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Modified PROMETHEE Approach for Solving Multi-Criteria Location Problems with Complex Criteria Functions

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Abstract: The specific problem that occurs in multi-criteria decision-making (MCDM) processes is ranking a number of alternatives using complex criteria functions (the hierarchical structure of criteria) whose values must consider the impacts of all-important characteristics and parameters of alternatives. The problem becomes more complex by increasing the number of levels of sub-criteria functions (degree of decomposition). This paper proposes an extended procedure based on the mean values conversion of the net outranking flow of sub-criterion functions obtained by modified PROMETHEE methods. The actual value of criterion functions is used only at the last level, and transformed values of the net outranking flow for generating a final rank of alternatives are introduced at other levels. This procedure provides a more objective comparison of the impact of various individual criteria to rank the alternatives and easier making of unique solution, where the impact of decision-maker (DM) experience and subjective estimation is minimised in the selection. Applicability and practicability of the presented procedure for solving the selection problem of a logistics warehouse location are demonstrated in the analysis of a case study example.

Keywords: criterion function; decision-making; location problem; multi-criteria optimization; PROMETHEE

1 INTRODUCTION

Multiple alternatives and criteria in location problem create conflict conditions within decision-making process and not taking into account all relevant factors could cause wrong decisions and long-term consequences. Evaluation criteria in the selection of a logistics warehouse location can be generated and classified according to various aspects of observation of the system and the DM.

Farahani et al. [1] present a review of location problems formulated as multiple criteria decision problems. Decision-making based upon multi-criteria is a process that requires the definition of objectives at multiple stages, criteria selection, specification of alternatives, weights assignment to the criteria and application of certain mathematical procedures to rank the alternatives [2]. A significant part of multi-criteria methods belongs to outranking methods [3] because of their adaptability to real problems. In addition, these methods introduce new generalized criteria to express DM preferences concerning particular criteria of the resolved issue.

The procedures of solving various tasks and multi-criteria location problems with conventional PROMETHEE and modified approaches are discussed in many papers [4-12]. The mentioned methods do not introduce parameter values of complex alternative characteristics directly into the model, so there can exist a hierarchical structure of criteria. For example, at the first level they combine them in an intuitive way (e.g. legal-regulatory alternative characteristic combines the following features (sub-criteria): integration into regional and urban development plans, prospect of land and objects ownership regulation, etc.). The consequences of this approach (flat structure of criteria) may be a number of criteria on the one side and the problems of determining the relative weight of each criterion on the other side. Theoretically, there is no limit for most of the methods and modern information technology allows relatively easy and quick obtaining of results. Therefore, in order to make an accurate and flexible decision in that case, some studies developed solutions for considering interaction among criteria [13, 14] as well as criterion reduction [15, 16] which is an appropriate method to extract useful knowledge from large amounts of information. In addition, the consideration of preference relations at each level of the hierarchy, which constitutes a base for the discussion with the DM, is discussed in [17]. Generally, the decomposition of criteria facilitates the preference evocation regarding pairwise comparisons of criteria considering relative significance.

In this paper, we proposed a modified and extended outranking method, taking into account the hierarchical structure of criteria. The modified algorithm with the development of appropriate software tool simplifies the process of ranking, provides a more objective comparison of the impact of various individual criteria to rank the alternatives and reduces the impact of experience and subjective estimation of the DM. The algorithm developed in this paper allows that this problem could be solved by conventional methods of multi-criteria optimization, so that through several stages of iteration it is relatively easy and quick to make unique solutions. The proposed procedure recommends that the number of criteria at the 1-st level of ranking and sub-criteria on the r-th level does not exceed 12. The impact of subjective estimation and experience of the DM is minimised through change of current generalized criterion and implementation of a new one. In addition, the change of the selection procedure of generalized criteria and mean values conversion of the outranking flow additionally reduce this impact.

2 PRINCIPLES OF MODIFIED PROMETHEE APPROACH

The family of PROMETHEE methods [18], and their modifications [7, 9, 19] are built upon criterion concept generalization by utilisation of generalized criterion functions (types I, II, ..., VI) and mathematical formulations for alternatives ranking. Ranking of m alternatives $A = \{a_1, ..., a_i, ..., a_m\}$ includes concept generalization of n criteria $f = \{f_1, ..., f_k, ..., f_n\}$.

