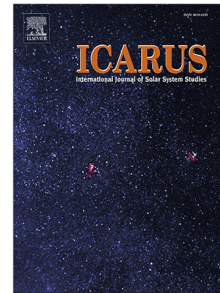


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Gravitational re-accumulation as the origin of most contact binaries and other small body shapes

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1 Gravitational Re-accumulation as the Origin of Most
2 Contact Binaries and Other Small Body Shapes

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12 **Abstract**

13 Asteroids show a variety of shapes, ranging from roundish to elongated
14 to binary systems and contact binaries like (25143) Itokawa, the target of the
15 Hayabusa mission (JAXA). These bodies spend most of their time within a
16 collisional system, the asteroid belt, where impact processes are relatively
17 frequent. Speculations on the origin of asteroid shapes invoke mechanisms
18 such as collisions and spin-up effects. N-body numerical simulations of frag-
19 ment evolution following catastrophic collisions have been recently carried
20 out (Campo Bagatin *et al.*, 2018). In this study the idea that the stochastic
21 process of gravitational re-accumulation may be responsible for many ob-
22 served asteroid shapes is introduced. Asteroid contact binaries are shown to
23 be regularly produced by the gravitational re-accumulation process following
24 catastrophic impact. Similar processes may have occurred in the case of some
25 comets and Trans-Neptunian Objects.

26 *Keywords:* Asteroids, Collisional physics, Contact binaries, Asteroid
27 shapes

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28 1. Introduction

29 Up to the late 1980s asteroids were detected only as small spots of light
30 moving in the sky across the stars. At that time, the study of the dynamical
31 parameters of their orbits and of their photometric colours were—together
32 with light curves—almost the only available direct knowledge about them.
33 That situation began to change dramatically with the first radar observations
34 and especially since October 29th, 1991, when the *Galileo* probe took the first
35 image of a main belt asteroid, *951 Gaspra*. It was the first time that we had
36 the chance to see what an asteroid looks like, examine its shape and directly
37 measure its size. Less than a dozen asteroids have been resolved by space
38 probes and over one hundred—especially NEAs (Near Earth Asteroids)—have
39 been observed by radiotelescopes since then, allowing the beginning of a new
40 era in the study of asteroids. This has allowed improvement to the study
41 of surface composition, morphology of craters, internal structure, rotation
42 states, and shapes.

43 Speculations on the origin of asteroid shapes invoke mechanisms such as
44 collisions and spin-up effects. The case of contact binaries is particularly in-
45 teresting, that is elongated bodies in which two parts can be clearly identified:
46 a body where most of the mass is, and a head, both resting on each other in a
47 stable configuration. Tens of objects have been identified to be contact bina-
48 raries by radar, spacecraft images and light curves, half of which were identified
49 in the last 5 years. They include 6 comets, 10 TNOs (trans-neptunian ob-
50 jects) and almost 70 asteroids (<http://johnstonsarchive.net/astro/index.html>).
51 In the case of NEAs, 12% are contact binaries according to radar detection
52 statistics. Some simulations show unstable binary dynamics leading to con-
53 tact binaries (Taylor & Margot, 2014), but Jacobson & Scheeres (2011)
54 and Boldrin *et al.* (2016) showed that YORP spin-up may instead have led
55 many NEA contact binaries to eventually evolve to binary systems or aster-
56 oid pairs. Regarding bilobated comets, Hirabayashi *et al.* (2016) identified a
57 cyclic mechanism for fission and re-combination of comets, nevertheless not
58 implying their origin.

59 Direct spacecraft images are available for four comets, one TNO and one
60 asteroid, including comet 67P (Rosetta, ESA) and asteroid (25183) Itokawa
61 (Hayabusa, JAXA). In addition, the New Horizons space probe recently re-
62 vealed of TNO 2014 MU69 to be a contact binary with components of ap-
63 parently roundish shape, but oblate. Dynamical mechanisms invoking non-
64 gravitational effects like YORP (Rubincam, 2000) and BYORP (Steinberg

65 & Sari, 2011), acting on asteroid spins, have been proposed to explain the
66 origin of contact binaries (Ćuk & Nesvorný, 2010). Such explanations may
67 work in the case of some NEAs, due to their small size and proximity to
68 the Sun, making YORP torque efficient. However, NEAs represent 70% of
69 the observed contact binary sample, making a general explanation neces-
70 sary to explain the morphology of any kind of small solar system bodies. In
71 fact, the reason why some asteroids look roundish while others look elon-
72 gated is not currently understood either, and no overall process responsible
73 for such shapes has been identified to date. The fate of asteroid shaping is
74 likely related to their collisional history and internal structure. Asteroids
75 are formed inside the asteroid belt, where relative encounter speeds are dis-
76 tributed around 5.8 km/s and collisions are mostly catastrophic (Farinella
77 *et al.*, 1992). Unfortunately, no direct measurement of asteroid interiors has
78 been possible yet. Notwithstanding, experimental, theoretical, statistical,
79 and numerical studies have been carried out over the last four decades and
80 may help us to understand the processes that affect such bodies and may
81 influence their structure and shape.

82 This study is part of a wide investigation about the process of fragment
83 re-accumulation that follows high-speed impacts between asteroids. The de-
84 tailed description of methodology of the overall study and quantitative results
85 regarding asteroid density and porosity and their implications are in Campo
86 Bagatin *et al.* (2018). Here we report about the results regarding morphology
87 of asteroids and how different shapes are produced as a natural process in
88 the gravitational re-accumulation of fragments.

89 2. Methodology

90 Most asteroids smaller than some 100 km in diameter are believed to
91 be gravitational aggregates (Campo Bagatin *et al.*, 2001; Richardson *et al.*,
92 2002) formed in collisional processes in the main asteroid belt. **Many argu-
93 ments support this assumption, from both observational and the-
94 oretical, as summarized in Campo Bagatin *et al.* (2018).** For that
95 reason, we perform numerical simulations of the collisional and dynamical
96 evolution of irregularly shaped rigid fragments interacting under their mu-
97 tual gravitational forces after a collision takes place. We briefly recall here the
98 methodology followed in Campo Bagatin *et al.* (2018). Each rigid fragment
99 (usually referred to as fragment or component) is modeled as a packing of tens
100 to hundreds of rigid particles whose mutual distance is kept constant. Such

101 fragments cannot deform nor break, so they move under rigid-body mechani-
 102 cal laws and can experience partially inelastic collisions with other fragments.
 103 Such simulations were performed using a soft-sphere discrete-element model
 104 N-body numerical code (PKDGRAV) (Richardson *et al.*, 2000; Stadel, 2001;
 105 Schwartz *et al.*, 2012). Re-accumulation may happen right after a shattering
 106 collision of a coherent body or a previous gravitational aggregate has taken
 107 place. The overall idea is to concentrate our efforts on the phase in which
 108 some fragments have escaped the system and the remaining are beginning to
 109 re-accumulate. The shattering phase itself is out of the scope of our study
 110 (for more discussion, see Jutzi *et al.* (2015)) and our single fragments are not
 111 formed through clumping of smaller scale fragments, which is typical of for-
 112 mer studies (e.g. Michel *et al.* (2004), Benavidez *et al.* (2018)). We draw the
 113 mass distribution of fragments, as well as their shapes, from results of labo-
 114 ratory shattering experiments carried out by Durda *et al.* (2015). Fragments
 115 are scaled and randomly placed in space with initial velocities and spins. In
 116 this way we build 36 rigid fragments, each made of tens to hundreds of spher-
 117 ical particles, with total ~ 5000 — 10000 particles. The largest fragment of
 118 the distribution is placed in a central location, in analogy to what is observed
 119 in the outcome of laboratory experiments (as we show in Sec. 2.2). The ini-
 120 tial conditions setup is repeated for each of the simulations, investigating the
 121 effect of different fragment mass density and total mass of the system. The
 122 mass distributions of fragments obtained in this way are in agreement with
 123 those describing post-fragmentation states of catastrophic disruption simu-
 124 lations by Jutzi *et al.* (2009, 2010). The final size of the aggregates range
 125 from ~ 0.5 to ~ 10 km. Simulations consist of allowing all fragments to
 126 gravitationally interact with each other and undergo mutual collisions. Each
 127 system eventually comes to rest in a permanent configuration with no further
 128 relative motion between components and with the overall spin correspond-
 129 ing to the angular momentum of the system, which ranges from low to high
 130 values and is conserved in the simulations.

