It is nonetheless useful to verify that the use of the alternative method wouldn't lead 589 us to different conclusions. Repeating the analysis with this method gives no significant precession 590 amplitude estimates among orbitally-tuned records between 3 and 2 Ma, four significant estimates 591

for tuned records between 2 and 1 Ma, and one significant estimate for depth-derived records between 592 2 and 1 Ma, similar to the zero, five, and two significant estimates, respectively, from the method 593 we use. The slightly more conservative results in the alternative method arises from the fact that 594 the χ^2 distribution is asymmetric, so that the difference between its 2.5th and 50th percentile is 595 less than between its 50th and 97.5th percentile. Because confidence is assessed on the basis of an 596 upper confidence level for the noise floor in the alternative method and a lower confidence level for 597 the power spectrum in the method we use, the latter will tend to indicate significance more often. 598 In our case, results from both methods are similar because the 24 degrees of freedom used makes 599 the resulting χ^2 distribution close to symmetric. 600

⁶⁰¹ Appendix C. Empirical Nonlinear Orbital Fitting (ENOF) algorithm

ENOF's parameter search for A_p , A_o , Δt_p and Δt_p is undertaken using a Matlab routine, "lsqcurvefit", which uses a trust-region algorithm for optimization. The trust-region method is preferred over a Levenberg-Marquardt method because it allows us to place bound constraints on the parameter values. Δt_o and Δt_p are capped at ± 10 ky to avoid overfitting by an arbitrary phase assignment. Initial estimates are set at 0 for all parameters.

Being a gradient-descent method, the trust-region algorithm is not a global search of all possible parameter combinations so does not theoretically guarantee a global minimum. Such an exhaustive search is computationally prohibitive. Nonetheless, it is useful to check that the parameter space is likely smooth and without multiple local minima that could cause a gradient-descent method to converge to the wrong values. A "brute force" parameter search on a select 50 ky window reveals a smooth, elliptical parameter space with respect to the residuals, and converges toward the parameter values selected by nonlinear least squares (Figure C1).



Figure C1: Comparison of nonlinear least-squares estimates to an exhaustive parameter search. (a) Estimates for A_p and A_o , where Δt_p and Δt_o are at the values estimated by nonlinear least-squares. The white star represents the nonlinear least-squares estimate of the best-fitting combination of parameters, and the contours represent the the sum of squared residuals computed by the brute force search. The fit is estimated over a 50ky period starting at 1.8 Ma in L2H19's smoothed rate-of-change. (b) Same as (a), but for estimates of Δt_o and Δt_p , where A_p and A_o are at the values estimated by nonlinear least-squares.

⁶¹⁴ Appendix C.1. Parameter uncertainties and confidence intervals

The 95% confidence intervals for A_p and A_o are estimated using the Matlab routine "nlparci". They 615 are approximated from a linearization of the nonlinear regression problem under an assumption that 616 the estimator is asymptotically normally distributed. The approximate confidence interval is given 617 by $\hat{\theta}_{CI} = \hat{\theta} \pm S_E T_{inv}(1-\alpha/2, n-p)$, where $\hat{\theta}$ is the vector of nonlinear regression parameter estimates. 618 S_E is the estimated standard error, and T_{inv} the Student's T inverse cumulative distribution that 619 is evaluated at a confidence level α with degrees of freedom equal to the number of observations, 620 *n*, minus the number of model parameters, *p*. Here, $S_E(\hat{\theta}_i) = s \| \mathbf{r}^i \|$, where $s = \sqrt{\frac{SSR(\hat{\theta})}{n-p}}$. SSR 621 is the sum of squared residuals, and r^i is the *i*th row of the matrix R^{-1} , which originates from a 622 QR-factorization of the Jacobian matrix, J = QR (see Bates and Watts, 1988). To prevent negative 623 amplitudes, confidence intervals are truncated below zero. 624

It is useful to verify the coverage of ENOF's approximate 95% confidence intervals, specifically that they are not likely to underestimate the uncertainty. A measure of validity is the fraction of the synthetic tests, described in Appendix D.1, for which the true orbital amplitude falls within the range of the estimate's confidence interval. For synthetic time-series featuring orbital variance with SNR= 0 (pure noise), 0.075, 0.25, 0.5, 1, and 2, the 95% confidence interval for the precession amplitude contains the correct amplitude in 94.5%, 97.3%, 98.9%, 99.4%, 99.5%, and 99.6% of tests, suggesting that the confidence intervals are conservative.

