Note on the absence of R^2 corrections to Newton's potential

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We consider Einstein gravity with the addition of R^2 and $R^{\mu\nu}R_{\mu\nu}$ interactions in the context of effective field theory, and the corresponding scattering amplitudes of gravitons and minimally coupled heavy scalars. First, we recover the known fact that graviton amplitudes are the same as in Einstein gravity. Then we show that all amplitudes with two heavy scalars and an arbitrary number of gravitons are also not affected by these interactions. We prove this by direct computations, using field redefinitions known from earlier applications in string theory, and with a combination of factorization and power-counting arguments. Combined with unitarity, these results imply that, in an effective field theory approach, the Newtonian potential receives neither classical nor quantum corrections from terms quadratic in the curvature.

I. INTRODUCTION

Much work has been devoted recently to studying the effects of possible modifications of Einstein-Hilbert (EH) gravity, see [1,2] for recent reviews. Apart from adding a cosmological constant, the conceptually simplest modifications consist in adding terms with higher powers of the curvature to the EH action. Quadratic and cubic corrections make an appearance in the effective gravitational action for closed strings in [3–6], and as counterterms at one loop in gravity coupled to matter [7] and at two loops in pure gravity [8,9]. At the quadratic level, the independent operators can be taken to be R^2, $R^{\mu\nu}R_{\mu\nu}$ and the Gauß-Bonnet (GB) combination $R^{\mu\nu\rho\lambda}R_{\mu\nu\rho\lambda} - 4R^{\mu\nu}R_{\mu\nu} + R^2$. The analysis of [3–5] showed that R^2 or $R^{\mu\nu}R_{\mu\nu}$ cannot be probed by looking at scattering amplitudes, since they can be removed by field redefinitions without influencing the S-matrix as a consequence of the S-matrix equivalence theorem, reviewed later, while the GB term, being topological in four dimensions, can be discarded.[1]

A related question is whether and how higher-derivative corrections affect the Newtonian potential. The setup here is that of considering the elastic scattering amplitude of two heavy scalars minimally coupled to the gravitational field, from which the form of the gravitational potential can then be extracted [12–17]. The recent works [18,19] addressed the effect of terms cubic in the curvature on the Newtonian potential and particle bending angle,[2] and in this note we assess the effect of quadratic terms. The coupling of the massive scalars to the gravitational field is different than that of the dilaton in string theory, hence the question should be reassessed. Some of these results are probably known but given the renewed interest in the connections between scattering amplitudes and gravitational physics it seems timely to collect these insights also in the light of modern amplitude methods.

We model the two heavy bodies probing the gravitational potential by massive scalars, and the relevant action is

$$S = \int d^D x \sqrt{-g}\left[-\frac{2}{\kappa^2}R + aR^{\mu\nu}R_{\mu\nu} + bR^2 \right.$$
$$\left. + \frac{1}{2}\sum_{i=1}^{2}(\partial_\mu\phi_i\partial^\mu\phi_i - m_i^2\phi_i^2) \right]. \tag{1.1}$$

Note that the scalars are not allowed to propagate in loops, as their sole purpose is to act as massive sources.

One approach, not followed here, is to treat the higher-derivative corrections exactly, i.e., to all orders in a and b. The analysis carried out in [3,6,22–25] shows the presence

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[1]In $D = 4 - 2\epsilon$ dimensions, the latter gives at one loop only finite (quantum) terms which are local and thus do not contribute to the gravitational potential. At higher loops the issue should be reconsidered, see [10,11] for a discussion of the physical (ir) relevance of evanescent terms.

[2]See [20,21] for an alternative way to extract the two-body Hamiltonian from the scattering amplitude.
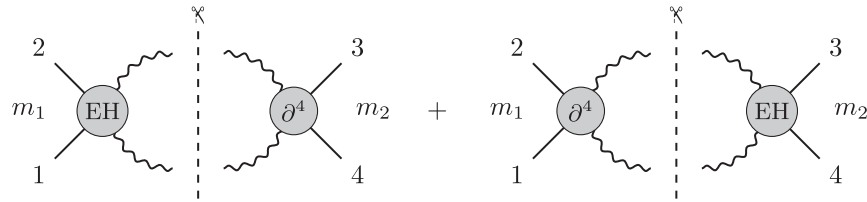
FIG. 1.   The one-loop unitarity cut in the q^2-channel contributing to the massive scalar scattering. Here the ∂^4 blob denotes the amplitude with one insertion of either R^2 or $R^{\mu\nu}R_{\mu\nu}$.