If $f_j(a)$ is the value of alternative a in relation to the criterion j, the parameters that characterize the family of PROMETHEE methods are:

$P_j(a, b)$ - preference function which represents the intensity of preferences of the alternative a regarding alternative b.

$$P_j\left(a,b\right)=\begin{cases}0,\ \text{if}\ f\left(a\right)\le f\left(b\right)\\ P_j\left[f\left(a\right)-f\left(b\right)\right]=P_j\left[d\left(a,b\right)\right]\end{cases} \tag{1}$$

p - strict preference threshold
q - indifference threshold
ω_j - relative importance (weight) of the j criterion (j = 1, …, n)
$\Pi(a, b)$ - multi-criteria preference index of a in relation to b.

$$\prod(a,b)=\frac{\sum_{j=1}^{n}\omega_j f_j(a,b)}{\sum_{j=1}^{n}\omega_j} \tag{2}$$

entering (negative), leaving (positive) and net outranking flows

$$\begin{aligned}&\Phi^{+}(a)=\sum_{\forall b\in A}\prod(a,\,b)\\&\Phi^{-}(a)=\sum_{\forall b\in A}\prod(b,\,a)\\&\Phi(a)=\Phi^{+}(a)-\Phi^{-}(a)\end{aligned} \tag{3}$$

Generally, the alternative is better if the leaving flow is higher and the entering flow is lower [20].

$$\begin{cases}aP^{+}b\ \text{iff}\ \Phi^{+}(a)>\Phi^{+}(b)\\ aI^{+}b\ \text{iff}\ \Phi^{+}(a)=\Phi^{+}(b)\end{cases} \tag{4}$$

$$\begin{cases}aP^{-}b\ \text{iff}\ \Phi^{-}(a)>\Phi^{-}(b)\\ aI^{-}b\ \text{iff}\ \Phi^{-}(a)=\Phi^{-}(b)\end{cases} \tag{5}$$

In Eqs. (4) and (5), P and I represent preference and indifference. PROMETHEE I determines the partial preorder (P^{I}, I^{I}, R) within the set of alternatives A, which fulfilled the principle:

$$\begin{aligned}&aP^{\mathrm{I}}b\ (a\ \text{outranks}\ b),\ \text{if}=\begin{cases}aP^{+}b\ \text{and}\ aP^{-}b\\ aP^{+}b\ \text{and}\ aI^{-}b\\ aI^{+}b\ \text{and}\ aP^{-}b\end{cases}\\&aI^{\mathrm{I}}b\ (a\ \text{is indifferent to}\ b),\ \text{if}\ aI^{+}b\ \text{and}\ aI^{-}b\\&aRb\ (a\ \text{and}\ b\ \text{are incomparable}),\ \textit{otherwise.}\end{aligned} \tag{6}$$

Furthermore, PROMETHEE II gives a complete preorder (P^{II}, I^{II}), induced by the net flow and determined by:

$$\begin{aligned}&aP^{\mathrm{II}}b\ (a\ \text{outranks}\ b),\ \text{iff}\ \Phi(a)>\Phi(b)\\&aI^{\mathrm{II}}b\ (a\ \text{is indifferent to}\ b),\ \text{iff}\ \Phi(a)=\Phi(b)\end{aligned} \tag{7}$$

Each action a is associated with an interval $[x_a, y_a]$ and a complete interval order (P^{III}, I^{III}) is defined by PROMETHEE III as follows:

$$\begin{aligned}&aP^{\mathrm{III}}b\ (a\ \text{outranks}\ b),\ \text{iff}\ x_a>y_b\\&aI^{\mathrm{III}}b\ (a\ \text{is indifferent to}\ b),\ \text{iff}\ x_a\le y_b\ \text{and}\ x_b\le y_a\end{aligned} \tag{8}$$

where n is the number of actions and $\alpha > 0$ in general. The interval $[x_a, y_a]$ is defined by:

$$\begin{cases}x_a=\bar{\Phi}\left(a\right)-\alpha\sigma_a\\ y_a=\bar{\Phi}\left(a\right)+\alpha\sigma_a\end{cases} \tag{9}$$

$$\bar{\Phi}\left(a\right)=\frac{1}{n}\sum_{b\in A}\left(\Pi\left(a,b\right)-\Pi\left(b,a\right)\right)=\frac{1}{n}\Phi\left(a\right) \tag{10}$$

$$\sigma_a^2=\frac{1}{n}\sum_{b\in A}\left(\Pi\left(a,b\right)-\Pi\left(b,a\right)-\bar{\Phi}\left(a\right)\right)^2 \tag{11}$$

The application specifies the choice of a. To prevent large number of indifferences, mean value of intervals length should be less than average distance between two consecutive values of mean flows ($\alpha \approx 0.15$), [20].

Particular task in multi-criteria optimization is to rank the alternatives by usage of more complex criteria functions, which are represented by sub-criteria functions where the level of decomposition of a function can go to a certain (r-th) level. The relative weight of criteria or sub-sub-criteria, etc., which exert their influence on the further course of the ranking must be determined at each level. A formal record of such problems is given in Tab. 1, where are:
m - alternatives number
n - criteria function number at r^{th} level
s - number of criteria function at $(r-1)^{\text{th}}$ level
l - number of criteria function at 2^{nd} level
k - number of criteria function at 1^{st} level.