⁶³² Appendix C.2. Sensitivity to choice of window length and shift

It is important to verify that our results are not qualitatively sensitive to the choice of window τ , the 633 fitting window length, and k, the distance between each overlapping windowed fit. Orbital ampli-634 tudes are estimated for L2H19 between 3 and 1 Ma when ENOF is run with different combinations 635 of values of τ and k, respectively ranging from 50 to 100 ky and 1 to 6 ky. Using a larger value of τ 636 tends to slightly reduce both the precession and obliquity estimates as the algorithm has less flexi-637 bility to fit against observations when attempting to fit a single amplitude value over a wider range. 638 Changing τ from 50 ky to 100 ky reduces the precession amplitude estimate from 23.5 meg⁻¹ky⁻¹ to 639 19.7 meg⁻¹ky⁻¹, and the obliquity estimate from 37.0 meg⁻¹ky⁻¹ to 34.0 meg⁻¹ky⁻¹. Alternative 640 values of τ ranging from 60 to 90 ky give proportional reductions in amplitude estimates relative 641 to $\tau = 50$ ky. Results are mostly insensitive to changes in k, with both precession and obliquity 642 estimates changing by 0.1 meg⁻¹ky⁻¹ or less for values of k ranging from 1 to 6 ky. 643

We also repeat the eccentricity-independent test for precession, described in section 5.1, with different combinations of values of τ and k. Wider windows produce a slightly higher correlation with eccentricity because they produce smoother variations. Using $\tau = 50, 60, 70, 80, 90, \text{ and } 100 \text{ ky}$ results in correlations of A_p with eccentricity of 0.46, 0.47, 0.49, 0.50, 0.54, and 0.55. Results are mostly insensitive to the choice of k, and repeating the tests with values of k ranging from 1 to 6 ky for each value of τ yields a change of 0.01 or less in the correlation of A_p with eccentricity. We repeated the 10⁴ phase-randomized trials described in section 5.1 for $\tau = 100$ ky, which give a 99% significance level of 0.51, compared to a correlation of 0.55 in L2H19, indicating that the significance of the eccentricity-independent test for precession is likely not sensitive to reasonable to choices of τ or k.

⁶⁵⁴ Appendix C.3. Additional figures



Figure C2: Trends in orbital forcing in $d\delta^{18}O/dt$ owing to precession and obliquity over the Pleistocene and late Pliocene, using data from L2H19. (a) Envelope of the precession contribution from 3 to 0 Ma (red line) and orbital eccentricity (black line).(b) same as (a), but for obliquity, and where the black line represents the envelope of obliquity. Envelopes are calculated as the magnitude of the Hilbert transform of the ENOF-estimated precession and obliquity contributions, and for the calculated values of obliquity. Shaded areas represent the 95% confidence intervals, and black trend-lines indicate a linear least-squares fit.

⁶⁵⁵ Appendix D. Sensitivity tests for ENOF and spectral analysis

To quantify the sensitivity of orbital amplitude estimates from both ENOF and spectral analysis to (i) the presence of background noise, (ii) the presence of age error, and (iii) differences in sampling resolution, we apply both methods to a common set of synthetic time-series with properties expected to be similar to Pleistocene δ^{18} O records.