131 2.1. Fragment mass distribution

132 The mass and shape distributions—in terms of aspect ratios—obtained in
 133 the laboratory experiments carried out at NASA Ames in 2015 (Durda *et al.*,
 134 2015) were the starting point to build random distributions of masses and
 135 shapes of the synthetic components in numerical simulations. From each
 136 of the six collisional experiments at the NASA Ames Vertical Gun Range
 137 (AVGR), we worked out a relative mass (m_i/M) distribution and the aspect

138 ratio for the largest 36 fragments obtained in shattering experiments (m_i is
139 the mass of a generic fragment, M the mass of the target).

140 For any given simulation we run, we draw at random a number of frag-
141 ments from the corresponding experimental distribution. We build our syn-
142 thetic, irregularly shaped, components by extracting them from of a parent
143 body that was obtained by randomly assembling a cloud of randomly dis-
144 tributed 5000 spherical particles that was allowed to collapse by gravitational
145 re-accumulation. A suitable density is assigned to the whole parent body
146 so that it will be the density of the extracted components. **This density**
147 **corresponds to the meteorite analog density of the asteroid type to**
148 **be simulated (e.g $\rho = 3.5 \text{ g/cm}^3$ for S-type and 2.5 g/cm^3 for C-type**
149 **asteroids).** Each extracted component is a rigid aggregate made of spherical
150 particles and it has a tentative 3D ellipsoidal shape whose axes ratios are
151 randomly taken from the experimental distributions, as described in detail in
152 Campo Bagatin *et al.* (2018). Specifically, for any given experimental frag-
153 ment distribution, we draw at random mass ratios m_i/m_{LF} (m_{LF} is the mass
154 of the largest fragment) from the corresponding experimental relative mass
155 distribution and extract sets of aspect ratios from the values obtained from
156 the empirical distributions of shapes. In this way we have—for each new gen-
157 erated component—a different set of axis ratios corresponding to each mass
158 ratio. This procedure can be repeated as many times as needed depending
159 on the number of components to be built. Finally the whole distribution
160 is scaled to a convenient mass, keeping the density of components constant.
161 Our nominal case is such that all components have an equivalent spherical
162 diameter of ≈ 2 km altogether.

163 In any given simulation, components have to be located in space under
164 suitable initial configurations. The largest component of the distribution is
165 placed at the center of the coordinate system and the rest are randomly lo-
166 cated in space freely or within a given limiting volume. Overlaps are avoided
167 in the set up process by suitable random spacing. Different initial fragment
168 distributions are shown in Fig. 6 and movies 1 to 3 (online supplementary
169 material).

170 Different values for the limiting overall volume were considered to check
171 the dependence of the results on initial conditions. We chose volumes in
172 power of 2 relative to the aggregate volume (V_e). V_e is the volume of the
173 equivalent sphere of the total mass of the components, assuming it has the
174 same density of the components.

175 This choice corresponds to five different initial boundary spherical vol-

176 umes to contain the created components. Volumes are set in such a way that
 177 they double with respect to each other: $V_4 = 2V_3 = 4V_2 = 8V_1 = 16V_e$, where
 178 V_e is the volume of the sphere equivalent to the aggregate volume.

179 The velocities of components are directed towards the center of mass and
 180 a spin vector is assigned randomly to each component within given ranges
 181 quantified below. The speed distribution is taken as uniform up to values
 182 smaller than the escape speed (typically a few tens of cm/s for km-size ob-
 183 jects, depending on the mass of the system). In this way, initial conditions
 184 are a snapshot of the dynamical situation of the components that are bound
 185 gravitationally, once they have reversed the direction of their velocity vector
 186 and are on their way back to the center of mass of the system. The veloc-
 187 ity distribution at that point is largely unknown for real re-accumulation
 188 processes. Also, fragments do not reverse their direction of velocity at the
 189 same time in real re-accumulation events. Therefore, assuming any kind of
 190 distribution at a given time is indeed arbitrary; for that reason we chose a
 191 simple uniform distribution of speed values. No mass-velocity dependence is
 192 assumed in this phase.

193 The rotation period of each component was also drawn from a flat distri-
 194 bution spin period, in the range 0-12 h. Again, there is little knowledge of the
 195 spin distribution of fragments resulting from shattering experiments, there-
 196 fore any assumption is arbitrary. Main Belt asteroids are collisionally evolved,
 197 which causes their spin periods to approximately match a maxwellian distri-
 198 bution (Farinella *et al.*, 1981) averaged at about 6 h. In our case, the
 199 spin distribution resulting from of shattering events is not necessarily non-
 200 uniform, however it is certainly not collisionally evolved. Therefore, we as-
 201 sumed a simple flat distribution for the spin rate of components centered on
 202 the average value of Main Belt asteroid spin rates. Once radial velocities
 203 and spins are set, it is possible to change the value of the overall angular
 204 momentum to match specific situations.

205 **Additional angular momentum can be injected in the system as**
 206 **a whole at the end of the fragment distribution set up. That was**
 207 **done in all simulations labelled as ‘Stage 2’ (Table 2 and 3) of the**
 208 **first part of the study (Campo Bagatin *et al.*, 2018).**

209 In this way, we are simulating the initial conditions of a mass distribu-
 210 tion of fragments with irregular shapes that are at the beginning of the re-
 211 accumulation phase following a catastrophic disruption where the fragments
 212 with ejection speeds larger than the escape limit have already left the system.
 213 Many different initial conditions were created corresponding to each of the

214 experimental mass and shape distributions so that 104 numerical simulations
215 were run, 89 of which were successful in producing stable gravitational ag-
216 gregates around 2 km in size (the rest had too large angular momentum to
217 produce single aggregates). Another set of 40 simulations was run to extend
218 the results to the 0.5–10 km asteroid size range and to check the effect of
219 simulation parameters, as reported and discussed in Campo Bagatin *et al.*
220 (2018).

221 PKDGRAV allows the system to gravitationally and collisionally evolve
222 until stabilization. When the simulation is over, volume, density and porosity
223 are calculated by a suitable algorithm developed for this purpose.

224 2.2. Location of largest component

225 The numerical simulations performed in the frame of this research share
226 a common assumption: the largest fragment occupies a central position in
227 the space distribution of components at the start of each simulation. This
228 is based on experimental evidence as is illustrated in movies 4 and 5 (online
229 supplementary material). Even if not often explicitly stated in the litera-
230 ture, nor even usually quantified, this was a common result in collisional
231 laboratory experiments since the 1980s. However, it was difficult to assess at
232 that time due to the lack of high-quality, high-speed cameras and suitable
233 software. Pictures and video recording of hyper-velocity impact fragmenta-
234 tion experiments can now show this is a usual pattern. The experimental
235 results of Durda *et al.* (2015), which have been taken as a starting point of
236 this study, show this pattern again. Fig. 1 shows frames from three impact
237 shattering experiments performed at NASA AVGR. The target (a & c) and
238 the situation a few milliseconds after the collision (b & d) are shown for 2
239 different views with the shape and position of the unshattered target to show
240 the relative position of the largest fragment resulting from shattering. It is
241 evident that—in all cases—the largest fragment is the closest to the center of
242 the original target, with low speed relative to the center of mass of the sys-
243 tem (also see Table 1). The rest of the fragments are always ejected at larger
244 speeds. If this experimental behavior can be assumed at asteroid scale, then
245 the largest component of the initial distribution of the re-accumulation phase
246 shall generally occupy a central position, which is not necessarily coincident
247 with the center of mass of the system.