Table 1 A formal record of the problem of multi-criteria analysis

Criterion level					Alternatives				Relative weights		
1	2	…	$r-1$	r	A_1	A_2	…	A_m	r	...	1
K_1^2	K_1^2	…	K_1^{r-1}	K_1^r	C_{11}	C_{21}	…	C_{m1}	ω_1^r	...	ω_1^1
				K_2^r	C_{12}	C_{22}	…	C_{m2}	ω_2^r	...	
			K_2^{r-1}	K_3^r	C_{13}	C_{23}	…	C_{m3}	ω_3^r	...	
				K_4^r	C_{14}	C_{24}	…	C_{m4}	ω_4^r	...	
	K_2^2	…	K_3^{r-1}	K_5^r	…	…	…	…	ω_5^r	...	
	K_3^2	…	K_4^{r-1}	K_6^r	…	…	…	…	ω_6^r	...	
			K_5^{r-1}	K_7^r	…	…	…	…	ω_7^r	...	
				K_8^r	…	…	…	…	ω_8^r	...	
…	…	…	…	…	…	…	…	…	…	...	
K_k^1	…	…	…	…	…	…	…	…	…	...	ω_k^1
	…	…	…	…	…	…	…	…	…	...	
	K_l^2	…	…	…	…	…	…	…	…	...	
		…	K_s^{r-1}	…	…	…	…	…	…	...	
				K_1^2	C_{1n}	C_{2n}	…	C_{mn}	ω_n^r	...	

For example, the basic criterion (first level of observation) could be expressed with criterion function at 1^{st} level K_k^1, i.e. the sub-criterion function at the 2^{nd} level could be expressed with K_l^2, etc.

Each criterion has its relative weight ω_n^r and requirement for minimization or maximization of the function (criterion). It is not necessary that the sub-criteria functions of a certain level should have the same requirement regarding minimization and maximization.

The algorithm of the procedure proposed for solving multi-criteria optimization with criteria functions at multiple levels that contain sub-criteria functions is shown in Fig. 1. In this approach of problem solving, it is recommended that the level of significance criteria or the levels of ranking according to these criteria-sub-criteria should be limited to 3. The implementation procedures of modified PROMETHEE methods [21], extended with complex criterion function consist of the following:

Phase 1: Formation of the matrix of the criterion values for specific alternatives and standard deviation for each criterion

Step 1.1: Formation of the matrix of values

$$\begin{bmatrix} C_{11} & C_{12} & \dots & C_{1j} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2j} & \dots & C_{2n} \\ . & . & \dots & . & \dots & . \\ C_{i1} & C_{i2} & \dots & . & \dots & C_{in} \\ . & . & \dots & . & \dots & . \\ C_{m1} & C_{m2} & \dots & C_{m2} & \dots & C_{mn} \end{bmatrix} \quad (12)$$

where the values f_{ij} of each considered criterion f_j for each of the possible alternatives a_i are $f_{ij} = f_j(a_i) = C_{ij}$, $i = 1, 2, ..., m$; $j = 1, 2, ..., n$.

To switch the criteria form from minimum to maximum it is necessary to perform proper conversion.

$$C_{ij} = \begin{cases} C_{ij}, \text{ for } K_j \text{ is } max \text{ type} \\ \max(C_{ij}) - C_{ij}, \text{ for } K_j \text{ is } min \text{ type} \end{cases} \quad (13)$$