⁶⁶⁰ Appendix D.1. Sensitivity to noise

Noise introduces bias in both spectral and ENOF estimates. To quantify this bias, we study estimates 661 from both methods when applied to synthetic data. Two competing scenarios are considered in which 662 estimates for a pure noise signal are compared with estimates for synthetic time-series featuring 663 orbital forcing. We adopt a null hypothesis, H_0 , that orbital variability is absent, and estimate 664 H_0 's distribution with a Monte Carlo approach, applying both methods to 2.5×10^4 realizations 665 of noise. Red noise is initially formed following a power law of 2 with a decorrelation frequency 666 of $1/150 \text{ ky}^{-1}$, then differenced to produce a mostly white continuum consistent with observations, 667 as evident in Figure 1. To form probability distributions associated with the alternate hypothesis 668 that orbital variability is present, we evaluate 2.5×10^4 synthetic time-series spanning 2 to 1 Ma 669 featuring climatic precession and obliquity in an average amplitude ratio of 0.55, which is close to 670 the estimated precession-to-obliquity amplitude ratio in orbitally-tuned records between 2 and 1 Ma. 671 Noise with the same properties as in H_0 is then added to the synthetic signal. Six cases, denoted 672 H_1 to H_6 , or H_{1-6} , are considered, for signal-to-noise ratios of 0.075, 0.25, 0.5, 1, 2, and with no 673 noise. Signal-to-noise ratio is defined here as the variance ratio of the orbital and noise components. 674 The variance of the realized time series, comprising both signal and noise, is fixed such that each 675 distribution features different orbital amplitudes which approach zero as the signal-to-noise ratio 676 approaches zero. 677



 $- - H_0$, pure noise $- H_1$, SNR=0.075 $- H_2$, SNR=0.25 $- H_3$, SNR=0.5 $- H_4$, SNR=1 $- H_5$, SNR=2 $- H_6$, no noise

Figure D1: ENOF estimates for obliquity and precession amplitudes in tests on synthetic time-series with different signal-to-noise ratios. (a) Estimates for precession, taken as the square root of the variance of the first term in equation 1, before correcting for noise-induced bias, where each distribution represents 2.5×10^4 realizations. (b) Same as (a), but for the obliquity estimates, taken as the square root of the variance of the second term in equation 1. (c) Same as (a), but after applying the bias correction. (d) Same as (b), but after applying the bias correction.

In the absence of noise, H_6 , all estimates in both methods give the true amplitude. In H_{1-5} the median ENOF estimate exceeds the true precession amplitude by 70%, 28%, 15%, 7%, and 3%, and obliquity by 29%, 10%, 5%, 3%, and 1% (Figure D1a,b). Spectral estimates give similar biases if the noise floor is not accounted for, with the median estimate exceeding the true precession amplitude by 82%, 29%, 15%, 8%, and 4% and obliquity by 29%, 9%, 5%, 2%, and 1% (Figure D2a,b). These biases are corrected before evaluating H_{1-6} against H_0 .

The bias associated with ENOF is a function of the amplitude of noise relative to the orbital signal. Determining the correction requires an estimate of the noise fraction of the total amplitude,



 $- - H_0$, pure noise $- - H_1$, SNR=0.075 $- - H_2$, SNR=0.25 $- - H_3$, SNR=0.5 $- - H_4$, SNR=1 $- - H_5$, SNR=2 $- - H_6$, no noise

Figure D2: Spectral estimates of obliquity and precession amplitudes in synthetic tests on time-series with different signal-to-noise ratios. (a) Estimates for precession, as described in section 3.1, before correcting for noise-induced bias, where each distribution represents 2.5×10^4 realizations. (b) Same as (a), but for the obliquity estimates. (c) Same as (a), but after applying the bias correction. (d) Same as (b), but after applying the bias correction.

which is estimated by fitting the background continuum of the power spectrum as done for spectral estimates, then computing the area below the noise level as a fraction of the total area under the power spectrum and taking the square root of this value. A correction amplitude is determined by linearly interpolating the noise fraction with the mean bias from synthetic tests.

In spectral analysis, estimates are bias-corrected by subtracting the noise floor as described in Appendix B. If the variance estimate is negative, an amplitude of zero is assigned to avoid obtaining a complex amplitude estimate when taking the square root of the variance. This, and the distortion introduced by taking the square root of values close to zero, account for the shape of H_0 and the upticks at zero in H_0 and H_1 in Figures D2c and D2d. This distortion does not occur in ENOF estimates because the bias correction is applied after variance is converted to amplitude.