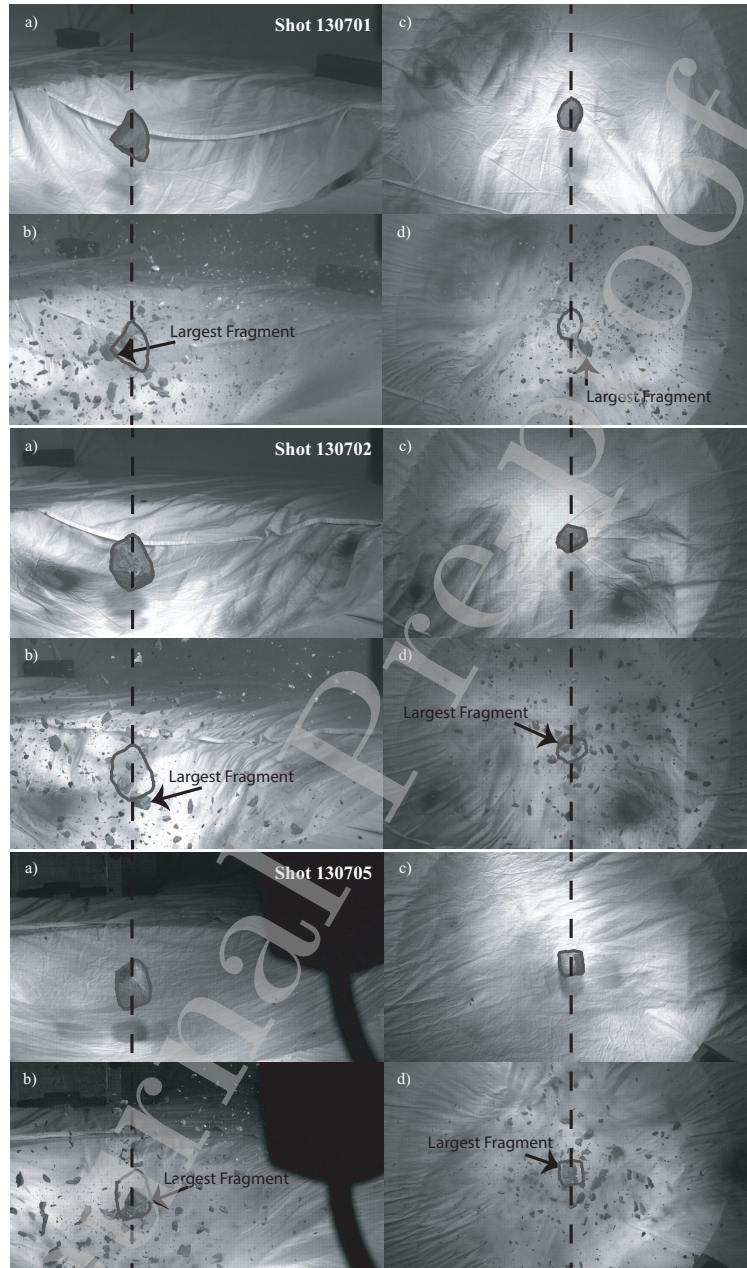


Figure 1: Snapshots of three shattering experiments. For each shot, side (a, b) and azimuthal (c, d) views show the position of the largest fragment 14, 24 and 20 ms respectively after the projectile impact. The original shape and position of the target are marked as a reference.

248 *2.3. Mass distribution reliability*

249 A comparison between the synthetic mass distributions obtained in our
 250 numerical simulations and published distributions of numerical simulations
 251 of asteroid shattering by SPH (Smoothed Particle Hydrodynamics) may be
 252 useful to assess the validity and compatibility of our results with different
 253 approaches to the problem. Jutzi *et al.* (2009) performed SPH numerical
 254 simulations of high-speed shattering of given targets and compared them to
 255 the laboratory results on targets with the same mass, material and the same
 256 impact speed. The size distributions obtained numerically were in reason-
 257 able agreement with the experimental ones. They showed that as a general
 258 trend their cumulative size distributions stay slightly below the corresponding
 259 curve for experimental results (their figures 3, 4, 5, 7, 9, 10, 11, 12, 13). For
 260 comparison, in Campo Bagatin *et al.* (2018), **figures 3, 4 and 5** show the
 261 cumulative mass distributions that we obtain in the generation of our frag-
 262 ments. The trend for those distributions is very similar to Jutzi *et al.* (2009):
 263 most stay slightly below the experimental curve. We can also compare the
 264 slopes of the experimental distributions in Durda *et al.* (2015)—which we use
 265 as a reference to build our synthetic distributions both in Campo Bagatin *et al.*
 266 *et al.* (2018) and in this paper—and the size distributions obtained by former
 267 SPH fragmentation models, namely Jutzi *et al.* (2010). The latter reported
 268 cumulative size distributions with variable slope in two nominal cases in the
 269 $\alpha = (2.21, 2.24)$ range. This is in very good agreement with the slopes for
 270 the cumulative mass distribution reported in Durda *et al.* (2015), that were
 271 in the $\beta = (0.75, 0.82)$ range for two of the four experimental outcomes used.
 272 The relation between size and cumulative mass distribution slopes is $\alpha = 3\beta$,
 273 which allows a direct comparison between the two sets of distributions. In
 274 particular, the experimental cumulative mass distribution that was used to
 275 produce the numerical simulation that generates a contact binary similar to
 276 the shape of asteroid Itokawa is $\beta = 0.75$. In conclusion, we can state that the
 277 mass distributions used in our simulations—directly derived from experimen-
 278 tal distributions—are in very good agreement with those found in shattering
 279 simulations, making our results compatible with the SPH approach shown in
 280 Jutzi *et al.* (2009, 2010).

281 **3. Results**

282 The final shapes of the end-state aggregate structures are generally irreg-
 283 ular. Such structures typically take 3 to 5 hours to settle down. Different

284 mass distributions, irregular fragment shapes and different angular momenta
 285 drive each system to particular configurations mainly driven by a stochas-
 286 tic process. Nevertheless, common patterns can be identified. Small initial
 287 separation between components favors formation of preservation of some-
 288 what roundish configurations (81% of cases), as fragments can only travel
 289 a short distance before colliding with nearby components. These situations
 290 may correspond to relatively low-energy shattering collisions. In that case,
 291 the residual kinetic energy for fragments would be small allowing for small
 292 fragments displacement with respect to their original location in the parent
 293 body.

294 Contrary to what is commonly assumed to be natural for fragment re-
 295 accumulation, despite beginning at central location, the largest fragment
 296 ends up buried into the nucleus of the final aggregate only in 14% of our
 297 simulations. This is an unexpected result of our research and is fundamental
 298 in the explanation of the aggregate shapes as well as in the observation of
 299 asteroids and comets. The formation of relatively elongated objects with
 300 shapes very similar to the observed contact binaries is found in 23.6% of the
 301 simulations carried out, where two parts can be clearly identified as the head
 302 and the body of the object.

303 3.1. Shape classification

304 In order to analyze the results presented here we produce visual descrip-
 305 tions of the *pkdgrav* outcome. For any numerical simulation, each output
 306 corresponds to the time evolution of the physical quantities (size, mass, po-
 307 sition, velocity, and spin vectors) of each of the 5000-10000 particles used,
 308 grouped into rigid aggregates at a given time step. Those are constructed
 309 into images that are eventually stitched into a movie using auxiliary code,
 310 including the public-domain ray-tracer POV-Ray. Different views are pro-
 311 duced so that the qualitative morphologies of each end state can be suitably
 312 classified. Fig. 2 shows different aggregate morphology, ranging from rounded
 313 to elongated and contact binaries. Irregular shapes cannot be parameterized
 314 in a simple way, but still some rough classifications can be constructed, as
 315 follows.