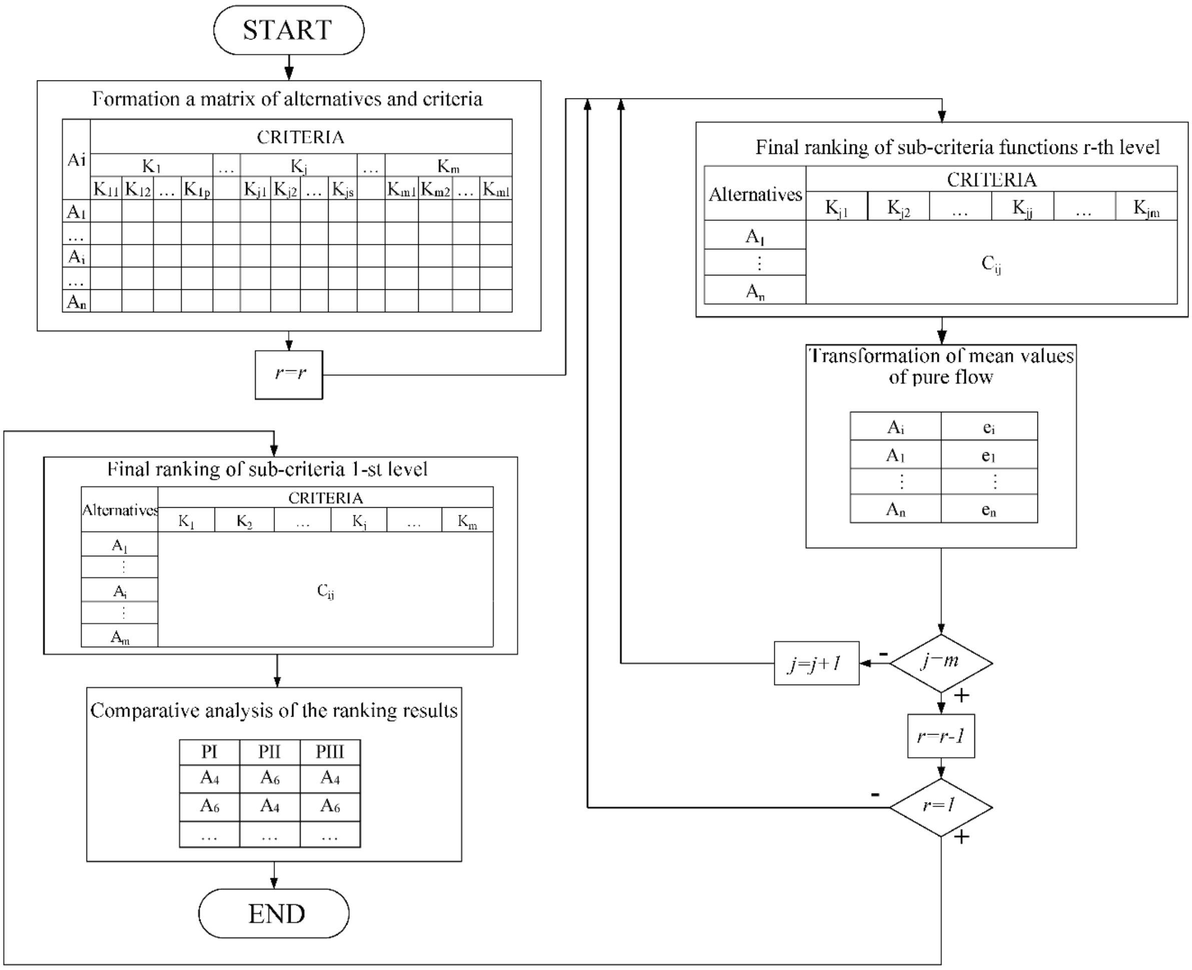


Figure 1 The algorithm method to solve multi-criteria problems with complex criteria functions

Step 1.2: Calculation of criterion K_j standard deviation

$$\sigma_j = \sqrt{\frac{1}{n} \sum_{i=1}^{m} \left(C_{ij} - \overline{C_j} \right)^2} \quad (14)$$

Phase 2: Ranking of alternatives based on sub-criteria functions at a k-level

The ranking of alternatives at a k-level is carried out in several steps:

Step 2.1: The choice of the preference function types

Each criterion is assigned with a single preference function $P_j(a_i, a_k)$. Generalized criterion function (with a value within the interval 0 ÷ 1) is chosen in relation to preference functions. It is needed to form the tables of values (distance of performance value) $(d_{ik})_j = f_j(a_i) - f_j(a_k)$ and positive differences $(d_{ik})_j > 0$ and rank them by size, where $l = 1, ..., s$. The indifference threshold - q and the strict preference threshold - p are determined by the expressions:

$$q_j = \frac{1}{3} d_j \max$$
$$p_j = \frac{2}{3} d_j \max \tag{15}$$

The series of preference function empirical values y_{jl} (where $0 < y_{jl} > 1$) have to be calculated and written as:

$$\begin{bmatrix} y_{j1} \\ \cdot \\ y_{jl} \\ \cdot \\ y_{js} \end{bmatrix}$$

An error of approximation $\varepsilon_l = [p_j(x_{jl}) - y_{jl}]$ is determined for all values $(d_{ik})_j$, where the maximum number of them can be $m(m - 1)/2$. Among all generalized criterion functions for the given pair set $\{(d_{ik})_j, P_{jik}(d_{ik})_j\}$, a function with the lowest sum of squares is selected:

$$S = \sum_{l=1}^{s} \varepsilon_l^2 \tag{16}$$

The error ε_l is evaluated for the alternatives a_i and a_k for which $(d_{ik})_j = f_j(a_i) - f_j(a_k) > 0$, i.e.

$$P_{jik}\left[\left(d_{ik}\right)_j\right] = \begin{cases} 0, \text{ for } \left(d_{ik}\right)_j < 0 \\ P_j\left[\left(d_{ik}\right)_j\right], \text{ for} \left(d_{ik}\right)_j > 0 \end{cases} \tag{17}$$

where is $P_j(d_{ik})_j$ - the chosen function of the generalized criterion.