of additional poles in the propagator of the linearized metric tensor field $h_{\mu\nu}$: a massive ghost/tachyon and a massive scalar appear in the spectrum because of the addition of the $R^{\mu\nu}R_{\mu\nu}$ and R^2 terms. At tree level, this leads to the following corrections to the Newtonian potential [22,23,26,27]:

$$V(r) = -\frac{\kappa^2}{32\pi}\frac{M}{r}\left(1 - \frac{4}{3}e^{-m_2 r} + \frac{1}{3}e^{-m_0 r}\right), \quad (1.2)$$

where

$$m_2 = \frac{1}{\kappa}\sqrt{-\frac{2}{a}}, \qquad m_0 = \frac{1}{\kappa}\frac{1}{\sqrt{3b+a}}. \qquad (1.3)$$

However, here we take a different route, first advocated in [14], and treat the Lagrangian (1.1) as that of an effective field theory [28–30], with the dimensionless parameters a and b considered as small. In this case, the masses of the new particles alluded to earlier would be above the cutoff of our theory, hence these particles should not be included as genuinely propagating states.[3] The effect of these terms in our treatment will be that of introducing new vertices, including two-point vertices, which give rise to local interaction terms, with the spectrum being unmodified compared to that of EH gravity. In particular, in the effective field theory approach the new Yukawa potentials induced by the quadratic terms are absent at tree level [30]. This is best seen in momentum space, where the massive propagators are replaced by a polynomial in the momentum transfer squared, which in turn leads to local terms which give no contribution to long-range physics.

Importantly, this is not the end of the story since further classical (and of course quantum) corrections can emerge from loop diagrams [31]. A great simplification stems from the fact that we are interested only in effects on low-energy physics—the classical and quantum corrections to the potential. These can only arise from terms in the scattering amplitude that are nonanalytic in the momentum transfer squared $q^2 := (p_1 + p_2)^2$ between the two massive

scalars [14,31] and can be efficiently captured using unitarity-based methods [32,33]. The latter approach was used efficiently in [16,17] and [34–37] to extract the classical and quantum corrections at $\mathcal{O}(G_N^2)$ to the Newton potential and particle bending angle, respectively, where $G_N := \kappa^2/(32\pi)$ is Newton's constant. We also note the recent works [38,39], where the conservative Hamiltonian for binary systems was extracted at $\mathcal{O}(G_N^3)$ from two-loop amplitude computations. Therefore we only need to focus on unitarity cuts in this channel.

At one loop, we have to consider a two-particle cut such as that in Fig. 1. As the figure shows, at this loop order there are two building blocks: the tree-level two-scalar/two-graviton amplitudes in EH, and the same amplitudes with one insertion of the quadratic corrections to the action, see Fig. 1.

There are only two Feynman diagrams contributing to the latter and it turns out that their sum is zero both for $R^{\mu\nu}R_{\mu\nu}$ and R^2, see Fig. 2. This result implies that the contribution to the scalar potential to first order in a or b is also zero at 2PM, since the amplitude with one insertion of the quadratic corrections vanishes.

This argument can be extended straightforwardly to higher loops. In practice, the focus will be on cut diagrams such as the one depicted in Fig. 3. One of the amplitudes in the cut is in the background of a quadratic correction in the curvature, while the other is a standard EH amplitude. The types of amplitudes that will be needed are: amplitudes with two scalars and an arbitrary number of gravitons, and amplitudes only made of gravitons.[4]

The result of this note is that such amplitudes in a four-derivative background are all zero, hence do not affect Newton's potential to any order in G_N. We prove this in three ways:

(1) Using a redefinition of the metric, in conjunction with the S-matrix equivalence theorem, similarly to what was done in [3–5]. Results obtained here are valid up to linear order in the small parameters a, and b, which is perfectly sufficient from an effective field theory point of view. This argument is valid in D dimensions, hence it lends itself to an application of D-dimensional unitarity;

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[3]Even for a and b being of $\mathcal{O}(1)$, these masses would be of the order of the Planck mass, where a whole tower of higher-derivative terms would have to be included.