⁶⁹⁶ Appendix D.2. Sensitivity to age error

The introduction of age model error, which disperses orbital energy to nearby frequencies, is expected to cause underestimation bias in both methods. ENOF allows for variable time offsets from orbital variations, but typically cannot fit age error that exceeds the bounds on Δt_o and Δt_p , though it may recover some amplitude if the age model is incorrect by close to a full orbital cycle. The default bounds of ± 10 ky allow ENOF to fit almost any precession phase, but only covers half of an obliquity cycle.

 10^4 synthetic time-series are generated, with age error, Δt , introduced. Background noise is added 703 with the same properties as described in section Appendix D.1, and scaled so that time-series have a 704 signal-to-noise ratio of 0.5. Variability in the sedimentation rate, s(t), is first modeled as a random 705 walk, i.e., $s_{n+1} = s_n + \eta_n$ where η_n is drawn from a Gaussian distribution (Huybers and Wunsch, 706 2004), then integrated to give variations in depth with age. Synthetic time-series span 2 to 1 Ma 707 and, following Huybers and Wunsch (2004), age error is modeled as following a 'Brownian bridge', 708 tapering to zero at each endpoint to simulate the increasing amplitude of age error with time away 709 from age-control points. Five cases, S_{0-4} , are considered. S_0 , having no age error, is identical to 710 H_3 in Appendix D.1. In S_{1-4} , η_n is scaled such that age model error has a standard deviation of 0, 711 2, 4, 5, and 10 ky. 712

There is little to no attenuation in obliquity amplitude estimates for either ENOF or spectral analysis in S_{1-3} . The average ENOF obliquity amplitude estimates for S_{1-3} are within 1%, 1%, and 2.2% of that for S_0 . For spectral analysis, the average obliquity attenuation relative to S_0 is less than 1% for S_{1-3} . The ENOF obliquity estimate is dramatically reduced in S_4 , however, because a standard deviation of 10ky implies age variations of ~ ±20 ky, double the range over which Δt_o and Δt_p are allowed to vary.

ENOF estimates of precession amplitude are generally more sensitive to age error than spectral 719 estimates. Mean estimates for S_{1-3} are 1%, 15%, and 25% less than for spectral estimates. A 720 regression of spectral estimates of precession amplitude against ENOF estimates in depth-derived 721 records produces a slope of 1.07 ± 0.35 , where the range indicates the 95% confidence interval, and 722 is consistent with the relationship predicted by S_3 , which gives a slope of 1.33 (Figure D4). Note 723 that the age error in S_3 is lower than was independently estimated for the depth-derived age model 724 (Huybers, 2007), but that such smaller age errors are also implied by the difference between this age 725 model and orbitally-tuned records as well as the amplitude of the obliquity signal in depth-derived 726 records. 727



Figure D3: ENOF and spectral estimates of orbital amplitudes in synthetic time-series disrupted with age error. **a** ENOF precession amplitude estimate with age-error standard deviation ranging from 0 ky (dashed black line), to 10 ky (dotted blue line). Each distribution represents 10^4 realizations, and the vertical black line represents the true amplitude. **b** Same as (a), but for obliquity. **c** Same as (a), but where amplitudes are estimated using spectral analysis. **d** Same as (c), but for obliquity.



Figure D4: Expected and observed relationship between ENOF and spectral estimates of orbital amplitudes in the presence of age model error. (a) Regression of spectral estimates of precession amplitude against ENOF estimates in 10⁴ tests on synthetic time-series with SNR=0.5 and with age error having a standard deviation of 5 ky (thick gray line). For comparison, also shown are the ENOF and spectral estimates of precession amplitude in δ^{18} O records listed in Table 1 on the depth-derived timescale between 2 and 1 Ma (markers) and their 95% confidence intervals (thin gray bars), as well as the 95% confidence interval for a linear least-squares fit through the markers (dashed lines). Regressions shown here are forced to have zero intercepts. (b) same as (a), but for obliquity.