Step 2.2: Determination of the preference index (Π)
The preference index is calculated as:

$$\prod (a_i, a_k) = \frac{\sum_{j=1}^{n} \omega_j P_i(a_i, a_k)}{\sum_{j=1}^{n} \omega_j} \tag{18}$$

Step 2.3: Flow values calculation
The input, output and outranking flows are obtained in agreement with Eq. (3). The mean values conversion of the net outranking flow from a level k, which are used to rank the alternatives at a level $(k - 1)$ is performed by the expression:

$$e_i = \frac{\overline{\Phi}(a_i) - \min \overline{\Phi}}{R} \tag{19}$$

where "range" R is

$$R = \max \overline{\Phi} - \min \overline{\Phi} \tag{20}$$

Therefore, in the first iteration, the multi-criteria analysis starts from the last r^{th} level, where the actual (real or actual) values of the criterion functions are used.

In the next iteration with criteria functions at an $r - 1$ level, the transformed values of the net outranking flows Φ obtained in the previous stage are used. Thus, calculated results represent the values of the new matrix of alternatives and the criteria by which the criteria are broken down into $r - 1$ levels. The procedure is repeated until the 1st (primary) level.

Phase 3: Final rank of alternatives
The final ranking of alternatives at the 1st level is conducted in accordance with the PROMETHEE family of methods (I, II and III).

The addition of new generalized criteria, along with the change of predefined ones is the advantage of the proposed procedure. Generalized criteria of the types I, II, IV and VI are retained. The criteria III and V are substituted by the criterion that is linear, with parameters obtained by means of linear regression. The parameters of criteria types III, V and VI, obtained by regression analysis, are shown in Fig. 2, [21].

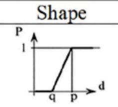
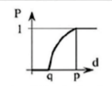
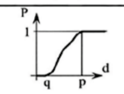
TYPE OF GENERALIZED CRITERION			$P_j(x)$
Type	Name	Shape	
III	Criterion with linear preference		$P_j(x) = \begin{cases} 0, & d < q \\ b_0 + b_1 x, & q \le d < p \\ 1. & d \ge p \end{cases}$
V	Square criterion		$P_j(x) = \begin{cases} 0, & d < q \\ b_0 + b_1 x + b_2 x^2, & q \le d < p \\ 1. & d \ge p \end{cases}$
VI	Cube criterion		$P_j(x) = \begin{cases} 0, & d < q \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3, & q \le d < p \\ 1. & d \ge p \end{cases}$

Figure 2 Changes of generalized criteria (q - the threshold or indifference, p - strict preference threshold, σ - standard deviation and b_0, b_1, b_2, b_3 - the coefficients of the regression line)

The impact of DM subjective estimation and experience on the selection of generalized criteria is decreased by this modified procedure. Developed software tool is used for the selection of generalized criterion with the smallest sum of squared deviations between the experimental data and the theory curve. Through the comparative analysis of results, the DM receives the information that could assist in selecting the best alternative.

3 CASE STUDY

The problem of strategic decision-making is shown in the case study example of location for a new logistics warehouse (region of Šumadija and Western Serbia) using complex criteria functions, which are broken down into several levels. The main factors, whose influence is considered in the definition of potential locations, are the share of imports, existing transportation infrastructure and connections in the observed region. Let us assume, from a macro point of view, that the DM has to analyse possible locations (zones) in the mentioned region $A_1 \div A_5$ (A_1 - Kragujevac, A_2 - Kraljevo, A_3 - Užice, A_4 - Kruševac and A_5 - Čačak).

The criteria can be generated and classified according to various aspects of monitoring systems and the DM. For this purpose, the selection criteria and sub-criteria are generated through a process of literature review and discussion with experts and stakeholders. After discussion and voting, academic experts reviewed and finally chose a list of 20 criteria. Tabs. 2 and 3 present a list of criteria and sub-criteria for evaluation [22], including their relative weights and a brief description of the contents of each criterion and different dimensions considered. In our example, in order to simplify the problem, the decomposition of complex criterion functions is performed only up to the second level.

Table 2 Criteria for evaluating the alternatives

Criteria	Label of criteria	Relative weights	Label of subcriteria	Relative weights of subcriteria
Technological	K_I	20%	K_{11}	40%
			K_{12}	40%
			K_{13}	20%
Social/ labour	K_{II}	20%	K_{21}	15%
			K_{22}	20%
			K_{23}	25%
			K_{24}	25%
			K_{25}	15%
Legal-regulatory framework	K_{III}	10%	K_{31}	30%
			K_{32}	30%
			K_{33}	20%
			K_{34}	20%
Economical	K_{IV}	25%	K_{41}	30%
			K_{42}	30%
			K_{43}	30%
			K_{44}	10%
Technical	K_V	25%	K_{51}	10%
			K_{52}	30%
			K_{53}	30%
			K_{54}	30%