316 The elongation parameter (Campo Bagatin *et al.*, 2018) is calculated
 317 from numerical output as a semi-quantitative measure of the separation of
 318 the largest component relative to the other fragments in each simulation.
 319 This is a measure of off-center mass distribution of the re-accumulated body
 320 and is calculated as the distance between the position of the center of mass of

321 the largest component, \vec{r}_{LC} , and the position of the center of mass of the rest
 322 of components, \vec{r}_{RC} , normalized by the radius of the equivalent sphere of the
 323 aggregate (the sphere whose volume is equal to the volume of the aggregate
 324 itself), R_e .

$$E = \frac{|\vec{r}_{LC} - \vec{r}_{RC}|}{R_e}.$$

325 E discriminates between objects whose largest component occupies a central
 326 position (small $E > 0$ values) and those for which the largest component
 327 is away from the center. In this way it is possible to discriminate the separation
 328 of the largest component from the center of the distribution even in the
 329 case of roundish bodies. The stable gravitational aggregates obtained at the
 330 end of the 89 simulation were classified accordingly into the following classes:

- 331 • RC (Roundish-Centred): Morphologically roundish aggregates are characterized
 332 by low (< 0.4) values of the elongation parameter. That
 333 implies that the largest fragment is buried inside the aggregate, surrounded
 334 by smaller fragments. 14.6% of our simulations belong to that
 335 class.
- 336 • R (Roundish): Morphologically roundish aggregates characterized by
 337 values of the elongation parameter larger than in the RC case. That
 338 corresponds to largest fragments displaced with respect to the center
 339 during the re-accumulation process, showing up in the external part of
 340 the aggregate. These represent 28.1% of our simulations.
- 341 • E (Elongated): These aggregates have no roundish shape, fragments
 342 form a generically elongated object. The elongation parameter may be
 343 of no help in this case, as the largest fragment may occupy any position
 344 in the aggregate. However, most cases show an off-center position for
 345 the largest fragment. 25.8% of the cases show that morphology.
- 346 • CB (Contact-Binary): These shapes are analogous to some asteroid
 347 (or comet) contact binaries. That is, a main body formed by all the
 348 fragments but the largest, and a head (the largest fragment) in contact
 349 with one of the body ends. The elongation parameter typically takes
 350 large values for CB. 23.6% of our simulations end up that way.
- 351 • S (Satellite formation): When the shape of the gravitational aggregate
 352 is not very elongated, a fragment may have enough angular momentum

353 to detach from the structure and orbit the main body. The stability
 354 of the satellite was not studied here. Only 3 simulations produced this
 355 result (3%).

- 356 • C (Clustered fragments): In some case, small aggregates of similar size
 357 form as independent bodies (not bound to each other). Mass ratios
 358 between any small aggregate and the main one were 0.5–0.2 (4.5% of
 359 simulations).
- 360 • L (Lost fragments): A few fragments (1 to 5) depart from the main
 361 formed aggregate at low speeds (a few tens of m/s to a few m/s) (4%
 362 of cases).

363 In summary, roundish shapes (class RC and R, 42.7%) and globally elon-
 364 gated (E and CB, 49.4%) are very common while satellite systems (3.4%) and
 365 similar-size clusters (4.5%) are seldom outcomes of our simulations. Loss of
 366 a few fragments happens in about half of the simulations generating E and
 367 CB morphologies. C and L cases may go on to be asteroid pairs or clusters,
 368 but this was not studied in detail. Morphological classes are not mutually
 369 exclusive, e.g., an L class may be in some case an E class for the main ag-
 370 gregate.

371 In Campo Bagatin *et al.* (2018) the Dynamically Equivalent Equal–Volume
 372 Ellipsoid (DEEVE) method was used to calculate the volume of our synthetic
 373 aggregates. This method identifies the triaxial ellipsoid whose volume is dy-
 374 namically equivalent to that of the aggregate. A proof of this useful method
 375 is provided in the Appendix. We therefore derive the aspect ratio of our
 376 synthetic aggregates from the DEEVE (c/a and b/a , where a , b and c are
 377 the DEEVE axes, from largest to smallest) and compare them to those of
 378 the few asteroids and comets for which acceptable estimation of aspect ratios
 379 are available from spacecraft and radar observation. That includes mostly
 380 spacecraft visited and radar observed asteroids, a few observed comets and
 381 the only small TNO for which a close observation is available (New Horizons,
 382 NASA, on the 1st of January, 2019). Fig. 3 shows how aspect ratios are dis-
 383 tributed as compared to observed small bodies. Following Campo Bagatin *et*
 384 *al.* (2018), we consider two classes of simulated aggregates corresponding to
 385 two different density values of their components. They correspond to the two
 386 most common asteroid spectroscopic classes: S–type (high density, silicate
 387 composition) and C–type (low density, carbonaceous composition).

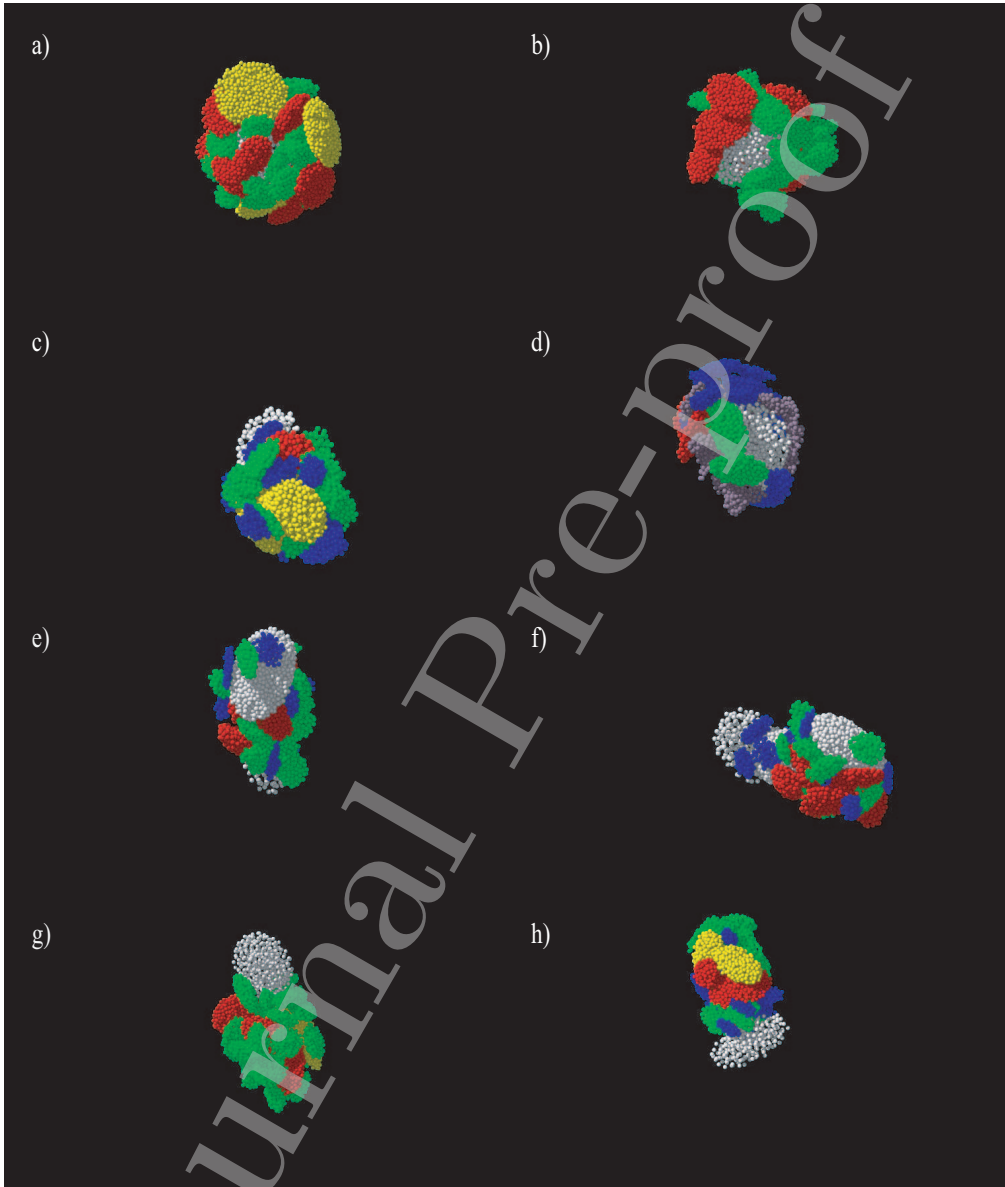


Figure 2: Snapshots of the end state of numerical simulations of 8 representative gravitational aggregates showing shape diversity. Different colors correspond to different masses of the discrete rigid aggregate components, as explained in Campo Bagatin *et al.* (2018).