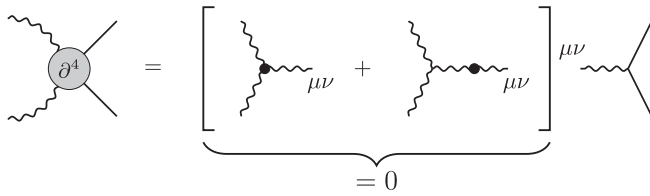[4]See Fig. 1 of [38] for sample two-loop cut diagrams.

FIG. 2.   The sum of the two diagrams contributing to the two-scalar two-graviton amplitudes to first order in R^2. The same result holds for $R^{\mu\nu}R_{\mu\nu}$. All external on-shell states are in D dimensions, the legs labeled by Lorentz indices are off-shell. The relevant Feynman rules can be found in the Appendix.
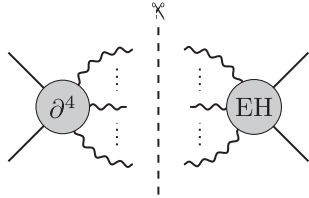

FIG. 3.   An example of higher-loop cut diagram contributing to the Newtonian potential. The ∂^4 symbol denotes an insertion of either R^2 or $R^{\mu\nu}R_{\mu\nu}$.

(2) Using a combination of dimensional analysis and little-group scaling. This argument is valid in four dimensions, as far as the gravitons are concerned;

(3) Using a diagrammatic argument, which turns out to be valid for any number of insertions of the higher-derivative couplings.

These three approaches will be discussed in turn in the three following sections. We also include an Appendix, containing the Feynman rules needed in the calculations.

II. TAMING QUADRATIC TERMS WITH FIELD REDEFINITIONS

In this section we show that n-point gravitons and two-scalar/n-graviton amplitudes in the background of a term quadratic in the curvature are zero. A key ingredient in our proof is the S-matrix equivalence theorem.[5] According to this theorem, fairly generic field redefinitions do not alter the S-matrix. In the context of the effective action of string theory, this has been used to show that terms quadratic in R or $R_{\mu\nu}$ [3–5] or containing any power of R or $R_{\mu\nu}$ (apart from the EH term) [6] do not affect the S-matrix—they can be redefined away. With a similar logic, we introduce the following local field redefinition of the metric:

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[5]This beautiful theorem has a long-winded history that we will not attempt to retrace here. An incomplete list of relevant works include [7,40–46].

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \alpha_1 g_{\mu\nu} R + \alpha_2 R_{\mu\nu} + \sum_{i=1}^{2} \beta_1^{(i)} \partial_\mu \phi_i \partial_\nu \phi_i$$
$$+ \beta_2^{(i)} g_{\mu\nu} \partial_\sigma \phi_i \partial^\sigma \phi_i + \beta_3^{(i)} g_{\mu\nu} \phi_i^2. \tag{2.1}$$

The main point here is that we can fix the α parameters by requiring the vanishing of the coefficients of the R^2 and $R^{\mu\nu}R_{\mu\nu}$ interactions, while the β parameters can be fixed in such a way that no nonminimal coupling between gravity and the scalar fields are generated. To first order in the parameters a and b these nonminimal interactions have the form $R\partial_\mu\phi_i\partial^\mu\phi_i$, $R^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i$ and $R\phi_i^2$. To first order in a and b, the solution is

$$\alpha_1 = \frac{a+2b}{2(D-2)}\kappa^2, \qquad \alpha_2 = -\frac{a\kappa^2}{2}, \tag{2.2}$$

and

$$\beta_1^{(i)} = -\frac{a\kappa^4}{8}, \qquad \beta_2^{(i)} = \frac{a+2b}{8(D-2)}\kappa^4,$$
$$\beta_3^{(i)} = -\frac{a+Db}{2(D-2)^2}m_i^2\kappa^4. \tag{2.3}$$