Table 3 Sub-criteria for evaluating the alternatives

Label	Sub-criteria - 2nd level
K_{11}	Road transport system-distance from highway / km
K_{12}	Effective railway transport system / points
K_{13}	Airport access-min distance / km
K_{21}	Unemployment rate / points
K_{22}	Alleviate unemployment / %
K_{23}	Availability of specialized technicians / points
K_{24}	Availability of trained technical labours / points
K_{25}	Availability of untrained technical labours / points
K_{31}	Availability of land / points
K_{32}	Prospect of land and objects ownership regulation / points
K_{33}	Adjustment to the urban planning / points
K_{34}	Adjustment to the environmental legislation / points
K_{41}	Costs of location activation / euro/m^2
K_{42}	Average cost of infrastructure (water/sewerage system) / euro/m^3
K_{43}	Construction costs of access roads and infrastructure / points
K_{44}	Period of return on funds / months
K_{51}	Geological characteristics of the location / points
K_{52}	Technical conditions for joining the railway infrastructure / points
K_{53}	Technical conditions for joining the water transport infrastructure / points
K_{54}	Technical conditions for joining the road infrastructure / points

The alternative values for certain criteria, necessary for the implementation of algorithms and multi-criteria analysis, are given in Tab. 4. Each criterion has a relative weight that expresses the weighting coefficient and the requirement for minimizing or maximizing functions (criteria).

According to the proposed procedure for pre-defined criteria, the multi-criteria analysis should be conducted in two phases:

Phase 1: Multi-criteria analysis of the criteria functions at a 2nd level by the PROMETHEE III method. The procedure will be illustrated for the functions K_{11}, K_{12} and K_{13}, which represent sub-criteria functions of the criterion at the 1st level - K_I. Tab. 5 shows the input data related to these sub-criteria functions.

Table 4 Input data for multi-criteria optimization

Criteria		Alternatives					ω_j	max or min
		A_1	A_2	A_3	A_4	A_5		
K_I	K_{11}	26	75	164	80	24	0.08	min
	K_{12}	4	5	2	3	3	0.08	max
	K_{13}	60	10	109	35	70	0.04	min
K_{II}	K_{21}	23	21.1	39	16.6	21	0.03	min
	K_{22}	0.6	0.65	0.8	0.65	0.6	0.04	max
	K_{23}	4	3	2	3	2	0.05	max
	K_{24}	3	3	2	3	3	0.05	max
	K_{25}	3	3	3	3	3	0.03	max
K_{III}	K_{31}	4	3	2	2	2	0.03	max
	K_{32}	3	2	2	3	3	0.03	max
	K_{33}	4	3	2	3	3	0.02	max
	K_{34}	3	3	2	3	3	0.02	max
K_{IV}	K_{41}	54	60	56	55.2	40.8	0.075	min
	K_{42}	0.82	0.93	0.64	0.36	1.3017	0.075	min
	K_{43}	2	3	4	3	2	0.075	min
	K_{44}	60	84	120	72	72	0.025	min
K_V	K_{51}	3	3	2	3	3	0.025	max
	K_{52}	4	4	2	4	4	0.075	max
	K_{53}	2	4	1	3	3	0.075	max
	K_{54}	4	3	2	4	3	0.075	max

Table 5 Input data for ranking the alternatives based on sub-criteria functions K_{11}, K_{12} and K_{13}

Criteria		Alternatives					ω_j	max/min
		A_1	A_2	A_3	A_4	A_5		
K_I	K_{11}	26	75	164	80	24	0.4	min
	K_{12}	4	5	2	3	3	0.4	max
	K_{13}	60	10	109	35	70	0.2	min

Tab. 6 shows the procedure for selection of generalized criteria functions for the criteria K_{11}, K_{12} and K_{13}.

Table 6 Selected generalized criterion for criteria K_{11}, K_{12} and K_{13} with standard deviation

Sub – criteria	σ	*S* - sum of squared deviations							Chosen genera-lized criterion
		Type							
		I	II	III	IV	V	VI	VII	
K_{11}	50.874	2.850	2.050	0.063	0.250	0.057	0.043	0.087	Type VI
K_{12}	1.020	2.519	0.852	0.160	0.519	0.148	2.519	0.177	Type V
K_{13}	33.391	2.850	2.050	0.041	0.300	0.022	0.012	0.013	Type VI

The alternatives final ranking is done through obtaining the values of preference index and flows by developed software tool in accordance with Eq. (18) (Tab. 7).