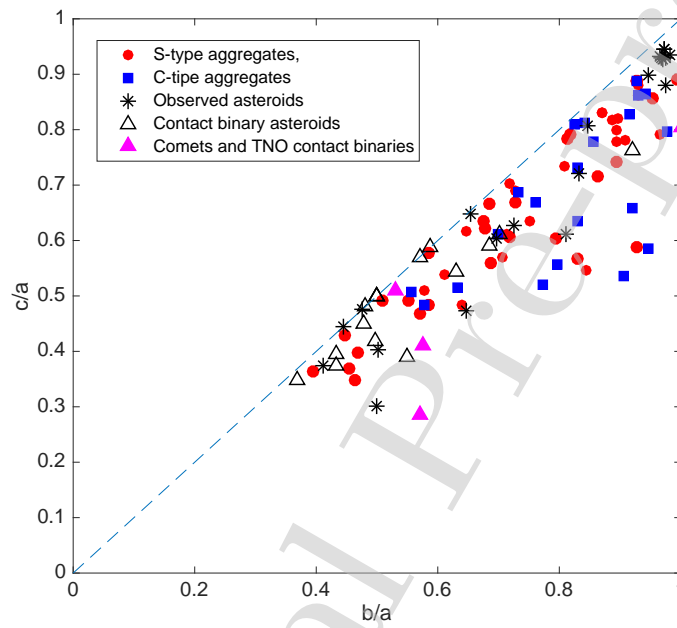


Figure 3: Aspect ratios c/a vs. b/a of the simulated aggregates. Full circles stand for S-type synthetic aggregates, full squares for C-types, according to the Campo Bagatin *et al.* (2018) classification. Asterisks stand for spacecraft and ground-based observed asteroids. Open triangles identify observed contact binary asteroids and full triangles observed comets and TNO 2014 MU69.

388 *3.2. Dependence on initial conditions*

389 Visual inspection of many simulated re-accumulation processes raises sus-
 390 picion about the occurrence of roundish shapes being more frequent when the
 391 overall volume of the initial mass distribution is small, with few cases corre-
 392 sponding to elongated shapes. Therefore we checked the dependence of final
 393 aggregate shape on initial mass distribution in space. In order to do so, we
 394 calculated the component of the diagonalized inertia tensor—of both the ini-
 395 tial and final distribution of mass—corresponding to the moment of inertia
 396 with respect to the shortest principal axis of inertia, in each case. That axis
 397 corresponds, in most cases, to the same direction of the angular momentum
 398 vector. In some cases the body is precessing about that axis so that those
 399 two directions are not necessarily coincident.

400 We compared the initial and final largest moments of inertia for all
 401 the simulated systems to check for dependence on initial mass distribution
 402 (Fig.4). In order to do so, we had to select only those simulations for which
 403 the mass of the bound system was conserved. As reported in Sec. 3.1, in
 404 some simulation a small number of fragments do not re-accumulate. It is
 405 necessary to have the same mass at the beginning and the end of each sim-
 406 ulation in order to fairly compare initial and final space mass distributions.
 407 This selection preserves 52 simulations with equal final aggregate mass equal
 408 to the mass of the initial distribution. The initial largest moment of inertia,
 409 $(I_3)_0$, is normalized to MR^2 where M is the mass of the system and R is
 410 the radius of the DEEVE volume, $\bar{I}_0 = (I_3)_0/(MR^2)$. Such normalization
 411 implies that a sphere has a normalized value of the moment of inertia equal
 412 to 0.4. The moment of inertia of the final aggregate, $I_3 = m_e (a^2 + b^2) / 5$, is
 413 instead suitably normalized to $m_e R^2$, where m_e is the mass resulting from
 414 the DEEVE calculation, $\bar{I} = I_3/(m_e R^2)$. In the Appendix we shortly show
 415 that the mass m_e is a dependent parameter in such calculation, that gener-
 416 ally does not take the same value than the “real” mass M of the irregular
 417 aggregate itself. In fact, the DEEVE method finds the semi-axes of the el-
 418 lipsoid with the same volume and the same principal moments of inertia of
 419 the irregular shaped aggregate and this requires that the value of the mass
 420 is derived accordingly to fit the DEEVE.

421 This analysis reveals that 2/3 of the simulations corresponding to $\bar{I}_0 < 10$
 422 result in $\bar{I} < 0.5$, corresponding to roundish shapes. Instead, only 1/3 of
 423 simulations for which $\bar{I}_0 > 10$ result in $\bar{I} < 0.5$. This confirms a trend towards
 424 tight initial distributions preferring final roundish shapes, while less confined
 425 initial distributions give rise to any kind of final shape. In the latter case, the

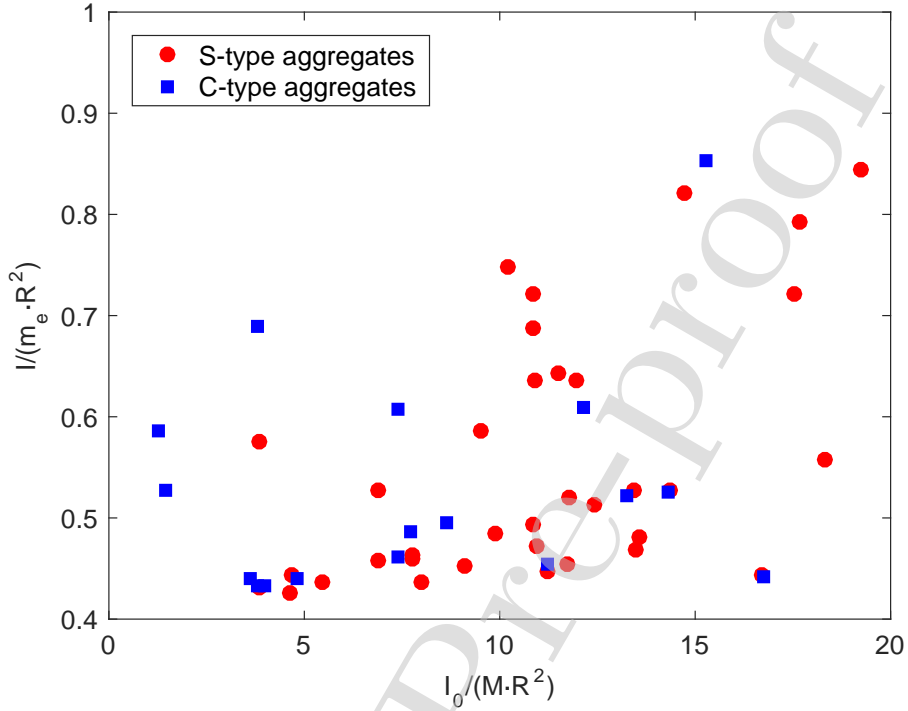


Figure 4: Normalized largest moment of inertia of initial (I_0) and final (I) mass distributions. As a reference, an exactly spherical distribution would give a value of the moment of inertia equal to 0.4.

426 re-accumulation process loses memory of the initial distribution, its evolution
 427 is dominated by hundreds of low-speed collisions between components with
 428 stochastic final configurations.