Estimates of orbital amplitudes are not bias-corrected toward higher values to account for age model error, but are bias corrected toward lower values to account for the presence of noise. The overall result is therefore a conservative estimate of orbital amplitude, particularly for precession on account of its being more sensitive to age errors. Although in principle it would be possible to correct estimates for age error, such a correction would inevitably be quite uncertain, and we prefer to make a more conservative test of whether orbital variability is present.

⁷³⁴ Appendix D.3. Sensitivity to sampling interval

To test the sensitivity of orbital amplitude estimates from both methods to sampling resolution, 735 L2H19 is resampled between 2 and 1 Ma in the depth domain at resolutions ranging from 0.25 ky 736 to 10 ky, then placed on U1308's tuned age model. Sampling resolution in time is determined using 737 Site U1308's average sedimentation rate of 8 cm ky^{-1} (Hodell and Channell, 2016). ENOF and 738 spectral analysis are repeated for each resampled version of the time series. A clear attenuation 739 of the estimated precession amplitude is observed as the sampling interval increases (Figure D5). 740 Precession amplitude estimates are more sensitive to coarsening of sampling resolution than obliquity 741 on account of its shorter period and its being more heavily amplitude- and frequency- modulated. 742 Thus, coarsening leads to a deterioration of the estimated precession-to-obliquity amplitude ratio 743 (Figure D5 b,d). Both methods are similarly sensitive to this problem, with ENOF and spectral 744 analysis respectively giving reductions of 14% and 13% in the ratio when resampling L2H19 at the 745 resolution of DSDP 607. 746



Figure D5: Effect of coarse sampling on the estimation of orbital amplitudes in benthic δ^{18} O records. (a) Spectral estimates of precession (red) and obliquity (blue) amplitudes and their 95% confidence intervals (red and blue shading) in $d\delta^{18}$ O/dt between 2 and 1 Ma, using L2H19 on the orbitally-tuned timescale, when L2H19 is resampled in the depth domain at different resolutions. Also indicated are the mean sampling resolution of the records listed in Table 1 over 2-1 Ma (vertical dashed lines). (b) The precession-toobliquity amplitude ratio (black) and its 95% confidence interval (gray shading). (c) Same as (a), but using ENOF amplitude estimates. (d) Same as (b), but using ENOF amplitude estimates.

747 Appendix E. Phase randomization

⁷⁴⁸ Phase randomization follows the approach described in Schreiber and Schmitz (2000), which is ⁷⁴⁹ repeated here. The phase-randomized (surrogate) time-series, \tilde{s}_n , is given by:

$$\tilde{s}_n = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{i\alpha_k} |S_k| e^{-i2\pi kn/N}$$
(E.1)

where $0 < \alpha_k \leq 2\pi$ are uniform random numbers, S_k are from the discrete Fourier transform,

$$|S_k|^2 = \left|\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_n e^{i2\pi k n/N}\right|^2,$$
 (E.2)

 $_{751}$ and s_n is the original time series.