Table 7 The values of preference index and flows

P(*a*, *b*) - the values of preference index							
		Alternatives					
		A_1	A_2	A_3	A_4	A_5	Output flow
Alternatives	A_1	0	0.166557	0.81871	0.318029	0.15015	1.45345
	A_2	0.262117	0	0.909819	0.42359	0.467766	2.06329
	A_3	0	0	0	0	0	0.00000
	A_4	0.053772	0	0.61532	0	0.083227	0.75232
	A_5	0.056857	0.173748	0.608255	0.192082	0	1.03094
Input flow		0.372746	0.340305	2.952105	0.933701	0.701143	

The order of alternatives is obtained based on interval order using the PROMETHEE III method. The transformed values of net outranking flow for the K_{I} criterion are obtained using Eqs. (19) and (20) and given in Tab. 8. In the same way, the values of net outranking flow are obtained for the remaining criteria functions at the 1st level - K_{II}, K_{III}, K_{IV} and K_{V}. The values are shown in Tab. 9.

Table 8 Transformed net outranking flows of criteria K_{I} obtained in Phase 1

A_i	C_{ij}
A_1	0.86262
A_2	1.00000
A_3	0.00000
A_4	0.59266
A_5	0.70200

Table 9 Input data for multi-criteria optimization in Phase 2

Criteria	Alternatives					ω_j	max/ min
	A_1	A_2	A_3	A_4	A_5		
K_{I}	0.862615	1.000000	0.000000	0.592656	0.701998	0.20	max
K_{II}	0.928637	0.853619	0.000000	1.000000	0.430345	0.20	max
K_{III}	1.000000	0.450924	0.000000	0.613741	0.613741	0.10	max
K_{IV}	1.000000	0.000000	0.047434	0.933870	0.958466	0.25	max
K_{V}	0.870650	0.960054	0.000000	1.000000	0.845921	0.25	max

Phase 2: Multi-criteria analysis of the criteria functions at the 1st level, where the input data are transformed values of the net outranking flows obtained in the previous stage (Tab. 10).

Table 10 Transformed values of the net outranking flows obtained in the previous stage - Phase 1

	Transformed values of the net outranking flows				
	A_1	A_2	A_3	A_4	A_5
K_{I}	0.862615	1.000000	0.000000	0.592656	0.701998
K_{II}	0.928637	0.853619	0.000000	1.000000	0.430345
K_{III}	1.000000	0.450924	0.000000	0.613741	0.613741
K_{IV}	1.000000	0.000000	0.047434	0.933870	0.958466
K_{V}	0.870650	0.960054	0.000000	1.000000	0.845921

The determined values of standard deviation value for each criterion (K_{I}, K_{II}, K_{III}, K_{IV} and K_{V}) and chosen generalized criterion functions are shown in Tab. 11. Generating the final ranking of alternatives based on the criteria K_{I}, K_{II}, K_{III}, K_{IV} and K_{V} is done with the family of methods PROMETHEE I, II and III ($A_1 > A_2 > A_4 > A_5 > A_3$).

Table 11 Selected generalized criterion for criteria K_{I}, K_{II}, K_{III}, K_{IV} and K_{V} with standard deviation

Sub - Criteria	σ	S - sum of squared deviations							Chosen genera-lized criterion
		Type							
		I	II	III	IV	V	VI	VII	
K_{I}	0.345	2.850	1.450	0.041	0.650	0.018	0.007	0.134	Type VI
K_{II}	0.377	2.850	1.450	0.031	0.250	0.030	0.029	0.115	Type VI
K_{III}	0.323	2.519	2.519	0.064	0.352	0.051	0.022	0.067	Type VI
K_{IV}	0.461	2.850	0.850	0.179	0.850	0.102	0.010	0.515	Type VI
K_{V}	0.372	2.850	1.050	0.143	1.050	0.128	0.012	0.783	Type VI

Based on the comparative analysis of the ranking of a potential location of the logistics warehouse, by the proposed procedure and the family of PROMETHEE methods, the DM makes the final decision - a conceptual solution (alternative 1-A_1) fulfils all set constraints and comes out as the best solution. Multi-criteria analysis results are summarized in the form of a report that provides comparative analysis of the final ranking of alternatives by the family of methods PROMETHEE I, II, III and the diagrams of interval order (Fig. 3a).