Fig. 5 shows no evidence of dependence of the final shape of aggregates—
 in terms of the largest moment of inertia, \bar{I} —on specific angular momentum
 values of the system, $\langle L \rangle$, defined as

$$\langle L \rangle = L / (GM^3R)^{1/2},$$

429 where M , R and L are, respectively, the mass, equivalent radius, and angular
 430 momentum of the object. The large amount of low-speed collisions between
 431 irregularly shaped components going on during the re-accumulation process
 432 seem to completely cancel the effect of initial angular momentum, at least for
 433 non-critical values ($\langle L_0 \rangle > 0.015$). This range was not explored in detail in

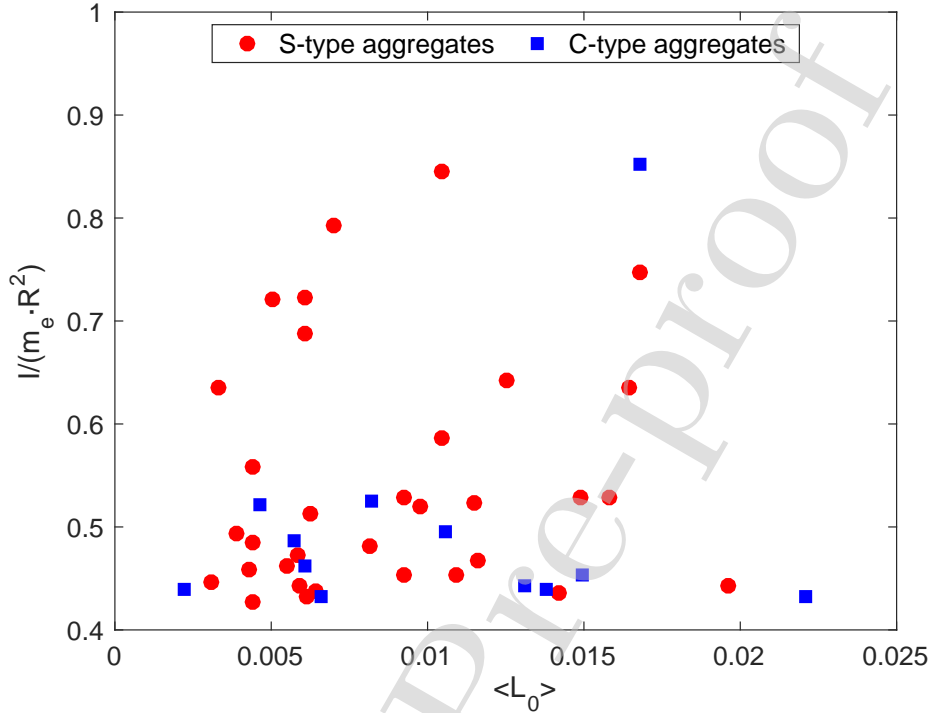


Figure 5: Normalized largest moment of inertia for final (\bar{I}) mass distributions corresponding to normalized values of normalized angular momentum, $\langle L_0 \rangle$.

our investigation). This result is interesting as it implies that elongated asteroid shapes are not necessarily the result of initial high angular momentum configurations.

Further asteroid evolution due to spin-up or cratering collisions may change asteroid shapes and probably form “top” shapes, like in the case of the primary of many binary near—Earth asteroids and NEAs Ryugu and Bennu recently visited by the Hayabusa 2 (JAXA) and OSIRIS-REx (NASA) space missions. However, the explanation of evolved shapes is beyond the scope of the present work.

3.3. Benchmark Itokawa

We used a study carried out by Lowry *et al.* (2014) on the morphology and mass distribution of asteroid Itokawa as a benchmark for our own study. That group determined that the best fit to YORP measurements of Itokawa

447 corresponds to a density ratio between the “body” and the “head” of 0.61
 448 ± 0.14 and corresponding mass ratio of 0.21 ± 0.05 . Mass ratios can be
 449 easily constructed by suitably setup of initial conditions in our simulations
 450 but density ratios depend on the mass distribution of fragments and their
 451 final arrangement. In our case, the comparison was done by calculating the
 452 average value of the ratios of the body density to the largest fragment den-
 453 sity in the simulations that show contact binary structures. Our simulations
 454 have average density ratios of 0.57 ± 0.03 , indicating a mass distribution of
 455 the body components quite in agreement with the estimate by Lowry *et al.*
 456 (2014) for Itokawa. The mass density of each part was determined using the
 457 following procedure. The head is removed from the output file containing the
 458 physical parameters of each particle of the whole body. The inertia tensor of
 459 the whole of the remaining fragments (the “body”) is calculated and then di-
 460 agonalized in order to set its principal axes of inertia along suitable reference
 461 system axes. At that point, the DEEVE can be employed to calculate the
 462 volume of the body and therefore its density is worked out (its mass is easily
 463 calculated as the sum of the masses of its particles). The same procedure is
 464 applied to the head. Finally the density ratios of the two parts are derived.

465 The average spin period for our synthetic CB types is 12.3 h, but the spin
 466 range spans a wide range from relatively fast (3.7 h) to slow (145 h) rotation;
 467 the median value is 10.3 hr. For comparison, asteroid Itokawas spin period
 468 is 12.1 hr. Specific angular momentum has an average value of 0.168 for our
 469 CB types. Values are quite dispersed so that the median (0.147) is a better
 470 estimate. For comparison, $\langle L \rangle$ for asteroid Itokawa can be calculated as
 471 0.158 from the Breiter *et al.* (2009) estimation of its moment of inertia and
 472 the Abe *et al.* (2006) and Fujiwara *et al.* (2006) estimations for the mass and
 473 size of the asteroid. Fig. 6 shows a comparison of the end state of a sample
 474 simulated contact binary morphology compared to asteroid Itokawa.

475 4. Discussion and conclusions

476 The visual analysis of the 89 movies corresponding to successful sim-
 477 ulations reveals general patterns for the shaping of asteroid gravitational
 478 aggregates. We obtain many different shapes for gravitational aggregates,
 479 ranging from rounded to elongated and contact-binary. Contrary to what is
 480 generally imagined, only about 15% of simulated aggregates belong to RC
 481 class, that is roundish shape with the largest fragment in central position. R
 482 (roundish) bodies with the largest fragment located in non-central position

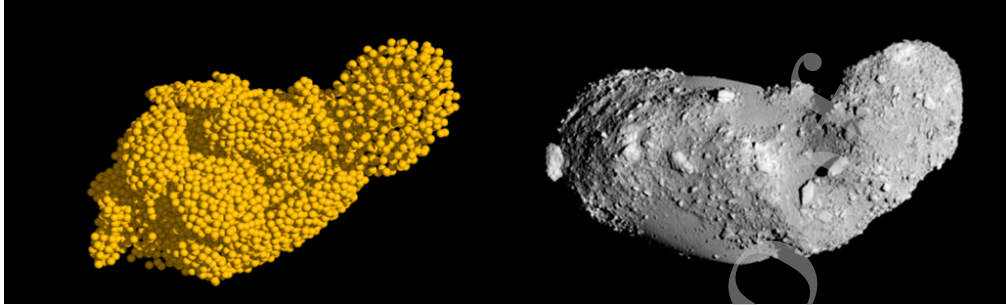


Figure 6: End state of contact binary (CB) morphology (left) compared to asteroid Itokawa as observed by the Hayabusa (JAXA) spacecraft (right). The largest fragment shows a slightly larger spacing between its spherical basic elements than the rest of fragments. This is due to the way in which the scaling from the synthetic largest fragment that matches laboratory experiments to km-size objects is made. However, the mass and overall size of the largest fragment is suitably scaled and the dynamics is not affected. Michel & Richardson (2013) show a similar numerical result for the morphology of asteroid Itokawa.

483 almost double (28%) RC class bodies. E (elongated) to CB (contact binary)
 484 aggregates are roughly half of the outcome of all our simulations, among
 485 them, a remarkable 24% belong to the latter class.

486 As a general conclusion, we suggest that the gravitational re-accumulation
 487 process is largely stochastic. It is dominated by low-speed multiple collisions
 488 between irregular fragments, generally losing memory of initial conditions.
 489 We identified a general mechanism leading to elongated and—in particular—
 490 to contact-binary structure. In most simulations, at the beginning of the re-
 491 accumulation process, some component close to the largest fragment nudges
 492 it at low speed (tens of cm/s), forcing it slowly away from its central posi-
 493 tion, while the remaining fragments continue their fall towards the center of
 494 mass of the system (Fig. 7, movies 1 to 3: online supplementary material).
 495 Therefore, when re-accumulation is over (this process typically lasts 4 to 6
 496 hours of real time), fragments are not clustered around the largest one, since
 497 it was removed from the center at the beginning. Instead, the largest frag-
 498 ment ends up at one end of the aggregate, for example as the "head" of a
 499 contact binary.