Under the field redefinitions specified above, the original action (1.1) becomes

$$S' = \int \mathrm{d}^D x \sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\sum_{i=1}^{2}(\partial_\mu\phi_i\partial^\mu\phi_i - m_i^2\phi_i^2)\right.$$
$$+ \kappa^4 \frac{D(a+Db)}{4(D-2)^2}\left(\sum_{i=1}^{2}m_i^2\phi_i^2\right)^2$$
$$- \kappa^4 \frac{a+Db}{4(D-2)}\left(\sum_{i=1}^{2}m_i^2\phi_i^2\right)\left(\sum_{i=1}^{2}\partial_\mu\phi_i\partial^\mu\phi_i\right)$$
$$\left. + \kappa^4 \frac{a+b}{16}\left(\sum_{i=1}^{2}\partial_\mu\phi_i\partial^\mu\phi_i\right)^2 + \mathcal{O}(a^2, b^2, ab)\right]. \tag{2.4}$$

By the equivalence theorem, S and S' lead to the same S-matrix. From the new action (2.4) it is now manifest that no corrections to the EH (two-scalar) n-graviton amplitudes are generated.

It is also interesting to note that the field redefinition introduces contact terms for the four-scalar amplitude. One can easily check that the result for this quantity from (1.1) matches exactly the new four-point vertex:

$$\frac{\delta^4 S'}{\delta\phi_1\delta\phi_1\delta\phi_2\delta\phi_2} = \quad \text{(diagram)} \quad = \quad \text{(diagram)} \quad + \quad \text{(diagram)} \tag{2.5}$$

where the dot and the cross denote the insertion of R^2 and $R^{\mu\nu}R_{\mu\nu}$, respectively.

We truncated the action S' to linear order in a and b. Keeping higher orders in these parameters would imply that higher-derivative terms such as $R\Box R$, R^3 and $RR^{\mu\nu}R_{\mu\nu}$ appear in the new action. These could in turn be eliminated by adding further terms (involving more derivatives) to the field redefinition. Note that the above-mentioned contact terms of the scalar fields (which do not affect the computation of corrections to the Newton potential), and contractions of three or more Riemann tensors (which lead to genuine modifications of the Newtonian potential [18,19]), cannot be eliminated in this way. Finally, we note that the field-redefinition argument we discussed can in principle be applied to a wider class of terms including nonminimal couplings of the scalars to gravity. In particular, one can show that terms of the form $(R_{\mu\nu\rho\sigma})^n(R_{\mu\nu})^m R^l \partial\phi\partial\phi$ can be removed by field redefinition, unlike nonminimal couplings involving only a combination of Riemann tensors of the form $(R_{\mu\nu\rho\sigma})^n \partial\phi\partial\phi$.

III. TAMING QUADRATIC TERMS WITH AMPLITUDE TECHNIQUES

In this section we address the question of the absence of two-scalar/n-graviton and n-graviton amplitudes induced by terms quadratic in R or $R_{\mu\nu}$ from a modern amplitude perspective. This viewpoint allows to address this question to higher orders in the four-derivative couplings, and furthermore has the advantage of treating all four-derivative interactions in (1.1) in one go. For this reason, in this section we will refer in general to any of the four-derivative interactions as R^2 and to any of the two associated couplings as $\tilde{b}$.

The argument is two-fold. First, we show the absence of possible factorizations for an amplitude with two scalars and two gravitons. Next, we show that no two-scalar/n-graviton contact terms, unseen by factorization, are present. Together, these imply the absence of two-scalar/n-graviton amplitudes with one insertion of R^2. We address these two parts in turn.

A. Absence of factorization channels

A two-scalar/two-graviton amplitude in an R^2 background could factorize onto an EH three-point scalar-scalar-graviton amplitude and a three-graviton amplitude

produced by an R^2 interaction. However it is elementary to show that in four dimensions R^2 couplings cannot modify the three-graviton amplitude, and all three-point amplitudes arise from either EH gravity or a six-derivative modification involving $R^{\alpha\beta}{}_{\mu\nu}R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$.