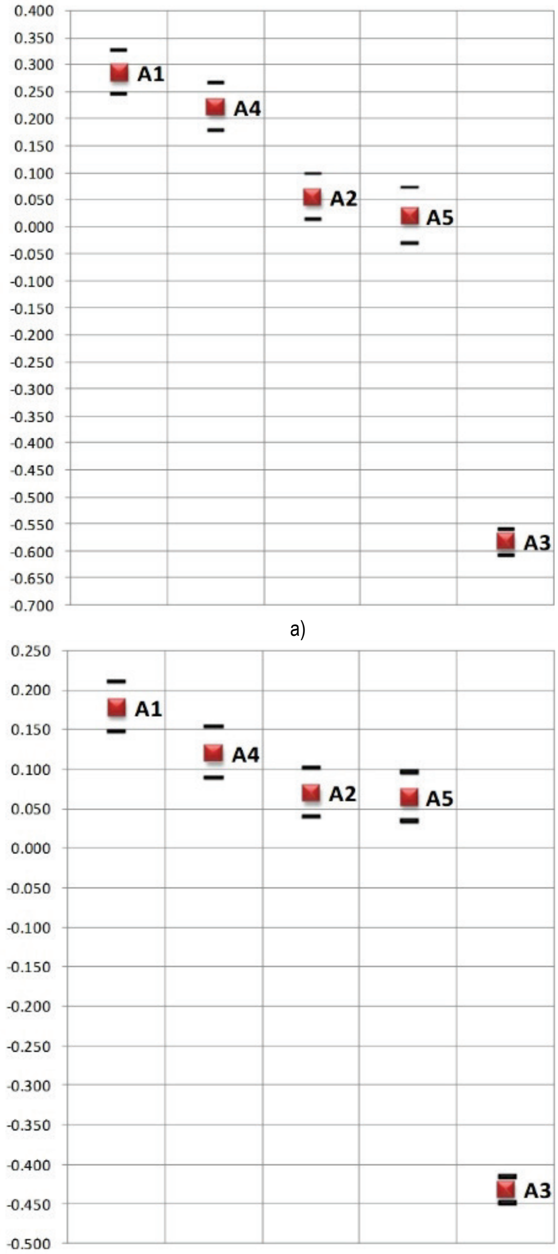


Figure 3 Diagram of interval order of alternatives: a) using criteria functions at multiple levels; b) without criteria functions at multiple levels

In order to present the effectiveness and feasibility of the proposed procedure, the same problem is analysed with the modified procedure using multi-criteria analysis of the problem without the criterion functions at multiple levels (Fig. 3b).

Table 12 Final ranking of alternatives without criteria functions at multiple levels

PROMETHEE I		PROMETHEE II		PROMETHEE III	
Rank		Rank		Rank	
4	A_1	4	A_1	3	A_1
3	A_4	3	A_4	1	A_4
2	A_2	2	A_2	1	A_2
1	A_5	1	A_5	1	A_5
0	A_3	0	A_3	0	A_3

Now, by ranking the alternatives with twenty criteria (Tab. 3), it is possible to notice a significant correlation and the stability of the final rank of alternatives between the proposed procedure and the procedure without criteria functions at multiple levels. Results are shown in Tab. 12.

So, this issue is important from practical and theoretical points of view. From the theoretical aspect, the particular problem should be defined as realistically as possible and properly solved by utilisation of efficient and exact methods of multi-criteria analysis. On the practical side, the outcomes of different criteria have to be compared objectively. Responding to these demands, the proposed procedure gives the solution for a distinctive problem which occurs in multi-criteria optimization: more objective analysis of various criteria influences on the final ranking through utilisation of multiple levels criteria functions and decrease of impact of DM.

4 CONCLUSION

Business conditions are becoming increasingly complex and require a multi-criteria approach to the process of solving location problems, to ensure an objective comparison of a larger number of alternatives that are usually expressed in different units with different factors of significance. The conventional methods of multi-criteria analysis do not introduce parameter values of complex alternative characteristics directly into the model, but they combine them in an intuitive way.

The algorithm developed in this paper allows that this problem could be solved by conventional methods of multi-criteria optimization, so that through several stages of iteration it is relatively easy and quick to make unique solutions. The modified algorithm uses the criteria functions at multiple levels or, more precisely, transforms the mean values of the pure flow, obtained by the method PROMETHEE III, introduces new types of generalized criteria in selection process. This case study shows the selecting process of a future regional logistics warehouse location. The power of the algorithm lies in the simplification of ranking, while respecting all the characteristics and parameters of an alternative, more objective comparison of the impact of various individual criteria for ranking alternatives and reducing them to a common goal function.

Relative weights of criteria can sometimes have a decisive influence on the solution, and the proposed procedure could analyse the impact of weights on the behaviour of the final multi-criteria decision analysis. The final order of alternatives depends on the multi-criteria decision-making technique and particularly on the process of defining the criteria for evaluation, the transformation (normalization) of criteria values and determination of their relative importance. It should be noted that this study does not take into account, to a sufficient extent, the process of determining the relative importance and impact of weight changes. Therefore, the criteria can be expressed both in quantitative and qualitative forms and, in this sense, further research must be focused on the use of elements of the theory of fuzzy sets and new approaches in determining the relative weight of criteria and further integration in the proposed methods.

Hence, further research on this topic would lead to the development of an advanced tool for dealing with great number of real life problems. Finally, this would cause a noticeable reduction of the impact of DM experience and subjectivity in case of the criteria hierarchy.

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