500 **Our study is mainly focused on asteroids. Other populations of**
 501 **small bodies (comets that originate in the trans-Neptunian region**
 502 **or TNOs themselves) may not share a similarly intense collisional**
 503 **history. Therefore, extrapolation of the interpretation of results**

504 **for asteroids always has to be done with caution.** Our results on
505 the morphology of gravitational aggregates and contact binary formation
506 are independent of fragment material density, as was expected; in fact no
507 meaningful difference is found when density is changed from 3500 kg/m^3
508 to 2500 kg/m^3 (corresponding respectively to S-type and C-type asteroid
509 meteorite analogues, as explained in Sec. 2). For this reason we suggest
510 that similar process may take place in collisional events also in the trans-
511 neptunian region, where most of the observed contact binary comets were
512 likely generated. This genesis may be complementary to the mechanism
513 proposed for the formation of comet 67P by Jutzi & Benz (2017) and in
514 agreement with Schwartz *et al.* (2018), who also considered full collisional
515 physics. In the trans-neptunian region—as in the asteroid belt—relative
516 encounter speeds are currently in the catastrophic regime for the constituent
517 materials (Dell’Oro *et al.*, 2013). Other comets, like Borrelly and Hartley 2,
518 also show contact-binary shapes. Further debate on small body formation
519 in the outer Solar System arose when TNO 2014 MU69 was observed by
520 a fly-by of the New Horizons (NASA) space probe. 2014 MU69 is a 30
521 km size body formed by two clearly distinct components resting on each
522 other. We can speculate that a re-accumulation origin of such body could be
523 potentially possible by the mechanism described here. A collisional origin for
524 such objects would need a relatively high impact rate at some point in the
525 trans-Neptunian region, that cannot be presently ruled out. However, a close
526 binary evolving to touching by some dissipative mechanism could also explain
527 this object: a primordial origin for such a contact-binary structure (Jutzi *et*
528 *al.*, 2015) would imply a soft collisional evolution of individual components
529 in a depopulated primordial environment.

530 It is also interesting to notice that binary systems arise spontaneously in
531 a few simulations (e.g., movie 3 of the online supplementary material). For
532 critical values of the angular momentum of the system, one of the fragments
533 detaches from the spinning aggregate at the end of the re-accumulation stage
534 and becomes a satellite around the central aggregate. Further evolution of
535 the system was beyond the scope of this study.

536 Some NEA asteroid primaries and single bodies observed by radar and
537 spacecraft observations show equatorial marks that can be suspected to be
538 the former location of possible detached components (Tardivel *et al.*, 2018).
539 The described binary formation mechanism implies that our synthetic satel-
540 lites are denser than the corresponding gravitational aggregate primaries.
541 One of the very few estimates of density of both components of asteroid bi-

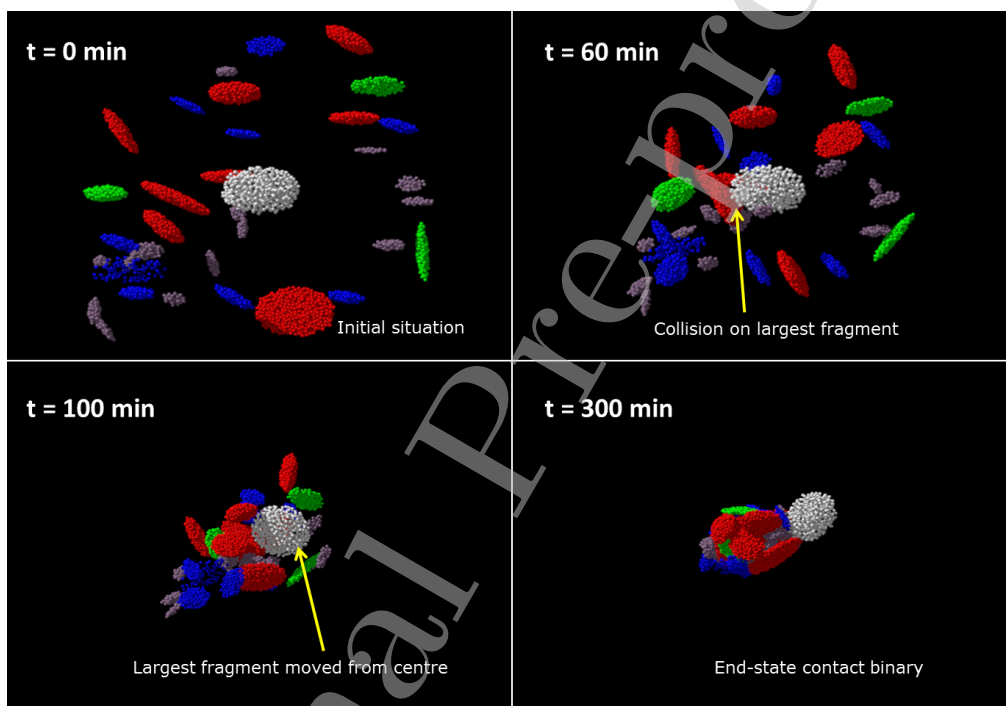


Figure 7: Schematic representation of the mechanism that drives the largest fragment away from its central position, leading to contact binary shape.

542 nary systems (1999 KW4) (Ostro *et al.*, 2006) indicates that the primary is
543 in fact less dense than the satellite.

544 When angular momentum is larger than the critical value necessary for
545 the formation of a satellite, loss of one or many components of the system
546 occurs, with relative speeds on the order of several m/s in our simulations.
547 The mass ratio of escaping components with respect to the rest of the body
548 is in the 0.1 to 0.001 range, corresponding to the size ratios estimated for
549 “asteroid pairs” and “asteroid clusters” (Pravec *et al.*, 2018). These are pairs
550 and small groups of asteroids that have very similar orbital elements, whose
551 orbits—once integrated back in time—lead to a common origin. Most of the
552 clusters found by Pravec *et al.* (2018) can be explained by rotational fission
553 due to spin up of the parent body. However, clusters (18777) Hobson and
554 (22280) Mandragora are in the main belt and they likely need an alterna-
555 tive explanation for spin-up fission as the YORP effect is not viable in this
556 case according to the authors. Our simulations suggest that a collision with
557 injection of extra angular momentum to the system, followed by partial re-
558 accumulation of part of the parent body mass and the escape of fragments
559 during the re-accumulation process may explain those systems. However, a
560 dedicated study should be made to match the known characteristics of this
561 those systems. It is worth reminding that formation of asteroid pairs and
562 clusters is a different mechanism with respect to the formation of asteroid
563 families: the discriminating parameter is the ejection speed of fragments. In
564 the case of asteroid families, fragments are ejected at speeds far larger than
565 the escape speed, typically on the order of hundreds of m/s. In the case
566 of asteroid clusters, speeds are small, barely above the escape speed. Our
567 simulations show that fragments initially bound escape eventually due to the
568 excess of angular momentum of the system. The study of the size distribu-
569 tion, spins and speeds of the escaping fragments in our simulations is beyond
570 the scope of this paper and shall be investigated in future work.

571 The process of gravitational re-accumulation of fragments following as-
572 teroid collisions offers a general mechanism to explain asteroid (and possibly
573 comet) morphology, including contact binaries and some asteroid pairs and
574 clusters, while it suggests a possible scenario for the formation of asteroid
575 binary systems. Collisions were a key element during the formation and
576 shaping of planetesimals in the primordial Solar System. Our results may
577 contribute to the understanding of the early collisional processes that led to
578 the building of the early rocky planets and the leftovers of that formation
579 phase.