Little-group scaling, combined with considerations of the mass-dimension of the couplings, constrains the most general form of the three-graviton amplitudes. We begin by considering the three-graviton amplitude $A_3(1^{++}, 2^{++}, 3^{--})$. It is well known that there are only two possible helicity structures for this amplitude,

$$A_3(1^{++}, 2^{++}, 3^{--}) \sim \frac{[1\,2]^6}{[2\,3]^2[3\,1]^2} \quad \text{or}$$

$$\tilde{A}_3(1^{++}, 2^{++}, 3^{--}) \sim \frac{\langle 2\,3\rangle^2\langle 3\,1\rangle^2}{\langle 1\,2\rangle^6}. \tag{3.1}$$

Purely on dimensional grounds, the first amplitude arises from a two-derivative interaction, such as EH gravity, while the latter would require a nonlocal interaction in the theory and should be discarded. Next consider the all-plus helicity configuration $A_3(1^{++}, 2^{++}, 3^{++})$. Here one has two possibilities,

$$A_3(1^{++}, 2^{++}, 3^{++}) \sim [1\,2]^2[2\,3]^2[3\,1]^2 \quad \text{or}$$

$$\tilde{A}_3(1^{++}, 2^{++}, 3^{++}) \sim \frac{1}{\langle 1\,2\rangle^2\langle 2\,3\rangle^2\langle 3\,1\rangle^2}. \tag{3.2}$$

We can immediately discard the second one from the request of locality. As for the first, it arises from six-derivative interaction terms such as $R^{\alpha\beta}{}_{\mu\nu}R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$ or $R^{\alpha}{}_{\mu}{}^{\beta}{}_{\nu}R^{\mu}{}_{\rho}{}^{\nu}{}_{\sigma}R^{\rho}{}_{\alpha}{}^{\sigma}{}_{\beta}$, but not from those terms which can be eliminated by a field redefinition, e.g., $R\Box R$ or $RR^{\mu\nu}R_{\mu\nu}$. As a consequence, the addition of a four-derivative interactions to the action with mass-dimension zero coupling $\tilde{b}$ cannot generate a three-point amplitude in four dimensions. Hence, any two-scalar/two-graviton amplitude in the R^2 background can only be a contact term.

B. Absence of contact terms

We still have the possibility of a two-scalar/two-graviton contact term, which via factorization would give rise to nontrivial higher-point amplitudes. In the following we will

show that such contact terms are absent for amplitudes with any number of gravitons and up to two scalars.

First we consider pure graviton amplitudes at tree level, starting from four external particles. We showed that the three-graviton amplitude is unaffected by any R^2 insertion, hence no factorization channel for the four-point amplitude is available. The remaining task is to exclude potential contact terms. The latter can in general be written schematically at a given multiplicity n and order B in $\tilde{b}$ as

$$\mathcal{M}_{\text{contact}} \propto \tilde{b}^B \kappa^{n-2+2B} \prod_{i=1}^{n} \lambda_i^{\otimes a_i} \tilde{\lambda}_i^{\otimes s_i}. \qquad (3.3)$$

The κ^{2B} comes from the additional powers in κ carried by each of the R^2 insertions (with respect to EH) along with a single power of $\tilde{b}$.[6] Dimensional analysis implies that

$$c + \frac{1}{2} \sum_{i=1}^{n} (a_i + s_i) = 4 - n, \qquad (3.4)$$

where $c = 2 - n - 2B$ is the overall dimension of the couplings. This can be rewritten as

$$\sum_{i=1}^{n} (a_i + s_i) = 4 + 4B. \qquad (3.5)$$

A further constraint comes from little-group scaling, which requires

$$-a_i + s_i = 2h_i, \qquad (3.6)$$

where h_i is the helicity of particle i. From (3.6) we also have the constraint that either $a_i \geq 4$ or $s_i \geq 4$, corresponding to the helicity of graviton i being minus or plus two, respectively.

Specializing now to $n = 4$ we see that the latter constraint cannot be satisfied along with (3.5) for $B < 3$, thus it is not possible to build contact terms with $B = 1, 2$. This means that up to second order in $\tilde{b}$ it is impossible to write down any contact term contribution to the four-graviton amplitude coming from an R^2 interaction. At $B = 3$ the argument breaks down because three insertions of R^2 terms can be mimicked by one insertion of the Riemann tensor to the fourth power, which gives rise to a nonvanishing amplitude.