⁷⁵² Appendix F. Ice sheet and energy balance model parameters

Symbol	Value	Unit	Parameter
ρ_i	900	${ m kg}~{ m m}^{-3}$	Density of ice
$ ho_w$	1000	${ m kg}~{ m m}^{-3}$	Density of water
$ ho_m$	3300	${ m kg}~{ m m}^{-3}$	Density of mantle
$ ho_a$	1.5	${ m kg} { m m}^{-3}$	Density of air
k_i	4	$\mathrm{J}~\mathrm{m}^{-1}~\mathrm{K}~\mathrm{s}$	Thermal conductivity of ice
k	0.03	${ m m}^2/{ m kg}^{-1}$	Mass absorption coefficient, water
g	9.8	ms^{-2}	Gravitational acceleration
σ	5.67×10^{-8}	${ m W}~{ m m}^{-2}~{ m K}^{-4}$	Stefan Boltzmann constant
C_d	0.011		Drag coefficient
L_v	2.5×10^{6}	$\rm J~kg^{-1}$	Latent heat of vaporization
L_m	3.34×10^{5}	$\rm J~kg^{-1}$	Latent heat of melting
L_s	2.84×10^{6}	$\rm J~kg^{-1}$	Latent heat of sublimation
a	6.37×10^{6}	m	Radius of earth
K	273.15	Κ	Melting temperature of water
C_p	2100	J kg $^{-1}$ K $^{-1}$	Specific heat capacity, water
C_{air}	1.5	J kg $^{-1}$ K $^{-1}$	Specific heat capacity, air
C_{ss}	$10 \ \rho_i C_p$	$J m^{-2} K^{-1}$	Heat capacity of subsurface layer
C_s	$5 \rho_i C_p$	$J m^{-2} K^{-1}$	Heat capacity of surface
C_a	5000 $\rho_a C_{air}$	$J m^{-2} K^{-1}$	Heat capacity of atmosphere
K_s	5	J/K	Sensible heat flux coefficient
H_m	5200	m	Starting thickness of lower atmospheric layer
H_{tm}	2000	m	Thickness of upper atmospheric layer
α_l	0.3	-	Land albedo
$lpha_i$	0.8	-	Ice albedo
A	0.2	-	Atmosphere absorption
R	0.3	-	Atmosphere reflection
T	0.5	-	Atmosphere transmission
ϵ_a	0.85	-	Longwave emissivity

P	1	m/a	Precipitation rate	
Γ_m	6.35	m K/km	Moist adiabatic lapse rate	

Table F.3: Energy balance constants and parameters for the model described in section 6.

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Symbol	Value	Unit	Parameter
n	3	-	Glen's law exponent
m	1.25	-	Stress-strain law exponent
A_{ice}	7.71×10^{-29}	${\rm Pa}^{-3} {\rm ~s}^{-1}$	Ice deformability
T_b	5000	years	Bed depression time
H_{eq}	0		Equilibrium surface height
$ ho_s$	2390	${ m kg}~{ m m}^{-3}$	Bulk sediment density
$ ho_b$	3370	${ m kg}~{ m m}^{-3}$	Bedrock density
u_o	3×10^{9}	$Pa \cdot s$	Reference sediment viscosity
h_s	10	m	Thickness of sediment layer
D_o	2.5×10^{-14}	s^{-1}	Reference deformation rate
ϕ_{sed}	22	degrees	Angle of internal friction

Table F.4: Ice sheet and sediment layer parameters for the model described in section 6.

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⁷⁵⁵ Appendix F.1. Sensitivity tests

Ten model simulations spanning 3 to 1 Ma with no background trend in the climate, but different mean climate states, were conducted to test the effect of the background state on the partitioning of orbital energy in simulated ice volume. Supplemental files include all ten runs, with atmospheric radiation emission levels ranging from 6.5 km to 7.2 km. Three runs are displayed in Figure F1, and demonstrate a stronger expression of precession when the mean temperature is cooler and the ice sheet terminus lies further south.

Note that other mechanisms not accounted for in the EBM may also influence an ice sheet's response to orbital forcing. For example, a more robust treatment of the temperature dependence of precipitation, which the EBM treats as constant, may influence the net mass balance. Some subglacial mechanisms, such as basal melting, are also neglected, as is the possible role of a partial marine ⁷⁶⁶ margin and associated ice mass loss by calving.



Figure F1: Model ice volume between 3 and 1 Ma under varying background conditions. (a) Simulated ice volume when a constant emission level of 7.20 km is used, with other parameters as listed in Table F.4. Bold numbers represent the latitude of the southernmost extent of the ice sheet terminus. (b-c) Same as (a), but for emission levels of 6.81 km and 6.50 km. (d-f) Power spectral density for the simulated ice volume curves in panels (a)-(c), normalized by the power at $1/41 \text{ ky}^{-1}$, with dashed lines indicating the 1/41, 1/23, and $1/19 \text{ ky}^{-1}$ frequencies.

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