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684 **Appendix: Dynamically Equivalent Equal Volume Ellipsoid (DEEVE)**

Let B be a rigid body with principal moments of inertia I_1 , I_2 and I_3 such that $I_1 \leq I_2 \leq I_3$, whose corresponding principal central axes coincide respectively with the axes X , Y and Z of a Cartesian frame $OXYZ$. The rigid body B has a mass M spanning over a region \mathbb{V} with volume $V > 0$, so that, at each point of \mathbb{V} with coordinates (x, y, z) in $OXYZ$, its volume mass density is $\rho(x, y, z) > 0$. With this notation we have that

$$I_1 = \iiint \rho(x, y, z) [y^2 + z^2] dx dy dz,$$

$$I_2 = \iiint \rho(x, y, z) [x^2 + z^2] dx dy dz$$

and

$$I_3 = \iiint \rho(x, y, z) [x^2 + y^2] dx dy dz,$$

685 from which it readily follows that:

$$I_1 + I_2 - I_3 = 2 \iiint \rho(x, y, z) z^2 dx dy dz \quad (1)$$

686

$$I_2 + I_3 - I_1 = 2 \iiint \rho(x, y, z) x^2 dx dy dz \quad (2)$$

687

$$I_1 + I_3 - I_2 = 2 \iiint \rho(x, y, z) y^2 dx dy dz \quad (3)$$

688 It is clear that $\rho(x, y, z) z^2 > 0$ for every (x, y, z) in the region formed
 689 by \mathbb{V} excluding the plane $z = 0$, and the volume of such region is $V > 0$
 690 (same as that of \mathbb{V}) because the plane $z = 0$ has a null volume¹. This implies
 691 that $\rho(x, y, z) z^2$ is positive in a region of positive volume and null elsewhere,
 692 so the integral of Eq. 1 is strictly positive, thus being $I_1 + I_2 - I_3 > 0$.
 693 With analogous arguments for $\rho(x, y, z) x^2$ and $\rho(x, y, z) y^2$ the following
 694 relationships are obtained:

$$I_1 + I_2 - I_3 > 0, \quad I_2 + I_3 - I_1 > 0 \quad \text{and} \quad I_1 + I_3 - I_2 > 0 \quad (4)$$

¹From a mathematical viewpoint, the “volume” of a region is its Lebesgue measure in the space \mathbb{R}^3 . In particular, the Lebesgue measure in \mathbb{R}^3 of any plane is zero.

695 The *Dynamically Equivalent Equal-Volume Ellipsoid* (DEEVE) of the
 696 rigid body B is a uniform ellipsoid that has the same volume V and the
 697 same principal moments of inertia I_1, I_2, I_3 as B . If m_e is the mass of the
 698 DEEVE and a, b, c are the semi-axes of the DEEVE contained in its principal
 699 central axes associated to I_1, I_2, I_3 respectively, then we have that:

$$I_1 = \frac{m_e}{5} (b^2 + c^2); \quad I_2 = \frac{m_e}{5} (a^2 + c^2); \quad I_3 = \frac{m_e}{5} (a^2 + b^2) \quad (5)$$

700

$$V = \frac{4\pi}{3} abc \quad (6)$$

701 From Eqs. 4 and 5 it readily follows that:

$$0 < I_2 + I_3 - I_1 = \frac{2m_e}{5} a^2 \quad (7)$$

702

$$0 < I_1 + I_3 - I_2 = \frac{2m_e}{5} b^2 \quad (8)$$

703

$$0 < I_1 + I_2 - I_3 = \frac{2m_e}{5} c^2 \quad (9)$$

By multiplying Eqs. 7 to 9:

$$(2m_e/5)^3 a^2 b^2 c^2 = (I_2 + I_3 - I_1)(I_1 + I_3 - I_2)(I_1 + I_2 - I_3) > 0,$$

which according to Eq. 6 leads to

$$(2m_e/5)^3 [3V/(4\pi)]^2 = (I_2 + I_3 - I_1)(I_1 + I_3 - I_2)(I_1 + I_2 - I_3) > 0.$$

Taking positive cubic roots² in the previous relationship it follows that

$$(2m_e/5) \cdot [3V/(4\pi)]^{\frac{2}{3}} = [(I_2 + I_3 - I_1)(I_1 + I_3 - I_2)(I_1 + I_2 - I_3)]^{\frac{1}{3}} > 0,$$

704 and thus:

$$m_e = \frac{5}{2} \left[\frac{4\pi}{3V} \right]^{\frac{2}{3}} [(I_2 + I_3 - I_1)(I_1 + I_3 - I_2)(I_1 + I_2 - I_3)]^{\frac{1}{3}} > 0, \quad (10)$$

705 whereas taking positive square roots in Eqs. 7-9 the following expressions
 706 for the semi-axes are obtained:

²Thus discarding conjugate-complex cubic roots without physical sense for m_e .

$$a = \sqrt{\frac{5(I_2 + I_3 - I_1)}{2m_e}} > 0 \quad (11)$$

$$b = \sqrt{\frac{5(I_1 + I_3 - I_2)}{2m_e}} > 0 \quad (12)$$

$$c = \sqrt{\frac{5(I_1 + I_2 - I_3)}{2m_e}} > 0 \quad (13)$$

709 Eqs. 10-13 provide the parameters m_e , a , b , and c of the DEEVE in terms
710 of the parameters I_1 , I_2 , I_3 and V of the initially considered rigid body B .

711 It should be remarked that the condition $V > 0$ and its consequence
712 given in Eq. 4 are necessary to avoid divisions by zero in Eqs. 10 to 13.
713 Furthermore, the condition $I_1 \leq I_2 \leq I_3$ implies that³ $I_2 + I_3 - I_1 \geq I_1 +$
714 $I_3 - I_2 \geq I_1 + I_2 - I_3$, which—according to Eqs. 11 to 13—readily leads to
715 $a \geq b \geq c$.

716 Notice that the DEEVE is the solution to the problem of finding a three-
717 axial ellipsoid with semi-axes a , b , c , having the same principal moments of
718 inertia and the same volume than those of some given body B (under the
719 conditions specified above) with mass M . This leads to a rearrangement and
720 suitable scaling of mass, that is now a parameter m_e which depends on V ,
721 I_1 , I_2 and I_3 , and is not—in general—coincident with the physical mass M
722 of B . In fact, mass m_e has to be fit into the ellipsoid with the same volume
723 V , so that the moments of inertia also coincide with the original ones. It
724 is straightforward to check that for a parallelepiped P . Let P have a mass
725 M and sizes h , k , l , such that its volume is $V = h \cdot k \cdot l$, Eq. 10 gives
726 $m_e = \frac{5}{2} \left(\frac{4\pi}{3}\right)^{2/3} \frac{M}{6} \neq M$.

727 Therefore, when calculating the physical density ρ_B of a given body B ,
728 the expression $\rho_B = M/V$ has to be utilized.

³In fact $2(I_2 - I_1) \geq 0$ and $2(I_3 - I_2) \geq 0$, so $I_2 + I_3 - I_1 = I_1 + I_3 - I_2 + 2(I_2 - I_1) \geq$
 $I_1 + I_3 - I_2$ and in turn $I_1 + I_3 - I_2 = I_1 + I_2 - I_3 + 2(I_3 - I_2) \geq I_1 + I_2 - I_3$.

Shot	M (g)	m_p (g)	v (km/s)	$v_{rel_{LF}}$ (m/s)	$v_{rel_{per}}$ (m/s)
130701	433.0	0.1587	4.73	4.19	31.9
130702	534.6	0.1587	4.45	2.10	34.3
130705	479.1	0.1587	3.68	0.86	17.5

Table 1: Comparison of relative velocity of the largest fragment and a generic peripheral fragment, both relative to the center of mass of the system. The number of each experimental collisional shot—as in Durda *et al.* (2015)—is indicated in the first column. The mass of the target and the projectile and the impact speed are indicated in the second, third and fourth columns respectively. Relative velocity of the largest and that of a peripheral fragment, with respect to the center of mass, are indicated respectively in the fourth and fifth columns.

- Contact binary asteroids can be formed by gravitational re-accumulation.
- Rubble-pile asteroid shape formation is ruled by stochastic low speed collisions.
- Asteroid satellites formed during re-accumulation may be single shards.
- Largest fragments are not necessarily in the center of asteroid rubble-piles.

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