The final step is to recursively extend the argument to $n > 4$. We only need to exclude a contact term. It is immediate to realize that the combined constraints (3.5) and (3.6) cannot be satisfied for $B < n - 1$, hence contact terms up to order $n - 2$ in $\tilde{b}$ are ruled out. Since the

recursive argument starts from three- and four-graviton amplitudes, we conclude that all n-graviton amplitudes are unaffected by R^2 contributions up to order 2 in $\tilde{b}$.

Dimensional analysis was sufficient to show that up to $\mathcal{O}(\tilde{b}^2)$ there are no R^2 corrections to n-graviton amplitudes. In order to push our considerations to even higher orders in $\tilde{b}^2$ we need to invoke additional diagrammatic arguments, which will be detailed in the next section.

The discussion presented so far can be generalized with ease to two-scalar/n-graviton amplitudes. Once again the three-point minimal interaction involving a single graviton and two scalars is unaffected by the R^2 couplings, which rules out factorization channels in the four-point amplitude. The general form of the contact term now becomes

$$\mathcal{M}_{\text{contact}} \sim \tilde{b}^B \kappa^{n-2+2B} \text{M}(q_i) \prod_{i=3}^{n} \lambda_i^{\otimes a_i} \tilde{\lambda}_i^{\otimes s_i}, \qquad (3.7)$$

where M is a quadratic combination of the momenta q_i of the scalars, labeled by $i = 1, 2$, arising from their minimal coupling to gravity. Then $[\text{M}] = 2$, and repeating the same dimensional analysis as before we find

$$\sum_{i=1}^{n-2} (a_i + s_i) = 4B. \qquad (3.8)$$

For $B = 1$ the constraints (3.8) and (3.6) cannot be satisfied simultaneously, for any number of gravitons. As a side remark, note that this argument does not prevent the appearance of amplitudes with four scalars, because these amplitudes require a double insertion of $\text{M}(q_i)$ with $i = 1, …, 4$. This is also in complete agreement with our approach based on field redefinitions—indeed (2.4) does generate a four-scalar amplitude.

In summary, we have shown the absence of R^2 corrections at linear order in the coupling $\tilde{b}$ for all tree amplitudes with n gravitons and up to two scalars. As argued in the Introduction, this implies via unitarity the absence of all R^2 corrections to the Newtonian potential, both at the classical and quantum level.

IV. BEYOND LINEAR ORDER IN a AND b VIA DIAGRAMMATICS

As discussed earlier, from an on-shell perspective it is not possible to push our general considerations further than the first and second order in the R^2 and $R^{\mu\nu} R_{\mu\nu}$ couplings for two-scalar/n-graviton or n-graviton amplitudes, respectively (again collectively denoted as ∂^4 in the figures below). In this section we show how, combining on-shell arguments with diagrammatic insights, we can refine our earlier discussion to include higher-order corrections in a and b. We consider first the two-scalar two-graviton amplitude at order $\tilde{b}$:

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[6]Recall that we are interested in the effect of the four-derivative interactions on the amplitude, hence $B > 0$.

$$
\text{(diagram)} \stackrel{(\tilde{b})}{=} \sum_{\text{perm}} \left(\frac{1}{4} \text{(diagram)} + \frac{1}{4} \text{(diagram)} \right) = 0 \,, \tag{4.1}
$$

and the four-graviton amplitude at order $\tilde{b}^2$

$$
\text{(diagram)} \stackrel{(\tilde{b}^2)}{=} \sum_{\text{perm}} \left(\frac{1}{8} \text{(diagram)} + \frac{1}{4} \text{(diagram)} + \frac{1}{8} \text{(diagram)} \right) = 0, \tag{4.2}
$$

where the sum runs over all possible permutations of the external legs and appropriate symmetry factors have been associated to each diagram. The vanishing of these amplitudes is ensured by the on-shell argument given in Sec. III. To simultaneously satisfy (4.1) and (4.2), one needs

$$
\text{(diagram)}_{\mu\nu} + \text{(diagram)}_{\mu\nu} = 0 \,, \tag{4.3}
$$

where the line carrying the Lorentz indices is off shell, whereas the other two lines are on shell. Here we recovered the diagrammatic identity originally found by direct computation which is displayed in Fig. 2, from a purely on-shell argument.

We can use this identity as a replacement rule to turn insertions of R^2-type vertices into propagator insertions, or the other way around. Such replacements lead to great simplifications and are in some cases sufficient to prove the vanishing of entire classes of amplitudes. Examples of this situation are the four-graviton amplitude for $B > 2$ and the five-graviton amplitude for $B > 3$, whose vanishing is not guaranteed by the on-shell argument presented earlier; by drawing all possible diagrams one can immediately see that the identity (4.3) implies the vanishing of these amplitudes to all orders in $\tilde{b}$.

As an example of how to obtain further diagrammatic relations from (4.3) and the known vanishing amplitudes, consider the four-graviton amplitude at $\mathcal{O}(\tilde{b})$. We know that

$$
\text{(diagram)} \stackrel{(\tilde{b})}{=} \sum_{\text{perm}} \left(\frac{1}{8} \text{(diagram)} + \frac{1}{4} \text{(diagram)} + \frac{1}{4!} \text{(diagram)} \right) = 0 \,. \tag{4.4}
$$

Using (4.3), we obtain a further identity:

$$
\sum_{\text{perm}} \left(\text{(diagram)} + \frac{1}{3} \text{(diagram)} \right) = 0 \,, \tag{4.5}
$$

where all external legs are on-shell.

To find a new off-shell identity one has to look at the two-scalar/three-graviton amplitude at $\mathcal{O}(\tilde{b})$ or the six-graviton amplitude at $\mathcal{O}(\tilde{b}^2)$. After applying (4.3) to cancel as many terms as possible, we are left with a sum of four diagrams, which we know must vanish, i.e.,

$$
\sum_{\text{perm}} \left(\frac{1}{3} \text{(diagram)}_{\mu\nu} + \frac{1}{3} \text{(diagram)}_{\mu\nu} + \text{(diagram)}_{\mu\nu} + \text{(diagram)}_{\mu\nu} \right) = 0 \,, \tag{4.6}
$$

where the permutations are over the on-shell legs. As a consistency check, notice that this identity reduces to (4.5) once we put the fourth leg on-shell.

The identity (4.6) combined with (4.3) is sufficient to prove that the corrections to the six-graviton and two-scalar/three-graviton amplitudes induced by the four-derivative couplings vanish to all orders in $\tilde{b}$. Iterating this procedure at higher multiplicity and power of the coupling $\tilde{b}$, one will find additional identities involving a higher and higher number of particles. We expect the combination of all of these identities to be sufficient to guarantee the vanishing of the R^2 modified n-graviton and two-scalar/n-graviton amplitudes for any n and any power of $\tilde{b}$.

V. CONCLUSIONS

In summary we have shown from different but complementary angles that amplitudes contributing to the computation of the Newtonian potential receive no corrections from curvature squared terms in four dimensions. As we have seen in the previous section, this statement not only applies to linear order in the couplings a and b, as appropriate for an effective field theory treatment, but also continues to hold to higher orders in the couplings. It would be very interesting to settle this question for arbitrarily high

orders, and we expect that amplitude techniques may provide an alternative, more efficient method than field redefinitions, which become quickly very cumbersome at high orders in the parameters. Similarly it would be interesting to revisit the case of terms cubic (or higher) in the curvature, and test whether there exist appropriate field redefinitions that remove terms involving the Ricci scalar and tensor while preserving the minimal coupling of the heavy scalars (up to contact terms involving four or more scalars). Also for this case the amplitudes/on-shell techniques employed in Sec. III may prove useful. Given the results of [6], we expect a positive answer to this question, which would leave the cubic corrections computed in [18,19] as the first higher-derivative corrections to EH gravity that can modify the Newtonian potential.

ACKNOWLEDGMENTS

APPENDIX: FEYNMAN RULES

We collect in this Appendix the relevant Feynman rules used in the text. The three-point graviton vertex in EH with two on-shell legs is

$$
\begin{aligned}
\alpha\beta = i\kappa\,\epsilon_1 \cdot \epsilon_2 &\left[\epsilon_1 \cdot \epsilon_2 \left(\frac{3}{2}\,p_1 \cdot p_2\,\eta^{\alpha\beta} - p_1^\alpha p_1^\beta - p_1^{(\alpha}p_2^{\beta)} - p_2^\alpha p_2^\beta \right) \right. \\
&\left. - \epsilon_1 \cdot p_2\,\epsilon_2 \cdot p_1\,\eta^{\alpha\beta} + 2\epsilon_1 \cdot p_2\,\epsilon_2^{(\alpha}p_2^{\beta)} + 2\epsilon_2 \cdot p_1\,\epsilon_1^{(\alpha}p_1^{\beta)} - 2p_1 \cdot p_2\,\epsilon_1^{(\alpha}\epsilon_2^{\beta)} \right] \\
&- i\kappa\big(\epsilon_2 \cdot p_1\,\epsilon_1^\alpha - \epsilon_1 \cdot p_2\,\epsilon_2^\alpha \big)\big(\epsilon_2 \cdot p_1\,\epsilon_1^\beta - \epsilon_1 \cdot p_2\,\epsilon_2^\beta \big),
\end{aligned}
\tag{A1}
$$

The Feynman rules for insertions of R^2 (denoted by a bullet) and $R^{\mu\nu}R_{\mu\nu}$ (denoted by a cross) are:

$$
\alpha\beta = ib\,\kappa^3\,\epsilon_1 \cdot \epsilon_2\,\big(-3\epsilon_1 \cdot \epsilon_2\,p_1 \cdot p_2 + 2\epsilon_1 \cdot p_2\epsilon_2 \cdot p_1 \big)\big(k^2\,\eta^{\alpha\beta} - k^\alpha k^\beta \big),
\tag{A2}
$$

$$
= 2ib\,\kappa^2\,\big(k^2\,\eta^{\alpha\beta} - k^\alpha k^\beta \big)\big(k^2\,\eta^{\mu\nu} - k^\mu k^\nu \big),
\tag{A3}
$$

$$\begin{aligned}
{}_{2}\underset{1}{\times}\!\!\!\!\alpha\beta = ia\,\kappa^{3}\,\epsilon_{1}\cdot\epsilon_{2}&\left[\frac{k^{2}}{2}\left(\epsilon_{1}\cdot p_{2}\,\epsilon_{2}\cdot p_{1}-\frac{3}{4}\epsilon_{1}\cdot\epsilon_{2}\,k^{2}\right)\eta^{\alpha\beta}+\frac{1}{2}\epsilon_{1}\cdot\epsilon_{2}\,k^{2}\,p_{1}^{(\alpha}p_{2}^{\beta)}\right.\\
&+\left(\frac{1}{4}\epsilon_{1}\cdot\epsilon_{2}\,k^{2}-\epsilon_{1}\cdot p_{2}\,\epsilon_{2}\cdot p_{1}\right)k^{\alpha}k^{\beta}+k^{2}\left(\epsilon_{1}\cdot p_{2}\,\epsilon_{2}^{(\alpha}p_{2}^{\beta)}+\epsilon_{2}\cdot p_{1}\,\epsilon_{1}^{(\alpha}p_{1}^{\beta)}\right)\\
&\left.-\frac{k^{4}}{2}\epsilon_{1}^{(\alpha}\epsilon_{2}^{\beta)}\right]-ia\,\frac{k^{2}}{2}\left(\epsilon_{2}\cdot p_{1}\,\epsilon_{1}^{\alpha}-\epsilon_{1}\cdot p_{2}\,\epsilon_{2}^{\alpha}\right)\left(\epsilon_{2}\cdot p_{1}\,\epsilon_{1}^{\beta}-\epsilon_{1}\cdot p_{2}\,\epsilon_{2}^{\beta}\right),
\end{aligned}\tag{A4}$$

$$\overset{}{\underset{}{\times}} = ia\,\kappa^{2}\,\mathrm{Sym}\left[k^{\alpha}k^{\beta}k^{\mu}k^{\nu}-\frac{1}{2}\eta^{\alpha\beta}k^{\mu}k^{\nu}k^{2}-\eta^{\alpha\mu}k^{\beta}k^{\nu}k^{2}+\frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}k^{4}+\frac{1}{2}\eta^{\alpha\mu}\eta^{\beta\nu}k^{4}\right],\tag{A5}$$

where the symmetrization is both over the indices $\mu\leftrightarrow\nu$, $\alpha\leftrightarrow\beta$ and $(\mu,\nu)\leftrightarrow(\alpha,\beta)$.

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